Axial meson exchange and the $Z_c(3900)$ and $Z_{cs}(3985)$ resonances as heavy hadron molecules

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Early speculations about the existence of heavy hadron molecules were grounded on the idea that light-meson exchange forces could lead to binding. In analogy to the deuteron, the light mesons usually considered include the pion, sigma, rho and omega, but not the axial meson $a_1(1260)$. Though it has been argued in the past that the coupling of the axial meson to the nucleons is indeed strong, its mass is considerably heavier than that of the vector mesons and thus its exchange ends up being suppressed. Yet, this is not necessarily the case in heavy hadrons molecules; we find that even though the contribution to binding from the axial meson is modest, it cannot be neglected in the isovector sector where vector meson exchange cancels out. This might provide a natural binding mechanism for molecular candidates such as the $Z_c(3900)$, $Z_c(4020)$, or the more recently observed $Z_{cs}(3985)$. However, the $Z_{cs}(3985)$ is more dependent on a mixture of different factors, which (besides axial meson exchange) include η exchange and the nature of scalar meson exchange. Together they point towards the existence of two $Z_{cs}(3985)$-like resonances instead of one, while the observations about the role of scalar meson exchange in the $Z_{cs}(3985)$ might be relevant for the $P_{cs}(4459)$. Finally, the combination of axial meson exchange and flavor-symmetry breaking effects indicates that the isovector $J^{PC} = 0^{++}$ $D^*\bar{D}^*$ and the strange $J^P = 2^+$ $D^*\bar{D}_s^*$ molecules are the most attractive configurations and thus the most likely molecular partners of the $Z_c(3900)$, $Z_c(4020)$, and $Z_{cs}(3985)$.

I. INTRODUCTION

Heavy hadron molecules were originally theorized as an analogy to the deuteron [1,2]. The argument is that the same type of forces binding two nucleons together might bind other hadrons as well. Since then a continuous inflow of ideas from nuclear physics has enriched our understanding of heavy molecular states, ranging from phenomenological approaches such as light-meson exchanges [3–6] to modern effective field theory (EFT) formulations 11–7]]. This is not at all surprising; in both cases we are dealing with hadrons, where nucleons happen to be the most well studied of all hadrons.

The origin and derivation of nuclear forces has itself a tortuous and winding history, in which many competing ideas have been proposed but few have succeeded [12]. The reasons behind the failures are important though, as they might be specific to nucleons. If we focus on light-meson

exchange forces, the idea is that the nuclear potential can be derived from the exchange of a few light mesons, which usually include the pion, the sigma, the rho, and the omega, i.e., the one boson exchange model (OBE) [13,14]. Mesons heavier than the nucleon are generally not expected to have a sizable contribution to the nuclear force; their Compton wavelength is shorter than the size of the nucleon and the forces generated by their exchange are heavily suppressed.

A prominent example is the axial meson $a_1(1260)$, which is expected to have a considerably strong coupling to the nucleons [15–17]. It is also heavier than the vector mesons; the ratio of the masses of the axial and rho mesons, m_{a1} and m_ρ, is $m_{a_1}/m_\rho \sim 1.6$. In fact it is even heavier than the nucleon and its influence on the description of the nuclear force has turned out to be rather limited [18]. However, this is not necessarily the case for heavy hadrons; on one hand, they are heavier than axial mesons and nucleons, and on the other vector-meson exchange cancels out in a few specific molecular configurations, which increases the relative importance of the axial meson.

The axial meson has a particularly interesting feature; its quantum numbers $I^G(J^{PC}) = 1^-(1^{++})$ indicate that it can mix with the axial current of the pions. That is, we can modify the axial pion current by including a term proportional to the axial meson

$$\partial_\mu \pi \to \partial_\mu \pi + \lambda_1 m_{a1} a_{1\mu}, \qquad (1)$$

where π and $a_{1\mu}$ are the pion and axial meson fields, respectively, and λ_1 is a proportionality constant which we expect to be in the $\lambda_1 \sim (1.6\text{--}2.1)$ range. From this, the coupling of the axial meson with the charmed hadrons D and D^* will be proportional to their axial coupling to the pions, g_1

$$g_{Y(a_1)} = \frac{1}{\sqrt{3}} \lambda_1 \frac{m_{a1}}{f_\pi} g_1 \sim (8-11)g_1, \qquad (2)$$

where the coupling is defined by matching the axial meson exchange potential to a Yukawa [see Eqs. (15) and (22)]. Depending on the configuration of the two-hadron system under consideration, this exchange potential might be remarkably attractive and even explain binding if other light-meson exchanges are suppressed.

The reason we are interested in the axial meson is because of a specific difficulty when explaining the isovector hidden-charm $Z_c(3900)$ [19], $Z_c^*(4020)$ [20], and $Z_{cs}(3985)$ [21] resonances as hadronic molecules: while their closeness to the $D^*\bar{D}$, $D^*\bar{D}^*$, and $D^*\bar{D}_s - D\bar{D}_s^*$ thresholds suggest the molecular nature of the Z_c's [22–26] and the Z_{cs} [27–32]—the reason why their interaction is strong remains elusive. It happens that the rho and omega exchange cancel out for the Z_c and Z_c^*, which in turn requires a binding mechanism not involving the vector mesons (where this cancellation does not happen with axial mesons, as we will see). Possible explanations include one-pion and sigma exchange [33–35], two-pion exchanges (the correlated part of which is sometimes interpreted as a sigma exchange), and charmonium exchanges [36–39]. Here we will investigate how the axial meson exchange works as a binding mechanism within the OBE model.

The manuscript is structured as follows: in Sec. II we will derive the axial meson exchange potential for the charmed meson-antimeson system. In Sec. III we will review scalar and vector meson exchange and the potentials they generate, which are still an important part of the OBE potential. In Sec. IV we will investigate how the inclusion of the axial meson makes the simultaneous description of the $X(3872)$ and the $Z_c(3900)$ more compatible with each other. In Sec. V we will consider how the previous ideas apply to the $Z_{cs}(3985)$ and how the OBE potential can be made compatible with the expectations from SU(3)-flavor symmetry. Finally in Sec. VI we will explain our conclusions.

II. AXIAL MESON EXCHANGE

First we will derive the potential generated by axial meson exchange. We will begin with the interaction Lagrangian between heavy mesons and pions and from it we will derive the Lagrangian and potential for axial mesons.

The quark content of the heavy mesons is $Q\bar{q}$, with $Q = c$, b a heavy quark and $q = u$, d, s a light quark. The properties of heavy mesons and their interactions are expected to be independent of the heavy-quark spin, which is usually referred to as heavy-quark spin symmetry (HQSS) [40,41]. The consequences of HQSS for heavy-hadron molecules are important and have been extensively explored in the literature [9–11,23,42–46]. For S-wave heavy mesons (e.g., the D and D^* charmed mesons) the standard way to take into account HQSS is to define a superfield H_Q as

$$H_Q = \frac{1}{\sqrt{2}}[P\mathbf{1} + \vec{\sigma} \cdot \vec{P}^*], \qquad (3)$$

with P and $\vec{P}^*$ the $J^P = 0^-$ and 1^- heavy mesons, $\mathbf{1}$ the 2×2 identity matrix, and $\vec{\sigma}$ the Pauli matrices, where our definition of H_Q corresponds to the nonrelativistic limit of the superfield defined in Ref. [47]. This field has good properties with respect to heavy-quark spin rotation, i.e., the heavy-quark transformation $|Q\rangle \to e^{-i\vec{S}_H \cdot \theta}|Q\rangle$ induces the superfield transformation

$$H_Q \to e^{-i\vec{S}_H \cdot \theta} H_Q, \qquad (4)$$

from which it is clear that $H_Q^\dagger H_Q \mathcal{O}$ field combinations, with $\mathcal{O}$ some operator in the form of a 2×2 matrix, will be independent of heavy-quark spin rotations. With this formalism, the interaction of S-wave heavy mesons with the pion can be written as

$$\mathcal{L} = \frac{g_1}{\sqrt{2}f_\pi} \text{Tr}[H_Q^\dagger H_Q \vec{\sigma} \cdot \vec{a}], \qquad (5)$$

where $g_1 = 0.6$ is the axial coupling (a value which is compatible with $g_1 = 0.59 \pm 0.01 \pm 0.07$ as extracted from the $D^* \to D\pi$ decay [48,49]), $f_\pi \simeq 132$ MeV the pion weak decay constant and $\vec{a}$ is the (reduced) axial current, which traditionally only includes the pion

$$\vec{a} = \vec{\nabla}\pi, \qquad (6)$$

where we implicitly include the SU(2)-isospin indices in the pion field, i.e., $\pi = \tau_c \pi^c$ with c an isospin index.

Alternatively, instead of grouping the P and $\vec{P}^*$ into a single superfield with good heavy-quark rotation properties, we notice that the heavy-quark spin degrees of freedom do not come into play in the description of heavy-light hadron interactions. This allows to write interactions in terms of a fictitious light-quark subfield—a heavy field with the quantum numbers of the light quark within the heavy meson [50]. If we call this effective field q_L, the corresponding Lagrangian will read

$$\mathcal{L} = \frac{g_1}{\sqrt{2}f_\pi} q_L^\dagger \vec{\sigma}_L \cdot \vec{a} q_L, \tag{7}$$

with $\vec{\sigma}_L$ the spin operators (Pauli matrices) as applied to the light-quark spin. When this operator acts on the light-quark degrees of freedom it can be translated into the corresponding spin operator acting on the heavy meson field with the rules

$$\langle P|\vec{\sigma}_L|P\rangle = 0, \tag{8}$$

$$\langle P|\vec{\sigma}_L|P^*\rangle = \vec{\epsilon}, \tag{9}$$

$$\langle P^*|\vec{\sigma}_L|P^*\rangle = \vec{S}, \tag{10}$$

with $\vec{\epsilon}$ the polarization vector of the P^* heavy meson and $\vec{S}$ the spin-1 matrices. From now on we will work in this notation.

The previous light-quark subfield Lagrangian leads to the nonrelativistic potential

$$V_\pi(\vec{q}) = -\zeta \frac{g_1^2}{2f_\pi^2} \vec{\tau}_1 \cdot \vec{\tau}_2 \frac{\vec{\sigma}_{L1} \cdot \vec{q} \vec{\sigma}_{L2} \cdot \vec{q}}{\vec{q}^2 + m_\pi^2},$$

$$= -\zeta \frac{g_1^2}{6f_\pi^2} \vec{\tau}_1 \cdot \vec{\tau}_2 \frac{\vec{\sigma}_{L1} \cdot \vec{\sigma}_{L2} \vec{q}^2}{\vec{q}^2 + m_\pi^2}$$

$$- \zeta \frac{g_1^2}{6f_\pi^2} \vec{\tau}_1 \cdot \vec{\tau}_2 \frac{(3\vec{\sigma}_{L1} \cdot \vec{q} \vec{\sigma}_{L2} \cdot \vec{q} - \vec{\sigma}_{L1} \cdot \vec{\sigma}_{L2} q^2)}{\vec{q}^2 + m_\pi^2}, \tag{11}$$

with $\vec{q}$ the exchanged momentum, $m_\pi \simeq 138$ MeV the pion mass and $\zeta = \pm 1$ a sign which is $+1(-1)$ for the meson-meson (meson-antimeson) potential (which comes from the G-parity of the pion). In the second and third lines we separate the potential into its S-wave and S-to-D-wave components (i.e., spin-spin and tensor pieces); owing to the exploratory nature of the present manuscript, we will be only concerned with the S-wave components of hadronic molecules and will ignore the D waves.

Now, to include the axial meson we simply modify the axial current $\vec{a}$ as follows:

$$\vec{a} = \vec{\nabla}\pi + \lambda_1 m_{a1} \vec{a}_1, \tag{12}$$

with the isospin indices again implicit, i.e., $a_1 = \tau_c a_1^c$, and λ_1 a parameter describing how the axial meson mixes with the pion-axial current (which value we will discuss later). This readily leads to the potential

$$V_{a_1}(\vec{q}) = -\zeta \lambda_1^2 \frac{g_1^2 m_{a1}^2}{2f_\pi^2} \vec{\tau}_1 \cdot \vec{\tau}_2 \left[\frac{\vec{\sigma}_{L1} \cdot \vec{\sigma}_{L2}}{\vec{q}^2 + m_{a_1}^2} \right.$$

$$\left. + \frac{1}{m_{a_1}^2} \frac{\vec{\sigma}_{L1} \cdot \vec{q} \vec{\sigma}_{L2} \cdot \vec{q}}{\vec{q}^2 + m_{a_1}^2} \right]$$

$$= -\zeta \lambda_1^2 \frac{g_1^2 m_{a1}^2}{2f_\pi^2} \vec{\tau}_1 \cdot \vec{\tau}_2 \frac{\vec{\sigma}_{L1} \cdot \vec{\sigma}_{L2}}{\vec{q}^2 + m_{a_1}^2} \left(1 + \frac{\vec{q}^2}{3m_{a_1}^2} \right) + \cdots, \tag{13}$$

where in the last line we isolate the S-wave component.

Finally we are interested in the r-space expressions of the pion and axial-exchange potentials. For this we Fourier transform into r space, which in the pion case yields

$$V_\pi(\vec{r}) = \zeta \frac{g_1^2 m_\pi^2}{6f_\pi^2} \vec{\tau}_1 \cdot \vec{\tau}_2 \vec{\sigma}_{L1} \cdot \vec{\sigma}_{L2} \frac{e^{-m_\pi r}}{4\pi r} + \cdots, \tag{14}$$

where the dots represent tensor (i.e., S-to-D-wave) and contact-range (i.e., Dirac delta) terms (which we also ignore owing to their short-range nature). For the axial meson exchange we have instead

$$V_{a_1}(\vec{r}) = -\zeta \lambda_1^2 \frac{g_1^2 m_{a1}^2}{3f_\pi^2} \vec{\tau}_1 \cdot \vec{\tau}_2 \vec{\sigma}_{L1} \cdot \vec{\sigma}_{L2} \frac{e^{-m_{a1} r}}{4\pi r} + \cdots, \tag{15}$$

where the dots indicate again contact and tensor terms.

The coupling of the axial meson to the hadrons depends on λ_1, which could be deduced from the matrix elements of the axial current $A_{5\mu}$

$$\langle 0|A_{5\mu}|\pi\rangle = f_\pi q_\mu, \qquad \langle 0|A_{5\mu}|a_1\rangle = f_{a_1} m_{a_1} \epsilon_\mu, \tag{16}$$

with q_μ the momentum of the pion, f_π and f_{a_1} the weak decay constants of the pion and axial meson, m_{a_1} the mass of the axial meson and ϵ_μ its polarization vector. From Eq. (1) we arrive at the identification

$$\lambda_1 = \frac{f_{a_1}}{f_\pi}, \tag{17}$$

but f_{a_1} is not particularly well known. Different estimations exist, of which a few worth noticing are:

(i) The Weinberg sum rules [51] or the Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin (KSFR) relations [52,53], both of which lead to $m_{a_1} = \sqrt{2}m_\rho = 1.09$ GeV, $f_{a_1} = f_\pi$, and $\lambda_1 = 1$.

(ii) The $\tau \to 3\pi\nu_\tau$ decay involves the axial meson as an intermediate state, and has been used in the past to determine f_{a_1}:

(a) Three decades ago Ref. [54] obtained

$$m_{a_1} f_{a_1} = (0.25 \pm 0.02) \text{ GeV}^2, \tag{18}$$

for $m_{a_1} = 1.22$ GeV, which translates into $\lambda_1 = 1.55 \pm 0.12$. Later Ref. [55] made the

observation that $m_{a_1}f_{a_1}$ shows a simple dependence on the $\tau \to 3\pi\nu_\tau$ branching ratio, from which it updated the previous value to $m_{a_1}f_{a_1} = (0.254 \pm 0.20)$ GeV^2, yielding $\lambda_1 = 1.58 \pm 0.12$.

(b) Ref. [17] noticed a result from Ref. [56], which contains a phenomenological relation between $m_{a_1}f_{a_1}$ and the relative branching ratios for the $\tau \to 2\pi\nu_\tau$ and $\tau \to 3\pi\nu_\tau$ decay. This led the authors of Ref. [17] to the estimation[1]

$$m_{a_1}f_{a_1} = 0.33 \text{ GeV}^2, \qquad (19)$$

which is equivalent to $\lambda_1 = 2.05$.

(c) Chiral Lagrangian analyses of the $\tau \to 3\pi\nu_\tau$ decay [57,58], usually yield $\lambda_1 \sim 1.4$–1.5 but with $m_{a_1} \sim 1.1$ GeV, which is somewhat light.

(iii) The lattice QCD calculation of Ref. [55] gives

$$m_{a_1}f_{a_1} = (0.30 \pm 0.02) \text{ GeV}^2, \qquad (20)$$

and $m_{a_1} = 1.25 \pm 0.08$ GeV, from which we extract $\lambda_1 = 1.82 \pm 0.08 \pm 0.12$ where the first and second error refer to the uncertainties in $m_{a_1}f_{a_1}$ and m_{a_1}, respectively.

(iv) Ref. [59] uses QCD sum rules to obtain

$$f_{a_1} = (238 \pm 10) \text{ MeV}, \qquad (21)$$

that is, $\lambda_1 = 1.80 \pm 0.08$.

From the previous, it is apparent that the uncertainties in λ_1 are large. But we can reduce its spread if we concentrate on the determinations of λ_1 for which m_{a_1} is close to its value in the Review of Particle Physics (RPP) [60], i.e., $m_{a_1} = 1.23$ GeV (which is also the value we will adopt for the mass of the axial meson). In this case we end up with the $\lambda_1 \sim (1.55 - 2.05)$ window, which we will approximate by $\lambda_1 = 1.8 \pm 0.3$. This is the central value and uncertainty we will use from now on.

At this point it is interesting to compare the strengths of the resulting Yukawa-like piece of the previous potentials

$$V_Y(\vec{r}) = \pm \frac{g_Y^2}{4\pi}\mathcal{O}_I\mathcal{O}_S\frac{e^{-mr}}{r}, \qquad (22)$$

where g_Y is an effective Yukawa-like coupling, $\mathcal{O}_I = 1$ or $\vec{\tau}_1 \cdot \vec{\tau}_2$ and $\mathcal{O}_S = 1$ or $\vec{\sigma}_{L1} \cdot \vec{\sigma}_{L2}$ the usual isospin and spin operators, while m is the mass of the exchanged meson. For the pion and axial meson exchange potentials we have that the strength of the effective Yukawas are

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[1]We notice that Ref. [17] uses the $f_\pi \sim 93$ MeV normalization for the decay constants, i.e., a $\sqrt{2}$ factor smaller than ours. Thus we have adapted their results to our normalization.

$$\frac{g_{Y(\pi)}^2}{4\pi} \simeq 6.6 \times 10^{-2} \quad \text{and} \quad \frac{g_{Y(a_1)}^2}{4\pi} \simeq 2.0 - 3.5, \qquad (23)$$

which gives an idea of the relative strength of axial meson exchange with respect to the pion. Provided it is attractive, the condition for this effective Yukawa-like potential to bind is

$$\frac{2\mu}{m}\frac{g_Y^2}{4\pi}|\langle\mathcal{O}_I\rangle\langle\mathcal{O}_S\rangle| \geq 1.68, \qquad (24)$$

with μ the reduced mass of the two-hadron system. If we consider the $I^G(J^{PC}) = 1^+(1^{+-})D^*\bar{D}$ system, which is the usual molecular interpretation of the $Z_c(3900)$, the potential is indeed attractive and the previous condition is fulfilled for $\lambda_1 \geq 1.1$ (if $m_{a_1} = 1.23$ GeV).

III. SCALAR AND VECTOR MESON EXCHANGE

Besides the pion and the axial mesons, usually the other important exchanged light mesons in the OBE model are the scalar σ and the vector mesons ρ and ω. In the following we will discuss the potentials they generate.

A. Scalar meson

For the scalar meson we write a Lagrangian of the type

$$\mathcal{L}_S = g_{\sigma 1}\text{Tr}[H_Q^\dagger H_Q]\sigma \qquad (25)$$

$$= g_{\sigma 1}q_L^\dagger\sigma q_L, \qquad (26)$$

depending on the notation (superfield/subfield in first/second line), with $g_{\sigma 1}$ the coupling of the scalar meson to the charmed hadrons. From this Lagrangian we derive the potential

$$V_\sigma(\vec{q}) = -\frac{g_{\sigma 1}^2}{\vec{q}^2 + m_\sigma^2}, \qquad (27)$$

which is attractive and where m_σ is the scalar meson mass. Finally, if we Fourier transform into coordinate space we will arrive at

$$V_\sigma(\vec{r}) = -g_{\sigma 1}^2\frac{e^{-m_\sigma r}}{4\pi r}. \qquad (28)$$

The parameters in this potential are the coupling $g_{\sigma 1}$ and the mass m_σ. For the coupling we will rely on the linear sigma model (LσM) [61], which we briefly review here as it will prove useful for the discussion on the $Z_{cs}(3985)$ later. The LσM is a phenomenological model in which originally we have a massless nucleon field that couples to a combination of four boson fields, i.e., this model contains a nucleon interaction term of the type

$$\mathcal{L}_{\text{int}}^{\text{N}\sigma\text{L}} = g\bar{\psi}_N(\phi_0 + i\gamma_5\vec{\tau}\cdot\vec{\phi})\psi_N, \qquad (29)$$

where ψ_N is the relativistic nucleon field and g a coupling constant. By means of spontaneous symmetry breaking we end up with three massless bosons $\vec{\phi}$, which might be interpreted as pions, while the isospin scalar ϕ_0 acquires a vacuum expectation value ($\langle\phi_0\rangle = f_\pi/\sqrt{2}$) which also provides the nucleons with mass. The σ field is defined as a perturbation of the ϕ_0 field around its vacuum expectation value ($\phi_0 = f_\pi/\sqrt{2} + \sigma$). This model provides a relation between f_π, the nucleon mass $M_N \simeq 940$ MeV and the couplings of the scalar mesons and the pions to the nucleon, where $g = g_{\sigma NN} = g_{\pi NN} = \sqrt{2}M_N/f_\pi = 10.2$.

Nowadays we know that the pion coupling is derivative (as required by chiral symmetry), yet if we are considering one-pion exchange only this derivative coupling can be matched to a nonderivative one as in both cases the same potential is obtained. In this case the pion coupling is given by $g_{\pi NN} = g_A\sqrt{2}M_N/f_\pi$ with $g_A = 1.26$, which means that the linear sigma model is off by about a 26% (or a 30% once we take into account the Goldberger-Treiman discrepancy). Thus this is the expected uncertainty that we should have for $g_{\sigma NN}$. For comparison purposes, the LσM gives $g_{\sigma NN}^2/4\pi = 8.3$ while the OBE model of nuclear forces [13,14] prefers slightly larger values $g_{\sigma NN}^2/4\pi = 8.5$–8.9 (which are still compatible with the LσM). For the charmed mesons, which contain only one light quark, we will assume the quark model relation $g_{\sigma 1} = g_{\sigma NN}/3 \simeq 3.4$ [62] (though we notice that Ref. [62] advocates a slightly larger coupling of $g_{\sigma qq} = \sqrt{2}m_q^{\text{con}}/f_\pi \simeq 3.6$, with m_q^{con} the constituent $q = u, d$ quark mass).

For the mass of the sigma the OBE model of nuclear forces uses $m_\sigma = 550$ MeV, but it is also common to find $m_\sigma = 600$ MeV in a few recent implementations of the OBE model for hadronic molecules [33–35,63,64]. Nowadays the RPP designation of the σ is $f_0(500)$ and the mass is in the 400 MeV–550 MeV range [60]. However this does not necessarily imply that the mass of the $f_0(500)$ pole should be used for the scalar meson exchange, owing in part to its large width and in part to its relation with correlated two-pion exchange, as has been extensively discussed [65–68]. Direct fits of $g_{\sigma NN}$ and m_σ can also lead to more than one solution, though they are usually compatible with the RPP mass range of the sigma and with the expected 30% uncertainty for the coupling in the LσM. For instance, a renormalized OBE fit to NN data [69] leads to two solutions, one with $m_\sigma = 477$ MeV, $g_{\sigma NN} = 8.76$, and another with $m_\sigma = 556$ MeV, $g_{\sigma NN} = 13.04$. What we will do then is to investigate binding as a function of the σ mass.

B. Vector mesons

The interaction of the vector mesons with hadrons is analogous to that of the photons and it can be expanded in a multipole expansion. For the S-wave charmed mesons the spin of the light-quark degree of freedom is $S_L = \frac{1}{2}$, which admits an electric charge ($E0$) and magnetic dipole ($M1$) moment, from which the Lagrangian reads

$$\begin{aligned}\mathcal{L}_V &= \mathcal{L}_{E0} + \mathcal{L}_{M1} \\ &= g_{V1}\text{Tr}[H_Q^\dagger H_Q]V^0 \\ &\quad + \frac{f_{V1}}{2M}\epsilon_{ijk}\text{Tr}[H_Q^\dagger H_Q\sigma_i]\partial_j V_k \qquad (30)\end{aligned}$$

$$= q_L^\dagger\left[g_{V1}V^0 + \frac{f_{V1}}{2M}\epsilon_{ijk}\sigma_{Li}\partial_j V_k\right]q_L, \qquad (31)$$

depending on the notation (superfield or subfield), where g_{V1} and f_{V1} are the electric- and magnetic-type couplings with the S-wave charmed mesons, M is a mass scale (it will prove convenient to choose this mass scale equal to the nucleon mass, i.e., $M = M_N$) and $V^\mu = (V^0, \vec{V})$ the vector meson field. For notational convenience we have momentarily ignored the isospin factors. From this the vector-meson exchange potentials are also expressible as a sum of multipole components

$$V_V(\vec{q}) = V_{E0}(\vec{q}) + V_{M1}(\vec{q}), \qquad (32)$$

which read

$$V_{E0}(\vec{q}) = +\frac{g_{V1}^2}{\vec{q}^2 + m_V^2}, \qquad (33)$$

$$\begin{aligned}V_{M1}(\vec{q}) &= -\frac{f_{V1}^2}{4M^2}\frac{(\vec{\sigma}_{L1}\times\vec{q})\cdot(\vec{\sigma}_{L2}\times\vec{q})}{\vec{q}^2 + m_V^2} \\ &= -\frac{f_{V1}^2}{6M^2}\vec{\sigma}_{L1}\cdot\vec{\sigma}_{L2}\frac{\vec{q}^2}{\vec{q}^2 + m_V^2} + \cdots, \qquad (34)\end{aligned}$$

with m_V the vector meson mass and where the second line of the M1 contribution to the potential isolates its S-wave component. After Fourier transforming into coordinate space we end up with

$$V_V(\vec{r}) = [g_{V1}^2 + f_{V1}^2\frac{m_V^2}{6M^2}\vec{\sigma}_{L1}\cdot\vec{\sigma}_{L2}]\frac{e^{-m_V r}}{4\pi r} + \cdots, \qquad (35)$$

where the dots indicate contact-range and tensor terms, which we are ignoring. If we particularize for the ρ meson, we will have to include isospin factors

$$V_\rho(\vec{r}) = \vec{\tau}_1\cdot\vec{\tau}_2[g_{\rho 1}^2 + f_{\rho 1}^2\frac{m_\rho^2}{6M^2}\vec{\sigma}_{L1}\cdot\vec{\sigma}_{L2}]\frac{e^{-m_\rho r}}{4\pi r} + \cdots. \qquad (36)$$

For the ω no isospin factor is required, but there is a sign coming from the negative G-parity of this meson

$$V_\omega(\vec{r}) = \zeta[g_{\omega 1}^2 + f_{\omega 1}^2 \frac{m_\omega^2}{6M^2} \vec{\sigma}_{L1} \cdot \vec{\sigma}_{L2}] \frac{e^{-m_\omega r}}{4\pi r} + \ldots, \quad (37)$$

where, as usual, $\zeta = +1$ (-1) for the meson-meson (meson-antimeson) potential.

The determination of the couplings with the vector mesons follows the same pattern we have used for the axial mesons. The neutral vector mesons, the ω and the ρ^3 (where ^{3} refers to the isospin index, i.e., the neutral ρ), have the same quantum numbers as the photon and thus can mix with the electromagnetic current. It is convenient to write down the mixing in the form

$$\rho_\mu^3 \to \rho_\mu^3 + \lambda_\rho \frac{e}{g} A_\mu, \quad (38)$$

$$\omega_\mu \to \omega_\mu + \lambda_\omega \frac{e}{g} A_\mu, \quad (39)$$

with e the electric charge of the proton and $g = m_V/2f_\pi \simeq 2.9$ the universal vector meson coupling constant. These two substitution rules effectively encapsulate Sakurai's universality and vector meson dominance [52,53,70].

The proportionality constants can be determined from matching with the electromagnetic Lagrangian of the light-quark components of the hadrons. To illustrate this idea, we can apply the substitution rules to the E0 piece of the Lagrangian describing the interaction of the neutral vector mesons with the charmed antimesons, i.e.,

$$\mathcal{L}_{E0} = \text{Tr}[H_{\bar{c}}^\dagger(g_{\rho 1}\tau_3\rho_0^3 + g_{\omega 1}\omega_0)H_{\bar{c}}], \quad (40)$$

where we have chosen the antimesons because they contain light quarks. After applying Eqs. (38) and (39), we end up with

$$\begin{aligned} \mathcal{L}_{E0}^{\text{e.m.(L)}} &= e\text{Tr}\left[H_{\bar{c}}^\dagger\left(\frac{g_{\rho 1}}{g}\lambda_\rho\tau_3 + \frac{g_{\omega 1}}{g}\lambda_\omega\right)H_{\bar{c}}\right]A_0 \\ &= e\text{Tr}[H_{\bar{c}}^\dagger(\lambda_\rho\tau_3 + \lambda_\omega)H_{\bar{c}}]A_0. \end{aligned} \quad (41)$$

where in the second line we have used that $g_{\rho 1} = g_{\omega_1} = g$. This is to be matched with the contribution of the light quarks to the E0 electromagnetic Lagrangian

$$\mathcal{L}_{E0}^{\text{e.m.}} = e\text{Tr}[H_{\bar{Q}}^\dagger(Q_H + Q_L)H_{\bar{Q}}]A^0, \quad (42)$$

where Q_H and Q_L are the electric charges of the heavy antiquark and light quarks in the isospin basis of the superfield $H_{\bar{Q}}$, of which only Q_L is relevant for matching purposes.

$$Q_L = \begin{pmatrix} \frac{2}{3} & 0 \\ 0 & -\frac{1}{3} \end{pmatrix}. \quad (43)$$

which implies that $\lambda_\rho = 1/2$ and $\lambda_\omega = 1/6$. Alternatively we could have determined λ_ρ and λ_ω from the nucleon couplings to the vector mesons ($g_\rho = g$, $g_\omega = 3g$) and their electric charges, leading to the same result.

Given λ_ρ and λ_ω and repeating the same steps but now for the M1 part of the Lagrangian, we can readily infer the magnetic-type coupling f_{V1} of the charmed antimesons with the vector mesons, which turn out to be

$$f_{V1} = g\kappa_{V1} \quad \text{with} \quad \kappa_{V1} = \frac{3}{2}\left(\frac{2M}{e}\right)\mu_L(D^{*0}), \quad (44)$$

where $\mu_L(D^{*0})$ refers to the light-quark contribution to the magnetic moment of the D^{*0} charmed antimeson, which in the heavy-quark limit will coincide with the total magnetic moment of the heavy meson. From the quark model we expect this magnetic moment to be given by the u-quark, i.e.,

$$\left(\frac{2M}{e}\right)\mu_L(D^{*0}) = \left(\frac{2M_N}{e}\right)\mu_u \simeq 1.85, \quad (45)$$

where we have taken $M = M_N \simeq 940$ MeV (i.e., the nucleon mass) so as to express the magnetic moments in units of nuclear magnetons.

The outcome is $g_{V1} = 2.9$ and $\kappa_{V1} = 2.8$, which are the values we will use here. Besides this determination, the vector meson dominance model of Ref. [71] leads to $g_{V1} = 2.6$ and $\kappa_{V1} = 2.3$ (as explained in more detail in Ref. [35]), i.e., compatible with our estimates within the 20% level. For the particular case of the E0 coupling, there is a lattice-QCD calculation for the heavy mesons [72] yielding $g_{V1} = 2.6 \pm 0.1 \pm 0.4$ in the heavy-quark limit (i.e., compatible within errors with $g_{V1} = 2.9$).

IV. DESCRIPTION OF THE $X(3872)$ AND $Z_c(3900/4020)$

Now we will consider the $X(3872)$, $Z_c(3900)$ and $Z_c(4020)$ from the OBE model perspective. The problem we want to address is can they be described together with the same set of parameters? We will find that

(i) the axial meson indeed favors the compatible description of the X and Z_c resonances,

(ii) the effect of axial mesons depends on the choice of a mass for the scalar meson in the OBE model.

In general, lighter scalar meson masses will diminish the impact of axial meson exchange and eventually even vector meson exchange, leading to the binding of both the X and Z_c for $m_\sigma \to 400$ MeV. This is not necessarily a desired feature, as the Z_c in the molecular picture is not necessarily a bound state but more probably a virtual state or a resonance [26,27]. That is, we expect the strength of the charmed meson-antimeson potential to be short of binding for the Z_c and Z_c^*. However, as the mass of the scalar meson

increases and reaches the standard values traditionally used in the OBE model, $m_\sigma \sim 500$ MeV–600 MeV, the importance of the axial meson becomes clearer, where the a_1 meson might be the difference between a virtual state close to the threshold or not.

A. Molecular degrees of freedom: Which interpretation to choose?

The nature of the $X(3872)$ and the $Z_c(3900/4020)$ is still an open problem. Here we will assume that they are molecular, that is, that they are two-meson states. This requires us to identify two-meson thresholds close to the masses of the $X(3872)$ and $Z_c(3900/4020)$ states and compatible with their quantum numbers. The most obvious candidates are the different charmed meson-antimeson combinations, e.g., the $D^*\bar{D}$ for the $X(3872)$ and $Z_c(3900)$, and the $D^*\bar{D}^*$ for the $Z_c(4020)$. This will be the choice we will make in the present work.

However, this is not the only possibility. For instance, some interpretations assume that the $X(3872)$ contains a $J/\psi\omega$ component [73], to which we may include $J/\psi\rho$ if isospin breaking effects are explicitly considered. For the $Z_c(3900/4020)$, if we assume that their quantum numbers are indeed $I^G(J^{PC}) = 1^+(1^{+-})$, they could also contain a $\eta_c\rho$ component [37] (though this channel is located further from the Z_c's than the charmed meson-antimeson components). If we extend this argument to the $Z_{cs}(3985)$, besides the charmed meson-antimeson $D_s^*\bar{D}$ and $D_s\bar{D}^*$ components, we could also add $\eta_c K^*$ (or even $J/\psi K$) [32].

Though these degrees of freedom have been explicitly considered in other works, we will not include them. The reason is their expected relative strength and range when compared to the other meson exchanges considered here. To illustrate this idea, we might consider the $\eta_c\rho$ component in the $Z_c(3900)$, for which the $\eta_c\rho \to D^*\bar{D}$ transition potential is mediated by charm vector meson exchange. The form of this potential can in principle be deduced in a way analogous to the vector meson exchange potential, leading to

$$\langle \eta_c\rho | V_{D^*}(\vec{q}) | D^*\bar{D}(Z_c)\rangle = \frac{h_1 h_2}{\mu_{D^*}^2 + \vec{q}^2}, \qquad (46)$$

where for simplicity we have only considered the E0 component; "(Z_c)" indicates that we are already projecting into the $Z_c(3900)$ channel, $h_{1(2)}$ are the relevant coupling constants in the vertices 1(2), μ_D^* is the effective mass of the exchanged D^* meson, which is somewhat lighter than its physical mass owing to the fact that the D^* meson has a nontrivial zeroth component of its four-momentum (check Sec. IV F for a more detailed explanation). This potential is relatively short ranged, but (light) vector meson exchange cancels out in the Z_c channel, meaning that D^* exchange could be more important than expected.

But this conclusion still depends on the absence of other light-meson exchange contributions that could mask the D^* exchange. In this work we have at least two of these contributions; scalar and axial meson exchange. If we compare the expected strength of a_1 and D^* exchanges at low momenta (i.e., the potentials of Eqs. (13) and (46) at $|\vec{q}^2| \to 0$), their ratio will be

$$\lim_{\vec{q}^2 \to 0} \left| \frac{V_{D^*}(\vec{q}; Z_c)}{V_{a_1}(\vec{q}; Z_c)} \right| = \frac{h_1 h_2}{\mu_{D^*}^2} \frac{2f_\pi^2}{3\lambda_1^2 g_1^2} \simeq 0.04, \qquad (47)$$

where following Ref. [32] we have taken $h_1 = \sqrt{2}g_V(m_D + m_{\eta_c})/\sqrt{2m_D}\sqrt{2m_{\eta_c}} \simeq 1.45g_V$ and $h_2 = g_V$, while for the effective mass we have used $\mu_{D^*} \simeq \sqrt{m_{D^*}^2 - (m_{\eta_c}^2 - m_D^2)} \simeq 1.67$ GeV (check Sec. IV F). This preliminary comparison indicates that the contribution from the D^* exchange can be probably neglected if we have already included axial meson exchange. However if neither scalar nor axial meson exchanges are included or are expected to be weaker than here, it will make sense to include the D^* exchange and the $\eta_c\rho$ channel. It is also worth mentioning that there is another factor reducing the potential importance of the aforementioned $\eta_c\rho$ channel; it is located about 200 MeV below the $Z_c(3900)$. As a consequence, iterations of the D^*-exchange potential will be further suppressed owing to this mass gap. Yet, a caveat is in place; the previous argument does not take into account the effect of form factors (see Sec. IV D), which could be very different in the a_1 and D^* exchange cases, or all the possible channels or meson exchanges involved. These effects could increase the importance of channels other than charmed meson-antimeson.

B. General structure of the potential

Before considering the light-meson exchange potential in detail, we will review the general structure of the S-wave potential. For the $D^{(*)}\bar{D}^{(*)}$ system there are two relevant symmetries—SU(2) -isospin and HQSS—from which we decompose the potential into

$$V = (V_a + \tau W_a) + (V_b + \tau W_b)\vec{\sigma}_{L1} \cdot \vec{\sigma}_{L2}, \qquad (48)$$

with $\tau = \vec{\tau}_1 \cdot \vec{\tau}_2$. In this notation, the X and Z potentials read

$$V_X = (V_a - 3W_a) + (V_b - 3W_b), \qquad (49)$$

$$V_Z = (V_a + W_a) - (V_b + W_b). \qquad (50)$$

However, it will be more useful to define the isoscalar and isovector contributions to the potential as follows:

$$V_a^{(0)} = V_a - 3W_a, \qquad (51)$$

$$V_b^{(0)} = V_b - 3W_b, \qquad (52)$$

$$V_a^{(1)} = V_a + W_a, \qquad (53)$$

$$V_b^{(1)} = V_b + W_b, \qquad (54)$$

from which the general structure of the potential is the one shown in Table I. In the following we will explain what the contributions of each light-meson to the potential are, yet we can advance that

(i) $V_a^{(I)}$ receives contributions from σ and E0 ρ/ω and is attractive for $D^{(*)}\bar{D}^{(*)}$ molecules.

(ii) $V_b^{(I)}$ receives contributions from the pion, the axial meson and M1 ρ/ω exchange:

(a) $V_b^{(0)}$ is dominated by M1 vector meson exchange and its sign is negative, making the $X(3872)$ the most attractive isoscalar molecular configuration.

(b) $V_b^{(1)}$ is dominated by axial meson exchange and its sign is positive, implying that the $Z_c(3900)$ and $Z_c(4020)$ are among the most attractive isovector molecular configurations.

(c) The most attractive isovector configuration should be the $I^G(J^{PC}) = 1^-(0^{++})D^*\bar{D}^*$ molecule, though no molecular state has been found yet with these quantum numbers.

Finally, the potentials for the $Z_c(3900)$ and $Z_c(4020)$ are identical, which explains the evident observation that they come in pairs [23]. For this reason, from now on we will ignore the $Z_c(4020)$ and concentrate in the $Z_c(3900)$, as results in the latter automatically apply to the former.

C. The OBE potential

In the OBE model the $H_c H_c'$ and $H_c \bar{H}_c'$ potentials (where H_c, H_c' represent the S-wave charmed mesons) can be written as the sum of each light-meson contribution

$$V_{\text{OBE}} = V_\pi^{(\zeta)} + V_\sigma + V_\rho + V_\omega^{(\zeta)} + V_{a_1}^{(\zeta)}, \qquad (55)$$

where the individual contributions have been already discussed in this paper (for the approximation in which the hadrons are pointlike):

(i) $V_\pi^{(\zeta)}$ and $V_{a1}^{(\zeta)}$ in Eqs. (14) and (15)

(ii) V_σ in Eq. (28)

(iii) V_ρ and $V_\omega^{(\zeta)}$ in Eqs. (36) and (37).

We have included the superscript $^{(\zeta)}$ as a reminder that the contributions stemming from exchange of negative G-parity light mesons (π, ω, a_1) change sign depending on whether we are considering the meson-meson ($\zeta = +1$) or meson-antimeson ($\zeta = -1$) systems. These signs have been already included in the definition of the potential contributions, i.e., in Eqs. (14), (15), and (37). For convenience we review our choice of couplings in Table II.

Here we consider only the S-wave component of the light-meson exchange potential, i.e., we ignore the tensor (S-to-D-wave) components. This choice allows a simpler analysis of the factors involved in binding.

Finally, we will assume the OBE model to be a fairly complete description of the charmed meson-antimeson potential. Even though there are shorter-range components of the potential, e.g., the previously discussed transition potentials into the charmonium—(light) meson channels or the vector charmonium exchange potentials considered in Refs. [36–39]—if we follow the arguments of Sec. IV A. these pieces of the potential should be suppressed with respect to (light) meson exchange. Nonetheless, they could

TABLE I. SU(2)-isospin and HQSS structure of the S-wave potential in the heavy meson-antimeson molecules. "System" indicates the specific charmed meson-antimeson molecule, $I(J^{PC})$ its quantum numbers, "Potential" the potential and "Candidate" refers to known experimental resonances that might be explained by the specific configuration considered. $V_a^{(I)}$ and $V_b^{(I)}$ are the central and spin-spin pieces of the potential, with $I = 0, 1$ referring to the isospin. From light-meson exchanges we expect $V_a^{(0)} < 0$ and $V_b^{(0)} < 0$ in the isoscalar sector, which makes the 1^{++} and 2^{++} the most promising configurations for binding (in the absences of other binding factors, e.g., coupled channels, nearby charmonia, etc.). For the isovector sector we expect $V_a^{(1)} < 0$ and $V_b^{(1)} > 0$, from which the 0^{++} and 1^{+-} configurations are the most promising. However $V_b^{(1)}$ is really weak, making this conclusion contingent on other factors (e.g., isospin breaking in vector meson exchange).

System	$I^G(J^{PC})$	Potential	Candidate	System	$I^G(J^{PC})$	Potential	Candidate
$D\bar{D}$	$0^+(0^{++})$	$V_a^{(0)}$	$\cdots$	$D\bar{D}$	$1^-(0^{++})$	$V_a^{(1)}$	$\cdots$
$D^*\bar{D}$	$0^-(1^{+-})$	$V_a^{(0)} - V_b^{(0)}$	$\cdots$	$D^*\bar{D}$	$1^+(1^{+-})$	$V_a^{(1)} - V_b^{(1)}$	$Z_c(3900)$
$D^*\bar{D}$	$0^+(1^{++})$	$V_a^{(0)} + V_b^{(0)}$	$X(3872)$	$D^*\bar{D}$	$1^-(1^{++})$	$V_a^{(1)} + V_b^{(1)}$	$\cdots$
$D^*\bar{D}^*$	$0^+(0^{++})$	$V_a^{(0)} - 2V_b^{(0)}$	$\cdots$	$D^*\bar{D}^*$	$1^-(0^{++})$	$V_a^{(1)} - 2V_b^{(1)}$	$\cdots$
$D^*\bar{D}^*$	$0^-(1^{+-})$	$V_a^{(0)} - V_b^{(0)}$	$\cdots$	$D^*\bar{D}^*$	$1^+(1^{+-})$	$V_a^{(1)} - V_b^{(1)}$	$Z_c(4020)$
$D^*\bar{D}^*$	$0^+(2^{++})$	$V_a^{(0)} + V_b^{(0)}$	$\cdots$	$D^*\bar{D}^*$	$1^-(2^{++})$	$V_a^{(1)} + V_b^{(1)}$	$\cdots$

TABLE II. Couplings of the light mesons we are considering in this work (π, σ, ρ, ω and a_1) to the charmed mesons. For the masses of the light mesons we will use $m_\pi = 138$ MeV, $m_\sigma = 550$ MeV, $m_\rho = 770$ MeV, $m_\omega = 780$ MeV and $m_{a_1} = 1230$ MeV. For the vector mesons we use the scaling mass $M = 938$ MeV. For the charmed mesons we will consider their isospin-averaged masses, $m_D = 1867$ MeV and $m_{D^*} = 2009$ MeV.

Coupling	Value	Relevant to meson(s)
g_1	0.60	π, a_1
g_σ	3.4	σ
g_{V1}	2.9	ρ, ω
κ_{V1}	2.8	ρ, ω
λ_1	1.8	a_1

be easily included as a contact-range potential following the general structure of Table I. This type of improved OBE with contacts has been investigated in the two-nucleon system [74], where its main advantage is that it allows for the renormalization of the OBE model, i.e., it becomes possible to generate observable results that are independent of meson form factors and cutoffs. However, the detailed study of these effects lies beyond the scope of the present work; for instance, it will involve the determination of four independent coupling constants corresponding to the four independent potential components of Table I, or a minimum of four molecular candidates to calibrate said couplings.

D. Form factors and regulators

As mentioned, the previous form of the potential assumes pointlike hadrons. The finite size of the hadrons involved can be taken into account with different methods, e.g., form factors. The inclusion of form factors amounts to multiply each vertex involving a heavy hadron and light meson by a function of the exchanged momentum, i.e.,

$$\mathcal{A}_R(H \to HM(q)) = f_M(q)\mathcal{A}(H \to HM(q)), \quad (56)$$

where $\mathcal{A}$ and $\mathcal{A}_R$ are the pointlike and regularized amplitudes, respectively, and $f(q)$ the form factor. In terms of the potential, the inclusion of a form factor is equivalent to the substitution rule

$$V_M(\vec{q}) \to f_M^2(\vec{q})V_M(\vec{q}). \quad (57)$$

Here we will use multipolar form factors, i.e.,

$$f_M(q) = \left(\frac{\Lambda^2 - m^2}{\Lambda^2 - q^2}\right)^{n_P}, \quad (58)$$

with Λ the form-factor cutoff, $q^2 = q_0^2 - \vec{q}^2$ the exchanged four-momentum of the meson M, m the mass of the meson, and n_P the multipole momentum. In general this procedure requires that $\Lambda > m$.

The form-factor cutoff can be different for each of the exchanged mesons, which is what happens for instance in the meson theory of nuclear forces [13,14]. However, we will also consider the simplification of a single cutoff for all exchanged mesons; this choice is popular within OBE descriptions of hadronic molecules as it entails less free parameters. In contrast with the two-nucleon system, the number of actual data for the different two-hadron systems is usually limited to a few bound state candidates at most. This indeed favors theoretical simplifications such as a single cutoff, but it is important to stress that there is no compelling phenomenological reason why this should have to be the case. For the inclusion of the axial meson, we advance that the assumption of a single cutoff entails $n_P \geq 2$ (otherwise, the cutoff will be smaller than the axial meson mass). If we allow for each meson to have its own cutoff, there will be no constraints on the polarity of the form factor.

It is also worth mentioned that in a first approximation we will assume the cutoffs to be identical in the different isospin, flavor and heavy/light-quark spin channels. However this assumption only holds if the previous symmetries are perfectly preserved, which is not the case. We will later discuss how the breaking effects of these symmetries (in particular flavor and HQSS) might play a role in the coherent description of the $X(3872)$, $Z_c(3900)$, and also the $Z_{cs}(3985)$.

Multipolar form factors are local regulators and thus they still generate a local potential for which the Fourier transform is analytic. The expressions can be a bit convoluted though, particularly for the dipolar and higher-momentum form factors. Here we only consider the S-wave piece of the light-meson exchange potentials where the contact-range contributions have been removed, which slightly simplifies the analytic expressions. First, for each meson we have a Yukawa-like potential of the type

$$V_M(\vec{r}) = \frac{g_Y^2}{4\pi}\left(\mathcal{O}_I \sum_i c_i \mathcal{O}_S^i\right)\frac{e^{-mr}}{r}$$

$$= g_Y^2\left(\mathcal{O}_I \sum_i c_i \mathcal{O}_S^i\right)m W_Y(mr), \quad (59)$$

with g_Y the effective Yukawa coupling, $\mathcal{O}_I$ and $\mathcal{O}_S$ isospin and spin operators, m the mass of the exchanged light meson, and where the exact potential could involve a sum of different spin operators with c_i their coefficients. In the second line we have included the dimensionless function

$$W_Y(x) = \frac{e^{-x}}{4\pi x}, \quad (60)$$

which is the only thing that changes when a multipolar form factor is included

$$W_Y(x) \rightarrow W_Y(x, \lambda; k_P), \tag{61}$$

where $\lambda = \Lambda/m$, $k_P = 2n_P$ and with

$$W_Y(x, \lambda, k_P) = \int \frac{d^3 z}{(2\pi)^3} \left(\frac{\lambda^2 - 1}{\lambda^2 + z^2} \right)^{k_P} \frac{e^{i\vec{z} \cdot \vec{x}}}{1 + z^2}. \tag{62}$$

The general form of W_Y for integer $k_P \geq 1$ can be found by recursion. If we redefine W_Y as

$$W_Y(x, \lambda; k_P) = (\lambda^2 - 1)^{k_P} I_Y(x, \lambda; k_P), \tag{63}$$

then I_Y follows the recursive relation

$$I_Y(x, \lambda; 1) = \frac{1}{\lambda^2 - 1} [W_Y(x) - \lambda W_Y(\lambda x)], \tag{64}$$

$$I_Y(x, \lambda; k_P > 1) = \frac{1}{2\lambda k_P} \frac{d}{d\lambda} [I_Y(x, \lambda; k_P - 1)], \tag{65}$$

from which we can find the form of the potential for arbitrary multipolar form factors.

For the choice of the polarity n_P, if we decide to follow the simplifying assumption of a single cutoff for all exchanged mesons, we will have to choose at least a dipolar form factor ($n_P \geq 2$). Otherwise the form factor cutoff will be lighter than the axial meson, rendering it impossible the inclusion of said meson with a multipolar form factor. In particular, for $n_P = 1$ we have to remove the axial meson to correctly reproduce the $X(3872)$, in which case the necessary cutoff is $\Lambda = 1.00$ GeV (compatible with $\Lambda = 1.01^{+0.18}_{-0.10}$ GeV in the OBE of Ref. [35], which also uses a monopolar form factor).

E. Finite meson width effects

Previously we have simply assumed that the light mesons generating the OBE potential are narrow states. However, though this assumption is well justified in the case of the pion, the omega and to a lesser extent the rho, the scalar and axial mesons are rather broad actually, where in the RPP [60] their widths are listed to be

$$\Gamma_\sigma = 400\text{--}800 \text{ MeV} \quad \text{and} \quad \Gamma_{a_1} = 250\text{--}600 \text{ MeV}. \tag{66}$$

Naturally this raises the question of how these large widths can be taken into account within the OBE model.

This problem being rather conspicuous, particularly in the case of the σ, has been investigated in the past. The basic idea is to substitute the narrow meson propagator by a propagator averaged over the actual mass distribution of the meson

$$\frac{1}{m^2 + \vec{q}^2} \rightarrow \int_{m_{\text{th}}}^\infty \frac{\rho(\mu^2) d(\mu^2)}{\mu^2 + \vec{q}^2}, \tag{67}$$

where $\rho(\mu^2)$ is the spectral distribution of the wide meson and m_{th} the threshold mass of the particles into which this meson can decay. The evaluation of this integral depends on the form of $\rho(\mu^2)$, which in turn has led to different approximations for handling wide mesons. Five decades ago Ref. [75] proposed a really practical solution for wide mesons decaying into two pions (i.e., this solution is tailored for the σ), which amounts to a two-pole approximation of the previous integral

$$\int_{m_{\text{th}}}^\infty \frac{\rho(\mu^2) d(\mu^2)}{\mu^2 + \vec{q}^2} \approx \frac{\alpha_1}{m_1^2 + \vec{q}^2} + \frac{\alpha_2}{m_2^2 + \vec{q}^2}, \tag{68}$$

where α_1 and α_2 are positive numerical coefficients such that $\alpha_1 + \alpha_2 = 1$ and m_1, m_2 are the masses of the two poles which obey the relation $m_1 < m$ and $m_2 > m$. This two-pole approximation has been used for instance in the Nijmegen high-precision potentials [76].

Later, Ref. [77] proposed a more detailed solution, which begins by considering the exchange of a narrow meson generating a Yukawa-like potential of the type

$$V_Y(r) = -\frac{g_Y^2}{4\pi} \frac{e^{-mr}}{r}, \tag{69}$$

with g_Y the coupling. When this meson acquires a width, the evaluation of the integral over the spectral mass distribution [i.e. Eq. (67)] results in a potential that is the sum of a few contributions, where we refer to Ref. [77] for details. The two most important of these contributions are the following: close to $mr \sim 1$, the original Yukawa potential is modified into a potential of the type

$$V_Y(r) \approx -\frac{g^2}{4\pi} \frac{e^{-mr}}{r} \left(1 - \frac{\Gamma r}{\pi} - \frac{\Gamma}{\pi m} \right), \tag{70}$$

with Γ the width of the meson. In practical terms this implies that at short distances the potential for a broad meson is weaker than for a narrow one, where it is interesting to notice that one can still use the pole mass of the meson. The other relevant contribution to the potential of a broad meson comes from this meson decaying into two lighter mesons of mass $2M$, which will generate an additional attractive longer-range contribution to the potential at distances $2Mr \sim 1$. In the case of the σ meson, we would expect the appearance of a two-pion exchange contribution, while for the a_1 meson this procedure will give us a $\pi\rho$ exchange potential. To summarize, in comparison to a narrow meson, a wide meson exchange potential will be weaker at short distances and stronger at long distances.

At this point it is worth noticing that though the analysis of Ref. [77] has not been explicitly used for the construction of meson exchange potentials, it nonetheless explains the features of previous OBE models. For instance, the

relation between two-pion and sigma exchange [65–68] (or $\pi\rho$ and a_1 exchange [15,78]) is well known and has been extensively discussed in the past.

Yet, the actual effect of a meson is related both to its coupling and range. In this regard, it might be possible to simply disregard the complications coming from the width of the meson in favor of considering the mass and coupling of the meson as effective parameters. This idea seems to be compatible with the practice of OBE models, where we can compare the Bonn-A/B [13,14] and CD-Bonn [79] models for illustrative purposes. The Bonn-A and Bonn-B models are traditional OBE potentials where the exchanged bosons are effectively treated as narrow; for the case of the σ meson it features a mass and a coupling of $m_\sigma = 550$ MeV and $g_\sigma^2/4\pi \simeq 8.7$–8.8 (which basically coincides with coupling expected in the LσM, $g_\sigma^2/4\pi \simeq 8.3$). In contrast the CD-Bonn model includes two-boson exchange (e.g., two-pion exchange) and the mass and couplings of the σ meson are $m_\sigma = 452$ MeV and $g_\sigma^2/4\pi \simeq 4.25$ instead. This actually illustrates a few of the ideas of Ref. [77]; in practice the strength of the σ exchange potential in the CD-Bonn model is about 50% weaker than in the Bonn-A/B model, but this is counterbalanced by the presence of two-pion exchange in CD-Bonn. In the end, the two types of OBE potentials are roughly equivalent at the observable level, yielding comparable predictions.

From this later observation, in this work we will simply treat the wide scalar and axial mesons as narrow, with their masses and couplings being considered as effective instead of fundamental. Obviously there is a relation between the effective and physical values, which is what is discussed in Refs. [75,77], but no unique solution for taking into account the finite width effects. This in turn explains why the parameters of these mesons can take such different values depending on the approach. What we will do is to consider how predictions change with different values of the masses and couplings of the scalar and axial mesons.

F. Mass gaps and effective meson masses

When the light-meson exchange potential entails a transition between two hadrons of different masses, i.e., there is a vertex of the type

$$H \rightarrow H'M, \tag{71}$$

with H, H' the initial and final hadrons and M the light meson, we will have to modify the effective mass of the in-flight light meson. In this case the light-meson exchange potential is not diagonal and entails a transition between the HH' and $H'H$ configurations

$$V_M(\vec{q}) = V_M(\vec{q}, HH' \rightarrow H'H), \tag{72}$$

for which the light-meson propagator in the exchange potential will change to

$$\frac{1}{m^2 + \vec{q}^2} \rightarrow \frac{1}{\mu^2 + \vec{q}^2}, \tag{73}$$

where μ is the effective mass of the light meson, i.e.,

$$\mu^2 = m^2 - \Delta^2, \tag{74}$$

with $\Delta = m(H') - m(H)$.

If we are dealing with a charmed meson-antimeson system, this correction will only have to be taken into account for the $D^*\bar{D}$ and $D\bar{D}^*$ systems, i.e., for the $X(3872)$ and $Z_c(3900)$. Only the spin-spin part of the potential will be affected, as the central part cannot generate a transition between the D and D^* charmed mesons. For the pion and axial meson exchange potentials, the correction is trivial

$$V_\pi(\vec{r}) = \zeta \frac{g_1^2 \mu_\pi^2}{6 f_\pi^2} \vec{\tau}_1 \cdot \vec{\tau}_2 \vec{\sigma}_{L1} \cdot \vec{\sigma}_{L2} \frac{e^{-\mu_\pi r}}{4\pi r}, \tag{75}$$

$$V_{a_1}(\vec{r}) = -\zeta \lambda_1^2 \frac{g_1^2 m_{a1}^2}{3 f_\pi^2} \vec{\tau}_1 \cdot \vec{\tau}_2 \vec{\sigma}_{L1} \cdot \vec{\sigma}_{L2} \frac{e^{-\mu_{a1} r}}{4\pi r}, \tag{76}$$

with μ_π and μ_{a1} the effective pion and axial meson masses, where we notice that for axial meson exchange the $m_{a1} \rightarrow \mu_{a1}$ substitution is limited to the long-range decay exponent (but not the m_{a1}^2 factor involved in the strength of the potential). For the vector meson exchange potential the correction only affects its spin-spin piece

$$V_V(\vec{r}) = g_{V1}^2 \frac{e^{-m_V r}}{4\pi r} + f_{V1}^2 \frac{\mu_V^2}{6M^2} \vec{\sigma}_{L1} \cdot \vec{\sigma}_{L2} \frac{e^{-\mu_V r}}{4\pi r}. \tag{77}$$

Finally for the sigma meson no modification is required.

If we combine this modification with a multipolar form factor, from direct inspection of Eq. (58) we find that besides modifying the effective mass of the light-meson, we also have to modify its cutoff

$$m \rightarrow \mu \quad \text{and} \quad \Lambda \rightarrow \sqrt{\Lambda^2 - \Delta^2}. \tag{78}$$

Taking into account that $m(D^*) - m(D) \sim 140$ MeV, the only light meson that is substantially affected by this change is the pion, for which its spin-spin contribution essentially vanishes for the $X(3872)$ and $Z_c(3900)$ molecules.

G. Bound state equation

For obtaining predictions with the S-wave OBE potential of Eq. (55), we plug it into the reduced Schrödinger equation

$$-u''(r) + 2\mu_{HH} V_{\text{OBE}}(r) u(r) = -\gamma^2 u(r), \tag{79}$$

where $u(r)$ is the reduced wave function, μ_{HH} is the reduced mass of the particular charmed meson-antimeson

TABLE III. Cutoffs required to reproduce the $X(3872)$ and to bind a molecular Z_c at threshold in a OBE model with and without axial mesons for different masses of the scalar meson. We use a multipolar form factor with polarity $n_P = 2, 3, 4$ at each vertex, Eq. (58). $\Lambda(X/Z_c)$ shows the $X(3872)$ and Z_c cutoffs for the OBE model including the axial meson, while for the axial-less case we add the superscript $\cancel{a}_1$. Finally we show the ratio between the $X(3872)$ and Z_c cutoffs. The central value represents $m_\sigma = 550$ MeV, the error in the central values arise from the uncertainty in $g_\sigma = 3.4 \pm 1.0$ and the intervals (in parentheses) correspond to $m_\sigma = 450$–600 MeV.

Polarity (n_P)	$\Lambda(X)$	$\Lambda^{\cancel{a}_1}(X)$	$\Lambda(Z_c)$	$\Lambda^{\cancel{a}_1}(Z_c)$	$R(Z_c/X)$	$R^{\cancel{a}_1}(Z_c/X)$
2	$1.37^{+0.08}_{-0.09}$ (1.27–1.41)	$1.37^{+0.09}_{-0.09}$ (1.27–1.41)	$1.82^{+0.62}_{-0.43}$ (1.37–1.99)	$1.99^{+\infty}_{-0.60}$ (1.37–2.38)	$1.33^{+0.35}_{-0.24}$ (1.08–1.41)	$1.45^{+\infty}_{-0.36}$ (1.08–1.69)
3	$1.65^{+0.10}_{-0.11}$ (1.53–1.69)	$1.65^{+0.11}_{-0.11}$ (1.53–1.70)	$2.19^{+0.76}_{-0.51}$ (1.67–2.40)	$2.44^{+\infty}_{-0.75}$ (1.68–2.92)	$1.33^{+0.36}_{-0.24}$ (1.09–1.42)	$1.48^{+\infty}_{-0.38}$ (1.10–1.72)
4	$1.95^{+0.11}_{-0.13}$ (1.82–2.00)	$1.97^{+0.13}_{-0.14}$ (1.83–2.03)	$2.72^{+0.96}_{-0.65}$ (2.10–2.96)	$3.26^{+\infty}_{-1.16}$ (2.16–4.02)	$1.39^{+0.39}_{-0.26}$ (1.15–1.48)	$1.65^{+\infty}_{-0.51}$ (1.18–1.98)

system under consideration, and γ is the wave number of the bound state which is related to the two-body binding energy (B_2) by $B_2 = \frac{\gamma^2}{2\mu_{HH}}$. For the $X(3872)$, we will consider it to be a $D^*\bar{D}$ system bound by 4 MeV in the isospin symmetric limit (i.e., we will be using the isospin-averaged D and D^* masses). For the $Z_c(3900)$ we will be interested in determining the cutoff for which it becomes a bound state at threshold, that is, the $\gamma = 0$ limit, as we will explore in the following lines.

H. The $X(3872)$ and $Z_c(3900)$ cutoffs

Now we will explore whether the $X(3872)$ and $Z_c(3900)$ can be explained with the same set of parameters in a simplified OBE model with a single cutoff for all exchanged mesons. This is not necessarily the most realistic assumption—the OBE model in the two-nucleon sector has different cutoffs for each of the exchanged mesons [13,14]—but it allows for a simpler analysis. Here, what we are interested in is whether it is plausible to explain these two states with compatible parameters in the molecular picture, which seems to be the case.

As previously discussed, the single cutoff OBE model requires a polarity of $n_P \geq 2$ for the form factor. With a dipolar form factor $(n_P = 2)$ we can reproduce the mass of the X with

$$\Lambda(X) = 1.37^{+0.08}_{-0.09}(1.27–1.41)\ \text{GeV}. \qquad (80)$$

The central value corresponds to $m_\sigma = 550$ MeV, which is the σ mass used in the original OBE model for nuclear forces [13,14], the error comes from the uncertainty in the scalar coupling $g_\sigma = 3.4 \pm 0.1$, while the spread represents the mass range $m_\sigma = 450$ MeV $- 600$ MeV (with $g_\sigma = 3.4$), which covers most of the plausible choices for its mass. For $m_\sigma = 450$ MeV the dipolar cutoff is merely a bit above the axial meson mass, $m_{a1} = 1.23$ GeV. In contrast the cutoff for which the Z_c binds at threshold is

$$\Lambda(Z_c) = 1.82^{+0.62}_{-0.43}(1.37–1.99)\ \text{GeV}, \qquad (81)$$

for which we get the ratio

$$\frac{\Lambda(Z_c)}{\Lambda(X)} = 1.33^{+0.35}_{-0.24}(1.08–1.41). \qquad (82)$$

Now if we assume that the cutoffs for the X and the Z_c are the same, modulo HQSS violations (the size of which is about Λ_{QCD}/m_Q, i.e., of the order of 15% for $m_Q = m_c$, where we have taken $\Lambda_{QCD} \sim 200$ MeV), this ratio should be one within the aforementioned HQSS uncertainty

$$\frac{\Lambda_Z}{\Lambda_X} = 1 \pm 0.15 = (0.85–1.15), \qquad (83)$$

which means that the existence of the X is compatible within one standard deviation with a Z_c binding at the threshold for the lower σ meson mass range. This lower σ mass range basically gives cutoffs that are barely larger than the axial meson mass, which indicates that we should consider larger dipolar momenta for a better comparison. This is done in Table III, where we extend the comparison to the $n_P = 3, 4$ (i.e., tripolar and quadrupolar) cases; yet, we consistently end up about two standard deviations away. However this discrepancy is not troubling; a molecular Z_c is expected to be a virtual state or a resonance near the threshold, which means that the amount of attraction in the $1^+(1^{+-})D^*\bar{D}$ system is not enough to bind the charmed meson and antimeson at threshold. That is, the ratio should be larger than one [but still of $\mathcal{O}(1)$], though it is difficult to estimate how much larger. Thus the previous ratio and the ones in Table III are probably compatible with a molecular Z_c.

However, the most interesting comparison is against the OBE model without the axial meson, which will reveal the conditions under which the axial meson might be relevant. If we remove the axial meson, the cutoffs we get are

$$\Lambda^{\cancel{a}_1}_X = 1.37^{+0.09}_{-0.09}(1.27–1.41)\ \text{GeV}, \qquad (84)$$

$$\Lambda^{\cancel{a}_1}_Z = 1.99^{+\infty}_{-0.60}(1.37–2.38)\ \text{GeV}, \qquad (85)$$

where Z_c does not bind for the lower values of g_σ (we discuss this in a moment), hence the $^{+\infty}$ error, yielding the ratio

$$\frac{\Lambda_Z^{\phi_1}}{\Lambda_X^{\phi_1}} = 1.45^{+\infty}_{-0.36}(1.08\text{–}1.69), \qquad (86)$$

which results in larger relative cutoffs as the σ gets heavier, increasing the discrepancy with HQSS to the three standard deviation level (if we assume a molecular Z_c at the threshold, which is probably too restrictive). The ratios also grow larger for higher polarity n_P (see Table III). Again lighter σ masses result in cutoffs that are not completely satisfactory if we want to take the axial mesons into account. Finally, in the left panel of Fig. 1 we show the dependence of the cutoff ratio with the mass of the σ for a dipolar form factor, where we can see again that the impact of the axial meson increases with the mass of the scalar meson.

At first sight the comparison between the axial-full and axial-less OBE models indicates a modest contribution from the axial mesons. But the observation that the cutoff ratios increase with larger scalar meson masses, left panel of Fig. 1, and with it the compatibility of the molecular description of the X and the Z_c decreases, indicates that the previous conclusion depends on the strength of scalar meson exchange and the parameters used to describe the later. We actually do not know the coupling of the σ to the charmed baryons very precisely, but with considerable errors: the LoM and the quark model suggest $g_\sigma = 3.4 \pm 1.0$, where this uncertainty turns out to be important. If the attraction provided by the σ falls short of binding, the axial meson will be the difference between the charmed meson-antimeson interaction being weak or strong. Indeed, if there is no axial meson, the condition for the isovector $D^*\bar{D}$ system to bind is

$$g_\sigma \geq 2.45(2.22\text{–}2.56), \qquad (87)$$

which is within the expected uncertainties for the scalar coupling. That is, σ exchange is by itself no guarantee that the $Z_c(3900)$ can be explained in terms of the charmed meson-antimeson interaction alone. In the right panel of Fig. (1) we visualize the dependence of the $\Lambda(Z_c)/\Lambda(X)$ ratio as a function of g_σ, which further supports the previous interpretation of axial meson exchange as the factor guaranteeing the required molecular interaction necessary for the Z_c. Finally for a σ-less theory with axial mesons the Z_c will still bind for large cutoffs, with concrete calculations yielding

$$\Lambda^{\phi}(X) = 1.55 \text{ GeV} \quad \text{and} \quad \Lambda^{\phi}(Z_c) = 3.62 \text{ GeV}, \qquad (88)$$

for which the ratio is 2.33. In this later scenario the uncertainty of the factor involved in the mixing of the pion and axial meson current ($\lambda_1 = 1.8 \pm 0.3 = 1.5\text{–}2.1$) might be relevant, as this error induces the ratio to move within the 2.01–2.99 range.

The previous analysis of the scalar mass and coupling dependence is motivated in part by the width of the σ, which implies that its parameters as applied to the OBE model are in a sense effective and do not necessarily coincide with its physical parameters (see the discussion in Sec. IV E). This factor is also present for the axial meson, though in a lesser extent owing to a smaller width to mass ratio. Nonetheless it is relevant to study how the cutoff ratio depends on m_{a_1} and λ_1, which we do in Fig. 2. What we find is that the uncertainties in the $R(Z_c/X)$ ratio show a weaker dependence with the axial meson mass and
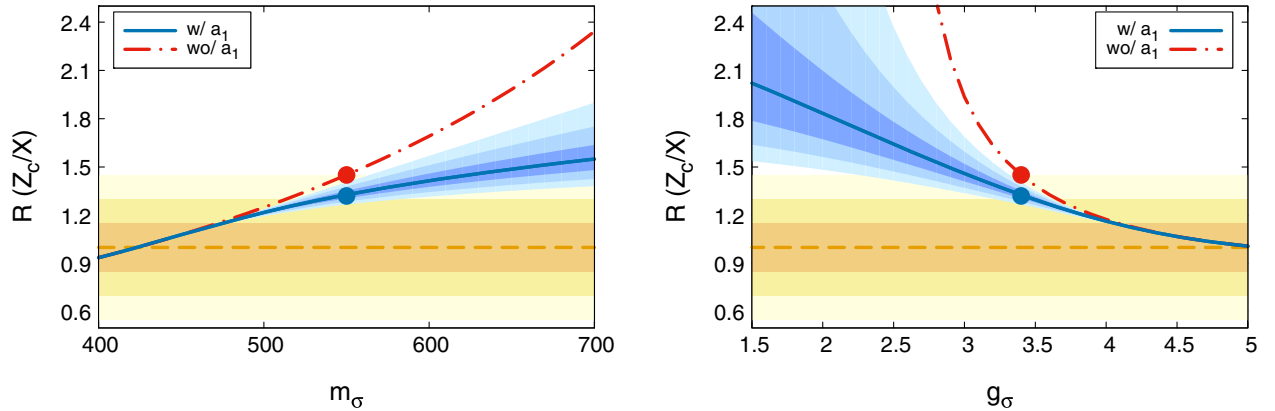

FIG. 1.   Cutoff ratios $R(Z_c/X)$ as a function of the mass and the coupling of the scalar meson for the OBE model with (solid lines) and without (dashed-dotted lines) axial meson exchange. $R(Z_c/X)$ is defined as the ratio of the cutoff for which a molecular Z_c will be a bound state at threshold over the cutoff for which the mass of the $X(3872)$ is reproduced as a $I^G(J^{PC}) = 0^+(1^{++})D^*\bar{D}$ bound state. If the Z_c were to be a bound state at threshold, the ratio would be expected to be $R(Z_c/X) = 1.0 \pm 0.15$, which we show in the figure as a dashed line and a series of bands representing one, two and three standard deviations (shown in increasingly light colors). When we vary the scalar meson mass (coupling), the scalar coupling (mass) is taken to be $g_\sigma = 3.4$ ($m_\sigma = 550$ MeV). The predictions for the preferred values of the scalar mass and couplings ($m_\sigma = 550$ MeV and $g_\sigma = 3.4$) are highlighted as round dots. For the axial meson, we use $m_{a_1} = 1230$ MeV and $\lambda_1 = 1.8 \pm 0.3$, where the uncertainty in the axial coupling is shown as a series of bands around the central predictions for the axial-full theory (where we show again the one, two, and three standard deviations bands in increasingly light colors).
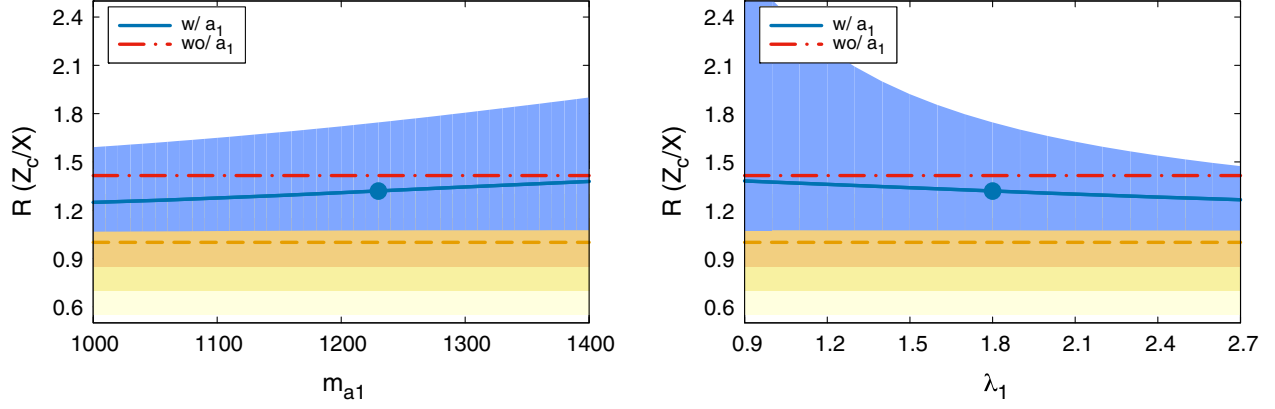
FIG. 2. Cutoff ratios $R(Z_c/X)$ as a function of the mass (left panel) and coupling (right panel) of the axial meson for the OBE model. $R(Z_c/X)$ is defined as in Fig. 1, with its expected value being $R(Z_c/X) = 1.0 \pm 0.15$, which we show as a dashed line and a series of bands representing one, two, and three standard deviations (shown in increasingly light colors). The solid and dashed-dotted lines represent the ratios for the OBE with and without axial meson exchange. The mass and coupling of the scalar meson are taken to be $m_\sigma = 550$ MeV and $g_\sigma = 3.4 \pm 1.0$, where in the case of the coupling we have added a 30% relative uncertainty, which is shown as the error band around the solid line. In contrast with Fig. 1, where we showed up to three standard deviations of the uncertainty in λ_1, for g_σ the errors are considerably larger and we show only one standard deviation instead. For the axial-less OBE model the ratio is constant, as there is no dependence with respect to the axial meson parameters, and calculated to be $R^{\not{a}_1} = 1.42^{+\infty}_{-0.34}$, where the error comes from the uncertainty of g_σ (not shown in the plots owing to its large spread). The $m_{a_1} = 1230$ MeV and $\lambda_1 = 1.8$ ratios for the axial-full theory are highlighted as a round dot. When we vary the axial mass (coupling), we set the axial coupling (mass) to its expected central value, i.e., $\lambda_1 = 1.8$ ($m_{a_1} = 1230$ MeV).

coupling with respect to the scalar meson. This result is in line with the previous observations that the importance of axial meson exchange is indeed subordinated to the scalar meson, which having a larger range naturally exerts a larger influence of the spectrum.

Finally, we recall here that we have worked under the assumption that the Λ_X and Λ_Z cutoffs should be identical. This would be true if the form-factor cutoffs of the OBE model are determined by the particles for which the potential is calculated (i.e., the charmed mesons), instead of by the light mesons being exchanged. However, this is not necessarily the case; in the OBE model of nuclear forces [13] the cutoffs of the light mesons are of the same order of magnitude, but not identical. This leads us to a second interpretation; the use of a unique cutoff could then be regarded as an average of the particular cutoffs of each of the light mesons. If this is the case, the Λ_X and Λ_Z cutoffs could differ from each other much more than naively expected, where a very important reason for this is that in the isovector molecules there is no vector meson exchange, giving a much larger weight for the axial meson cutoff in the Z_c-like molecules. In turn this will allows us to make independent predictions of the isoscalar and isovector charmed meson-antimeson molecules. We will explore this possibility in the following.

I. The HQSS partners of the $X(3872)$ and $Z_c(3900)$

Now we want to explore what type of molecular spectrum is derived when the exchange of the axial meson is included. For this we will consider a two cutoff model

where the isoscalar and isovector configurations are independent, i.e., we will use the Λ_X and Λ_Z cutoffs calculated in the previous section.

Ideally, the description of the two isospin channels would involve using a different cutoff for each of the exchanged meson, as done for instance in the meson theory of nuclear forces. However, instead of calibrating all these parameters—we only have two molecular candidates— here we will consider the interpretation in which the cutoff works as an effective parameter representing the different mix of mesons contributing in each channel. In particular the isovector $Z_c(3900)$ and $Z_c(4020)$ resonances do not exchange vector mesons, from which it is sensible to expect a different "effective" or "average" cutoff from the one in the $X(3872)$ case. Provided these two cutoffs are not too far away, which seems to be the case for most choices of parameters in the axial-full theory, this simplified description should be able to generate the qualitative features of the charmed meson-antimeson molecular spectrum.

The spectrum we obtain is summarized in Table IV for the axial-less and axial-full OBE models. While for the isoscalar configurations—the partners of the $X(3872)$— there is no practical difference between including or excluding the $a_1(1260)$, for the partners of the $Z_c(3900)$ there are important differences. We summarize our results as follows:

(i) In the isoscalar sector (the partners of the $X(3872)$) there is no significant difference between including or excluding axial meson exchange. The spectrum is

TABLE IV.　HQSS partners of the $X(3872)$ and $Z_c(3900)$ states in the molecular picture in the axial-less and axial-full theories. The spectrum is calculated with the OBE model proposed here and a dipolar form factor ($n_P = 2$), where the cutoff is determined independently in the isoscalar and isovector channels from the condition of reproducing the $X(3872)$ (as an isoscalar $J^{PC} = 1^{++}$ $D^*\bar{D}$ bound state 4 MeV below threshold) and the $Z_c(3900)$ (as an isovector $J^{PC} = 1^{+-}$ $D^*\bar{D}$ bound state at threshold). With these conditions we obtain the cutoffs $\Lambda_X^{\phi_1} = 1.37_{-0.09}^{+0.09}$ GeV and $\Lambda_Z^{\phi_1} = 1.99_{-0.60}^{+\infty}$ GeV in the axial-less model and $\Lambda_X^{a_1} = 1.37_{-0.09}^{+0.09}$ GeV and $\Lambda_Z^{a_1} = 1.82_{-0.43}^{+0.62}$ GeV in the axial-full model, where the errors correspond to the uncertainty in the σ meson coupling, $g_\sigma = 3.4 \pm 1.0$ (which is also propagated into the binding / virtual state energies). Owing to the absence of coupled channels, we only obtain bound or virtual states; we use the convention of a positive number for the binding energy of a bound state and a negative number for the energy of a virtual state, while a dash ("-") indicates the absence of a pole close to the threshold.

System (X-like)	I^G	J^{PC}	$B^{\phi_1}/E_V^{\phi_1}$	M^{ϕ_1}	$B^{a_1}/E_V^{a_1}$	M^{a_1}
$D\bar{D}$	0^+	0^{++}	$-0.0_{-0.7}^{+0.1}$	$3734.4_{-0.7}^{+0.0}$	$-0.0_{-0.8}^{+0.1}$	$3734.4_{-0.8}^{+0.0}$
$D^*\bar{D}$	0^+	1^{++}	Input	Input	Input	Input
$D^*\bar{D}$	0^-	1^{+-}	$-4.0_{-10.3}^{+3.6}$	$3871.8_{-10.3}^{+3.6}$	$-4.2_{-11.2}^{+3.9}$	$3871.6_{-11.2}^{+3.9}$
$D^*\bar{D}^*$	0^+	0^{++}	$-_{0.3}^{-}$	$\cdots$	$-_{0.3}^{-}$	$\cdots$
$D^*\bar{D}^*$	0^-	1^{+-}	$-1.0_{-1.8}^{+1.0}$	$4016.2_{-1.8}^{+1.0}$	$-1.0_{-1.9}^{+1.0}$	$4016.2_{-1.9}^{+1.0}$
$D^*\bar{D}^*$	0^+	2^{++}	$+3.5_{-0.1}^{+0.0}$	$4013.7_{-0.0}^{+0.1}$	$+3.5_{-0.1}^{+0.0}$	$4013.7_{-0.0}^{+0.1}$
System (Z_c-like)	I^G	J^{PC}	$B^{\phi_1}/E_V^{\phi_1}$	M^{ϕ_1}	$B^{a_1}/E_V^{a_1}$	M^{a_1}
$D\bar{D}$	1^-	0^{++}	$-0.1_{-\infty}^{+0.0}$	$3734.3_{-\infty}^{0.0}$	$-0.6_{-13.4}^{+6.9}$	$3733.8_{-13.4}^{+0.6}$
$D^*\bar{D}$	1^-	1^{++}	$-0.0_{-\infty}^{+0.0}$	$3875.8_{-\infty}^{+0.0}$	$-1.3_{-16.1}^{+6.0}$	$3874.5_{-16.1}^{+1.3}$
$D^*\bar{D}$	1^+	1^{+-}	Input	Input	Input	Input
$D^*\bar{D}^*$	1^-	0^{++}	$+0.0_{-\infty}^{+0.0}$	$4017.2_{-\infty}^{+0.0}$	$+0.3_{-0.3}^{+10.2}$	$4016.9_{-10.2}^{+0.3}$
$D^*\bar{D}^*$	1^+	1^{+-}	$+0.0_{-\infty}^{+0.0}$	$4017.2_{-\infty}^{+0.0}$	$+0.0_{-0.0}^{+0.0}$	$4017.2_{-0.0}^{+0.0}$
$D^*\bar{D}^*$	1^-	2^{++}	$+0.2_{-\infty}^{+0.0}$	$4017.0_{-\infty}^{+0.2}$	$-0.5_{-4.7}^{+0.7}$	$4016.7_{-4.7}^{+0.5}$

similar to the one predicted in Ref. [35] (an axial-less OBE model), though here we observe two additional states that are worth mentioning:

(a) A near threshold 0^{++} $D\bar{D}$ virtual state, which could correspond to the previously predicted $X(3700)$ state [11,80] or the recent 0^{++} bound state predicted in the lattice [81]. We note that this latter work includes the $D\bar{D} - D_s\bar{D}_s$ coupled-channel dynamics, which will provide additional attraction not present here.

(b) A negative C-parity (virtual state) partner of the $X(3872)$, which might be related to the observation of a state with these quantum numbers by COMPASS [82] with $M = 3860.0 \pm 10.4$ MeV. We notice that by taking $g_\sigma = 2.4$ we will predict the central value of the COMPASS observation. This might also indicate that smaller values of the scalar meson coupling are to be preferred.

(ii) In the isovector sector (the partners of the $Z_c(3900)$) there are interesting difference between the axial-less and axial-full models:

(a) In the axial-less model, all the isovector molecules have the same binding energy (except for perturbative corrections from pion exchanges), i.e., they are all located close to threshold. However this conclusion depends on $g_\sigma > 2.45$, which is within the error that we expect for the scalar coupling ($g_\sigma = 3.4 \pm 1.0$). If this condition is met, there should be a total of six isovector molecules or, equivalently, four unobserved partners of the $Z_c(3900)$ and $Z_c(4020)$. On the contrary, if g_σ is not strong enough isovector molecular states will not exist.

(b) In the axial-full model, a_1-exchange generates a spin dependent interaction that breaks this degeneracy. The most attractive configuration is the $I^G(J^{PC}) = 1^-(0^{++})D^*\bar{D}^*$ molecule, followed by the $Z_c(3900)$ and $Z_c(4020)$. However the size of this effect depends on g_σ; for the central value derived from the LσM, the hyperfine splitting will be of the order of merely 1 MeV, and the spectrum will be barely distinguishable from the axial-less model. But if g_σ is weak, the hyperfine splitting will become sizable. For instance, if $g_\sigma = 2.4$ the 0^{++} $D^*\bar{D}$ molecule will be more bound than the $Z_c(4020)$ by about 10 MeV.

In general, it is the isovector molecular spectrum which could provide more information about scalar and axial meson

exchange; the eventual observation of the HQSS partners of the $Z_c(3900)$ and $Z_c(4020)$ could determine which of the scenarios discussed here is the one chosen by nature—a degenerate spectrum would be compatible with a strong scalar meson exchange (or maybe with vector charmonia exchange [39]), while the discovery of a 0^{++} partner of the $Z_c(4020)$ will signal that axial meson exchange is a relevant part of the description of isovector molecules. However, if a 2^{++} partner happens to be discovered and it can be shown that requires more attraction than the $Z_c(4020)$, this will be difficult to accommodate in the previous molecular explanations, independently on whether it is grounded on scalar, axial or charmonium exchange.

V. DESCRIPTION OF THE NEW $Z_{cs}(3985)$

Finally we turn our attention to the $Z_{cs}(3985)$, recently discovered by BESIII [21]. The existence of this resonance can be readily deduced from the $Z_c(3900)$ and $Z_c(4020)$ and SU(3)-flavor symmetry, as the latter dictates that the $D^*\bar{D}$ interaction in the $I = 1$ isovector channel is identical to the one in the $D_s^*\bar{D}$ system [27,83]. However the realization of SU(3)-flavor symmetry in the OBE model is not automatic and depends on two conditions

   (i) the pseudo–Nambu-Goldstone meson current mixing with the octet part of the axial mesons,
   (ii) the scalar meson coupling with similar strengths to the $q = u$, d, s light quarks.

The first condition is required in order for the axial meson exchange to be nontrivial in the isovector molecules; if the axial mesons form a clear nonet with almost ideal decoupling of strange and nonstrange components, as happens with the vector mesons, then axial meson exchange will cancel out in both the $Z_c(3900)$ and $Z_{cs}(3985)$. We warn though that the status and nature of the axial mesons—$a_1(1260)$, $f_1(1285)$, $f_1(1420)$, $K_1(1270)$—is not clear; they might be composite [84], the $f_1(1420)$ might not exist [85,86]. Here we will not discuss these issues, but simply point out the conditions under which they will help explain the $Z_c(3900)$ and $Z_{cs}(3985)$.

The second condition is required for SU(3)-flavor symmetry to be respected between the $Z_c(3900)$ and $Z_{cs}(3985)$; if the scalar σ meson does not couple with the strange quarks, then a sizable part of the attraction in the $Z_c(3900)$ system will simply not be present in the $Z_{cs}(3985)$. As happened with the axial mesons, the nature of the scalar mesons is not clear either: they might be $q\bar{q}$ or tetraquark or a superposition of both, the mixing angle between the singlet and octet components is not known or it might violate the Okubo-Zweig-Iizuka (OZI) rule. We will note that the binding of the $Z_{cs}(3985)$ as a hadronic molecule requires a σ that couples with similar strength to the nonstrange and strange light quarks, which is not implausible.

A. Flavor structure of the potential

The $D^{(*)}$ and $D_s^{(*)}$ charmed mesons belong to the $\bar{3}$ SU(3)-flavor representation. From this the flavor structure of the $D_a^{(*)}\bar{D}_a^{(*)}$ potential, where $a = 1, 2, 3$ refers to the D^0, D^+, and D_s^+, is expected to be $3 \otimes \bar{3} = 1 \oplus 8$, i.e., the sum of a singlet and octet contributions

$$V(D_a^{(*)}\bar{D}_a^{(*)}) = \lambda^{(S)}V^{(S)} + \lambda^{(O)}V^{(O)}, \qquad (89)$$

with the superscript $^{(S)}$ and $^{(O)}$ referring to the singlet and octet and where the specific decomposition is shown in Table V. Of course, the flavor structure compounds with the HQSS structure, i.e., the singlet and octet potential can be further decomposed into a central and a spin-spin part

$$V^{(S)} = V_a^{(S)} + V_b^{(S)}\vec{\sigma}_{L1} \cdot \vec{\sigma}_{L2}, \qquad (90)$$

$$V^{(O)} = V_a^{(O)} + V_b^{(O)}\vec{\sigma}_{L1} \cdot \vec{\sigma}_{L2}. \qquad (91)$$

Finally, the relation between the singlet and octet components and the isospin components we previously defined for the $X(3872)$ and the $Z_c(3900)$ is

$$V^{(S)} = \frac{3}{2}V^{(I=0)} - \frac{1}{2}V^{(I=1)}, \qquad (92)$$

$$V^{(O)} = V^{(I=1)}. \qquad (93)$$

From the flavor decomposition of the potential (Table V) it is apparent that the potential for a molecular $Z_c(3900)$ and $Z_{cs}(3985)$ are identical [provided their tentative identifications with the $I^G(J^{PC}) = 1^+(1^{+-})D^*\bar{D}$ and $D^*\bar{D}_s - D\bar{D}_s^*$ systems are correct]. This in turn is compatible with their experimental masses, as can be deduced from the qualitative argument that their interaction is the same. This conclusion has indeed been checked by concrete EFT calculations [27], which do not make hypotheses about the binding mechanism but simply assume that the Z_c and Z_{cs} are bound states. The question we will explore now is

TABLE V. SU(3)-flavor structure of the charmed meson-antimeson potential. The charmed mesons (antimesons) belong to the $\bar{3}$ (3) representation of SU(3)-flavor symmetry, from which the potential accepts a $3 \otimes \bar{3} = 1 \oplus 8$ decomposition, which we show here. Notice that the SU(3)-flavor structure has to be combined with the HQSS structure in order to get the full S-wave potential [see Eqs. (90) and (91)].

System	I	S	V
$D^{(*)}\bar{D}^{(*)}$	0	0	$\frac{2}{3}V^{(S)} + \frac{1}{3}V^{(O)}$
$D_s^{(*)}\bar{D}_s^{(*)}$	0	0	$\frac{1}{3}V^{(S)} + \frac{2}{3}V^{(O)}$
$D^{(*)}\bar{D}^{(*)}$	1	0	$V^{(O)}$
$D^{(*)}\bar{D}_s^{(*)}$	$\frac{1}{2}$	-1	$V^{(O)}$

what are the conditions under which we expect the OBE model to respect this SU(3)-flavor structure.

B. Flavor structure of the light-meson exchanges

When extending the present formalism from SU(2)-isospin to SU(3)-flavor, a problem appears regarding the coupling of the charmed and light mesons; the isoscalar ($I = 0$) light mesons, being $q\bar{q}$ states, can be either in a flavor singlet or octet configuration. The singlet and isoscalar octet states, having the same quantum numbers, can mix and this mixing most often works out as to separate the $(u\bar{u} + d\bar{d})/\sqrt{2}$ and $s\bar{s}$ components of these two types of mesons almost perfectly. This is what happens for instance with the vector mesons ω and ϕ. However, the other light mesons we are considering here are further away from decoupling. The easiest case will be the pseudoscalar mesons, for which the singlet and octet almost do not mix. The axial meson case will be the most complex one, as it entails nontrivial mixing angles that have to be combined with the fact that the axial meson mixes with the pion current. In the following lines we will consider each case in detail.

1. Pseudoscalar meson octet

We will begin with the pseudoscalar mesons for which the singlet and octet (η_1 and η_8) members can be identified with the η' and η mesons, as the mixing angle is small. Thus in practice we can consider the pseudoscalar mesons as forming a standard octet

$$
M = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta}{\sqrt{6}} \end{pmatrix}. \tag{94}
$$

From this, the interaction term of the pseudoscalars with the charmed mesons can be written as

$$
\mathcal{L}_{\text{flavor}} = \frac{g_1}{f_\pi} \text{Tr}[H_{\bar{c}}^{a\dagger} \vec{\sigma} \cdot \vec{\partial} M_{ab} H_{\bar{c}}^b], \tag{95}
$$

where a, b are flavor indices, which are ordered as $\bar{D}^a = (\bar{D}^0, D^-, D_s^-)$, and $H_{\bar{c}}$ the heavy superfield for the charmed antimesons (as with this choice we have light quarks). Alternatively, if we use the light-subfield notation we will have

$$
\mathcal{L}_{\text{flavor}} = \frac{g_1}{f_\pi} q_L^{a\dagger} \vec{\sigma}_L \cdot \vec{\partial} M_{ab} q_L^b, \tag{96}
$$

with $q_L^a = (u_L, d_L, s_L)$ for $a = 1, 2, 3$.

2. Vector meson nonet

Next we consider the vector mesons, for which the nonstrange and strange components of the singlet (ω_1) and

octet (ω_8) decouple almost perfectly to form the ω and ϕ mesons. While the light-quark content of the singlet and isoscalar octet mesons is expected to be

$$
|\omega_1\rangle = \frac{1}{\sqrt{3}} [|u\bar{u}\rangle + |d\bar{d}\rangle + |s\bar{s}\rangle], \tag{97}
$$

$$
|\omega_8\rangle = \frac{1}{\sqrt{6}} [|u\bar{u}\rangle + |d\bar{d}\rangle - 2|s\bar{s}\rangle], \tag{98}
$$

for the physical ω and the ϕ meson we have

$$
|\omega\rangle \simeq \frac{1}{\sqrt{2}} [|u\bar{u}\rangle + |d\bar{d}\rangle], \tag{99}
$$

$$
|\phi\rangle \simeq |s\bar{s}\rangle, \tag{100}
$$

which means that the relation between the physical and SU(3) eigenstates is

$$
\begin{pmatrix} \phi \\ \omega \end{pmatrix} \simeq \begin{pmatrix} \sqrt{\frac{1}{3}} & -\sqrt{\frac{2}{3}} \\ \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} \end{pmatrix} \begin{pmatrix} \omega_1 \\ \omega_8 \end{pmatrix}. \tag{101}
$$

This matrix is actually a rotation, as can be seen by direct inspection.

In principle there should be two independent couplings for the singlet and the octet vector meson components, i.e., $g_V^{(1)}$ for ω_1 and $g_V^{(8)}$ for ω_8 and ρ [or $f_V^{(1)}$ and $f_V^{(8)}$ for the magnetic-type couplings]. These two couplings are reduced to one once we consider the OZI rule; the coupling of hadrons that do not contain strange quarks to the ϕ meson should be suppressed. This in turn generates a relation between $g_V^{(1)}$ and $g_V^{(8)}$ (there is only one independent coupling owing to the OZI rule) from which we can deduce the relation $g_\rho = g_\omega$.

Alternatively, if we consider that the mixing is indeed ideal, we can write down a vector meson nonet matrix

$$
V = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega^0}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega^0}{\sqrt{2}} & K^{*0} \\ \bar{K}^{*-} & \bar{K}^{*0} & \phi \end{pmatrix}, \tag{102}
$$

and notice that the structure of the interaction Lagrangian in flavor space will be

$$
\mathcal{L}_{\text{flavor}} \propto \bar{D}_a^\dagger V_{ab} \bar{D}_b, \tag{103}
$$

with a, b flavor indices and $\bar{D}_a$ the anticharmed meson field in flavor space, i.e., $\bar{D}_a = (\bar{D}^0, D^0, D_s^-)$ for $a = 1, 2, 3$. From this we end up with a unique g_V and f_V, and thus $g_\rho = g_\omega$ and $f_\rho = f_\omega$ automatically.

3. Axial meson octet vs nonet

The SU(3)-extension of the present formalism to the axial mesons will encounter three problems. The first is which are the flavor partners of the $a_1(1260)$ meson, which we will simply assume to be the $f_1(1285)$, $f_1(1420)$, and $K_1(1270)$ (we will further discuss this point later).

The second problem is the singlet and octet mixing; if we consider that the isoscalar partners of the a_1 are the $f_1(1285)$ and $f_1(1420)$, they will be a nontrivial mixture of a singlet and octet axial meson

$$\begin{pmatrix} f_1(1285) \\ f_1(1420) \end{pmatrix} = \begin{pmatrix} \cos\theta_1 & \sin\theta_1 \\ -\sin\theta_1 & \cos\theta_1 \end{pmatrix} \begin{pmatrix} f_1^1 \\ f_1^8 \end{pmatrix}, \quad (104)$$

where f_1^1 and f_1^8 are the singlet and octet components of the two f_1's. As a matter of fact, the $f_1(1285)$ and $f_1(1420)$ are relatively far away from the mixing angle which effectively separates them into nonstrange and strange components. If we define the decoupling mixing angle as $\theta_{\rm dec} = \mathrm{atan}(1/\sqrt{2}) \sim 35.3°$, the θ_1 angle can be expressed as

$$\theta_1 = \theta_{\rm dec} + \alpha_1, \quad (105)$$

where there is a recent determination of this angle by the LHCb, $\alpha_1 = \pm(24.0^{+3.1+0.6}_{-2.3-0.8})°$ [87] (which is the value we will adopt), and previously in the lattice $\alpha_1 = \pm(31 \pm 2)°$ [88].

Third, the axial neutral mesons are $J^{PC} = 1^{++}$ states but their strange partners do not have well-defined C-parity. Depending on their C-parity there are two possible types of axial mesons: $J^{PC} = 1^{++}$ and 1^{+-} mesons originate from $^3P_1$ and $^1P_1$ quark-antiquark configurations, where we have used the spectroscopic notation $^{S+1}L_J$, with S, L, and J being the spin, orbital, and total angular momentum of the quark-antiquark pair. For a quark-antiquark system the C-parity is $C = (-1)^{L+S}$, which translates into $C = +1(-1)$ for $^3P_1$ ($^1P_1$). The $J^{PC} = 1^{++}$ and 1^{+-} neutral axial mesons correspond with the a_1, f_1, and b_1, h_1, respectively, of which only the a_1, f_1 can mix with the pseudo–Nambu-Goldstone boson axial current. The strange partners of the a_1 and b_1 axial mesons are referred to as the K_{1A} and K_{1B}, but for the strange axial mesons C-parity is not a well defined number and the physical states are a mixture of the $^1P_1$ and $^3P_1$ configurations

$$\begin{pmatrix} K_1(1270) \\ K_1(1400) \end{pmatrix} = \begin{pmatrix} \cos\theta_K & \sin\theta_K \\ -\sin\theta_K & \cos\theta_K \end{pmatrix} \begin{pmatrix} K_{1B} \\ K_{1A} \end{pmatrix}, \quad (106)$$

with most determinations of θ_K usually close to either $30°$ or $60°$ [89–91].

The mixing of the axial pseudo–Nambu-Goldstone meson current [i.e., the SU(3) extension of the axial pion current] has to happen with the axial meson octet (instead of the physical axial mesons)

$$\partial_\mu M_{ab} \to \partial_\mu M_{ab} + \lambda_1 m_1 A^8_{1ab}, \quad (107)$$

where M_{ab} and A_{1ab} would be the pseudoscalar and axial meson octets, the first of which is given by Eq. (94) and the second by

$$A_1^8 = \begin{pmatrix} \frac{a_1^0}{\sqrt{2}} + \frac{f_1^8}{\sqrt{6}} & a_1^+ & K_{1A}^{+*} \\ a_1^- & -\frac{a_1^0}{\sqrt{2}} + \frac{f_1^8}{\sqrt{6}} & K_{1A}^{0*} \\ K_{1A}^{-*} & \bar{K}_{1A}^{0*} & -\frac{2f_1^8}{\sqrt{6}} \end{pmatrix}. \quad (108)$$

Thus, the specific relations for the contribution of the f_1^8 and K_{1A} to the axial meson currents, i.e.,

$$\partial_\mu\eta \to \partial_\mu\eta + \lambda_1 m_1 f_1^8, \quad (109)$$

$$\partial_\mu K \to \partial_\mu K + \lambda_1 m_1 K_{1A}, \quad (110)$$

have to be translated into the physical basis by undoing the rotations. For the f_1^8 we will get

$$\partial_\mu\eta \to \partial_\mu\eta + \lambda_1 m_1 (\sin\theta_1 f_1 + \cos\theta_1 f_1^*), \quad (111)$$

which determines the coupling of the f_1 and f_1^* with the D and D_s mesons, while for the K_{1A} we will get instead

$$\partial_\mu K \to \partial_\mu K + \lambda_1 m_1 (\sin\theta_K K_1 + \cos\theta_K K_1^*). \quad (112)$$

Owing to the form factors, the exchange of the heavier variants of the axial mesons (f_1^*, K_1^*) are expected to be suppressed with respect to the lighter ones (f_1, K_1). From this observation and the previous relations, the most important contribution for axial meson exchange will come from the couplings of the f_1 and K_1 mesons, which are proportional to $\sin\theta_1$ and $\sin\theta_K$, respectively.

The K_1 meson deserves a bit more of discussion as it can help us to get a sense of the accuracy of the previous relation from a comparison with the K_1 axial meson decay constant, which can be extracted from experimental information. The current mixing relation implies that the decay constant will be

$$\langle 0|A_{5\mu}|K_1\rangle = f_{K_1} m_{K_1} \quad \text{with} \quad f_{K_1} = \lambda_1 \cos\theta_K f_K, \quad (113)$$

with $\theta_K = 30°$–$35°$ or $55°$–$60°$ and $f_K = 160$ MeV yields $f_{K_1} = 110$ MeV–150 MeV or $f_{K_1} = 180$ MeV–230 MeV, respectively, which is to be compared with $f_{K_1} = 175 \pm 19$ MeV [92] (which in turn is extracted from the experimental data of Ref. [93]). The two possible mixing angles are in principle compatible with the previous determination of f_{K_1}; though it is possible to argue that the higher angle might be a slightly better choice, this is based on the assumption that $f_{K1A} \neq 0$ but $f_{K1B} = 0$. However, while the axial decay constant of the neutral $^1P_1$ axial mesons b_1

and h_1 have to be zero owing to their negative C-parity, i.e., $f_{b1} = 0$ and $f_{h1} = 0$, this is not true for the K_{1B} for which $f_{K1B} = 0$ is a consequence of chiral symmetry. In fact $f_{K1B} \neq 0$ owing to the finite strange quark mass. This effect, though small, is enough as to make the comparison of mixing angles more ambiguous [92,94].

Finally, it is worth stressing that the structure of the axial mesons is not particularly well known and there exist interesting conjectures about their nature in the literature. A few hypotheses worth noticing are: (i) the axial mesons might be dynamically generated (i.e., molecular) [84,95], (ii) the $K_1(1270)$ resonance might actually have a double pole structure [96,97], and (iii) the $f_1(1420)$ might simply be a $K\bar{K}$ decay mode of the $f_1(1285)$ [86]. All of them might potentially influence the theoretical treatment of the axial mesons; (i) actually was considered in Ref. [78] four decades ago for the $a_1(1260)$, where it was determined that it would not strongly influence the form of the potential. It is worth noticing that (iii) would imply that there are not enough axial mesons to form a nonet, but only an octet. This would be interesting, as in this scenario it might be plausible to identify the $f_1(1285)$ with f_1^8 in Eq. (104), leading to $\theta_1 = 90°$. However, though interesting, we will not consider the multiple ramifications of the previous possibilities in this work.

4. Scalar meson singlet vs nonet

The lightest scalar meson nonet is formed by the σ (or $f_0(500)$), $a_0(980)$, $f_0(980)$, and $K_0^*(700)$. If we are considering light-meson exchange the most important of the scalars will be the lightest one, i.e., the σ (see Ref. [98] for an extensive review about the status of this meson). While the a_0 and K_0^* are pure octets, the σ and $f_0(980)$ are a mixture of singlet and octet, i.e.,

$$\begin{pmatrix} f_0(500) \\ f_0(980) \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} S_1 \\ S_8 \end{pmatrix}, \quad (114)$$

where S_1 and S_8 represent the pure singlet and octet states. The meaning of S_1 and S_8 depends however on the internal structure of the scalar mesons; if the σ were to be a $q\bar{q}$ state, the light-quark content of S_1 and S_8 would be analogous to that of the vector mesons, i.e., to $|\omega_1\rangle$ and $|\omega_8\rangle$ in Eqs. (97) and (98). But if the σ were to be a $qq\bar{q}\bar{q}$ state, the light-quark content of S_1 and S_8 would be different (yet easily obtainable from the substitutions $u \to \bar{d}\bar{s}$, $d \to \bar{u}\bar{s}$, $s \to \bar{u}\bar{d}$, which assumes the diquark-antidiquark structure proposed by Jaffe [99], where the antidiquark and diquark are in a triplet and antitriplet configuration, respectively).

If g_1 and g_8 are the coupling of the charmed mesons to the singlet and octet scalar, respectively, we will have that the coupling of the σ to the nonstrange and strange charmed mesons will be

$$g_{\sigma 1} = g_{\sigma DD} = \cos\theta g_1 + \frac{1}{\sqrt{6}}\sin\theta g_8, \quad (115)$$

$$g'_{\sigma 1} = g_{\sigma D_s D_s} = \cos\theta g_1 - \frac{2}{\sqrt{6}}\sin\theta g_8. \quad (116)$$

Independently of whether the σ is a $q\bar{q}$ or $qq\bar{q}\bar{q}$ scalar, if we assume a mixing angle that decouples the nonstrange and strange components, we will end up with $g_{\sigma D_s D_s} = 0$ after invoking the OZI rule (though we will discuss this point later). In this case, σ meson exchange will badly break SU(3) symmetry between the $Z_c(3900)$ and $Z_{cs}(3985)$. However this conclusion depends on the previous assumptions, which are not necessarily correct. In the following we will discuss how the observed SU(3) symmetry can still be preserved with scalar meson exchange.

The most obvious solution would be a flavor singlet σ, as this would provide roughly the same attraction for a molecular $Z_c(3900)$ and $Z_{cs}(3985)$. In this regard it is relevant to notice Ref. [100], which analyzed the σ pole in unitarized chiral perturbation theory and obtained a mixing angle $\theta = 19 \pm 5°$. This would translate into a σ that is mostly a flavor singlet.

The interpretation of the σ as a singlet would also be compatible with the following naive extension of the LσM from SU(2)-isospin to SU(3)-flavor, in which an originally massless baryon octet interacts with a total of nine bosons by means of

$$\mathcal{L}_{\text{int}}^{\text{N}\sigma\text{L}'} = g\text{Tr}[\bar{B}_8(\phi_0 + i\gamma_5\lambda_a\phi_a)B_8], \quad (117)$$

with B_8 the baryon octet (N, Λ, Σ, Ξ), ϕ_0 and ϕ_a the bosonic fields, λ_a with $a = 1, …, 8$, the Gell-Mann matrices and g a coupling constant. In the standard LσM the nucleon field acquires mass owing to the spontaneous symmetry breaking and the subsequent vacuum expectation value of the ϕ_0 field. Here it is completely analogous, with $\langle\phi\rangle = f_P/\sqrt{2}$, the redefinition of $\phi_0 = f_P/\sqrt{2} + \sigma$ and the reinterpretation of the ϕ_a fields as the pseudoscalar octet (π, K, η). This procedure will give $g_{\sigma B_8 B_8} = g_{\phi B_8 B_8} = \sqrt{2}M_8/f_P$, with M_8 the averaged mass of the octet baryons and f_P representing either f_π, f_K, or f_η, which are all identical in the SU(3)-symmetric limit. The $g_{\sigma B_8 B_8}$ thus obtained is basically compatible with the previous SU(2) value for $g_{\sigma NN}$ ($\simeq 10.2$). Meanwhile the $F/(D+F)$-ratio would be $\alpha = 1/2$ (as the interaction term implies $F = D$), to be compared for instance with the SU(6) quark model value $\alpha = 2/5$ (i.e., a 20% discrepancy).

All this would suggest the use of approximately the same coupling of the σ to strange and nonstrange hadrons alike, resulting in the same attraction strength for both the $Z_c(3900)$ and $Z_{cs}(3985)$. In fact if we assume the relation $g_{\sigma qq} = \sqrt{2}m_q/f_P$ at the quark level, take $m_q = 336$, 340, 486 MeV for the $q = u, d, s$ constituent quark masses and choose $f_P = f_\pi$ for $q = u, d$ and $f_P = f_K$ for $q = s$, we

would get $g_{\sigma uu} \simeq g_{\sigma dd} \simeq 3.6$ and $g_{\sigma ss} \simeq 4.3$, leading to the counterintuitive conclusion that the coupling of the σ to the strange quark is larger than to the u, d quarks. However if we subtract the mass of the quarks $g_{\sigma qq} = \sqrt{2}(m_q^{\text{con}} - m_q)/f_P$ (with m_q^{con} and m_q the constituent and standard quark masses), we will obtain $g_{\sigma ss} \simeq 3.4$ instead (i.e., approximately identical to $g_{\sigma uu}$ and $g_{\sigma dd}$).

But the SU(3) extension of the LσM we have presented here is not the only possible one. In fact it could just be considered as a simplified version of the chiral quark model [101] in which the scalar octet is removed. It happens that the inclusion of the scalar octet in the chiral-quark model makes it perfectly possible to have a nonstrange σ and still explain the mass of the light baryons.

However the problem might not necessarily be whether the σ contains a sizable strange component or not, but whether it couples to the strange degrees of freedom. In this regard it has been suggested that the OZI rule does not apply in the scalar 0^{++} sector [102,103]. This in turn might be the most robust argument in favor of a sizable coupling of the σ meson with the strange-charmed mesons, as it does not depend on the flavor structure or the strange content of the σ. This in principle implies that the g_σ and g'_σ couplings to the $D^{(*)}$ and $D_s^{(*)}$ charmed mesons would be independent parameters.

Without the OZI rule there is no reason for the g_1 and g_8 couplings to have comparable sizes: while the application of the OZI rule implies that $g_8 = \sqrt{6}/2 \tan\theta g_1$ (which for the $q\bar{q}$ and $qq\bar{q}\bar{q}$ ideal decoupling angles would translate into $g_8 = \sqrt{3}/2 g_1 \simeq 0.87 g_1$ and $g_8 = -\sqrt{3} g_1 \simeq -1.73 g_1$, respectively)—without OZI g_1 and g_8 are independent parameters. Now, if it happens that $g_1 \gg g_8$ the result will be indistinguishable from a flavor singlet σ; the g_σ and g'_σ couplings can be approximated by $g_\sigma \simeq g'_\sigma \simeq g_1 \cos\theta$, resulting in approximately the same couplings to the $D^{(*)}$ and $D_s^{(*)}$ charmed mesons. As to whether the $g_1 \gg g_8$ condition is met or not, it happens that $g_1 \cos\theta$ can be identified with $g/3$ in the SU(3)-extension of the LσM of Eq. (117), giving it a relatively large value, while there is no reason why g_8 should be as large. Besides, $g_1 \gg g_8$ would also justify not including the scalar octet in the OBE model.

Yet, we might get a better sense of the sizes of g_1 and g_8 from a comparison with previous determinations of the σ coupling in the light-baryon sector. While the Nijmegen baryon-baryon OBE models originally considered a flavor-singlet σ [104], latter this idea was put aside in favor of a more standard singlet-octet interpretation for the σ [105]. Their description depended on a singlet and octet couplings, $g_{1B_8B_8}$ and $g_{8B_8B_8}$, the mixing angle θ and the $F/(F+D)$-ratio (which is necessary in the light-baryon octet but not for the antitriplet charmed mesons). They obtained $g_{8B_8B_8} = 0.22 g_{1B_8B_8}$, while for the later Nijmegen soft-core baryon-baryon OBE model [106] this ratio is $g_{8B_8B_8} = 0.34 g_{1B_8B_8}$. Thus it would not be a surprise if the relation $g_1 \gg g_8$ also happens for the charmed mesons.

The comparison of the coupling constants to the light baryons might provide further insights too. If we consider the Jülich hyperon-nucleon OBE model [65], their results are $g_{\sigma\Lambda\Lambda} \simeq 0.95(0.77) g_{\sigma NN}$ and $g_{\sigma\Sigma\Sigma} \simeq 1.13(1.05) g_{\sigma NN}$ in what is referred to as model A(B) in Ref. [65], where these couplings are supposed to represent correlated (uncorrelated) and uncorrelated (correlated) processes in the scalar channel. It is worth noticing that the Jülich model [65] predated the rediscovery of the σ as a pole in the pion-pion scattering amplitude [107,108], and consequently treated the σ as a fictitious degree of freedom. From a modern point of view in which the σ is not a fictitious meson, their results would be compatible (within the expected 30% error of the LσM) both with a σ that only couples with the nonstrange $q = u$, d quarks ($g_{\sigma\Lambda\Lambda}^{NS} = 0.67 g_{\sigma NN}^{NS}$ and $g_{\sigma\Sigma\Sigma}^{NS} = 0.67 g_{\sigma NN}^{NS}$, where $^{\text{NS}}$ indicate that it couples only to the nonstrange quarks) and with a σ that couples with equal strength to the $q = u$, d, s quarks ($g_{\sigma\Lambda\Lambda}^{\text{FS}} = g_{\sigma NN}^{\text{FS}}$ and $g_{\sigma\Sigma\Sigma}^{\text{FS}} = g_{\sigma NN}^{\text{FS}}$, where $^{\text{FS}}$ indicates that the coupling is flavor-symmetric).

In short, there are theoretical arguments in favor of a sizable coupling of the σ meson to the D_s and D_s^* charmed mesons, g'_σ. In what follows we will consider the problem form a phenomenological point of view, i.e., we will simply consider the g'_σ coupling to be a free parameter and discuss which are the values which allow for a simultaneous description of the Z_c and Z_{cs}, without regard as to which is the theoretical reason behind this.

C. Light-meson exchange for the Z_c and Z_{cs}

Now that we have reviewed the flavor structure of the pseudoscalar, scalar, vector and axial mesons, we can write down the resulting light-meson exchange potential for the Z_c and Z_{cs}. We will begin with the pseudoscalar mesons, for which the singlet and octet can be considered as effectively decoupled. For the Z_c there will be π- and η-exchange, while for the Z_{cs} only η-exchange will be possible. The pseudoscalar-exchange potential can be written as

$$V_P(D^*\bar{D}) = \zeta\vec{\tau}_1 \cdot \vec{\tau}_2 W_\pi(r) + \frac{1}{3} W_\eta(r), \qquad (118)$$

$$V_P(D_s^*\bar{D}) = -\frac{2}{3} W_\eta(r), \qquad (119)$$

where W_π and W_η are the π- and η-exchange potentials once we have removed the flavor and G-parity factors, i.e.,

$$W_\pi(r) = \frac{g_1^2}{6f_\pi^2}\vec{\sigma}_{L1} \cdot \vec{\sigma}_{L2} \mu_\pi W_Y(\mu_\pi r), \qquad (120)$$

$$W_\eta(r) = \frac{g_1^2}{6f_\eta^2}\vec{\sigma}_{L1} \cdot \vec{\sigma}_{L2} \mu_\eta W_Y(\mu_\eta r). \qquad (121)$$

In the flavor-symmetric limit we will have $m_\pi = m_\eta$ and $f_\pi = f_\eta$, leading to identical π- and η-exchange potentials,

$W_\pi = W_\eta$. However in the real world, $m_\eta > m_\pi$ and $f_\eta > f_\pi$, implying a suppression of the η-exchange potential relative to the pion-exchange one. In particular, we will take $f_\eta = 150$ MeV.

For the vector mesons there is instead an almost ideal mixing between the singlet and octet, from which the ω and ϕ are close to being purely nonstrange and strange, respectively. The structure of the potential will be

$$V_V(D^*\bar{D}) = \zeta\vec{\tau}_1 \cdot \vec{\tau}_2 W_\rho(r) + W_\omega(r), \quad (122)$$

$$V_V(D_s^*\bar{D}) = 0, \quad (123)$$

with W_ρ and W_ω the ρ- and ω-exchange potential once the flavor and G-parity factors have been removed

$$W_\rho = g_{\rho 1}^2 m_\rho W_Y(m_\rho r) + f_{\rho 1}^2 \frac{\mu_\rho^2}{6M^2}\vec{\sigma}_{L1} \cdot \vec{\sigma}_{L2}\mu_\rho W_Y(\mu_\rho r), \quad (124)$$

$$W_\rho = g_{\omega 1}^2 m_\omega W_Y(m_\omega r) + f_{\omega 1}^2 \frac{\mu_\omega^2}{6M^2}\vec{\sigma}_{L1} \cdot \vec{\sigma}_{L2}\mu_\omega W_Y(\mu_\omega r). \quad (125)$$

For the scalar meson we will consider it to generate two independent couplings for the nonstrange and strange charmed mesons

$$V_\sigma(D^*\bar{D}) = -g_{\sigma 1}^2 m_\sigma W_Y(m_\sigma r), \quad (126)$$

$$V_\sigma(D_s^*\bar{D}) = -g'_{\sigma 1} g_{\sigma 1} m_\sigma W_Y(m_\sigma r), \quad (127)$$

as this choice allows to explore the conditions for which we expect the Z_{cs} to bind, provided that the Z_c binds. We suspect that $g_{\sigma 1}$ and $g'_{\sigma 1}$ are of the same order of magnitude, yet provided $|g'_{\sigma 1}|$ is not much smaller than $|g_{\sigma 1}|$, the Z_c and Z_{cs} will be related to each other.

For the axial mesons, the $f_1(1285)$ and $f_1(1420)$ are probably a nonstandard mixture between a singlet and octet, where the mixing angles will have to be taken into account explicitly

$$V_A(D^*\bar{D}) = \zeta\vec{\tau}_1 \cdot \vec{\tau}_2 W_{a1}(r) + \frac{1}{3}[\sin\theta_1{}^2 W_{f1}(r) + \cos\theta_1{}^2 W_{f1^*}(r)], \quad (128)$$

$$V_A(D_s^*\bar{D}) = -\frac{2}{3}[\sin\theta_1{}^2 W_{f1}(r) + \cos\theta_1{}^2 W_{f1^*}(r)]. \quad (129)$$

The reduced W_{a1}, W_{f1} and W_{f1}^* potentials are given by

$$W_{a1} = -\lambda_1^2 \frac{g_1^2 m_{a1}^2}{3f_\pi^2}\vec{\sigma}_{L1} \cdot \vec{\sigma}_{L2}\mu_{a1}W_Y(\mu_{a1}r), \quad (130)$$

$$W_{f1} = -\lambda_1^2 \frac{g_1^2 m_{f1}^2}{3f_\eta^2}\vec{\sigma}_{L1} \cdot \vec{\sigma}_{L2}\mu_{f1}W_Y(\mu_{f1}r), \quad (131)$$

$$W_{f1}^* = -\lambda_1^2 \frac{g_1^2 m_{f1}^{*2}}{3f_\eta^2}\vec{\sigma}_{L1} \cdot \vec{\sigma}_{L2}\mu_{f1}^* W_Y(\mu_{f1}^* r), \quad (132)$$

with m_{a1}, m_{f1}, and m_{f1}^* the masses of the a_1, f_1, and f_1^* axial mesons (while μ_{a1}, μ_{f1} and μ_{f1}^* are their effective masses when there is a mass gap). In general the exchange of the f_1 and f_1^* mesons will be moderately suppressed owing to $f_\eta > f_\pi$.

D. The two Z_{cs}-like configurations

There is an interesting difference between the isovector (Z_c) and strange (Z_{cs}) sectors: in the first, C-parity is a good quantum number for the neutral component of the isospin triplet, i.e., for the Z_c^0, while in the second this is not the case. For the $D^*\bar{D}_s - D\bar{D}_s^*$ molecules, even if we consider these two channels to be degenerate (which we do here), the structure of the potential is still better understood as a coupled channel problem, i.e.,

$$V(Z_{cs}) = \begin{pmatrix} V_a^{(1)} & V_b^{(1)} \\ V_b^{(1)} & V_a^{(1)} \end{pmatrix}, \quad (133)$$

where $V_a^{(1)}$ and $V_b^{(1)}$ are the central and spin-dependent parts of the potential [see Eqs. (48)–(54)]. The two eigenvalues of the previous potential are

$$V(\tilde{Z}_{cs}) = V_a^{(1)} + V_b^{(1)}, \quad (134)$$

$$V(Z_{cs}) = V_a^{(1)} - V_b^{(1)}, \quad (135)$$

which would be the strange counterparts of the 1^{++} and 1^{+-} isovector configurations in Table I. In our C-parity convention the $\tilde{Z}_{cs}$ and Z_{cs} wave functions would be

$$|\tilde{Z}_{cs}\rangle = \frac{1}{\sqrt{2}}[|D^*\bar{D}_s\rangle + |D\bar{D}_s^*\rangle], \quad (136)$$

$$|Z_{cs}\rangle = \frac{1}{\sqrt{2}}[|D^*\bar{D}_s\rangle - |D\bar{D}_s^*\rangle], \quad (137)$$

respectively.[2] The most standard interpretation of the $Z_{cs}(3985)$ observed by BESIII is that it is indeed the strange partner of the $Z_c(3900)$ [27,29], that is, what we

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[2]Meanwhile, in the alternative C-parity convention the sign of $V_b^{(1)}$ in Eq. (133) changes and the same is true for the linear combinations in Eqs. (136) and (137). However the potentials in Eqs. (134) and (135) will remain the same.

have called the Z_{cs} configuration in Eqs. (135) and (137). This is the interpretation we will follow here.

However it is worth noticing that in the BESIII data [21] the $Z_{cs}(3985)^-$ is seen in the $D^{*0}D_s^-$ channel, while in the $D^0 D_s^{*-}$ what seems to be seen is a broader structure at a lower mass. This might be compatible with two nearby Z_{cs} and $\tilde{Z}_{cs}$ poles generating a constructive and destructive interference in the $D^*\bar{D}_s$ and $D\bar{D}_s^*$ channels. Alternatively, it might be a consequence of the production mechanism. The point though is that the existence of two Z_{cs} poles is not implausible and that we are not completely sure of which one corresponds to the state observed by BESIII. Indeed, we find that the existence of both the Z_{cs} and $\tilde{Z}_{cs}$ poles is a likely outcome of our present approach.[3]

E. The $Z_c(3900)$ and $Z_{cs}(3985)$ cutoffs

With the light-flavor structure of the OBE potential at hand, we simply have to choose the parameters (couplings, masses, and mixing angles), compare the cutoffs for which the $Z_c(3900)$ and $Z_{cs}(3985)$ bind and check whether they are compatible with each other. This is analogous to what we have already done with the $X(3872)$ and the $Z_c(3900)$, though now the focus is the preservation of SU(3)-flavor symmetry, from which we expect

$$\frac{\Lambda(Z_{cs})}{\Lambda(Z_c)} \simeq 1.0. \qquad (138)$$

Of course this relation is approximate; HQSS and SU(3)-flavor violations will generate deviations from this cutoff ratio. We expect HQSS and SU(3)-flavor breaking effects to be of the order of 15% and 20% (i.e., Λ_{QCD}/m_c and the difference between f_π and f_K), respectively, which add up to 25% if we sum them in quadrature, i.e.,

$$\frac{\Lambda(Z_{cs})}{\Lambda(Z_c)} = 1 \pm 0.25 = (0.75\text{--}1.25). \qquad (139)$$

For the SU(3)-flavor breaking, an extra factor to be taken into account is the relative sizes of the D and D_s mesons, which are not necessarily the same. If we use the electromagnetic radii as a proxy of the matter radii, although they have not been experimentally measured, there are theoretical calculations; in Ref. [111] they are estimated to be

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[3]We notice that it has also been suggested that the existence of two Z_{cs} states of similar mass might explain [109] the recently discovered $Z_{cs}(4000)^+$ by the LHCb [110] {in addition to the $Z_{cs}(3985)^-$ state observed by BESIII [21]}. This possibility is intriguing, but we do not consider it here because of the large width of the $Z_{cs}(4000)^+$ ($\Gamma \sim 130$ MeV), which is an order of magnitude larger than the width of the $Z_{cs}(3985)^-$ and thus difficult to explain if these two resonances were to be HQSS partners. This is not impossible though, as the Z_{cs} and $\tilde{Z}_{cs}$ contain different linear combinations of the $D^*\bar{D}_s$ and $D\bar{D}_s^*$ channels.

$\sqrt{r_{e.m}^2} \sim 0.43$ and 0.35 fm for the D and D_s, respectively. This indicates that the strange charmed meson D_s is 0.82 the size of its nonstrange partner, from which it would be expectable for the form-factor cutoff of a $D_s\bar{D}_s$ molecule to be a 22% larger than a $D\bar{D}$ molecule. This figure is in fact compatible with the f_K and f_π ratio we mentioned before, but indicates a bias in the flavor uncertainties; the naive expectation will be a larger cutoff for the Z_{cs} than for the Z_c. The $D_s\bar{D}$ molecules would be in between, with deviations at the 10% level expected for the cutoff (biased towards larger cutoffs), from which we might revise the range of acceptable cutoff ratios to

$$\frac{\Lambda(Z_{cs})}{\Lambda(Z_c)} \simeq 1.1 \pm 0.25 = (0.85\text{--}1.35), \qquad (140)$$

i.e., we have moved the expected central value from 1 to 1.1 to reflect on the smaller size of the strange charmed mesons. If we consider a σ that does not couple with the strange charmed meson $D_s^{(*)}$, for $n_P = 2$ the Z_c and Z_{cs} eventually bind for large enough cutoffs, though the ratio is too large

$$\left.\frac{\Lambda^{\sigma(\text{NS})}(Z_{cs})}{\Lambda^{\sigma(\text{NS})}(Z_c)}\right|_{\theta_1^+} = 3.69^{+1.16}_{-1.06}(3.35\text{--}4.92), \qquad (141)$$

$$\left.\frac{\Lambda^{\sigma(\text{NS})}(Z_{cs})}{\Lambda^{\sigma(\text{NS})}(Z_c)}\right|_{\theta_1^-} = 3.57^{+1.10}_{-1.00}(3.24\text{--}4.74), \qquad (142)$$

where $\theta_1^\pm = \theta_{\text{dec}} \pm \alpha_1$, from which it can be appreciated that the dependence on the θ_1 mixing angle is weak. The central value, its error and the bands corresponds to $m_\sigma = 550$ MeV, $g_\sigma = 3.4 \pm 1.0$, and $m_\sigma = 450\text{--}600$ MeV, check Eq. (80) and the explanations following it. If we instead consider a σ that couples with the same strength to the nonstrange and strange quarks, i.e., a σ with a flavor-symmetric coupling, we will get instead the ratios

$$\left.\frac{\Lambda^{\sigma(\text{FS})}(Z_{cs})}{\Lambda^{\sigma(\text{FS})}(Z_c)}\right|_{\theta_1^+} = 1.04^{+0.10}_{-0.04}(1.00\text{--}1.06), \qquad (143)$$

$$\left.\frac{\Lambda^{\sigma(\text{FS})}(Z_{cs})}{\Lambda^{\sigma(\text{FS})}(Z_c)}\right|_{\theta_1^-} = 1.06^{+0.14}_{-0.05}(1.00\text{--}1.09), \qquad (144)$$

which are basically independent of θ_1 and compatible with Eq. (140).

The conclusion is that some coupling of the σ meson to the strange charmed meson is required for a coherent molecular description of the Z_c and Z_{cs}. Thus we might consider the question of what is the g'_σ/g_σ ratio which is compatible with the upper bound for the cutoff ratio, i.e., with Eq. (140). This happens to be

$$\frac{g'_\sigma}{g_\sigma} \geq 0.66 - 0.70, \qquad (145)$$

which is weakly dependent on m_σ (that is why we do not include a bracket showing the m_σ spread) and somewhat dependent on θ_1, with $\theta_1^+(\theta_1^-)$ yielding 0.66 (0.70). This ratio is compatible with a few of the different interpretations of the σ we have discussed; provided the σ has a sizable coupling to the strange components, it should be a plausible outcome. For obtaining the cutoff ratio suggested by the strange and nonstrange charmed meson size comparison ($\simeq 1.1$) we will need instead

$$\frac{g'_\sigma}{g_\sigma} \simeq 0.91 - 0.94, \qquad (146)$$

which again is nearly independent of m_σ and weakly dependent on θ_1 [$\theta_1^+(\theta_1^-)$ gives 0.91(0.94)]. This ratio also falls within the realm of possibility, but is more stringent. The dependence of the cutoff ratio $R(Z_{cs}/Z_c)$ with the g'_σ/g_σ ratio is shown in Fig. 3, from which can see again that axial meson exchange becomes important if scalar meson exchange happens to be weak. Owing to the weak dependence of this ratio on θ_1, Fig. 3 only shows the θ_1^+ results

Finally, if we remove the sigma completely we will still get a ratio compatible with Eq. (140),

$$\left.\frac{\Lambda^{d}(Z_{cs})}{\Lambda^{d}(Z_c)}\right|_{\theta_1^+} = 1.50(1.33 - 4.19), \qquad (147)$$

$$\left.\frac{\Lambda^{d}(Z_{cs})}{\Lambda^{d}(Z_c)}\right|_{\theta_1^-} = 1.45(1.36 - 2.07), \qquad (148)$$

where the intervals now reflect the uncertainty in $\lambda_1 = 1.8 \pm 0.3$. σ-less molecular descriptions include most works which use vector-meson exchange (usually within the hidden-gauge approach) to predict molecular states—e.g., the $X(3872)$ [112], the hidden-charm pentaquarks [113,114] or, recently, more general descriptions of the molecular spectrum [39]. However, these descriptions traditionally require a different binding mechanism for the $Z_c(3900)$ resonance, which might include two-pion exchange or charmonium exchange [36–39]. Here we note that axial meson exchange could be a useful complementary addition to these models, but if we want these models to simultaneously reproduce the $X(3872)$ with the same set of parameters the σ is probably a required addition.

F. The HQSS partners of the $Z_{cs}(3985)$

Now we calculate the spectrum of the molecular partners of the $Z_{cs}(3985)$ within the axial-full OBE model. For simplicity we set the σ coupling to the nonstrange and strange mesons to be identical, i.e. $g_\sigma = g'_\sigma = 3.4 \pm 1.0$, and $\theta_1 = \theta_1^+$. We use a dipolar form factor ($n_P = 2$) where the cutoff is obtained from the condition of generating a pole in the $J^P = 1^+$ $(|D\bar{D}_s^*\rangle - |D^*\bar{D}_s\rangle)/\sqrt{2}$ scattering channel at threshold, yielding $\Lambda(Z_{cs}) = 1.88^{+1.02}_{-0.50}$ GeV.

The spectrum is shown in Table VI, where it is worth noticing the following:

(i) the details of the spectrum are less dependent on the strength of σ exchange and there is already a considerable $D^*\bar{D}_s^*$ hyperfine splitting for $g_\sigma = 3.4$.
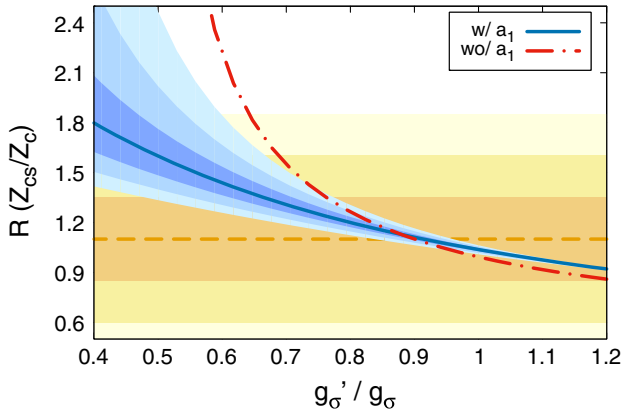

FIG. 3. Cutoff ratio between the $Z_{cs}(3985)$ and the $Z_c(3900)$ as a function of the ratio g'_σ/g_σ, where g_σ and g'_σ are the couplings of the scalar meson to the nonstrange and strange charmed baryons D and D_s. We assume $g_\sigma = 3.4$, its expected central value from the LσM and the quark model. We show the ratios in the OBE model with and without axial mesons, where we notice that for the axial-less case the Z_{cs}' does not bind for $g'_\sigma < 1.7$. We compare the cutoff ratio $R(Z_{cs}/Z_c)$ to the expected ratio derived from SU(3)-flavor, HQSS and corrections from the strange charmed meson size, $R(Z_{cs}/Z_c) \simeq 1.10 \pm 0.25$.

TABLE VI. HQSS partners of the $Z_{cs}(3985)$ as a $D\bar{D}_s^* - D^*\bar{D}_s$ molecule in the axial-full OBE model presented here. We use a dipolar form factor ($n_P = 2$) where the cutoff is determined from the condition of generating a pole at threshold in the $J^P = 1^+$ $D_s\bar{D}^* - D_s^*\bar{D}$ system, which for $g_\sigma = g'_\sigma = 3.4 \pm 1.0$ and $\theta_1 = \theta_1^+$ results in $\Lambda(Z_{cs}) = 1.88^{+1.02}_{-0.50}$ GeV. For the binding/virtual state energies we follow the same conventions as in Table IV, where positive (negative) numbers indicate a bound (virtual) state. Masses and bound/virtual state energies are in units of MeV.

System (Z_{cs}-like)	I	J^P	$B^{f_1}/E_V^{f_1}$	M^{f_1}
$D\bar{D}_s$	$\frac{1}{2}$	0^+	$+1.5^{+0.1}_{-0.7}$	$3834.1^{+0.7}_{-0.1}$
$\frac{1}{\sqrt{2}}(D^*\bar{D}_s + D\bar{D}_s^*)$	$\frac{1}{2}$	1^+	$+8.1^{+1.8}_{-6.5}$	$3970.1^{+6.5}_{-1.8}$
$\frac{1}{\sqrt{2}}(D^*\bar{D}_s - D\bar{D}_s^*)$	$\frac{1}{2}$	1^+	Input	Input
$D^*\bar{D}_s^*$	$\frac{1}{2}$	0^+	$-1.8^{+1.6}_{-1.2}$	$4119.0^{+1.2}_{-1.6}$
$D^*\bar{D}_s^*$	$\frac{1}{2}$	1^+	$+0.1^{+0.0}_{-0.0}$	$4120.8^{+0.0}_{-0.0}$
$D^*\bar{D}_s^*$	$\frac{1}{2}$	2^+	$+9.4^{+7.5}_{-2.4}$	$4111.4^{+2.4}_{-7.5}$

(ii) However, the hyperfine splitting has the opposite sign to that of the Z_c sector:

(a) In the $D^*\bar{D}_s - D\bar{D}_s^*$, the most attractive configuration is the $\tilde{Z}_{cs}$ one (instead of the Z_{cs}).

(b) The most attractive $D^*\bar{D}_s^*$ molecule is the $J^P = 2^+$ configuration (instead of the 0^+ one).

The reason for these features is pseudoscalar meson exchange: η-exchange is considerably stronger in the Z_{cs} sector than in the Z_c one, where at distances of the order of $m_\eta r \sim 1$ we have

$$V_P(r; Z_c) \approx -\frac{g_1^2}{6}\left(\frac{m_\pi^2}{f_\pi^3 r} - \frac{1}{3}\frac{m_\eta^2}{f_\eta^3 r}\right)\vec{\sigma}_{L1} \cdot \vec{\sigma}_{L2}, \qquad (149)$$

$$V_P(r; Z_{cs}) \approx -\frac{g_1^2}{6}\left(\frac{2}{3}\frac{m_\eta^2}{f_\eta^3 r}\right)\vec{\sigma}_{L1} \cdot \vec{\sigma}_{L2}. \qquad (150)$$

While in the flavor-symmetric limit these two potentials would be identical, when we input the physical pseudoscalar masses and decay constants it becomes apparent that pseudoscalar-meson exchange will be much more important in the Z_{cs} than in the Z_c (actually, by a factor of -2.8, where the minus sign is worth noticing). In addition, η- and σ-exchange have approximately the same range, which explains why it is not necessary to have a weak scalar coupling in order to have a sizable hyperfine splitting. Then, if we compare η-exchange with axial meson exchange, each of which generate hyperfine splittings of the opposite sign, the strength and range combination of the η clearly dominates over f_1 and f_1^* meson exchanges.

This generates opposite hyperfine splitting patterns in the nonstrange and strange $D^*\bar{D}^*$ and $D^*\bar{D}_s^*$ sectors, a characteristic of the spectrum that is unlikely to happen unless the Z_c and Z_{cs} are molecular. Depending on the nature of the Z_c and Z_{cs} resonances, this prediction might be trivial to check; for instance, if we assume all the $D^*\bar{D}^*$ and $D^*\bar{D}_s^*$ to be bound, the expected ordering of the masses of the states should be

$$M_B(Z_c^*, 0^{++}) < M_B(Z_c^*(4020)) < M_B(Z_c^*, 2^{++}),$$
$$M_B(Z_{cs}^*, 0^+) > M_B(Z_{cs}^*, 1^+) > M_B(Z_{cs}^*, 2^+), \qquad (151)$$

where M_B refers to the masses of the bound states. On the other hand, if they all happen to be virtual states the ordering would invert

$$M_V(Z_c^*, 0^{++}) > M_V(Z_c^*(4020)) > M_V(Z_c^*, 2^{++}),$$
$$M_V(Z_{cs}^*, 0^+) < M_V(Z_{cs}^*, 1^+) < M_V(Z_{cs}^*, 2^+), \qquad (152)$$

with M_V denoting their masses. Unfortunately, we do not know yet whether the Z_c and Z_{cs} are molecular, or whether they are bound/virtual states or resonances above the threshold [26,27]. But independently of this, the eventual observation of the HQSS partners of these two states, if accompanied by markedly different isospin splittings in the $D^*\bar{D}^*$ and $D^*\bar{D}_s^*$ sectors, would indeed reveal their molecular nature.

G. The scalar meson and the $P_{cs}(4459)$ pentaquark

The strange and nonstrange couplings of the scalar meson are not only important for a unified molecular description of the $Z_c(3900)$ and $Z_{cs}(3985)$, but also for the new strange hidden-charm pentaquark $P_{cs}(4459)$ [115] when considered in comparison to the other three molecular pentaquark candidates; the $P_c(4312)$, $P_c(4440)$, and $P_c(4457)$ [116].

The $P_{cs}(4459)$ is 19.2 MeV below the $\bar{D}^*\Xi_c$ threshold— 4478.0 MeV in the isospin symmetric limit, which is why the $P_{cs}(4459)$ has been conjectured to be a $\bar{D}^*\Xi_c$ molecule [117–121]. The charmed baryon Ξ_c is a flavor antitriplet with quark content csu (Ξ_c^+) and csd (Ξ_c^0), where the light-quark pair within the Ξ_c is in a $S_L = 0$ configuration, with S_L the total light-quark spin. As a consequence, pion exchange, axial meson exchange and the M1 part of vector meson exchange do not contribute to the $\bar{D}^*\Xi_c$ interaction. This observation also applies to the $\bar{D}\Sigma_c$ system [46,122–130], which is the most common molecular explanation for the $P_c(4312)$; the $P_c(4312)$ pentaquark is merely 8.9 MeV below the $\bar{D}\Sigma_c$ threshold.

The question is whether this is compatible with the expected binding of the $P_{cs}(4459)$ as a $\bar{D}^*\Xi_c$ molecule. Owing to the lack of explicit light-spin dependence, the only difference between the OBE descriptions of the $P_c(4312)$ and $P_{cs}(4459)$ is scalar meson exchange (the strength of vector meson being identical in both cases). While the Σ_c baryon contains two nonstrange light quarks, the Ξ_c contains only one and if we assume a σ that does not couple to the nonstrange light quarks we will have

$$g^{\rm NS}_{\sigma\Sigma_c\Sigma_c} \simeq \frac{1}{2}g^{\rm NS}_{\sigma\Xi_c\Xi_c}, \qquad (153)$$

which will translate into considerably less attraction (and binding) for the $P_{cs}(4459)$. In contrast, if the σ couples with approximately the same strength to the strange quark within the Ξ_c, we will have

$$g^{\rm FS}_{\sigma\Sigma_c\Sigma_c} \simeq g^{\rm FS}_{\sigma\Xi_c\Xi_c}, \qquad (154)$$

where the superscript $^{\rm FS}$ indicates that the coupling is now flavor-symmetric (in the sense of identical coupling strengths with the $q = u$, d, s quarks). That is, if the σ couples equally to all the light quarks, the binding of the $P_c(4312)$ and $P_{cs}(4459)$ molecules will be approximately the same.

However, the experimental determination of the mass of the $P_{cs}(4459)$ indicates that it is more bound than the $P_c(4312)$ by about 11.3 MeV, where the specific binding energies are

$$B(P_c) = 8.9 \text{ MeV} \quad \text{and} \quad B(P_{cs}) = 19.2 \text{ MeV}, \qquad (155)$$

for the $P_c(4312)$ and $P_{cs}(4459)$ pentaquarks, respectively. This could be interpreted as $g_{\sigma\Xi_c\Xi_c} > g_{\sigma\Sigma_c\Sigma_c}$, which would be somewhat surprising but still plausible. Yet, the comparison we have done does not take into account that there is a factor that generates spin dependence in the $P_{cs}(4459)$ pentaquark; the coupled channel dynamics with the nearby $\bar{D}\Xi_c'$ and $\bar{D}\Xi_c^*$ thresholds, where the $J = 1/2$ ($J = 3/2$) $\bar{D}^*\Xi_c$ molecule will mix with the $\bar{D}\Xi_c'$ ($\bar{D}\Xi_c^*$) channel. Owing to the relative location of the thresholds, this generates repulsion and attraction for the $J = 1/2$ and $3/2$ configurations, respectively. Ref. [118] computes this effect with a contact-range theory, yielding

$$\Delta B^{CC} = B^{CC}\left(\bar{D}^*\Xi_c, J = \frac{3}{2}\right) - B^{CC}\left(\bar{D}^*\Xi_c, J = \frac{1}{2}\right)$$
$$\approx (5 - 15)\text{MeV}, \qquad (156)$$

depending on the assumptions made to calculate this effect, where the superscript ^{CC} stands for "coupled channels". This hyperfine splitting is of the same order of magnitude as the aforementioned 11.3 MeV difference in binding between the P_c and P_{cs}. For comparison, a recent phenomenological calculation provides a similar hyperfine splitting [121] of $\Delta B^{CC} = (2.4$–$20.0)$ MeV.

Yet, it is interesting to notice that the LHCb collaboration [115] already explored the possibility that the $P_{cs}(4459)$ is actually composed of two peaks instead of one. For this two peak fit, the masses of the two P_{cs} pentaquarks will be $M(P_{cs1}) = 4454.9 \pm 2.7$ MeV and $M_2 = 4467.8 \pm 3.7$ MeV [115], yielding the following binding energies

$$B(P_{cs1}) = 23.1 \text{ MeV} \quad \text{and} \quad B(P_{cs2}) = 10.2 \text{ MeV}. \qquad (157)$$

We can describe this two-peak solution with the contact-range theory of Ref. [118], which provides us with an interesting advantage—we can explicitly turn off the coupled channel effects within the theory to predict what the energy of these two P_{cs} pentaquarks would have been in the absence of this effect, leading to

$$B^{SC}\left(\bar{D}^*\Xi_c, J = \frac{1}{2}, \frac{3}{2}\right) = (15.4$–$16.7) \text{ MeV}, \qquad (158)$$

with the superscript ^{SC} indicating "single channel" and where now the two-spin states are degenerate, but have binding energies that are still somewhat larger than the $P_c(4312)$ as a $\bar{D}\Sigma_c$ molecule.

The conclusion seems to be that, if the $P_{cs}(4459)$ pentaquark is a $\bar{D}^*\Xi_c$ molecule (and assume that the previous procedures effectively isolate the single- and coupled-channel contributions), it is probably more compatible with a σ that couples to all the light quarks with equal strength than with a σ that does not couple with the strange quark. If anything, it seems that there is more attraction in the $\bar{D}^*\Xi_c$ channel than in the $\bar{D}\Sigma_c$ one. But this conclusion still depends on the size of the coupled channel effects (they could have been underestimated) and the experimental uncertainties surrounding the $P_{cs}(4459)$ pentaquark. Thus, it might be possible that the $P_{cs}(4459)$ pentaquark might still be compatible instead with a sigma that does not couple to the strange degrees of freedom.

VI. CONCLUSIONS

In this paper we consider the problem of describing the $Z_c(3900)$, $Z_c(4020)$, and $Z_{cs}(3985)$ as heavy-hadron molecules from a phenomenological perspective. Of course, we do not know for sure whether they are molecular or not. Instead, we are in interested in what their binding mechanism is (provided they are molecular). Regarding the problem of their nature, the closeness of these resonances to the $D^*\bar{D}$, $D^*\bar{D}^*$, and $D^*\bar{D}_s$ thresholds suggest a molecular nature. The success of EFT formulations in describing the Z_c's [23,26,83] and Z_{cs} [27] further points towards the plausibility of the molecular nature. Yet, tetraquark explanations are also possible [131–134]. What is not trivial to explain though in the molecular picture is the binding mechanism; while the Z_c's should not be there in vector meson exchange models, OBE models usually require relatively large cutoffs for these two-body systems to bind 35–33]] (or might simply not bind depending on the choice of couplings), prompting other explanations such as two-pion exchange or charmonium exchange [36–39].

Here we consider a new factor in the molecular description of the Z_c's and Z_{cs}; axial meson exchange. The exchange of axial mesons is strongly suppressed in the two-nucleon system, partly owing to the fact that the axial meson mass is larger than the nucleon's, and partly owing to vector meson exchange being a more dominant factor than the axial mesons. But this is not necessarily the case for charmed mesons, prompting a reevaluation of the role of axial mesons. We find that the inclusion of the axial mesons makes the molecular description more plausible for the Z_c's, as they indeed provide additional attraction. But their importance depends on the strength of scalar meson exchange; if the coupling of the charmed hadrons to the scalar meson is smaller than suggested by phenomenological models, axial mesons will become the binding mechanism. Conversely, if the scalar coupling is large enough, axial mesons will become irrelevant. For molecular candidates in which vector meson exchange is strong, for instance the $X(3872)$, the axial meson exchange contribution is negligible. Thus we expect the relevance of axial meson exchange to be limited to molecules where the ρ- and ω-exchange cancel out, as is the case with the Z_c's.

Besides the axial meson, the nature of sigma exchange is probably the most important factor for a coherent molecular description of the $Z_c(3900)$ and $Z_{cs}(3985)$. If the sigma meson couplings breaks SU(3)-flavor symmetry to a large degree, the short-range of the axial meson, combined with its nontrivial SU(3)-flavor structure, might be insufficient to explain the Z_{cs} as the molecular SU(3) partner of the Z_c in the molecular picture. Thus a molecular Z_{cs} requires a nontrivial coupling of the strange charmed mesons D_s and D_s^* with the sigma meson. This is not improbable though, as there are theoretical reasons (in particular the suspicion that the OZI rule might not apply to the scalar mesons [102,103]) why the sigma meson could have a sizable coupling to the strange degrees of freedom. The bottom-line though is that a molecular Z_{cs} requires a non-negligible coupling of the sigma to the strange hadrons in the OBE model, independently of which is the origin of this coupling. This might not only be a requirement for the Z_{cs} to be molecular but also for the recently discovered $P_{cs}(4459)$, the interpretation of which as a $\bar{D}^*\Xi_c$ bound state might also require a coupling of the Ξ_c strange charmed baryon to the sigma similar to that of the nonstrange Σ_c.

Finally, the observable signature of axial meson exchange will be a particular type of hyperfine splitting for the isovector $D^*\bar{D}^*$ molecules, where attraction decreases with spin. If the isovector $D^*\bar{D}^*$ molecules are bound states or resonances, there will be a $J^{PC} = 0^{++}$ Z_c-like state that is probably 0 MeV–10 MeV lighter than the $Z_c(4020)$. This pattern is reversed for their strange partners, owing to flavor symmetry breaking effects that are specific to the molecular hypothesis. Thus, besides the standard prediction of a $Z_{cs}(4120)$ $J^P = 1^+$ $D^*\bar{D}_s^*$ partner of the $Z_{cs}(3985)$ [27], here we expect the existence of a lighter (about 10 MeV) $J^P = 2^+$ partner of the Z_{cs}^*. However if the Z_c's and Z_{cs} happen to be virtual states [26,27], the previous patterns will invert or become difficult to recognize, requiring a much more complex theoretical analysis to determine which are the most attractive spin configurations.

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