Doubly heavy baryon Ξ_{cc} production in $\Upsilon(1S)$ decay

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We study the doubly heavy baryon Ξ_{cc} production in $\Upsilon(1S)$ decay. The nonrelativistic framework is employed to describe the bound states for the calculation on the partial width. It is shown that the corresponding branching ratio can be significant and can be well measured as the $\Upsilon(1S)$ decay to $J/\Psi + c\bar{c} +$ anything process.

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I. INTRODUCTION

$\Upsilon(1S)$ [1] decay is a good arena to study QCD and hadron physics. Several instructive results have been obtained. For example, recent searches on the exotic XYZ hadrons via the inclusive channel $\Upsilon \to J/\Psi +$ anything [2] and on light tetraquark hadrons in several channels of Υ decay [3] have been made. Both reported negative results. As a matter of fact, in the energy region above J/Ψ mass at Beijing Electron Positron Collider and that above Υ mass at B factories, many exotic XYZ hadrons have been observed (for a recent review, see [4]). These exotic particles, except those directly couple to the virtual photon in e^+e^- annihilations, are all produced from the *decays* of either the exited $c\bar{c}$ bound states or the B hadrons. On the other hand, Υ decay is an environment significantly different from those where the exotic particle production is observed. Υ decays via the OZI-suppressed ways, i.e., the annihilation of the $b\bar{b}$ quarks. The dominant mode ($> 80\%$) is the hadronic one generally referred to as "3-gluon" decay [5], and the subsequent hadronization is a special case of multiproduction. The negative results [2,3] mentioned above can shed light on property of confinement and the unitarity of the hadronization in multiproduction processes as we have pointed out [6–9]. The experimental facts mentioned above confirm that the $c\bar{c}$ pair produced in perturbative process prefers to transfer into general hadrons like J/Ψ rather than exotic XYZ's in this multiproduction process; and that for light hadrons, it is also the similar case, i.e., the above negative experimental results on light exotic hadrons indicate that the dominant decay channels should be $\Upsilon \to h's$, with $h's$ referring to mesons as well as baryons. In one word, Υ generally decays to mesons and baryons, with exotic ones hardly possible to be observed. But the to-date measured decay channels of Υ are much far from exhausting the total decay width. Especially, almost no baryon channel is measured [5]. So measuring the baryon production is an important task for better understanding the dynamics in Υ decay.

Among all the baryons which can be produced in Υ decay, the doubly heavy baryon Ξ_{cc} is special. SELEX and LHCb have respectively reported the observations of this kind of baryons with different mass [10–12]. One of the possibilities can be that different SU(2) multi-states of Ξ_{cc} are observed by these two collaborations. To measure these multistates, and further to explore SU(3) multistates, can surely help to clarify and deepen our knowledge on the property and production mechanism of Ξ_{cc}. Υ decay can provide a clean platform for such measurements.

There is a further special reason that stands for the observation on Ξ_{cc} in Υ decay. It is noticed that most of the presented data of Υ decay are upper limits [5]. However, the decay channel $\Upsilon \to J/\Psi + X$ is well measured for several times by several collaborations and has attracted wide interests, which is important on the study of PQCD and NRQCD (for the full literature list, please see a recent review [13]). It was pointed out that, based on the soft J/Ψ spectrum by CLEO measurement which was quite rough at that time, and on the calculation of the partial width [14], the dominant contribution could be $\Upsilon(1S) \to J/\Psi + c\bar{c}g$. Then the spectrum and branching ratio is confirmed by CLEO II [15,16] and later by BELLE [2], though detailed calculations show that several competing sub-processes contribute [17,18]. This fact strongly implies that the perturbative production of $c\bar{c}c\bar{c}$ in Υ decay is significant. This leads to the fact that the double charm baryon is hence easily produced as argued by the color connection analysis

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[19]. For $c_1\bar{c}_2c_3\bar{c}_4g$ system from Υ decay, $c_1\bar{c}_2$ and $c_3\bar{c}_4$ respectively come from a virtual gluon. But $c_1\bar{c}_4$ and $c_3\bar{c}_2$ can respectively be in color singlet, i.e., the color space can be reduced as

$$(3_1 \otimes 3_4^*) \otimes (3_3 \otimes 3_2^*) = (1_{14} + 8_{14}) \otimes (1_{23} + 8_{23}). \quad (1)$$

This means that such combination of the pair can be color singlet and easy to translate to J/ψ for proper invariant mass. One can recognize that the color space can also be reduced as

$$(3_1 \otimes 3_3) \otimes (3_2^* \otimes 3_4^*) = (3_{13}^* + 6_{13}) \otimes (3_{24} + 6_{24}^*). \quad (2)$$

In such color states, the two-charm pair can combine with a light quark to become Ξ_{cc} [19–21] for proper invariant mass. This simple analysis implies that the production rate of $\Xi_{cc} + \bar{c}\,\bar{c}\,g$ is expected not small once the $J/\Psi + c\bar{c}g$ production rate is not small.

In this paper, we devote to study the production of Ξ_{cc} in Υ decay. We calculate the corresponding partial width and the momentum distribution of Ξ_{cc}. Multistates like Ξ_{cc}^+ or Ξ_{cc}^{++} could have different width and lead to quite different feasibility or difficulty in observing them, but their production mechanism is completely the same in Υ decay. Therefore we do not make any distinction for the investigation on the production. In the super B factory, once the center of mass energy is tuned on the Υ resonance, a large sample of Υ decay data can be obtained and could be employed for the measurement. The following calculations show that the branching ratio of Ξ_{cc} production can be order of 10^{-4}. For the Υ decay, the process with two charm pairs production is easy to be triggered by 3-jet like event shape and strangeness enhancement (e.g., the $\frac{K}{\pi}$ value) [16,22], of which some of the charm meson production events can be vetoed by lepton pair or hadron pair mass around J/Ψ mass. In this way, one can get a clean and large sample of events to study the doubly charm baryon multistates.

The calculation of the $\Upsilon \to \Xi_{cc} + \bar{c}\,\bar{c} + g$ suffers from the complexity that both the initial and final states contain bound states Υ and Ξ_{cc}, which need to be investigated respectively. In Sec. II, we describe the traditional nonrelativistic wave function method and apply it to the initial state Υ. However, for the doubly heavy baryon system, we have to factorize out the corresponding matrix elements via NRQCD method [23]. These are the contents of Sec. III, and we obtain the formulas to calculate the partial width. Then in Sec. IV, we investigate the numerical result with the estimation for the NRQCD parameters and give simple discussions on experimental feasibility.

II. INITIAL BOUND STATE

In the process $\Upsilon \to \Xi_{cc} + \bar{c}\,\bar{c}\,g$, both bottom and the charm quarks are heavy. For the initial bound state, the color singlet $b\bar{b}$ pair with $C = -1$, it directly leads to the nonrelativistic wave function formulations [24–27], where the relative momentum between b and $\bar{b}$ is vanishing, namely same as the case of positronium. For the final bound state, a factorization formulation within the NRQCD framework [28,29] is employed. One subtle point is that, the nonrelativistic formulations are investigated in the rest frame of each bound state, respectively; and then a corresponding covariant form of description is obtained, which can be employed in any frame. Here we start from the initial state: The differential width of the process $\Upsilon \to \Xi_{cc} + \bar{c}\,\bar{c}\,g$ can be formulated as [14]

$$\frac{d\Gamma}{dR} = \frac{|B_\Upsilon \langle \Xi_{cc}\bar{c}\,\bar{c}\,g|\mathcal{S}|b\bar{b}(^3S_1, 1)\rangle|^2}{T}, \quad (3)$$

where dR is the differential phase space volume element for Ξ_{cc} and $\bar{c}$, $\bar{c}$, g without the constrain of energy momentum conservation; $\mathcal{S}$ is the S-matrix; B_Υ is related to the wave function of Υ at origin as

$$B_\Upsilon = \frac{\Psi_\Upsilon(0)}{\sqrt{V}2m_b}. \quad (4)$$

For convenience, we normalize all final state particle states to be $2EV$ (where E is the particle's energy and V is the volume of the total space). This normalization is also used for all free quarks in bound states. For the initial state, B_Υ normalizes the state of Υ to be 1, so that the width can be directly written as above. In Eq. (3) the sum over all spin states for final particles and average of the 3 spin states for Υ are not explicitly shown and the "time" T is $2\pi\delta(0)$. We only consider the case of the initial state $b\bar{b}$ as color singlet, with a special "1" in the state ket to mark this.

For the factorization of the initial bound state, the width is written, based on the above equation, as

$$d\Gamma = dR' \frac{1}{3}\frac{1}{M_\Upsilon^2}|\Psi_\Upsilon(0)|^2 |\langle \Xi_{cc}\bar{c}\,\bar{c}\,g|\mathcal{T}|b\bar{b}(^3S_1, 1)\rangle|^2. \quad (5)$$

Here $dR' = dR(2\pi)^4\delta^{(4)}(P_i - P_f)$, the factors time T and volume V have been canceled by the $\delta^{(4)}(0)$. $\mathcal{T}$ is the $\mathcal{T}$-matrix with $S_{fi} = \delta_{fi} + (2\pi)^4\delta^{(4)}(P_i - P_f)\mathcal{T}_{fi}$. Sum over all spin states is indicated.

Employing the projection operator formulation (e.g., [24]), and the radial wave function R_Υ to describe the initial bound state, we get the decay amplitude $\mathcal{T}_{fi}$ as,

$$\mathcal{T}_{fi} = \frac{1}{2}\frac{1}{\sqrt{4\pi}}\frac{1}{\sqrt{M_\Upsilon}}R_\Upsilon(0)\mathrm{Tr}[O_0(\not{P} + M_\Upsilon)(-\not{\epsilon})]. \quad (6)$$

O_0 is the amplitude for $b\bar{b} \to \Xi_{cc}\bar{c}\,\bar{c}\,g$, with relative momentum of $b\bar{b}$ vanishing and without the $b\bar{b}$ legs. P and ϵ are 4-momentum and polarization vector of Υ, respectively.

III. FINAL BOUND STATE AND THE PARTIAL WIDTH

In the above Eq. (6), the final state of the Υ decay except the inclusively observed Ξ_{cc}, can be divided into a perturbative part X_P and a nonperturbative part X_N. To the lowest-order (tree level) in PQCD, the amplitude $\mathcal{T}_{fi}$ is obtained as

$$\mathcal{T}_{fi} = \int \frac{d^4 q_1}{(2\pi)^4} A_{ij}(k_1, k_2, P_1, P_2, P_3, k, k_N; q_1) \int d^4 x_1 e^{-iq_1 x_1} \times \langle \Xi_{cc}(k) + X_N | \overline{Q}_i(x_1) \overline{Q}_j(0) | 0 \rangle. \tag{7}$$

We assign $k_1, k_2, P_1, P_2, P_3, k$ as the momenta of the corresponding particles, $b, \bar{b}, \bar{c}, \bar{c}, g, \Xi_{cc}$, respectively, $k_1 = k_2 = P/2$. $A_{ij}(k_1, k_2, P_1, P_2, P_3, k, k_N; q_1)$, which includes the initial wave function and the perturbative contribution, can be directly read from Fig. 1. Both i and j are Dirac and color indices. In the matrix element, X_N represents the nonperturbative effects, with small total momentum k_N to fulfill the total energy-momentum conservation (k_N to be neglected in later derivations). $Q(x)$ is the Dirac field for charm quark. $q_1 x_1 \coloneqq q_{1\mu} x_1^\mu$ and we will always use this convention. Here we simply illustrate the derivation from the corresponding Wick terms in the S-matrix to get this result. This helps to expose the physical meaning of the q_1. After the field operators acting on the corresponding initial and final states, integrating on the spacetime variables which only appear in exponential (to get the δ function of vertex), as well as integrating on the fermion propagator four momenta and the corresponding δ function of vertex, one arrives to

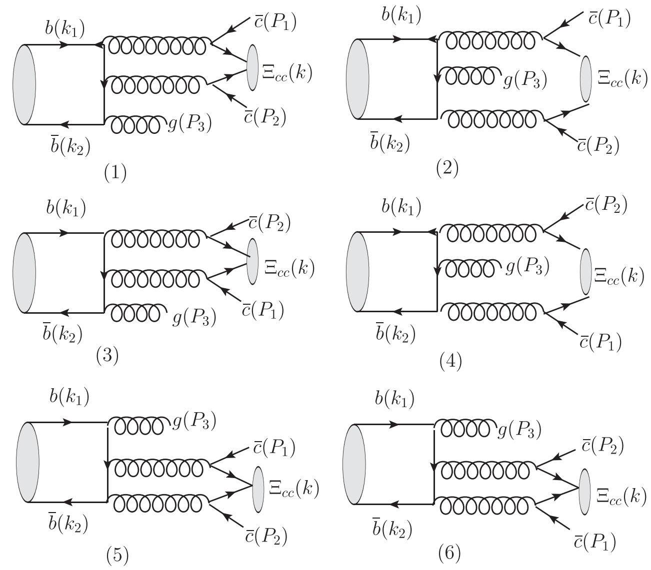

FIG. 1. Six Feynman diagrams for the "amplitude" A in Eq. (7). The g* g* g system are in the same color, angular momentum and charge conjugation states as those of Υ. The left bubble represents the wave function of Υ. A does not include the bubble of Ξ_{cc} and the two legs connected to it, which correspond to the matrix element in Eq. (7).

$$\int \frac{d^4 l_1}{(2\pi)^4} \frac{d^4 l_2}{(2\pi)^4} \int d^4 x d^4 y A(l_1, l_2) e^{-il_1 x} e^{iP_1 x} e^{-il_2 y} e^{iP_2 y} (2\pi)^4 \delta^4(P - l_1 - l_2 - P_3) \langle \Xi_{cc}(k) + X_N | \bar{Q}(x) \bar{Q}(y) | 0 \rangle. \tag{8}$$

With the help of the diagrams in Fig. 1, one can clearly understand the procedure: Since two charm field operators in the matrix acting on the final bound state, the result is unknown, hence the dependence of the x and y is unknown, and the integration on x and y is not yet done. We keep one δ function for easy to derive, since in this form we can keep both the integration on l_1 and l_2, the momenta of two gluon propagators. l_1 is the four-momentum of the propagator linking the vertex connecting the leg P_1, while l_2 of that linking P_2. $A(l_1, l_2)$ is the short for $A_{ij}(k_1, k_2, P_1, P_2, P_3, l_1, l_2)$. From the following derivation one sees how the l_1, l_2 are replaced and one gets $A_{ij}(k_1, k_2, P_1, P_2, P_3, k, k_N; q_1)$. The exponentials are those from the (configuration space) propagators and the field operators. The key physical point is the relation of the spacetime displacement invariance and the energy-momentum conservation. With the help of the spacetime displacement operator $e^{i\hat{P}y}$ ($\hat{P}$ is the four-momentum operator), Eq. (8) can be written as

$$\int \frac{d^4 l_1}{(2\pi)^4} \frac{d^4 l_2}{(2\pi)^4} d^4 x d^4 y A(l_1, l_2) e^{-il_1 x} e^{iP_1 x} e^{-il_2 y} e^{iP_2 y} (2\pi)^4 \delta^4(P - l_1 - l_2 - P_3) \langle \Xi_{cc}(k) + X_N | e^{i\hat{P}y} \bar{Q}(x-y) \bar{Q}(0) e^{-i\hat{P}y} | 0 \rangle. \tag{9}$$

$\langle \Xi_{cc}(k) + X_N |$ is the eigenstate of the Hermitian operator $\hat{P}$, so $\hat{P}^\mu | \Xi_{cc}(k) + X_N \rangle = (k^\mu + k_N^\mu) | \Xi_{cc}(k) + X_N \rangle$, $\hat{P}|0\rangle = 0$. At the same time, by taking $x - y = x_1$ and since $d^4 x = d^4 x_1$,

$$\int \frac{d^4 l_1}{(2\pi)^4} \frac{d^4 l_2}{(2\pi)^4} d^4 x_1 d^4 y A(l_1, l_2) e^{-il_1 x_1} e^{-il_1 y} e^{iP_1 x_1} e^{iP_1 y} e^{-il_2 y} e^{iP_2 y} e^{i(k+k_N)y} (2\pi)^4 \delta^4(P - l_1 - l_2 - P_3) \langle \Xi_{cc}(k) + X_N | \bar{Q}(x_1) \bar{Q}(0) | 0 \rangle. \tag{10}$$

Collecting the exponentials and integrals we have (now the $\langle \Xi_{cc}(k) + X_N | \bar{Q}(x_1) \bar{Q}(0) | 0 \rangle$ irrelevant from the following calculations)

$$\begin{aligned}
&\int \frac{d^4 l_1}{(2\pi)^4} d^4 x_1 d^4 y \frac{d^4 l_2}{(2\pi)^4} (2\pi)^4 \delta^4(P - l_1 - l_2 - P_3) e^{-i(l_1 + l_2 - P_1 - P_2 - k - k_N)y} A(l_1, l_2) e^{-i(l_1 - P_1)x_1} \\
&= \int \frac{d^4 l_1}{(2\pi)^4} d^4 x_1 d^4 y e^{-i(P - P_1 - P_2 - P_3 - k - k_N)y} A(l_1, l_2) e^{-i(l_1 - P_1)x_1} \\
&= \int \frac{d^4 l_1}{(2\pi)^4} d^4 x_1 (2\pi)^4 \delta^4(P - P_1 - P_2 - P_3 - k - k_N) A(l_1, l_2) e^{-i(l_1 - P_1)x_1} \\
&= (2\pi)^4 \delta^4(P - P_1 - P_2 - P_3 - k - k_N) \int \frac{d^4 q_1}{(2\pi)^4} d^4 x_1 A(q_1, k, k_N) e^{-iq_1 x_1}.
\end{aligned} \tag{11}$$

For the last line, $q_1 = l_1 - P_1$, $\int d^4 l_1 = \int d^4 q_1$. $(2\pi)^4 \delta^4(P - P_1 - P_2 - P_3 - k - k_N)$ is the total energy-momentum conservation in $S_{fi} = \delta_{fi} + (2\pi)^4 \delta^{(4)}(P_i - P_f) \mathcal{T}_{fi}$. Thus the $\mathcal{T}_{fi}$ is obtained as Eq. (7).

Taking the absolute square of the above amplitude, one gets

$$\begin{aligned}
d\Gamma = {}& \frac{1}{2M_\Upsilon} \sum_{X_N} \frac{d^3 k}{(2\pi)^3} \int \frac{d^3 P_1}{(2\pi)^3 2E_1} \frac{d^3 P_2}{(2\pi)^3 2E_2} \frac{d^3 P_3}{(2\pi)^3 2E_3} (2\pi)^4 \delta^4(P - P_1 - P_2 - P_3 - k) \times \frac{1}{3} \times \frac{1}{3} \times \frac{1}{2} \\
& \times \int \frac{d^4 q_1}{(2\pi)^4} \frac{d^4 q_3}{(2\pi)^4} A_{ij}(k_1, k_2, P_1, P_2, P_3, k; q_1) [\gamma^0 A^\dagger(k_1, k_2, P_1, P_2, P_3, k; q_3) \gamma^0]_{kl} \\
& \times \int d^4 x_1 d^4 x_3 e^{-iq_1 x_1 + iq_3 x_3} \langle 0 | Q_k(0) Q_l(x_3) | \Xi_{cc} + X_N \rangle \langle \Xi_{cc} + X_N | \bar{Q}_i(x_1) \bar{Q}_j(0) | 0 \rangle,
\end{aligned} \tag{12}$$

where the spin summation of the baryon Ξ_{cc}, and the polarization and color summation of two anticharm quarks are implied. Here we take nonrelativistic normalization for the baryon Ξ_{cc}. The momentum change in the nonperturbative process k_N is negligible in this order. For this approximation we can eliminate the sum over X_N with the unitary condition. In this way we can see that this matrix element is process-independent, i.e., independent from the details of X_N which could be different in various processes and energies. Employing the displacement operator again, while writing the δ function as spacetime integral, and defining the creation operator $a^\dagger(\mathbf{k})$ for Ξ_{cc} with the three momentum $\mathbf{k}$, we obtain in the vacuum saturation approximation

$$\begin{aligned}
d\Gamma = {}& \frac{1}{2M_\Upsilon} \frac{1}{18} \frac{d^3 k}{(2\pi)^3} \int \frac{d^3 P_1}{(2\pi)^3 2E_1} \frac{d^3 P_2}{(2\pi)^3 2E_2} \frac{d^3 P_3}{(2\pi)^3 2E_3} \int \frac{d^4 q_1}{(2\pi)^4} \frac{d^4 q_2}{(2\pi)^4} \frac{d^4 q_3}{(2\pi)^4} A_{ij}(k_1, k_2, P_1, P_2, P_3, k; q_1) \\
& \times [\gamma^0 A^\dagger(k_1, k_2, P_1, P_2, P_3, k; q_3) \gamma^0]_{kl} \int d^4 x_1 d^4 x_2 d^4 x_3 e^{-iq_1 x_1 + iq_3 x_3 - iq_2 x_2} \langle 0 | Q_k(0) Q_l(x_3) a^\dagger_{\mathbf{k}} a_{\mathbf{k}} \bar{Q}_i(x_1) \bar{Q}_j(x_2) | 0 \rangle,
\end{aligned} \tag{13}$$

with $q_2 = k - q_1$.

In Ξ_{cc} rest frame, the heavy quarks move with a small velocity v_c. Hence, the Fourier transformed matrix element can be expanded in v_Q with fields of NRQCD. The relation between NRQCD fields and Dirac fields $Q(x)$ in the rest frame is

$$Q(x) = e^{-im_c t} \begin{Bmatrix} \chi(x) \\ 0 \end{Bmatrix} + \mathcal{O}(v_c) + \cdots, \tag{14}$$

where $\chi(x)$ is NRQCD field. We will work at the leading order of v_c. In the following we introduce v as the four dimension velocity of Ξ_{cc} with $v^\mu = k^\mu / M_{\Xi_{cc}}$, to help to express our result of the matrix element in a covariant normalization. The matrix element in the rest frame is

$$v^0 \int d^4q_1 d^4q_2 d^4q_3 e^{-iq_1x_1-iq_2x_2+iq_3x_3} \times \langle 0|Q_k(0)Q_l(x_3)a^\dagger(\mathbf{k})a(\mathbf{k})\bar{Q}_i(x_1)\bar{Q}_j(x_2)|0\rangle = \int d^4q_1 d^4q_2 d^4q_3 e^{-iq_1x_1-iq_2x_2+iq_3x_3} \times \langle 0|Q_k(0)Q_l(x_3)a^\dagger(\mathbf{k}=0)a(\mathbf{k}=0)\bar{Q}_i(x_1)\bar{Q}_j(x_2)|0\rangle. \tag{15}$$

Using Eq. (14), the matrix element in Eq. (13) can be expanded with $\chi(x)$ and $\chi^\dagger(x)$. The spacetime dependence of the matrix element with NRQCD fields is controlled by the scale $m_c v_c$. At the leading order of v_c, i.e., $v_c = 0$, one can neglect the relative movement of the operators. The spacetime dependence of the operators are hence the same, and can be taken to origin because of displacement invariance. The mass of the baryon $M_{\Xi_{cc}}$ is approximated equaling to $2m_c$. With this approximation the matrix element in Eqs. (13), (15) is

$$\langle 0|\chi^{a_3}_{\lambda_3}(0)\chi^{a_4}_{\lambda_4}(0)a^\dagger a\chi^{a_1}_{\lambda_1}(0)\chi^{a_2}_{\lambda_2}(0)|0\rangle, \tag{16}$$

where we suppress the notation $\mathbf{k} = 0$ and it is always implied that NRQCD matrix elements are defined in the rest frame of Ξ_{cc}. The superscripts a_i ($i = 1, 2, 3, 4$) are used to label the color of quark fields, while the subscripts λ_i ($i = 1, 2, 3, 4$) for the quark spin indices. The above matrix element can be decomposed into two as following, with the color and spin bases explicitly written:

$$\langle 0|\chi^{a_3}_{\lambda_3}(0)\chi^{a_4}_{\lambda_4}(0)a^+ a\chi^{a_1}_{\lambda_1}(0)\chi^{a_2}_{\lambda_2}(0)|0\rangle = (\varepsilon)_{\lambda_4\lambda_3}(\varepsilon)_{\lambda_2\lambda_1}\cdot(\delta_{a_1a_4}\delta_{a_2a_3}+\delta_{a_1a_3}\delta_{a_2a_4})\cdot h_1 + (\sigma^n\varepsilon)_{\lambda_4\lambda_3}(\varepsilon\sigma^n)_{\lambda_2\lambda_1}\cdot(\delta_{a_1a_4}\delta_{a_2a_3}-\delta_{a_1a_3}\delta_{a_2a_4})\cdot h_3, \tag{17}$$

where σ^i ($i = 1, 2, 3$) are Pauli matrices. $\varepsilon = i\sigma^2$ is totally antisymmetric. The scalar matrix elements h_1 and h_3 are

$$h_1 = \frac{1}{48}\sum_{a_1,a_2}\langle 0|[\chi^{a_1}\varepsilon\chi^{a_2}+\chi^{a_2}\varepsilon\chi^{a_1}]a^\dagger a\chi^{a_2\dagger}\varepsilon\chi^{a_1\dagger}|0\rangle,$$
$$h_3 = \frac{1}{72}\sum_{a_1,a_2}\langle 0|[\chi^{a_1}\varepsilon\sigma^n\chi^{a_2}-\chi^{a_2}\varepsilon\sigma^n\chi^{a_1}]a^\dagger a\chi^{a_2\dagger}\sigma^n\varepsilon\chi^{a_1\dagger}|0\rangle. \tag{18}$$

$h_1(h_3)$ represents the probability for a cc pair in a ${}^1S_0({}^3S_1)$ state and in the color state of $6(\bar{3}*)$ to transform into the baryon. It is the Pauli exclusion principle determines that only these two kinds of combination of color and spin states, which are asymmetric, are possible [29]. With these results the space-time integration can be done, the momenta expressed with help of four-dimension velocity and the spin projectors replaced by the corresponding four-dimension ones, to recover the covariant form:

$$v^0\int d^4x_1 d^4x_2 d^4x_3 e^{-iq_1x_1-iq_2x_2+iq_3x_3}\langle 0|Q_k{}^{a_3}(0)Q_l{}^{a_4}(x_3)a^\dagger(\mathbf{k})a(\mathbf{k})\bar{Q}_i{}^{a_1}(x_1)\bar{Q}_j{}^{a_2}(x_2)|0\rangle = (2\pi)^4\delta^4(q_1-m_cv)(2\pi)^4\delta^4(q_2-m_cv)(2\pi)^4\delta^4(q_3-m_cv)[-(\delta_{a_1a_4}\delta_{a_2a_3}+\delta_{a_1a_3}\delta_{a_2a_4})(\tilde{P}_vC\gamma_5P_v)_{ji}(P_v\gamma_5C\tilde{P}_v)_{lk}h_1 + (\delta_{a_1a_4}\delta_{a_2a_3}-\delta_{a_1a_3}\delta_{a_2a_4})(\tilde{P}_vC\gamma^\mu P_v)_{ji}(P_v\gamma^\nu C\tilde{P}_v)_{lk}(v_\mu v_\nu - g_{\mu\nu})h_3+\cdots] \tag{19}$$

where $P_v = \frac{1+\gamma\cdot v}{2}$, $\tilde{P}_v = \frac{1+\tilde{\gamma}\cdot v}{2}$; $C = i\gamma^2\gamma^0$, the charge conjugation operator.

With the formula in this section and the above section, we obtain the decay width as following:

$$d\Gamma = \frac{16\pi^4\alpha_s^5|R_\Upsilon(0)|^2M_{\Xi_{cc}}}{9M_\Upsilon^2}\frac{1}{[(P_1+k/2)^2(P_2+k/2)^2]^2}\sum_{c=1}^{8}\left(\sum_{\xi=1}^{6}\bar{A}^{abc}_{\xi,\alpha\beta}\right)\left(\sum_{\zeta=1}^{6}\bar{A}^{*a'b'c}_{\zeta,\alpha'\beta'}\right)H^{\alpha\beta\alpha'\beta'}_{aba'b'} \times\frac{d^3k}{(2\pi)^32E_k}\prod_{i=1}^{3}\frac{d^3P_i}{(2\pi)^32E_i}(2\pi)^4\delta^4(P-P_1-P_2-P_3-k), \tag{20}$$

where

$$H^{\alpha\beta\alpha'\beta'}_{aba'b'} = -(\mathrm{Tr}[T^aT^{a'}T^bT^{b'}]+\mathrm{Tr}[T^aT^{a'}]\mathrm{Tr}[T^bT^{b'}])\times h_1\times B_1^{\alpha\beta\alpha'\beta'} + (\mathrm{Tr}[T^aT^{a'}T^bT^{b'}]-\mathrm{Tr}[T^aT^{a'}]\mathrm{Tr}[T^bT^{b'}])\times h_3\times B_2^{\alpha\beta\alpha'\beta'}, \tag{21}$$

$$B_1^{\alpha\beta\alpha'\beta'} = \mathrm{Tr}[\gamma^\alpha(\not{P}_2-m_c)\gamma^{\alpha'}P_v\gamma_5\tilde{P}_v\gamma^{\beta'}(-\not{P}_1-m_c)\gamma^\beta\tilde{P}_v\gamma_5P_v], \tag{22}$$

$$B_2^{\alpha\beta\alpha'\beta'} = \mathrm{Tr}[\gamma^\alpha(\not{P}_2-m_c)\gamma^{\alpha'}P_v\gamma_\mu\tilde{P}_v\gamma^{\beta'}(-\not{P}_1-m_c)\gamma^\beta\tilde{P}_v\gamma_\rho P_v](v^\mu v^\rho - g^{\mu\rho}). \tag{23}$$

The function $\bar{A}_\xi$ ($\xi = 1, 2, 3, 4, 5, 6$) are given in Appendix.

IV. NUMERICAL RESULT AND DISCUSSION

To get the numerical result, the parameters describing the bound states have to be set. The knowledge about them are varying. The radial wave function for Υ at origin can be obtained, e.g., by fitting its leptonic decay width which is well known. On the other hand, the value of h_1 and h_3, is difficult to be obtained. In principle they can be obtained by fitting data. There are yet no experiment results now. For diquark in color-triplet and spin triplet here we employ a potential model with the radial wave function $R_{cc}(r)$ at origin [30] to estimate the numerical value of h_3

$$h_3 = \frac{|R_{cc}(0)|^2}{4\pi}, \tag{24}$$

with its value to be 0.03 GeV^3. The key point for this simple model lies in that the quark degree of freedom of the matrix element h_3 only contains two charm quark fields, so it can be parameterized by a model of cc system. In the above factorization we only keep the leading contribution of the relative velocity. The simplest one is the nonrelativistic wave function as solution of the Schrödinger equation with the Hamiltonian containing certain potential. Suppose this averaged potential is irrelevant with the color and spin details, h_3 in Eq. (18) can be simplified as

$$h_3 = \lim_{x_1\to 0, x_2\to 0} \langle 0|\chi(x_1)\langle 0|\chi(x_2)|\Xi_{cc}\rangle\langle\Xi_{cc}|\chi^\dagger(x_2)|0\rangle\chi^\dagger(x_1)|0\rangle.$$

The $\chi(x)$ ($\chi^\dagger(x)$) is the nonrelativistic field which has the physical meaning of annihilate (create) a charm quark at spacetime point x. After solving the Schrödinger equation, one can get a static solution which can be written as

$$\begin{aligned}\langle 0|\chi(x_1)\langle 0|\chi(x_2)|\Xi_{cc}\rangle &= \psi(x_1 x_2)\\ &= \Psi(\vec{R})\psi(\vec{r})e^{-i(M+\delta E)t}\\ &= e^{i\vec{R}t}\psi(\vec{r})e^{-i(M+\delta E)t},\end{aligned}$$

with the difference of two clocks comoving with each charm quark neglected in this nonrelativistic approximation and only "inner" interaction potential between two charm quarks considered. $\vec{R}, \vec{r}$ are the center of mass position vector and relative position vector, respectively. Hence

$$h_3 = \lim_{x_1\to 0, x_2\to 0} |e^{i\vec{R}t}\psi(\vec{r})e^{-i(M+\delta E)t}|^2 = |\psi(0)|^2,$$

which is (24) for S wave. In this paper we just borrow the solution of the radial wave function obtained in [30]. On the other hand, there is no practical potential model for color-sextet state. An conceptual approximation can be taken with the similar potential model as color triplet. In this case h_1 is approximately the same order of magnitude as h_3. Here h_1 can be taken as a free parameter, the reason is explained in the following. In the numerical calculations, we take $\Psi_\Upsilon(0) = 2.194\ \mathrm{GeV}^{3/2}$, $M_\Upsilon = 9.46$ GeV, $M_\Xi = 3.621$ GeV, $m_b = 4.73$ GeV. m_c/m_b is taken to be parameter, since there is ambiguity at tree level calculations as well as in the models of the nonperturbative parameters. The dependence of branching ratio on m_c/m_b is studied as shown in Fig. 2, which is modest.

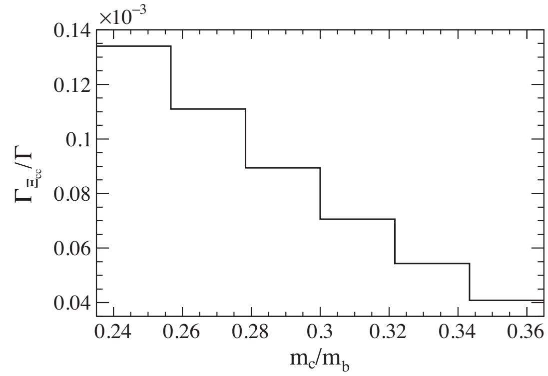

FIG. 2. Dependence of branching ratio on m_c/m_b.

With $m_c/m_b = 0.25$ and $\alpha_s(m_c) = 0.253$, the partial width is $\Gamma = (0.013h_1 + 0.24h_3)$ KeV. Here we see that the perturbative factor timing h_1 is much smaller than that of h_3. So if there is no specially large enhancement on h_1, this part of the contribution cannot be significant and the partial width is insensitive to the concrete value of h_1. So h_1 can be considered as free parameter. Since yet there is no practical model for this color and spin state, for simplicity we take $h_1 = h_3$, and the decay width is 7.3 eV, leading to the branching ratio as 1.3×10^{-4}. The main theoretical error/systematics on this estimation is coming from the final bound state parameters h_1 and h_3, and the scale in the strong coupling constant $\alpha_s(\mu)$. Taking into account a 10% error of the parameters h_1, h_3, the branching ratio can be $(1.3\pm 0.1)\times 10^{-4}$, since this is just a linear dependence. On the other hand, the dependence on the α_s is more sensitive. If we take the scale μ as $2m_c$, we obtain the branching ratio to be half of the value from taking the scale as m_c, i.e., $(7.0\pm 0.6)\times 10^{-5}$. The Ξ_{cc} and $\bar{c}$ momentum distributions are shown in Figs. 3 and 4, respectively. The

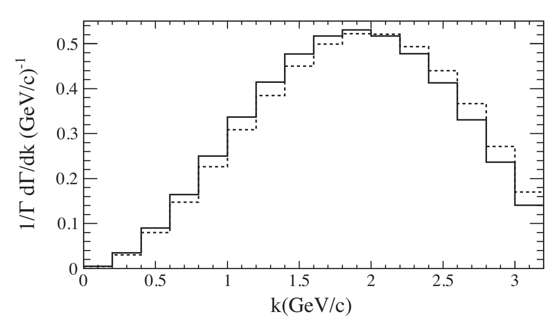

FIG. 3. The normalized momentum distribution of Ξ_{cc}; solid,$h_3 = 0$; dashed, $h_1 = 0$.

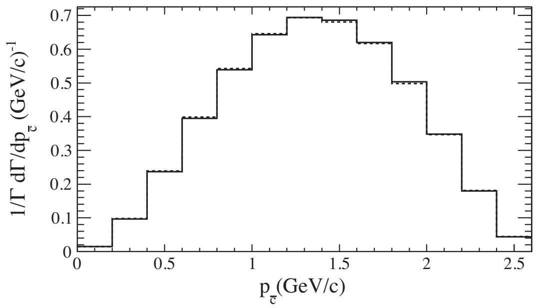

FIG. 4. The normalized momentum distribution of $\bar{c}$; solid,$h_3 = 0$; dashed, $h_1 = 0$.

line shape is mainly determined by the propagators of the perturbative diagrams, insensitive to the color and spin structures. These two anticharm quarks generally fragment to open charm hadrons, respectively. A model for the hadronization of these kind of final parton system can be found in [20,21].

The experiment of BELLE in 2016 has collected 102×10^6 Υ events [2,13]. So it is possible to make a scan on the Ξ_{cc} production. In the future, further precise measurement on the production of Ξ_{cc} can even be made with more large luminosity at BELLE2. Similar productions characteristic of the partonic state with four charm (anti)quarks (T_{cc}, di-J/Ψ resonance) can also be studied in Υ decay.

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APPENDIX

The functions $\bar{A}_\xi(\xi = 1, \ldots, 6)$ in the decay width are

$$
\begin{aligned}
\bar{A_1} &= \mathrm{Tr}^s[T^c T^b T^a] \frac{1}{[(q-P_3)^2 - m^2][(q-P_1-k/2)^2 - m^2]} \\
&\quad \times \mathrm{Tr}\left[\slashed{\epsilon}^*(P_3)(m + \slashed{P}_3 - \slashed{q})\gamma_\alpha \left(\slashed{q} - \slashed{P}_1 - \frac{\slashed{k}}{2} + m\right)\gamma_\beta (M + \slashed{P})\slashed{\epsilon}\right] \\
\bar{A_2} &= Tr^s[T^b T^c T^a] \frac{1}{[(P_2 + k/2 - q)^2 - m^2][(q-P_1-k/2)^2 - m^2]} \\
&\quad \times \mathrm{Tr}\left[\gamma_\alpha \left(m + \slashed{P}_2 + \frac{\slashed{k}}{2} - \slashed{q}\right)\slashed{\epsilon}^*(P_3)\left(\slashed{q} - \slashed{P}_1 - \frac{\slashed{k}}{2} + m\right)\gamma_\beta (M + \slashed{P})\slashed{\epsilon}\right] \\
\bar{A_3} &= \mathrm{Tr}^s[T^c T^a T^b] \frac{1}{[(P_3 - q)^2 - m^2][(q-P_2-k/2)^2 - m^2]} \\
&\quad \times \mathrm{Tr}\left[\slashed{\epsilon}^*(P_3)(m + \slashed{P}_3 - \slashed{q})\gamma_\beta \left(\slashed{q} - \slashed{P}_2 - \frac{\slashed{k}}{2} + m\right)\gamma_\alpha (M + \slashed{P})\slashed{\epsilon}\right] \\
\bar{A_4} &= \mathrm{Tr}^s[T^a T^c T^b] \frac{1}{[(P_1 + k/2 - q)^2 - m^2][(q-P_2-k/2)^2 - m^2]} \\
&\quad \times \mathrm{Tr}\left[\gamma_\beta \left(m + \slashed{P}_1 + \frac{\slashed{k}}{2} - \slashed{q}\right)\slashed{\epsilon}^*(P_3)\left(\slashed{q} - \slashed{P}_2 - \frac{\slashed{k}}{2} + m\right)\gamma_\alpha (M + \slashed{P})\slashed{\epsilon}\right] \\
\bar{A_5} &= \mathrm{Tr}^s[T^b T^a T^c] \frac{1}{[(P_2 + k/2 - q)^2 - m^2][(q-P_3)^2 - m^2]} \\
&\quad \times \mathrm{Tr}\left[\gamma_\alpha \left(m + \slashed{P}_2 + \frac{\slashed{k}}{2} - \slashed{q}\right)\gamma_\beta (\slashed{q} - \slashed{P}_3 + m)\slashed{\epsilon}^*(P_3)(M + \slashed{P})\slashed{\epsilon}\right] \\
\bar{A_6} &= \mathrm{Tr}^s[T^a T^b T^c] \frac{1}{[(P_1 + k/2 - q)^2 - m^2][(q-P_3)^2 - m^2]} \\
&\quad \times \mathrm{Tr}\left[\gamma_\beta \left(m + \slashed{P}_1 + \frac{\slashed{k}}{2} - \slashed{q}\right)\gamma_\alpha (\slashed{q} - \slashed{P}_3 + m)\slashed{\epsilon}^*(P_3)(M + \slashed{P})\slashed{\epsilon}\right] \qquad \text{(A1)}
\end{aligned}
$$

Here $\mathrm{Tr}^s[\ldots]$ means only keeping the symmetric part; $m = m_b$, $M = M_\Upsilon$, $q = P/2$. $\varepsilon(P_3)$ is the polarization vector of the gluon with momentum P_3.

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