Mass-spectra of singly, doubly, and triply bottom baryons

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The present work has been done on the baryons containing one, two, or three bottom quarks. In this article, the ground-state masses of Ω_b^-, Ξ_{bb}^0, Ξ_{bb}^-, Ω_{bb}^-, and Ω_{bbb}^{*-} baryons are calculated within the framework of Regge phenomenology. Further, the values of Regge slopes and Regge intercepts for singly, doubly, and triply bottom baryons are estimated in both the (J, M^2) and (n, M^2) planes to calculate the excited-state masses of these baryons. Here our attempt is to assign a possible spin parity to recently observed some singly bottom baryons and our results could provide useful information for future experimental searches. Our calculated masses are in agreement with the experimental observations where available and close to other theoretical predictions.

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I. INTRODUCTION

Investigations of heavy baryons properties have recently received much attention, both experimentally and theoretically. Studies of excited baryonic states are a crucial aspect of hadron spectroscopy and help to shed light on the mechanisms responsible for the dynamics of quarks and baryon formation. An ideal platform to understand the dynamics of QCD is to study the properties of baryons that contain a heavy quark [charm (c) or bottom (b)]. Significant experimental progress has been made in the field of singly bottom baryons in recent years. Until three years ago, there were only two excited bottom baryons, $\Lambda_b(5912)^0$ and $\Lambda_b(5920)^0$, which were observed by the LHCb and CDF Collaborations in 2012 [1,2]. In the past three years however, many excited singly beauty baryons have been observed:

(1) The lightest charged $\Sigma_b^\pm$ ($J^P = \frac{1}{2}^+$) and $\Sigma_b^{*\pm}$ ($J^P = \frac{3}{2}^+$) baryons have been observed by CDF Collaboration in the $\Lambda_b^0\pi^\pm$ spectrum [3,4]. Later, in 2018 the LHCb Collaboration reported two new resonances $\Sigma_b(6097)^\pm$ in the $\Lambda_b^0\pi^\pm$ channels [5] and also the new excited bottom baryon $\Xi_b(6227)^-$ was observed in both $\Lambda_b^0K^-$ and $\Xi_b^0\pi^-$ invariant mass spectra [6].
(2) In 2019 the LHCb Collaboration discovered two new states $\Lambda_b(6146)^0$ and $\Lambda_b(6152)^0$ in the $\Lambda_b\pi^+\pi^-$ invariant mass distribution [7]. Later, in 2020 the CMS Collaboration confirmed them and further reported an evidence for a broad excess of events in the $\Lambda_b\pi^+\pi^-$ mass spectrum in the range of 6040–6100 MeV [8]. Later, the LHCb confirmed this state to be $\Lambda_b(6072)^0$ [9].
(3) Recently, in 2020, four extremely narrow excited Ω_b^- states such as $\Omega_b(6316)^-$, $\Omega_b(6330)^-$, $\Omega_b(6340)^-$, and $\Omega_b(6350)^-$ decaying into $\Xi_b^0K^-$ were observed by the LHCb Collaboration [10].

However, none of the doubly and triply bottom baryons are experimentally observed yet. In our previous work, we explained why only five states were observed in the Ω_c^0 baryon and why they are so narrow [11]. Since there are exactly five $1P$ excitations with $J^P = 1/2^-$, $3/2^-$ for $S = 1/2$ and $J^P = 1/2^-$, $3/2^-$, $5/2^-$ for $S = 3/2$, if the ss diquark remains in its color triplet spin-1 ground state. But recently the LHCb [10] has observed four out of five extremely narrow excited Ω_b^- states, leaving the fifth to be predicted and observed. The experimentally missing state at the LHCb experiment may have $J^P = 5/2^-$, which is most likely due to the degeneracy with the nearby state at 6350 MeV. If this is the case, it may be hidden in the observed peak at 6350 MeV, which, although it appears consistent with a single resonance, is actually composed of two. In the recent assignment, the same $J^P = 5/2^-$ state is predicted as the unseen state, where the mass ranges from 6355 MeV to 6380 MeV [12].

The latest Particle Data Group (PDG) [13] listed all the singly bottom baryons with their masses, decay widths, spin parity (J^P), etc. Ground states of these baryons have been observed (with $J^P = \frac{1}{2}^+$, $\frac{3}{2}^+$) experimentally, except Ω_b^{*-} with $J^P = \frac{3}{2}^+$. Unlike other bottom baryons, Λ_b^0 have a

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TABLE I. Masses and J^P values of bottom baryons are listed in PDG [13]. The status is given as poor(*), fair(**), and likely(***).

Resonance	Mass (MeV)	J^P	Status
Λ_b^0	5619.60 ± 0.17	$\frac{1}{2}^+$	***
$\Lambda_b(5920)^0$	5920.09 ± 0.17	$\frac{3}{2}^-$	***
$\Lambda_b(6070)^0$	6072.3 ± 2.9	$?^?$	
$\Lambda_b(6146)^0$	6146.2 ± 0.4	$\frac{3}{2}^+$	***
$\Lambda_b(6152)^0$	6152.5 ± 0.4	$\frac{5}{2}^+$	***
Σ_b^+	5810.56 ± 0.25	$\frac{1}{2}^+$	***
Σ_b^-	5815.64 ± 0.27	$\frac{1}{2}^+$	***
Σ_b^{*+}	5830.32 ± 0.25	$\frac{3}{2}^+$	***
Σ_b^{*-}	5834.74 ± 0.30	$\frac{3}{2}^+$	***
$\Sigma_b(6097)^+$	6095.8 ± 1.7	$?^?$	***
$\Sigma_b(6097)^-$	6098.0 ± 1.8	$?^?$	***
Ξ_b^0	5791.9 ± 0.5	$\frac{1}{2}^+$	***
Ξ_b^-	5797.0 ± 0.6	$\frac{1}{2}^+$	***
$\Xi_b'(5935)^-$	5935.02 ± 0.05	$\frac{1}{2}^+$	***
$\Xi_b(5945)^0$	5952.3 ± 0.6	$\frac{3}{2}^+$	***
$\Xi_b(5955)^-$	5955.33 ± 0.13	$\frac{3}{2}^+$	***
$\Xi_b(6227)^-$	6227.9 ± 0.9	$?^?$	***
Ω_b^-	6046.1 ± 1.7	$\frac{1}{2}^+$	***
$\Omega_b(6316)^-$	6315.6 ± 0.6	$?^?$	*
$\Omega_b(6330)^-$	6330.3 ± 0.6	$?^?$	*
$\Omega_b(6340)^-$	6339.7 ± 0.6	$?^?$	*
$\Omega_b(6350)^-$	6349.8 ± 0.6	$?^?$	*

quite number of established states. The J^P values of recently observed excited singly bottom baryons are still missing, as shown in Table I. It is very crucial to assign the J^P values of the hadrons, which can help to probe their properties such as form factors, decay widths, branching ratios, hyperfine splitting, etc. The properties of heavy flavor baryons have been studied using various phenomenological as well as theoretical approaches in the last few years. Recently, a new scheme of state classification called Jls mixing coupling is applied to analyze the masses of the heavy baryons $\Omega_{c,b}$, $\Sigma_{c,b}$, and $\Xi'_{c,b}$ and interprets all the newly LHCb reported resonances of the excited $\Omega_{c,b}$ to be the P-wave negative-parity baryons [14]. The authors of Refs. [15–17] have employed a hypercentral constituent quark model for the calculation of excited-state masses of singly, doubly, and triply heavy baryons.

Ebert *et al.* [18] obtained the mass spectra of heavy baryons in the framework of the QCD-motivated relativistic quark model. Another approach, the quark-diquark model, was used to study the properties of singly heavy flavored baryons in which the structure of baryons is considered as the bound states of quark-diquark pairs instead of the usual three equivalent quarks. To this aim, the Bethe Salpeter equation, including the Yukawa potential between the constituent quarks of baryons, was used [19]. The authors of Ref. [20] used the Regge approach in the heavy quark-diquark picture and reexamine the orbitally excited spectrum of the charmed and bottom baryons. Their computations on spin-dependent mass splitting suggest that the newly observed baryons $\Xi_b(6227)^-$, $\Sigma_b(6097)^\pm$, $\Sigma_c(2800)$, and $\Xi'_c(2930)$ are all the $1P$-wave baryons with spin parity $J^P = \frac{3}{2}^-$ preferably. They have also shown the mass predictions of the unobserved Ξ_b baryon in the $1P$ and $1D$ states, providing useful information for future experiments. Reference [12] predicted the existence of recently observed four narrow excited Ω_b^- baryons with negative parity by assuming that these baryonic states were bound states of a b quark and a P-wave ss diquark.

Wei *et al.* have employed the Regge phenomenology and expressed the masses of unobserved ground-state doubly and triply bottom baryons as a function of masses of the well-established light baryons and singly bottom baryons. The values of Regge slopes and Regge intercepts are calculated to estimate the masses of the orbitally excited singly, doubly, and triply bottom baryons [21]. In the present work, we use the same approach of the Regge phenomenology with the assumption of linear Regge trajectories. We extract relations between the intercept, slope ratios, and baryon masses in both the (J, M^2) and (n, M^2) planes. Using these relations, we derived the expressions to calculate the ground-state masses of Ω_b^-, Ξ_{bb}^0, Ξ_{bb}^-, Ω_{bb}^-, and Ω_{bbb}^{*-} baryons. The Regge slopes and Regge intercepts of the $\frac{1}{2}^+$ and $\frac{3}{2}^+$ trajectories of singly, doubly, and triply bottom baryons are extracted and the masses of the baryon states lying on the $\frac{1}{2}^+$ and $\frac{3}{2}^+$ trajectories are estimated in both the (J, M^2) and (n, M^2) planes. Further we extend this scheme and try to calculate the remaining states other than natural and unnatural parity states in the (J, M^2) plane. We try to assign a possible spin parity to recently observed excited bottom baryons. This is the main motivation of this paper.

The remainder of this paper is organized as follows. In Sec. II, after a briefing of Regge theory, the ground-state masses of unobserved bottom baryons Ω_b^-, Ξ_{bb}^0, Ξ_{bb}^-, Ω_{bb}^-, and Ω_{bbb}^{*-} were extracted. After that we estimate the Regge slopes for singly, doubly, and triply bottom baryons for $\frac{1}{2}^+$ and $\frac{3}{2}^+$ trajectories, and then the orbitally excited-state masses are calculated in the (J, M^2) plane. In addition, we calculate the radial and orbital excited-state masses of these baryons in the (n, M^2) plane by estimating the Regge slopes and intercepts of particular Regge lines. A discussion of our results is given in Sec. III. We concluded our study in Sec. IV.

II. THEORETICAL FRAMEWORK

The plots of Regge trajectories of hadrons in the (J, M^2) plane are usually called Chew-Frautschi plots [22].

Also, hadrons lying on the same Regge line possess the same internal quantum numbers. The most general form of linear Regge trajectories can be expressed as [11,23]

$$J = \alpha(M) = a(0) + \alpha' M^2, \tag{1}$$

where $a(0)$ and α' represent the intercept and slope of the trajectory respectively. These parameters for different quark constituents of a baryon multiplet can be related by two relations [23–27],

$$a_{iiq}(0) + a_{jjq}(0) = 2a_{ijq}(0), \tag{2}$$

$$\frac{1}{\alpha'_{iiq}} + \frac{1}{\alpha'_{jjq}} = \frac{2}{\alpha'_{ijq}}, \tag{3}$$

where i, j, q represent quark flavors, $m_i < m_j$, and q denotes an arbitrary light or heavy quark. Using Eqs. (1) and (2) we obtain

$$\alpha'_{iiq} M^2_{iiq} + \alpha'_{jjq} M^2_{jjq} = 2\alpha'_{ijq} M^2_{ijq}. \tag{4}$$

After combining the relations (3) and (4) we obtain two pairs of solutions as

$$\frac{\alpha'_{jjq}}{\alpha'_{iiq}} = \frac{1}{2M^2_{jjq}} \times \left[(4M^2_{ijq} - M^2_{iiq} - M^2_{jjq}) \pm \sqrt{(4M^2_{ijq} - M^2_{iiq} - M^2_{jjq})^2 - 4M^2_{iiq}M^2_{jjq}}\right], \tag{5}$$

and

$$\frac{\alpha'_{ijq}}{\alpha'_{iiq}} = \frac{1}{4M^2_{ijq}} \times \left[(4M^2_{ijq} + M^2_{iiq} - M^2_{jjq}) \pm \sqrt{(4M^2_{ijq} - M^2_{iiq} - M^2_{jjq})^2 - 4M^2_{iiq}M^2_{jjq}}\right]. \tag{6}$$

This is the important relation we obtained between slope ratios and baryon masses. Many theoretical studies show that the slopes of Regge trajectories decrease with increasing quark masses [24,25,28–31]. For $m_j > m_i$, one can write $\alpha'_{jjq}/\alpha'_{iiq} < 1$. Consequently from Eq. (5), we can say that

$$\frac{1}{2M^2_{jjq}} \times \left[(4M^2_{ijq} - M^2_{iiq} - M^2_{jjq}) + \sqrt{(4M^2_{ijq} - M^2_{iiq} - M^2_{jjq})^2 - 4M^2_{iiq}M^2_{jjq}}\right] < 1, \tag{7}$$

and the above relation gives the inequality equation which is expressed as

$$2M^2_{ijq} < M^2_{iiq} + M^2_{jjq}. \tag{8}$$

Now to estimate the deviation of relation (8), we introduce a parameter called δ which is used to replace the sign of inequality with the equal sign. For baryons, it is denoted by $\delta^b_{ij,q}$ and given by

$$\delta^b_{ij,q} = M^2_{iiq} + M^2_{jjq} - 2M^2_{ijq}; \tag{9}$$

here also i, j, and q represent the arbitrary light or heavy quarks. Now from Eqs. (2) and (3) we can write

$$a_{iiq}(0) - a_{ijq}(0) = a_{ijq}(0) - a_{jjq}(0), \tag{10}$$

$$\frac{1}{\alpha'_{iiq}} - \frac{1}{\alpha'_{ijq}} = \frac{1}{\alpha'_{ijq}} - \frac{1}{\alpha'_{jjq}}; \tag{11}$$

based on these equations we introduce two parameters,

$$\lambda_x = a_{nnn}(0) - a_{nnx}(0), \qquad \gamma_x = \frac{1}{\alpha'_{nnx}} - \frac{1}{\alpha'_{nnn}}; \tag{12}$$

where n represents light nonstrange quark (u or d) and x denotes i, j, or q. From Eqs. (10)–(12) we have

$$a_{ijq}(0) = a_{nnn}(0) - \lambda_i - \lambda_j - \lambda_q,$$
$$\frac{1}{\alpha'_{ijq}} = \frac{1}{\alpha'_{nnn}} + \gamma_i + \gamma_j + \gamma_q. \tag{13}$$

Since in baryon multiplets for nnn and ijq states, we can write from Eq. (1),

$$J = a_{nnn}(0) + \alpha'_{nnn} M^2_{nnn},$$
$$J = a_{ijq}(0) + \alpha'_{ijq} M^2_{ijq}; \tag{14}$$

solving Eqs. (13) and (14) we have

$$M^2_{ijq} = (\alpha'_{nnn} M^2_{nnn} + \lambda_i + \lambda_j + \lambda_q)\left(\frac{1}{\alpha'_{nnn}} + \gamma_i + \gamma_j + \gamma_q\right). \tag{15}$$

Combining relations (9) and (15) we can prove that

$$\delta^b_{ij,q} = M^2_{iiq} + M^2_{jjq} - 2M^2_{ijq}$$
$$= 2(\lambda_i - \lambda_j)(\gamma_i - \gamma_j), \tag{16}$$

which says that $\delta^b_{ij,q}$ is independent of quark flavor q.

A. Ground-state masses of Ω_b^-, $\Xi_{bb}^{0,-}$, Ω_{bb}^-, and Ω_{bbb}^{*-} baryons

Equation (5), which we obtain above, can also be expressed as

$$\frac{\alpha'_{jjq}}{\alpha'_{iiq}} = \frac{\alpha'_{kkq}}{\alpha'_{iiq}} \times \frac{\alpha'_{jjq}}{\alpha'_{kkq}}; \tag{17}$$

here k can be any quark flavor. Thus we have

$$\frac{\left[(4M_{ijq}^2 - M_{iiq}^2 - M_{jjq}^2) + \sqrt{(4M_{ijq}^2 - M_{iiq}^2 - M_{jjq}^2)^2 - 4M_{iiq}^2 M_{jjq}^2}\right]}{2M_{jjq}^2} = \frac{\left[(4M_{ikq}^2 - M_{iiq}^2 - M_{kkq}^2) + \sqrt{(4M_{ikq}^2 - M_{iiq}^2 - M_{kkq}^2)^2 - 4M_{iiq}^2 M_{kkq}^2}\right]/2M_{kkq}^2}{\left[(4M_{jkq}^2 - M_{jjq}^2 - M_{kkq}^2) + \sqrt{(4M_{jkq}^2 - M_{jjq}^2 - M_{kkq}^2)^2 - 4M_{jjq}^2 M_{kkq}^2}\right]/2M_{kkq}^2}. \tag{18}$$

This is the general relation we have derived in terms of baryon masses which can be used to predict the mass of any baryon state if all other masses are known. In the present work, with the help of relation (18), we evaluate the ground-state masses of unobserved bottom baryons since Ω_b^- comprises two s quarks and one b quark. When we put $i = n$ (u or d), $j = s$, $q = b$, and $k = n$ in Eq. (18) we have

$$[(M_{\Sigma_b^-} + M_{\Omega_b^-})^2 - 4M_{\Xi_b'^-}^2] = \sqrt{(4M_{\Xi_b'^-}^2 - M_{\Sigma_b^-}^2 - M_{\Omega_b^-}^2)^2 - 4M_{\Sigma_b^-}^2 M_{\Omega_b^-}^2}. \tag{19}$$

Inserting the masses of Σ_b^- and $\Xi_b'^-$ ($J^P = \frac{1}{2}^+$) from PDG [13] into Eq. (19), we obtain the ground-state mass of Ω_b^- baryon as 6.054 GeV for $J^P = \frac{1}{2}^+$. Similarly we get $M_{\Omega_b^{*-}} = 6.074$ GeV for the $J^P = \frac{3}{2}^+$ state. In the same manner for doubly bottom baryons, in the case of Ξ_{bb}^0, which is composed of (ubb), we put $i = d$, $j = b$, $q = u$, $k = s$ and for Ω_{bb}^-, which is composed of (sbb), we put $i = u$, $j = s$, $q = s$, $k = b$ in Eq. (18) and obtain the mass expressions to calculate the ground-state masses of doubly bottom baryons Ξ_{bb}^0 and Ω_{bb}^{*-}, which are expressed as a function of well-established masses of light baryons and singly bottom baryons.

For Ξ_{bb}^0;

$$\frac{\left[(4M_{\Lambda_b^0}^2 - M_n^2 - M_{\Xi_{bb}^0}^2) + \sqrt{(4M_{\Lambda_b^0}^2 - M_n^2 - M_{\Xi_{bb}^0}^2)^2 - 4M_n^2 M_{\Xi_{bb}^0}^2}\right]}{2M_{\Xi_{bb}^0}^2} = \frac{\left[(4M_{\Sigma^0}^2 - M_n^2 - M_{\Xi^0}^2) + \sqrt{(4M_{\Sigma^0}^2 - M_n^2 - M_{\Xi^0}^2)^2 - 4M_n^2 M_{\Xi^0}^2}\right]}{\left[(4M_{\Xi_b^0}^2 - M_{\Xi_{bb}^0}^2 - M_{\Xi^0}^2) + \sqrt{(4M_{\Xi_b^0}^2 - M_{\Xi_{bb}^0}^2 - M_{\Xi^0}^2)^2 - 4M_{\Xi_{bb}^0}^2 M_{\Xi^0}^2}\right]} \tag{20}$$

For Ω_{bb}^{*-};

$$\frac{\left[(4M_{\Xi^{*0}}^2 - M_{\Sigma^{*+}}^2 - M_{\Omega^-}^2) + \sqrt{(4M_{\Xi^{*0}}^2 - M_{\Sigma^{*+}}^2 - M_{\Omega^-}^2)^2 - 4M_{\Sigma^{*+}}^2 M_{\Omega^-}^2}\right]}{2M_{\Omega^-}^2} = \frac{\left[(4M_{\Xi_b^{*0}}^2 - M_{\Sigma^{*+}}^2 - M_{\Omega_{bb}^{*-}}^2) + \sqrt{(4M_{\Xi_b^{*0}}^2 - M_{\Sigma^{*+}}^2 - M_{\Omega_{bb}^{*-}}^2)^2 - 4M_{\Sigma^{*+}}^2 M_{\Omega_{bb}^{*-}}^2}\right]}{\left[(4M_{\Omega_b^{*-}}^2 - M_{\Omega^-}^2 - M_{\Omega_{bb}^{*-}}^2) + \sqrt{(4M_{\Omega_b^{*-}}^2 - M_{\Omega^-}^2 - M_{\Omega_{bb}^{*-}}^2)^2 - 4M_{\Omega_{bb}^{*-}}^2 M_{\Omega^-}^2}\right]}. \tag{21}$$

Inserting the masses of Λ_b^0, neutron (n), Σ^0, Ξ^0, and Ξ_b^0 baryons from PDG [13] in Eq. (20), we get $M_{\Xi_{bb}^0} = 10.225$ GeV and also we can calculate $M_{\Xi_{bb}^-} = 10.230$ GeV, for the $J^P = \frac{1}{2}^+$ state. Similarly, putting the masses of Ξ^{*0}, Σ^{*+}, Ω^-, Ξ_b^{*0} from [13] and Ω_b^{*-} (calculated above) into Eq. (21), we get $M_{\Omega_{bb}^{*-}} = 10.449$ GeV for the $J^P = \frac{3}{2}^+$ state. Here in the calculation, we avoid using the masses of Δ^{++}, Δ^{+}, Δ^{0}, and Δ^{-} baryons because only the charge mixed states of $\Delta(1232)$ were assuredly measured, as mentioned in PDG [13]. It is already stated in the above relation (16) that $\delta^{b}_{ij,q}$ is independent of quark flavor q. Therefore, with the help of Eq. (19), we have the following relations:

(I) $i = u,\ j = s,\ q = u,\ b$

$$\begin{aligned}\delta^{(3/2)^+}_{us} &= M^2_{\Delta} + M^2_{\Xi^*} - 2M^2_{\Sigma^*}\\ &= M^2_{\Sigma^*_b} + M^2_{\Omega^*_b} - 2M^2_{\Xi^*_b};\end{aligned} \tag{22}$$

(II) $i = u,\ j = b,\ q = u,\ s$

$$\begin{aligned}\delta^{(3/2)^+}_{ub} &= M^2_{\Delta} + M^2_{\Xi^*_{bb}} - 2M^2_{\Sigma^*_b}\\ &= M^2_{\Sigma^*} + M^2_{\Omega^*_{bb}} - 2M^2_{\Xi^*_b};\end{aligned} \tag{23}$$

(III) $i = s,\ j = b,\ q = u,\ b$

$$\begin{aligned}\delta^{(3/2)^+}_{sb} &= M^2_{\Xi^*} + M^2_{\Xi^*_{bb}} - 2M^2_{\Xi^*_b}\\ &= M^2_{\Omega^*_b} + M^2_{\Omega^*_{bbb}} - 2M^2_{\Omega^*_{bb}};\end{aligned} \tag{24}$$

solving Eqs. (22) and (23) we have

$$(M^2_{\Omega^{*-}_{bb}} - M^2_{\Xi^{*0}_{bb}}) + (M^2_{\Xi^{*0}} - M^2_{\Sigma^{*+}}) = (M^2_{\Omega^{*-}_b} - M^2_{\Sigma^{*+}_b}); \tag{25}$$

similarly, its corresponding relation for $\frac{1}{2}^+$ multiplet is expressed as

$$(M^2_{\Omega^{-}_{bb}} - M^2_{\Xi^{0}_{bb}}) + (M^2_{\Xi^{0}} - M^2_{\Sigma^{+}}) = (M^2_{\Omega^{-}_b} - M^2_{\Sigma^{+}_b}). \tag{26}$$

Now using relations (25) and (26), we can obtain the mass expressions for Ξ^{*0}_{bb}, Ξ^{*-}_{bb}, and Ω^{-}_{bb} baryons. Again after substituting all other masses, we get the ground-state masses $M_{\Xi^{*0}_{bb}} = 10.330$ GeV, $M_{\Xi^{*-}_{bb}} = 10.333$ GeV, and $M_{\Omega^{-}_{bb}} = 10.350$ GeV. Further to calculate the ground-state mass of Ω^{*-}_{bbb} baryon, we extract the mass expression with the help of Eqs. (21) and (24), which is expressed as

$$\begin{aligned}&\frac{(6M^2_{\Xi^0_b} - 2M^2_{\Sigma^+} - M^2_{\Omega^-_b} - M^2_{\Omega^-_{bbb}} + M^2_{\Xi^0} + M^2_{\Xi^0_{bb}}) + \sqrt{(6M^2_{\Xi^0_b} - 2M^2_{\Sigma^+} - M^2_{\Omega^-_b} - M^2_{\Omega^-_{bbb}} + M^2_{\Xi^0} + M^2_{\Xi^0_{bb}})^2 - 8M^2_{\Sigma^+}(M^2_{\Omega^-_b} + M^2_{\Omega^-_{bbb}} - M^2_{\Xi^0} - M^2_{\Xi^0_{bb}} + 2M^2_{\Xi^0_b})}}{(4M^2_{\Xi^0} - M^2_{\Sigma^+} - M^2_{\Omega^-}) + \sqrt{(4M^2_{\Xi^0} - M^2_{\Sigma^+} - M^2_{\Omega^-})^2 - 4M^2_{\Sigma^+}M^2_{\Omega^-}}}\\ &= \frac{(7M^2_{\Omega^-_b} - 2M^2_{\Omega^-} - M^2_{\Omega^-_{bbb}} + M^2_{\Xi^0} + M^2_{\Xi^0_{bb}} - 2M^2_{\Xi^0_b}) + \sqrt{(7M^2_{\Omega^-_b} - 2M^2_{\Omega^-} - M^2_{\Omega^-_{bbb}} + M^2_{\Xi^0} + M^2_{\Xi^0_{bb}} - 2M^2_{\Xi^0_b})^2 - 8M^2_{\Omega^-}(M^2_{\Omega^-_b} + M^2_{\Omega^-_{bbb}} - M^2_{\Xi^0} - M^2_{\Xi^0_{bb}} + 2M^2_{\Xi^0_b})}}{2M^2_{\Omega^-}},\end{aligned} \tag{27}$$

by substituting the masses of Ξ^0_b, Σ^+, Ω^-_b, Ξ^0, Ω^-, and Ξ^0_{bb} ($J^P = \frac{3}{2}^+$), we get $M_{\Omega^{*-}_{bbb}} = 14.822$ GeV. Table II shows the comparison for the masses of Ω^-_b, Ξ^0_{bb}, Ξ^-_{bb}, Ω^-_{bb}, and Ω^{*-}_{bbb} baryons with $J^P = \frac{1}{2}^+$ and $\frac{3}{2}^+$ estimated in the present work and those predicted in other references.

B. Excited-state masses of singly, doubly, and triply bottom baryons

After evaluating the ground-state masses of unseen singly, doubly, and triply bottom baryons, in this section we calculate the excited-state masses of bottom baryons lying on $\frac{1}{2}^+$ and $\frac{3}{2}^+$ trajectories by obtaining the Regge slopes α'. For instance, using Eq. (6) we have

$$\frac{\alpha'^*_{nsb}}{\alpha'_{nns}} = \frac{1}{4M^2_{\Xi^*_b}} \times \left[(4M^2_{\Xi^*_b} + M^2_{\Sigma^*} - M^2_{\Omega^*_{bb}}) + \sqrt{(4M^2_{\Xi^*_b} - M^2_{\Sigma^*} - M^2_{\Omega^*_{bb}})^2 - 4M^2_{\Sigma^*}M^2_{\Omega^*_{bb}}}\right], \tag{28}$$

Putting the values of masses in the above equation, we get $\alpha'_{nsb}/\alpha'_{nns}$. With the help of Eq. (1), we have

$$\alpha' = \frac{(J+2) - J}{M^2_{J+2} - M^2_J}; \tag{29}$$

from above relation we have $\alpha'^*_{nns} = 0.9057\ \text{GeV}^{-2}$. So, we get $\alpha'^*_{nsb} = 0.2846\ \text{GeV}^{-2}$ for the $\frac{3}{2}^+$ trajectory. Similarly, with the aid of Eqs. (3), (5), and (6) we can find α'_{nnb}, α'_{ssb}, α'_{nbb}, α'_{sbb}, and α'_{bbb} for both $\frac{1}{2}^+$ and $\frac{3}{2}^+$ trajectories. According to Ref. [49], $\alpha'_{\Lambda_b} \simeq \alpha'_{\Sigma_b}$ and $\alpha'_{\Xi'_b} \simeq \alpha'_{\Xi_b}$. In this work, we take this approximation. The extracted Regge slopes α' and $\alpha^{*'}$ for the bottom baryons in the present work for $\frac{1}{2}^+$ and $\frac{3}{2}^+$ trajectories, respectively, are shown in Table III. In Ref. [21], the authors gave the values of the Regge slopes for singly, doubly, and triply bottom baryons, which are approximately the same to the corresponding values in this work. For example, $\alpha'_{nnb} = 0.295 \pm 0.022\ \text{GeV}^{-2}$ in Ref. [21], while $\alpha'_{nnb} = 0.2852\ \text{GeV}^{-2}$ in the present work.

Now from Eq. (1) one can have

$$M_{J+1} = \sqrt{M^2_J + \frac{1}{\alpha'}}; \tag{30}$$

TABLE II. Ground state ($J^P = \frac{1}{2}^+$ and $\frac{3}{2}^+$) masses of singly, doubly, and triply bottom baryons (in GeV).

	Ω_b^-		Ξ_{bb}^0/Ξ_{bb}^-		Ω_{bb}^-		Ω_{bbb}^{*-}
J^P	$\frac{1}{2}^+$	$\frac{3}{2}^+$	$\frac{1}{2}^+$	$\frac{3}{2}^+$	$\frac{1}{2}^+$	$\frac{3}{2}^+$	$\frac{3}{2}^+$
Present	6.054	6.074	10.225/10.230	10.330/10.333	10.350	10.449	14.822
PDG [13]	6.046						
Ref. [21]	6.048	6.069	10.199	10.316	10.320	10.431	14.788
Ref. [18,32]	6.064	6.088	10.202	10.237	10.359	10.389	
Ref. [33]	6.110	6.170	10.170	10.220	10.320	10.380	14.830
Refs. [15–17]	6.048	6.086	10.321	10.335	10.446	10.467	14.496
Ref. [34]	6.076	6.094	10.314	10.339	10.447	10.467	
Ref. [35]	6.081	6.102					
Ref. [36]	6.081	6.102	10.340	10.367	10.454	10.486	14.834
Ref. [37]		6.102	10.130	10.144	10.422	10.432	14.569
Ref. [38]	6.060	6.090	10.340	10.370	10.370	10.400	
Ref. [39]			10.272	10.337	10.369	10.429	
Ref. [40]	5.967	6.096	10.339	10.468	10.478	10.607	15.118
Ref. [41]	6.135	6.142	10.440	10.451	10.620	10.628	15.129
Ref. [42]			10.300	10.340	10.340	10.380	
Ref. [43]	6.056	6.085	10.143	10.178	10.273	10.308	14.366
Ref. [44]	6.024	6.084					
Ref. [45]		6.035		10.250		10.395	14.760
Ref. [46]	5.903	5.986	10.334	10.431	10.397	10.495	15.023
Ref. [47]	6.056	6.079	10.189	10.218	10.293	10.321	
Ref. [48]			10.197	10.236	10.260	10.297	

TABLE III. Regge slopes of ground state $\frac{1}{2}^+$ and $\frac{3}{2}^+$ trajectories of singly, doubly, and triply bottom baryons (in GeV^{-2}).

	Λ_b (nnb)	Σ_b (nnb)	Ξ_b (nsb)	Ξ'_b (nsb)	Ω_b (ssb)	Ξ_{bb} (nbb)	Ω_{bb} (sbb)	Ω_{bbb} (bbb)
α'	0.2852	0.2852	0.2795	0.2795	0.2740	0.1792	0.1656	$\cdots$
α'^*	$\cdots$	0.2906	0.2846	$\cdots$	0.2789	0.1695	0.1688	0.1216

using the Regge slope α', we obtain the orbitally excited-state masses of singly, doubly, and triply bottom baryons for both natural ($J^P = 1/2^+, 3/2^-, 5/2^+, \ldots.$) and unnatural ($J^P = 3/2^+, 5/2^-, 7/2^+, \ldots.$) parities in the (J, M^2) plane with the help of Eq. (30). Tables IV–XI show the estimated results for singly, doubly, and triply bottom baryons in the present work and those of other theoretical

TABLE IV. Masses of excited states of Λ_b^0 baryon in the (J, M^2) plane for natural parity states. The numbers in the boldface are the experimental values taken as the input [13] (in GeV).

$N^{2S+1}L_J$	Present	PDG [13]	[15]	[18]	[21]	[32]
$1^2S_{\frac{1}{2}}$	**5.620**	5.620	5.621	5.620	5.619	5.622
$1^2P_{\frac{3}{2}}$	5.924	5.920	5.988	5.942	5.913	5.947
$1^2D_{\frac{5}{2}}$	6.213	6.153	6.213	6.196	6.193	6.197
$1^2F_{\frac{7}{2}}$	6.489		6.432	6.411	6.461	6.405
$1^2G_{\frac{9}{2}}$	6.754			6.599	6.718	
$1^2H_{\frac{11}{2}}$	7.009					

TABLE V. Masses of excited states of $\Sigma_b^{\pm}$ baryon in the (J, M^2) plane for natural and unnatural parity states. The numbers in the boldface are the experimental values taken as the input [13] (in GeV).

	Present		Others				
$N^{2S+1}L_J$	Σ_b^+	Σ_b^-	PDG [13]	[18]	[21]	[32]	[34]
$1^2S_{\frac{1}{2}}$	**5.811**	**5.816**		5.808	5.813	5.805	5.823
$1^2P_{\frac{3}{2}}$	6.105	6.110	6.096 (Σ_b^+) 6.098 (Σ_b^-)	6.096	6.098	6.076	6.132
$1^2D_{\frac{5}{2}}$	6.386	6.390		6.284	6.369	6.248	6.397
$1^2F_{\frac{7}{2}}$	6.655	6.659		6.500	6.630		
$1^2G_{\frac{9}{2}}$	6.913	6.917		6.687	6.881		
$1^2H_{\frac{11}{2}}$	7.162	7.166					
$1^4S_{\frac{3}{2}}$	**5.830**	**5.835**		5.834	5.834	5.834	5.845
$1^4P_{\frac{5}{2}}$	6.118	6.123		6.084	6.117	6.083	6.144
$1^4D_{\frac{7}{2}}$	6.393	6.398		6.260	6.388	6.262	
$1^4F_{\frac{9}{2}}$	6.657	6.661		6.459	6.648		
$1^4G_{\frac{11}{2}}$	6.910	6.914		6.635	6.898		
$1^4H_{\frac{13}{2}}$	7.155	7.158					

TABLE VI. Masses of excited states of $\Xi_b^{0,-}$ baryon in the (J, M^2) plane for natural and unnatural parity states. The numbers in the boldface are the experimental values taken as the input [13] (in GeV).

	Present		Others			
$N^{2S+1}L_J$	Ξ_b^0	Ξ_b^-	[18]	[21]	[32]	[50]
$1^2S_{\frac{1}{2}}$	**5.792**	**5.797**	5.803	5.793	5.812	5.801
$1^2P_{\frac{3}{2}}$	6.093	6.098	6.130	6.080	6.130	6.106
$1^2D_{\frac{5}{2}}$	6.380	6.385	6.373	6.354	6.365	6.349
$1^2F_{\frac{7}{2}}$	6.654	6.659	6.581	6.616	6.558	6.559
$1^2G_{\frac{9}{2}}$	6.918	6.922	6.762	6.869		6.747
$1^2H_{\frac{11}{2}}$	7.712	7.176				
$1^4S_{\frac{3}{2}}$	**5.952**	**5.955**	5.963	5.952	5.963	
$1^4P_{\frac{5}{2}}$	6.240	6.243	6.226	6.232	6.218	
$1^4D_{\frac{7}{2}}$	6.516	6.518	6.414	6.499	6.390	
$1^4F_{\frac{9}{2}}$	6.780	6.782	6.610	6.756		
$1^4G_{\frac{11}{2}}$	7.034	7.036	6.782	7.003		
$1^4H_{\frac{13}{2}}$	7.279	7.281				

TABLE VII. Masses of excited states of $\Xi_b'^-$ baryon in the (J, M^2) plane for natural parity states. The numbers in the boldface are the experimental values taken as the input [13] (in GeV).

$N^{2S+1}L_J$	Present	PDG [13]	[18]	[21]	[32]	[36]
$1^2S_{\frac{1}{2}}$	**5.935**	5.935	5.936	5.935	5.937	5.970
$1^2P_{\frac{3}{2}}$	6.229	6.227	6.234	6.215	6.212	6.190
$1^2D_{\frac{5}{2}}$	6.510		6.432	6.486	6.377	6.393
$1^2F_{\frac{7}{2}}$	6.779		6.641	6.745		
$1^2G_{\frac{9}{2}}$	7.038		6.821	6.994		
$1^2H_{\frac{11}{2}}$	7.288					

TABLE VIII. Masses of excited states of Ω_b^- baryon in the (J, M^2) plane for natural and unnatural parity states (in GeV).

$N^{2S+1}L_J$	Present	PDG [13]	[15]	[18]	[21]	[34]
$1^2S_{\frac{1}{2}}$	6.054	6.046	6.048	6.064	6.048	6.076
$1^2P_{\frac{3}{2}}$	6.348	6.340	6.328	6.340	6.325	6.336
$1^2D_{\frac{5}{2}}$	6.629		6.567	6.529	6.590	6.561
$1^2F_{\frac{7}{2}}$	6.899		6.800	6.736	6.844	
$1^2G_{\frac{9}{2}}$	7.159			6.915	7.090	
$1^2H_{\frac{11}{2}}$	7.409					
$1^4S_{\frac{3}{2}}$	6.074		6.086	6.088	6.069	6.094
$1^4P_{\frac{5}{2}}$	6.362		6.320	6.334	6.345	6.345
$1^4D_{\frac{7}{2}}$	6.638		6.553	6.517	6.609	
$1^4F_{\frac{9}{2}}$	6.903		6.780	6.713	6.863	
$1^4G_{\frac{11}{2}}$	7.158			6.884	7.108	
$1^4H_{\frac{13}{2}}$	7.404					

TABLE IX. Masses of excited states of $\Xi_{bb}^{0,-}$ baryon in the (J, M^2) plane for natural and unnatural parity states (in GeV).

	Present		Others			
$N^{2S+1}L_J$	Ξ_{bb}^0	Ξ_{bb}^-	[21]	[36]	[51]	[34]
$1^2S_{\frac{1}{2}}$	10.225	10.230	10.199	10.340	10.322	10.314
$1^2P_{\frac{3}{2}}$	10.494	10.499	10.474	10.495	10.692	10.476
$1^2D_{\frac{5}{2}}$	10.757	10.761	10.742	10.676	11.002	10.592
$1^2F_{\frac{7}{2}}$	11.013	11.017	11.004			
$1^2G_{\frac{9}{2}}$	11.263	11.267	11.259			
$1^2H_{\frac{11}{2}}$	11.508	11.512				
$1^4S_{\frac{3}{2}}$	10.330	10.333	10.316	10.367	10.352	10.339
$1^4P_{\frac{5}{2}}$	10.612	10.615	10.588	10.731	10.695	10.759
$1^4D_{\frac{7}{2}}$	10.886	10.889	10.853	10.608	11.011	
$1^4F_{\frac{9}{2}}$	11.154	11.157	11.112			
$1^4G_{\frac{11}{2}}$	11.415	11.418	11.365			
$1^4H_{\frac{13}{2}}$	11.670	11.673				

TABLE X. Masses of excited states of Ω_{bb}^- baryon in the (J, M^2) plane for natural and unnatural parity states (in GeV).

$N^{2S+1}L_J$	Present	[16]	[21]	[36]	[32]	[34]
$1^2S_{\frac{1}{2}}$	10.350	10.446	10.320	10.454	10.359	10.447
$1^2P_{\frac{3}{2}}$	10.638	10.641	10.593	10.619	10.566	10.608
$1^2D_{\frac{5}{2}}$	10.918	10.792	10.858	10.720		10.729
$1^2F_{\frac{7}{2}}$	11.191	10.930	11.118			
$1^2G_{\frac{9}{2}}$	11.458		11.372			
$1^2H_{\frac{11}{2}}$	11.718					
$1^4S_{\frac{3}{2}}$	10.449	10.467	10.431	10.486	10.389	10.467
$1^4P_{\frac{5}{2}}$	10.729	10.637	10.700	10.766	10.798	10.808
$1^4D_{\frac{7}{2}}$	11.002	10.786	10.964	10.732		
$1^4F_{\frac{9}{2}}$	11.268	10.924	11.221			
$1^4G_{\frac{11}{2}}$	11.528		11.472			
$1^4H_{\frac{13}{2}}$	11.782					

TABLE XI. Masses of excited states of Ω_{bbb}^{*-} baryon in the (J, M^2) plane for unnatural parity states (in GeV).

$N^{2S+1}L_J$	Present	[17]	[21]	[36]	[33]	[52]
$1^4S_{\frac{3}{2}}$	14.822	14.496	14.788	14.834	14.830	14.371
$1^4P_{\frac{5}{2}}$	15.097	14.931				
$1^4D_{\frac{7}{2}}$	15.367	15.286	15.318	15.101		14.969
$1^4F_{\frac{9}{2}}$	15.632	15.631				
$1^4G_{\frac{11}{2}}$	15.893		15.831			
$1^4H_{\frac{13}{2}}$	16.150					

TABLE XII. Masses of excited states of Λ_b^0 baryon in (n, M^2) plane (in GeV). The masses from Ref. [15] are taken as input.

	$N^{2S+1}L_J$	Present	[18]	[34]	[35]	[53]	[36]
(S = 1/2)	$1^2S_{1/2}$	5.620	5.620	5.618	5.612	5.585	5.612
	$2^2S_{1/2}$	6.026 [15]	6.089		6.107		
	$3^2S_{1/2}$	6.406	6.455		6.338		
	$4^2S_{1/2}$	6.765	6.756				
	$5^2S_{1/2}$	7.106	7.015				
	$6^2S_{1/2}$	7.431	7.256				
(S = 1/2)	$1^2P_{3/2}$	5.924	5.942	5.939		5.920	5.941
	$2^2P_{3/2}$	6.304 [15]	6.333	6.273			
	$3^2P_{3/2}$	6.662	6.651	6.285			
	$4^2P_{3/2}$	7.002	6.922				
	$5^2P_{3/2}$	7.327	7.171				
(S = 1/2)	$1^2D_{5/2}$	6.213	6.196	6.212		6.165	6.183
	$2^2D_{5/2}$	6.527 [15]	6.531	6.530			
	$3^2D_{5/2}$	6.826	6.814				
	$4^2D_{5/2}$	7.113	7.063				
	$5^2D_{5/2}$	7.389					

TABLE XIII. Masses of excited states of $\Sigma_b^{\pm}$ baryon in (n, M^2) plane (in GeV). The masses from Ref. [15] are taken as input.

		Present		Others			
	$N^{2S+1}L_J$	Σ_b^+	Σ_b^-	[18]	[34]	[35]	[21]
(S = 1/2)	$1^2S_{1/2}$	5.811	5.816	5.808	5.823	5.833	5.813
	$2^2S_{1/2}$	6.275 [15]	6.262 [15]	6.213	6.343	6.294	
	$3^2S_{1/2}$	6.707	6.678	6.575	6.395		
	$4^2S_{1/2}$	7.113	7.070	6.869		6.447	
	$5^2S_{1/2}$	7.497	7.441	7.124			
	$6^2S_{1/2}$	7.862	7.795				
(S = 3/2)	$1^4S_{3/2}$	5.830	5.835	5.834	5.845		5.833
	$2^4S_{3/2}$	6.291 [15]	6.277 [15]	6.226	6.356	6.326	
	$3^4S_{3/2}$	6.720	6.690	6.583	6.393		
	$4^4S_{3/2}$	7.124	7.079	6.876		6.447	
	$5^4S_{3/2}$	7.506	7.447	7.129			
	$6^4S_{3/2}$	7.869	7.798				
(S = 1/2)	$1^2P_{3/2}$	6.105	6.110	6.096	6.132		6.098
	$2^2P_{3/2}$	6.506 [15]	6.484 [15]	6.430	6.141		
	$3^2P_{3/2}$	6.884	6.837	6.742	6.246		
	$4^2P_{3/2}$	7.242	7.174	7.009			
	$5^2P_{3/2}$	7.583	7.495				
(S = 3/2)	$1^4P_{5/2}$	6.118	6.123	6.084	6.144		6.117
	$2^4P_{5/2}$	6.489 [15]	6.468 [15]	6.421	6.592		
	$3^4P_{5/2}$	6.840	6.795	6.732	6.834		
	$4^4P_{5/2}$	7.174	7.108	6.999			
	$5^4P_{5/2}$	7.493	7.407				

(Table continued)

TABLE XIII. *(Continued)*

	$N^{2S+1}L_J$	Present		Others			
		Σ_b^+	Σ_b^-	[18]	[34]	[35]	[21]
(S = 1/2)	$1^2D_{5/2}$	6.386	6.390	6.284	6.397		6.396
	$2^2D_{5/2}$	6.778 [15]	6.746 [15]	6.612	6.402		
	$3^2D_{5/2}$	7.148	7.084				
	$4^2D_{5/2}$	7.501	7.407				
	$5^2D_{5/2}$	7.837	7.716				
(S = 3/2)	$1^4D_{7/2}$	6.393	6.398	6.260			6.388
	$2^4D_{7/2}$	6.751 [15]	6.721 [15]	6.590			
	$3^4D_{7/2}$	7.091	7.029				
	$4^4D_{7/2}$	7.415	7.324				
	$5^4D_{7/2}$	7.726	7.608				

TABLE XIV. Masses of excited states of Ξ_b^0 and Ξ_b^- baryon in (n, M^2) plane (in GeV). The masses from Ref. [15] are taken as input.

	$N^{2S+1}L_J$	Present		Others				
		Ξ_b^0	Ξ_b^-	[18]	[35]	[36]	[21]	[54]
(S = 1/2)	$1^2S_{1/2}$	5.792	5.797	5.803	5.806	5.806	5.793	5.825
	$2^2S_{1/2}$	6.203 [15]	6.189 [15]	6.266	6.230			
	$3^2S_{1/2}$	6.588	6.558	6.601	6.547			
	$4^2S_{1/2}$	6.952	6.907	6.913				
	$5^2S_{1/2}$	7.298	7.239	7.165				
	$6^2S_{1/2}$	7.629	7.556					
(S = 3/2)	$1^4S_{3/2}$	5.952	5.955	5.963		5.980	5.952	5.967
	$2^4S_{3/2}$	6.316 [15]	6.298 [15]					
	$3^4S_{3/2}$	6.660	6.623					
	$4^4S_{3/2}$	6.987	6.933					
	$5^4S_{3/2}$	7.300	7.230					
	$6^4S_{3/2}$	7.600	7.515					
(S = 1/2)	$1^2P_{3/2}$	6.093	6.098	6.130		6.093	6.080	6.076
	$2^2P_{3/2}$	6.460 [15]	6.437 [15]	6.502				
	$3^2P_{3/2}$	6.807	6.759	6.810				
	$4^2P_{3/2}$	7.138	7.066	7.073				
	$5^2P_{3/2}$	7.453	7.361	7.306				
(S = 3/2)	$1^4P_{5/2}$	6.240	6.243			6.201	6.232	
	$2^4P_{5/2}$	6.451 [15]	6.428 [15]					
	$3^4P_{5/2}$	6.655	6.608					
	$4^4P_{5/2}$	6.853	6.783					
	$5^4P_{5/2}$	7.046	6.953					
(S = 1/2)	$1^2D_{5/2}$	6.380	6.385	6.373		6.300	6.354	
	$2^2D_{5/2}$	6.687 [15]	6.696 [15]	6.655				
	$3^2D_{5/2}$	6.980	6.993					
	$4^2D_{5/2}$	7.262	7.278					
	$5^2D_{5/2}$	7.533	7.552					

(Table continued)

TABLE XIV. *(Continued)*

	$N^{2S+1}L_J$	Present Ξ_b^0	Ξ_b^-	Others [18]	[35]	[36]	[21]	[54]
(S = 3/2)	$1^4D_{7/2}$	6.516	6.518			6.395	6.499	
	$2^4D_{7/2}$	6.672 [15]	6.661 [15]					
	$3^4D_{7/2}$	6.824	6.801					
	$4^4D_{7/2}$	6.973	6.938					
	$5^4D_{7/2}$	7.119	7.073					

TABLE XV. Masses of excited states of $\Xi_b^{\prime -}$ baryon in (n, M^2) plane (in GeV). The masses from Ref. [15] are taken as input.

	$N^{2S+1}L_J$	Present	[21]	[36]	[54]
(S = 1/2)	$1^2S_{1/2}$	5.935	5.935	5.970	5.913
	$2^2S_{1/2}$	6.329 [15]			
	$3^2S_{1/2}$	6.700			
	$4^2S_{1/2}$	7.051			
	$5^2S_{1/2}$	7.386			
	$6^2S_{1/2}$	7.706			
(S = 1/2)	$1^2P_{3/2}$	6.229	6.215	6.190	6.157
	$2^2P_{3/2}$	6.605 [15]			
	$3^2P_{3/2}$	6.961			
	$4^2P_{3/2}$	7.299			
	$5^2P_{3/2}$	7.622			
(S = 1/2)	$1^2D_{5/2}$	6.510	6.486	6.393	
	$2^2D_{5/2}$	6.751 [15]			
	$3^2D_{5/2}$	6.984			
	$4^2D_{5/2}$	7.209			
	$5^2D_{5/2}$	7.427			

TABLE XVI. Masses of excited states of Ω_b^- baryon in (n, M^2) plane (in GeV). The masses from Ref. [15] are taken as input.

	$N^{2S+1}L_J$	Present	[18]	[34]	[35]	[44]	[21]
(S = 1/2)	$1^2S_{1/2}$	6.054	6.064	6.076	6.081	6.024	6.048
	$2^2S_{1/2}$	6.455 [15]	6.450	6.517	6.472	6.325	
	$3^2S_{1/2}$	6.832	6.804	6.561	6.593		
	$4^2S_{1/2}$	7.190	7.091		6.648		
	$5^2S_{1/2}$	7.531	7.338				
	$6^2S_{1/2}$	7.857					
(S = 3/2)	$1^4S_{3/2}$	6.074	6.088		6.102	6.084	6.069
	$2^4S_{3/2}$	6.481 [15]	6.461	6.528	6.478	6.412	
	$3^4S_{3/2}$	6.864	6.811	6.559	6.593		
	$4^4S_{3/2}$	7.226	7.096		6.645		
	$5^4S_{3/2}$	7.572	7.343				
	$6^4S_{3/2}$	7.902					

(Table continued)

TABLE XVI. *(Continued)*

	$N^{2S+1}L_J$	Present	[18]	[34]	[35]	[44]	[21]
(S = 1/2)	$1^2P_{3/2}$	6.348	6.340	6.336			6.325
	$2^2P_{3/2}$	6.662 [15]	6.705	6.344			
	$3^2P_{3/2}$	6.962	7.002	6.919			
	$4^2P_{3/2}$	7.249	7.258				
	$5^2P_{3/2}$	7.526					
(S = 3/2)	$1^4P_{5/2}$	6.362	6.334				6.345
	$2^4P_{5/2}$	6.653 [15]	6.700				
	$3^4P_{5/2}$	6.932	6.996				
	$4^4P_{5/2}$	7.200	7.251				
	$5^4P_{5/2}$	7.458					
(S = 1/2)	$1^2D_{5/2}$	6.629	6.529	6.561			6.590
	$2^2D_{5/2}$	6.659 [15]	6.846	6.566			
	$3^2D_{5/2}$	6.689					
	$4^2D_{5/2}$	6.719					
	$5^2D_{5/2}$	6.748					
(S = 3/2)	$1^4D_{7/2}$	6.638	6.517				6.609
	$2^4D_{7/2}$	6.643 [15]	6.834				
	$3^4D_{7/2}$	6.648					
	$4^4D_{7/2}$	6.653					
	$5^4D_{7/2}$	6.658					

TABLE XVII. Masses of excited states of Ξ_{bb}^0 and Ξ_{bb}^- baryon in (n, M^2) plane (in GeV). The masses from Ref. [16] are taken as input.

		Present		Others				
	$N^{2S+1}L_J$	Ξ_{bb}^0	Ξ_{bb}^-	[34]	[36]	[32]	[55]	[51]
(S = 1/2)	$1^2S_{1/2}$	10.225	10.230	10.314	10.340	10.202	10.185	10.322
	$2^2S_{1/2}$	10.609 [16]	10.612 [16]	10.571	10.576	10.441	10.751	10.551
	$3^2S_{1/2}$	10.979	10.981	10.612		10.630	11.170	
	$4^2S_{1/2}$	11.338	11.337			10.812		
	$5^2S_{1/2}$	11.685	11.683					
	$6^2S_{1/2}$	12.023	12.019					
(S = 3/2)	$1^4S_{3/2}$	10.330	10.333	10.339	10.367	10.237	10.216	10.352
	$2^4S_{3/2}$	10.617 [16]	10.619 [16]	10.592	10.578	10.428	10.770	10.574
	$3^4S_{3/2}$	10.896	10.897	10.593		10.673	11.184	
	$4^4S_{3/2}$	11.169	11.169			10.856		
	$5^4S_{3/2}$	11.435	11.434					
	$6^4S_{3/2}$	11.695	11.693					
(S = 1/2)	$1^2P_{3/2}$	10.494	10.499	10.476	10.495	10.408		10.692
	$2^2P_{3/2}$	10.765 [16]	10.766 [16]	10.704	10.713	10.607		
	$3^2P_{3/2}$	11.029	11.026	10.742		10.788		
	$4^2P_{3/2}$	11.287	11.281					
	$5^2P_{3/2}$	11.540	11.530					

(Table continued)

TABLE XVII. *(Continued)*

	$N^{2S+1}L_J$	Present Ξ_{bb}^0	Present Ξ_{bb}^-	Others [34]	[36]	[32]	[55]	[51]
(S = 3/2)	$1^4P_{5/2}$	10.612	10.615	10.759				10.695
	$2^4P_{5/2}$	10.776 [16]	10.763 [16]	10.973	10.713			
	$3^4P_{5/2}$	10.937	10.909	11.004				
	$4^4P_{5/2}$	11.097	11.053					
	$5^4P_{5/2}$	11.254	11.195					
(S = 1/2)	$1^2D_{5/2}$	10.757	10.761	10.592	10.676			11.002
	$2^2D_{5/2}$	10.901 [16]	10.901 [16]		10.712			
	$3^2D_{5/2}$	11.043	11.039					
	$4^2D_{5/2}$	11.183	11.176					
	$5^2D_{5/2}$	11.322	11.311					
(S = 3/2)	$1^4D_{7/2}$	10.886	10.889		10.608			11.011
	$2^4D_{7/2}$	10.896 [16]	10.896 [16]		11.057			
	$3^4D_{7/2}$	10.906	10.903					
	$4^4D_{7/2}$	10.916	10.910					
	$5^4D_{7/2}$	10.926	10.917					

TABLE XVIII. Masses of excited states of Ω_{bb}^- baryon in (n, M^2) plane (in GeV). The masses from Ref. [16] are taken as input.

	$N^{2S+1}L_J$	Present	[34]	[36]	[32]	[55]	[47]
(S = 1/2)	$1^2S_{1/2}$	10.350	10.447	10.454	10.359	10.271	10.293
	$2^2S_{1/2}$	10.736 [16]	10.707	10.693	10.610	10.830	10.604
	$3^2S_{1/2}$	11.109	10.744		10.806	11.240	
	$4^2S_{1/2}$	11.469	10.994				
	$5^2S_{1/2}$	11.819					
	$6^2S_{1/2}$	12.158					
(S = 3/2)	$1^4S_{3/2}$	10.449	10.467	10.486	10.389	10.289	10.321
	$2^4S_{3/2}$	10.743 [16]	10.723	10.721	10.645	10.839	10.622
	$3^4S_{3/2}$	11.029	10.730		10.843	11.247	
	$4^4S_{3/2}$	11.308	11.031				
	$5^4S_{3/2}$	11.580					
	$6^4S_{3/2}$	11.846					
(S = 1/2)	$1^2P_{3/2}$	10.638	10.608	10.619	10.566		
	$2^2P_{3/2}$	10.893 [16]	10.797	10.765	10.775		
	$3^2P_{3/2}$	11.142	10.805				
	$4^2P_{3/2}$	11.386					
	$5^2P_{3/2}$	11.624					
(S = 3/2)	$1^4P_{5/2}$	10.729	10.808	10.766	10.798		
	$2^4P_{5/2}$	10.888 [16]	11.028				
	$3^4P_{5/2}$	11.045	11.059				
	$4^4P_{5/2}$	11.199					
	$5^4P_{5/2}$	11.352					

(Table continued)

TABLE XVIII. *(Continued)*

	$N^{2S+1}L_J$	Present	[34]	[36]	[32]	[55]	[47]
(S = 1/2)	$1^2D_{5/2}$	10.918	10.729	10.720			
	$2^2D_{5/2}$	11.025 [16]	10.744	10.734			
	$3^2D_{5/2}$	11.131	10.937				
	$4^2D_{5/2}$	11.236					
	$5^2D_{5/2}$	11.340					
(S = 3/2)	$1^4D_{7/2}$	11.002					
	$2^4D_{7/2}$	11.021 [16]					
	$3^4D_{7/2}$	11.040					
	$4^4D_{7/2}$	11.059					
	$5^4D_{7/2}$	11.078					

approaches. Here the spectroscopic notations $N^{2S+1}L_J$ are used to represent the state of the particles, where N, L, S denote the radial excited quantum number, orbital quantum number, and intrinsic spin, respectively.

C. Masses of singly, doubly, and triply bottom baryons in the (n, M^2) plane

In this section, we evaluate the Regge parameters in the (n, M^2) plane to calculate the orbital and radial excited states of singly, doubly, and triply bottom baryons. The general equation for linear Regge trajectories in the (n, M^2) plane can be expressed as

$$n = \beta_0 + \beta M^2, \tag{31}$$

where $n = 1, 2, 3\ldots.$ is the radial principal quantum number, β_0, and β are the intercept and slope of the trajectories. The baryon multiplets lying on the single Regge line have the same Regge slope (β) and Regge intercept (β_0). Using relation (31), we calculate β and β_0 and with the help of these parameters we estimated the excited-state masses of singly, doubly, and triply bottom baryons lying on each Regge line for natural and unnatural parity states. For instance, using the slope equation, we have $\beta_{(S)} = 1/(M^2_{\Omega_b(2S)} - M^2_{\Omega_b(1S)})$ for the Ω_b^- baryon, where $M_{\Omega_b(1S)} = 6.054$ GeV (calculated above) and taking $M_{\Omega_b(2S)} = 6.455$ GeV from [15] for the $1/2^+$ trajectory, we get $\beta_{(S)} = 0.19935$ GeV^{-2}. From Eq. (31) we can write

$$1 = \beta_{0(S)} + \beta_{(S)} M^2_{\Omega_b(1S)},$$
$$2 = \beta_{0(S)} + \beta_{(S)} M^2_{\Omega_b(2S)}; \tag{32}$$

using the above relations, we get $\beta_{0(S)} = -6.30664$. With the help of $\beta_{(S)}$ and $\beta_{0(S)}$, we calculate the masses of the excited Ω_b^- baryon for $n = 3, 4, 5\ldots.$ Similarly, we can express these relations for P and D waves as

$$1 = \beta_{0(P)} + \beta_{(P)} M^2_{\Omega_b(1P)},$$
$$2 = \beta_{0(P)} + \beta_{(P)} M^2_{\Omega_b(2P)},$$
$$1 = \beta_{0(D)} + \beta_{(D)} M^2_{\Omega_b(1D)},$$
$$2 = \beta_{0(D)} + \beta_{(D)} M^2_{\Omega_b(2D)}. \tag{33}$$

In the same manner, we estimated the radial and orbital excited states of other singly, doubly, and triply bottom baryons for natural and unnatural parity states (see Tables XII–XIX).

D. Other states in the (J, M^2) plane

So far we have calculated the masses of singly, doubly, and triply bottom baryons for natural and unnatural parity

TABLE XIX. Masses of excited states of Ω^{*-}_{bbb} baryon in (n, M^2) plane (in GeV). The masses from Ref. [17] are taken as input.

	$N^{2S+1}L_J$	Present	[21]	[52]	[36]
(S = 3/2)	$1^4S_{3/2}$	14.822	14.788	14.371	14.834
	$2^4S_{3/2}$	15.163 [17]			
	$3^4S_{3/2}$	15.496			
	$4^4S_{3/2}$	15.823			
	$5^4S_{3/2}$	16.143			
	$6^4S_{3/2}$	16.456			
(S = 3/2)	$1^4P_{5/2}$	15.097			
	$2^4P_{5/2}$	15.425 [17]			
	$3^4P_{5/2}$	15.746			
	$4^4P_{5/2}$	16.061			
	$5^4P_{5/2}$	16.369			
(S = 3/2)	$1^4D_{7/2}$	15.367	15.318	14.969	15.101
	$2^4D_{7/2}$	15.776 [17]			
	$3^4D_{7/2}$	16.175			
	$4^4D_{7/2}$	16.564			
	$5^4D_{7/2}$	16.944			

states by using the conventional formulas. After the successful implementation of this model, now in this section, we try to obtain the remaining other states in the (J, M^2) plane by using the same method. Since we have calculated $1^2P_{\frac{3}{2}}$ and $1^4P_{\frac{5}{2}}$ states earlier, now here we first calculate the other three $1P$ states, i.e., $1^2P_{\frac{1}{2}}$, $1^4P_{\frac{1}{2}}$, and $1^4P_{\frac{3}{2}}$ by using Eq. (18). For the Ω_b^- baryon we put $i = u$, $j = s$, $q = b$, and $k = u$ in Eq. (18) and we have

$$[(M_{\Sigma_b} + M_{\Omega_b})^2 - 4M^2_{\Xi'_b}] = \sqrt{(4M^2_{\Xi'_b} - M^2_{\Sigma_b} - M^2_{\Omega_b})^2 - 4M^2_{\Sigma_b}M^2_{\Omega_b}}. \quad (34)$$

Because of the unavailability of experimental data, we have taken the masses of Σ_b and Ξ'_b baryons from Ref. [18]. Hence, after inserting M_{Σ_b} and $M_{\Xi'_b}$ for the $1^2P_{\frac{1}{2}}$ state in Eq. (34), we get $M_{\Omega_b^-} = 6.365$ GeV. Similarly we can obtain $M_{\Omega_b^-} = 6.359$ GeV and $M_{\Omega_b^-} = 6.360$ GeV for $1^4P_{\frac{1}{2}}$ and $1^4P_{\frac{3}{2}}$ states, respectively. In the same manner we can calculate the masses for doubly and triply bottom baryons Ξ_{bb}, Ω_{bb}^- and Ω_{bbb}^- as we have done earlier. Inserting the

TABLE XX. Masses of other excited states of Λ_b^0 baryon in the (J, M^2) plane. The numbers in the boldface are the theoretical values taken as the input [18] (in GeV).

$N^{2S+1}L_J$	Present	PDG [13]	[50]	[19]	[36]	[20]
$1^2P_{\frac{1}{2}}$	**5.930**	5.912	5.911	5.919	5.939	5.908
$1^2D_{\frac{3}{2}}$	6.128	6.146	6.147	6.199	6.181	6.144
$1^2F_{\frac{5}{2}}$	6.320		6.346	6.421	6.206	
$1^2G_{\frac{7}{2}}$	6.506					
$1^2H_{\frac{9}{2}}$	6.687					

TABLE XXI. Masses of other excited states of Σ_b baryon in the (J, M^2) plane. The numbers in the boldface are the theoretical values taken as the input [18] (in GeV).

$N^{2S+1}L_J$	Present	[19]	[57]	[34]
$1^2P_{\frac{1}{2}}$	**6.101**	6.122	6.095	6.127
$1^4P_{\frac{1}{2}}$	**6.095**		6.087	
$1^4P_{\frac{3}{2}}$	**6.087**		6.096	
$1^2D_{\frac{3}{2}}$	6.293	6.329		
$1^4D_{\frac{3}{2}}$	6.375			
$1^4D_{\frac{5}{2}}$	6.346			
$1^2F_{\frac{5}{2}}$	6.480	6.569		
$1^4F_{\frac{5}{2}}$	6.644			
$1^4F_{\frac{7}{2}}$	6.595			
$1^2G_{\frac{7}{2}}$	6.661			
$1^4G_{\frac{7}{2}}$	6.902			
$1^4G_{\frac{9}{2}}$	6.835			

TABLE XXII. Masses of other excited states of Ξ_b baryon in the (J, M^2) plane. The numbers in the boldface are the theoretical values taken as the input [18] (in GeV).

$N^{2S+1}L_J$	Present	[50]	[36]	[33]
$1^2P_{\frac{1}{2}}$	**6.120**	6.097	6.090	
$1^4P_{\frac{1}{2}}$	**6.227**			6.140
$1^4P_{\frac{3}{2}}$	**6.224**			
$1^2D_{\frac{3}{2}}$	6.316	6.344		
$1^4D_{\frac{3}{2}}$	6.508			
$1^4D_{\frac{5}{2}}$	6.484			
$1^2F_{\frac{5}{2}}$	6.506	6.555		
$1^4F_{\frac{5}{2}}$	6.777			
$1^4F_{\frac{7}{2}}$	6.734			
$1^2G_{\frac{7}{2}}$	6.690	6.743		
$1^4G_{\frac{7}{2}}$	7.036			
$1^4G_{\frac{9}{2}}$	6.974			

TABLE XXIII. Masses of other excited states of Ξ'_b baryon in the (J, M^2) plane. The numbers in the boldface are the theoretical values taken as the input [18] (in GeV).

$N^{2S+1}L_J$	Present	[36]	[21]
$1^2P_{\frac{1}{2}}$	**6.233**	6.305	
$1^2D_{\frac{3}{2}}$	6.425		
$1^2F_{\frac{5}{2}}$	6.612		
$1^2G_{\frac{7}{2}}$	6.794		
$1^2H_{\frac{9}{2}}$	6.971		

TABLE XXIV. Masses of other excited states of Ω_b^- baryon in the (J, M^2) plane.

$N^{2S+1}L_J$	Present	PDG [13]	[19]	[58]	[59]
$1^2P_{\frac{1}{2}}$	6.365		6.342	6.340	6.330
$1^4P_{\frac{1}{2}}$	6.359			6.340	
$1^4P_{\frac{3}{2}}$	6.360	6.350		6.350	6.350
$1^2D_{\frac{3}{2}}$	6.557		6.545		
$1^4D_{\frac{3}{2}}$	6.640				
$1^4D_{\frac{5}{2}}$	6.620				
$1^2F_{\frac{5}{2}}$	6.744		6.777		
$1^4F_{\frac{5}{2}}$	6.909				
$1^4F_{\frac{7}{2}}$	6.870				
$1^2G_{\frac{7}{2}}$	6.926				
$1^4G_{\frac{7}{2}}$	7.168				
$1^4G_{\frac{9}{2}}$	7.111				

masses of Λ_b, Σ, Ξ, and Ξ_b baryons for the $1^2P_{\frac{1}{2}}$ state taken from Ref. [18] and the mass of the N baryon taken from Ref. [56] into Eq. (20), we get $M_{\Xi_{bb}} = 10.399$ GeV for the $1^2P_{\frac{1}{2}}$ state. In this way, all the remaining $1P$ states for Ξ_{bb}, Ω_{bb}^- and Ω_{bbb}^- baryons can be obtained in the same way as previously done.

Once we have calculated the other three $1P$ states, $1^2P_{\frac{1}{2}}$, $1^4P_{\frac{1}{2}}$, and $1^4P_{\frac{3}{2}}$, Regge slopes for these trajectories can be evaluated for singly, doubly, and triply bottom baryons using Eqs. (3), (5), and (6) with the same procedure as we have done above. The further excited-state masses are calculated with the help of the equation below,

$$M_{J+1} = \sqrt{M_J^2 + \frac{1}{\alpha'}}. \quad (35)$$

TABLE XXV. Masses of other excited states of Ξ_{bb} baryon in the (J, M^2) plane.

$N^{2S+1}L_J$	Present	[60]	[34]	[47]	[32]
$1^2P_{\frac{1}{2}}$	10.399	10.310	10.476	10.406	10.368
$1^4P_{\frac{1}{2}}$	10.551	10.541			
$1^4P_{\frac{3}{2}}$	10.557	10.567			
$1^2D_{\frac{3}{2}}$	10.556				
$1^4D_{\frac{3}{2}}$	10.810				
$1^4D_{\frac{5}{2}}$	10.795				
$1^2F_{\frac{5}{2}}$	10.711				
$1^4F_{\frac{5}{2}}$	11.062				
$1^4F_{\frac{7}{2}}$	11.028				
$1^2G_{\frac{7}{2}}$	10.864				
$1^4G_{\frac{7}{2}}$	11.309				
$1^4G_{\frac{9}{2}}$	11.256				

TABLE XXVI. Masses of other excited states of Ω_{bb}^- baryon in the (J, M^2) plane.

$N^{2S+1}L_J$	Present	[34]	[32]	[47]	[61]
$1^2P_{\frac{1}{2}}$	10.546	10.607	10.532	10.519	
$1^4P_{\frac{1}{2}}$	10.678				
$1^4P_{\frac{3}{2}}$	10.688				10.513
$1^2D_{\frac{3}{2}}$	10.706				
$1^4D_{\frac{3}{2}}$	10.952				
$1^4D_{\frac{5}{2}}$	10.940				
$1^2F_{\frac{5}{2}}$	10.863				
$1^4F_{\frac{5}{2}}$	11.219				
$1^4F_{\frac{7}{2}}$	11.187				
$1^2G_{\frac{7}{2}}$	11.018				
$1^4G_{\frac{7}{2}}$	11.480				
$1^4G_{\frac{9}{2}}$	11.428				

TABLE XXVII. Masses of other excited states of Ω_{bbb}^- baryon in the (J, M^2) plane.

$N^{2S+1}L_J$	Present	[36]	[21]	[62]	[61]
$1^4P_{\frac{1}{2}}$	14.999	14.975			
$1^4P_{\frac{3}{2}}$	15.017	14.976	15.055	14.950	14.900
$1^4D_{\frac{3}{2}}$	15.267				
$1^4D_{\frac{5}{2}}$	15.263	15.101			
$1^4F_{\frac{5}{2}}$	15.530				
$1^4F_{\frac{7}{2}}$	15.506				
$1^4G_{\frac{7}{2}}$	15.789				
$1^4G_{\frac{9}{2}}$	15.745				

Tables XX–XXIV show our calculated results for the remaining other states for singly bottom baryons. We compared our estimated masses with other theoretical studies and our results are in accordance with them. Similarly, Tables XXV–XXVII show our predicted masses for doubly and triply bottom baryons. Very few results are available from previous theoretical studies and they are consistent with our estimated masses.

III. RESULTS AND DISCUSSION

In the present work, under the methodology of Regge phenomenology, we have obtained the ground-state masses of unseen Ω_b^-, Ξ_{bb}^0, Ξ_{bb}^-, Ω_{bb}^-, and Ω_{bbb}^{*-} baryons. Regge slopes of the singly, doubly, and triply bottom baryon trajectories were calculated in the (J, M^2) plane. With the aid of these Regge slopes, the masses of the orbitally excited bottom baryons were estimated for both natural and unnatural parity states. After that, the Regge slopes and intercepts were extracted for each Regge line in the (n, M^2) plane, and with the help of these parameters mass spectra of singly, doubly, and triply bottom baryons were obtained successfully.

(1) Λ_b^0 **baryon**: Experimentally, there are quite number of excited Λ_b^0 states reported recently; $\Lambda_b(5912)^0$, $\Lambda_b(5920)^0$, $\Lambda_b(6070)^0$, $\Lambda_b(6146)^0$, and $\Lambda_b(6152)^0$. Our calculated masses for Λ_b^0 baryon in the (J, M^2) plane are shown in Table IV for natural parity states. We observed that the experimentally found state $\Lambda_b(5920)^0$ having mass 5.920 GeV is close to our predicted mass 5.924 GeV with a slight mass difference of 4 MeV. So we confirmed this state as the $1P$ state with $J^P = \frac{3}{2}^-$. Also, the $\Lambda_b(6152)^0$ having mass 6.152 GeV is somewhat lower than our estimated mass 6.213 GeV with a mass difference of 61 MeV. So this state is confirmed to be the $1D$ state with $J^P = \frac{5}{2}^+$. Other than these two, one more state $\Lambda_b(6146)^0$ having mass 6.146 GeV is close to our calculated mass 6.128 GeV with a mass difference of 18 MeV. Hence, we confirmed this state as the $1D$

TABLE XXVIII. Masses of excited states of Ω_b^- baryon in the (J, M^2) plane (in GeV).

$N^{2S+1}L_J$	Present	PDG [13]	[15]	[18]	[21]	[34]	[19]	[58]	[59]
$1^2S_{\frac{1}{2}}$	6.054	6.046	6.048	6.064	6.048	6.076	6.098		
$1^4S_{\frac{3}{2}}$	6.074		6.086	6.088	6.069	6.094			
$1^2P_{\frac{1}{2}}$	6.365						6.342	6.340	6.330
$1^2P_{\frac{3}{2}}$	6.348	6.340	6.328	6.340	6.325	6.336			
$1^4P_{\frac{1}{2}}$	6.359							6.340	
$1^4P_{\frac{3}{2}}$	6.360	6.350						6.350	6.350
$1^4P_{\frac{5}{2}}$	6.362		6.320	6.334	6.345	6.345			
$1^2D_{\frac{3}{2}}$	6.557						6.545		
$1^2D_{\frac{5}{2}}$	6.629		6.567	6.529	6.590	6.561			
$1^4D_{\frac{3}{2}}$	6.640								
$1^4D_{\frac{5}{2}}$	6.620								
$1^4D_{\frac{7}{2}}$	6.638		6.553	6.517	6.609				
$1^2F_{\frac{5}{2}}$	6.744						6.777		
$1^2F_{\frac{7}{2}}$	6.899		6.800	6.736	6.844				
$1^4F_{\frac{5}{2}}$	6.909								
$1^4F_{\frac{7}{2}}$	6.870								
$1^4F_{\frac{9}{2}}$	6.903		6.780	6.713	6.863				
$1^2G_{\frac{7}{2}}$	6.926								
$1^2G_{\frac{9}{2}}$	7.159			6.915	7.090				
$1^4G_{\frac{7}{2}}$	7.168								
$1^4G_{\frac{9}{2}}$	7.111								
$1^4G_{\frac{11}{2}}$	7.158			6.884	7.108				
$1^2H_{\frac{11}{2}}$	7.409								
$1^4H_{\frac{13}{2}}$	7.404								

The excited-state masses of Ω_b^- baryon including natural, unnatural, and other states.

state with $J^P = \frac{3}{2}^+$ (see Table XX). Further, we compared excited-state masses with other theoretical predictions [15,18,21,32] and our results are in agreement with them. Similarly, we compared our results evaluated in the (n, M^2) plane and they are close to other theoretical and phenomenological studies [18,34] as shown in Table XII.

(2) $\Sigma_b^\pm$ **baryons**: Table V shows our calculated results for $\Sigma_b^\pm$ baryons for natural and unnatural parity states in the (J, M^2) plane. Experimentally, two resonances $\Sigma_b(6097)^+$ and $\Sigma_b(6097)^-$ were discovered recently having masses 6.096 and 6.098 GeV, respectively, which are very close to our estimated masses 6.105 GeV (Σ_b^+) and 6.110 GeV (Σ_b^-) with a mass difference of 9–12 MeV. Hence, $\Sigma_b(6097)^\pm$ states are identified as $1P$ states with $J^P = \frac{3}{2}^-$ for $S = 1/2$. Our estimated results are also in accordance with other theoretical predictions [18,21,34]. In the same manner we compared our calculated radial and orbital excited-state masses evaluated in the (n, M^2) plane for both natural and unnatural parity states (see Table XIII), and our results are consistent with other theoretical predictions.

(3) $\Xi_b^{0,-}$, $\Xi_b'^-$ **baryons**: Orbitally excited-state masses for $\Xi_b^{0,-}$ and $\Xi_b'^-$ baryons are shown in Tables VI and VII, respectively. We compared our calculated results with other theoretical predictions and experimental results. Experimentally found a new state $\Xi_b(6227)^-$ having mass 6.227 GeV is close to our predicted mass 6.229 GeV (shown in Table VII). This state can be a good candidate of the P wave and be identified with spin parity $J^P = \frac{3}{2}^-$ for $S = 1/2$. In the (n, M^2) plane our calculated results for $\Xi_b^{0,-}$, and $\Xi_b'^-$ baryons are consistent with the predictions of [18,21,36,54] (see Tables XIV and XV).

(4) Ω_b^- **baryon**: Our estimated ground- and excited-state masses of the Ω_b^- baryon in the (J, M^2) and (n, M^2) planes are shown in Tables VIII and XVI, respectively. Experimentally, only the ground state Ω_b^- is observed with spin parity $J^P = \frac{1}{2}^+$; Ω_b^{*-} with $J^P = \frac{3}{2}^+$ is still unseen. Our calculated ground state Ω_b^-, having a mass of 6.054 GeV, is very close to the

experimental data [13] with mass difference of 8 MeV. Also, for the $1^4S_{\frac{3}{2}}$ state, our predicted mass is in accordance with the results of Refs. [15,18,21,34]. After that, we compared our calculated orbital and radial excited states for natural and unnatural parity states in the (J, M^2) and (n, M^2) planes with other theoretical results. Our predicted masses are consistent with them. The experimentally observed state $\Omega_b(6340)^-$ having a mass of 6.340 GeV is close to our estimated mass of 6.348 GeV with a mass difference of 8 MeV. So, we assigned $\Omega_b(6340)^-$ as the $1P$ state with $J^P = 3/2^-$ for S = 1/2. Also, one more state $\Omega_b(6350)^-$ is found near to our calculated mass 6.360 GeV; hence, we can say that this state may belong to the $1P$ state with $J^P = 3/2^-$ for S = 3/2 (see Table XXIV). In the present work, the other two states, $\Omega_b(6316)^-$ and $\Omega_b(6330)^-$, are not identified.

(5) $\Xi_{bb}^{0,-}$ **baryons**: Table IX shows our calculated orbitally excited-state masses for Ξ_{bb}^0 and Ξ_{bb}^- baryons for both natural and unnatural parities in the (J, M^2) plane. Our estimated ground state $(1^2S_{\frac{1}{2}})$ masses are in good agreement with the prediction of Ref. [21] with a mass difference of 26–31 MeV. Similarly, for the $1^4S_{\frac{3}{2}}$ state, our predicted masses are close to the results of Refs. [21,51] with a slight mass difference of 9–17 MeV. Also, the mass splitting $M_{\Xi_{bb}^0} - M_{\Xi_{bb}^{*0}} = 105$ MeV is little big, however it is close to Refs. [21,40,46]. Further, the excited-state masses are also in accordance with the predictions of other theoretical studies [21,36,51]. In the same manner, we compared our evaluated excited-state masses in the (n, M^2) plane for natural and unnatural parities with other theoretical predictions and phenomenological studies as shown in Table XVII. Our results are consistent with the predictions of [32,34,36,51,55].

(6) Ω_{bb}^- **baryon**: Our calculated ground- and excited-state masses of the Ω_{bb}^- baryon for natural and unnatural parities in the (J, M^2) plane are shown in Table X. The ground-state $(1^4S_{\frac{1}{2}})$ mass is close to the masses of Refs. [21,32] and somewhat lower than the results of [16,34,36]. For the $1^4S_{\frac{3}{2}}$ state, our estimated mass is very well fitted with the results of Refs. [16,21,34]. Here also the mass splitting $M_{\Omega_{bb}^-} - M_{\Omega_{bb}^{*-}} = 99$ MeV is reasonable with Refs. [21,40,46]. For excited states, our predicted masses are reasonably close to the results of [21] and they are also in accordance with the other theoretical studies. Similarly, Table XVIII shows our estimated radial and orbital excited states in the (n, M^2) plane. Our results are consistent with the predictions of other theoretical approaches.

(7) Ω_{bbb}^{*-} **baryon**: For the triply bottom baryon, we obtained the ground-state and excited-state masses in both the (J, M^2) and (n, M^2) planes as shown in Tables XI and XIX, respectively. The ground-state $(1^4S_{\frac{3}{2}})$ mass for Ω_{bbb}^{*-} varies in the range 14.360–15.130 GeV predicted in other theoretical approaches and phenomenological studies (see Table II). Our estimated ground-state mass shows a few MeV differences with the results of [21,33,36]. For excited states, very limited results are available from previous theoretical studies and our estimated masses are in accordance with them.

IV. CONCLUSION

Here, our aim is satisfied for the determination of spin parity of experimentally observed unknown states: $\Sigma_b(6097)^+$, $\Sigma_b(6097)^-$, $\Xi_b(6227)^-$, $\Omega_b(6340)^-$, and $\Omega_b(6350)^-$. Also, we confirmed the spin parity of $\Lambda_b(5920)^0$, $\Lambda_b(6146)^0$, and $\Lambda_b(6152)^0$. This model is successful for the study of singly, doubly, and triply bottom baryons. The masses of the orbitally excited bottom baryons were estimated for both natural and unnatural parity states in the (J, M^2) and (n, M^2) planes and also the other states in the (J, M^2) plane. Our predictions will help future experimental studies at LHCb, CMS, and Belle II to identify these baryonic states. The reliability of this model is very much dependent on the availability of experimental data that we have taken as input.

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