Current and future constraints on single-field α-attractor models

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We study the observational constraints on single-field inflationary models achievable with the next generation of cosmic microwave background (CMB) experiments. We focus on a Stage IV (S4)-like experiment and forecast its constraints on inflationary parameters in the context of α-attractor inflation comprising a large class of single-field models. To tailor our forecasts, we use as a fiducial model the results obtained with current CMB and LSS data, assuming the α model *a priori*. We find that current CMB data are able to place a tight bound on the ratio of the tensor amplitude with the alpha parameter $r/\alpha = 3.87^{+0.78}_{-0.94} \times 10^{-3}$ and on the running of the scalar index $\alpha_S = -6.4^{+1.6}_{-1.3} \times 10^{-4}$ with a value of the scalar index consistent with current constraints. These tight constraints are the result of the strong bound imposed on n_s by the current cosmological data and the theoretical prior of α-attractor models on inflationary observables. This bound can also be translated into an upper bound for α. We find $\alpha < 25$ and $\alpha < 15$, given $r < 0.1$ and $r < 0.06$ for *Planck* 2018 and *Planck* 2018 and BICEP/Keck 2015 data, respectively. In the optimistic scenario of detection of primordial gravitational waves in the CMB B-mode polarization, CMB-S4 will be able to achieve a 15% bound on the value of the α parameter. This bound clearly shows the ability of CMB-S4 to constrain not only the energy scale of inflation but also the shape of its potential. Enlarging the baseline model to also include the neutrino sector merely reduces the accuracy on α by about 5%, and thus our main conclusions are still valid. Conversely, in the pessimistic scenario of no detection, a CMB-S4-like experiment will reduce the upper bound on α by around an order of magnitude, leading to a possible exclusion of the Starobinsky model at the level of 6 standard deviations.

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I. INTRODUCTION

In this paper we forecast the possible constraints that a future cosmic microwave background (CMB) Stage IV (hereafter, CMB-S4) experiment may impose on inflationary observables in the optimistic scenario of a detection of nonvanishing tensor anisotropies in the CMB polarization and temperature data. In general, the approach followed within the community (see, e.g., Refs. [1–5]) is to sample the inflationary parameters without assuming any specific inflationary model *a priori*. While this approach has the advantage of exploring the inflationary sector model independently, it does not allow for a complete sampling study of the parameter space in a specific model. Moreover, the assumption that the inflationary observables are independent of one another is in contrast with the prediction of any theory of inflation, which, for instance, assumes the validity of the slow-roll conditions (see, e.g., Refs. [6–8]). In this work, conversely to the current literature on the subject, we follow a model-dependent approach by imposing a specific model *a priori* and calculate the inflationary observables directly by imposing the slow-roll conditions on the inflationary potential.

The standard ΛCDM model is based on the simplest inflationary paradigm: canonical slow-roll single-field inflation. Inflation not only solves the need to fine-tune the initial conditions from the hot big bang scenario, but also provides an elegant mechanism to explain the origin of the scalar primordial perturbations that evolved into the current cosmic structures at large scales. Furthermore, quantum inflationary fluctuations are expected to source a stochastic background of gravitational waves—so-called primordial gravitational waves (PGWs)—sourcing fluctuations in the polarization of CMB photons at recombination leading to a very distinctive signature in the CMB B-mode power spectrum at large angular scales. Within this approach, the power spectra of scalar and tensor comoving curvature perturbations are parametrized as power laws:

$$P_S(k) = A_S \left(\frac{k}{k_\star^S}\right)^{n_S - 1 + \frac{\alpha_S}{2}\log k/k_\star^S}, \tag{1}$$

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$$P_T(k) = rA_S\left(\frac{k}{k_\star^T}\right)^{n_T}, \tag{2}$$

where $k_\star^S = 0.05\ \mathrm{Mpc}^{-1}$ and $k_\star^T = 0.002\ \mathrm{Mpc}^{-1}$, and the subscripts stand for scalar and tensor perturbations, respectively. The powers of the parametrizations are the scalar and tensor indices (n_S and n_T) and the running of the spectral index α_S. Statistical analysis of recent cosmological observations [*Planck* observations of the CMB [5,9,10] and large-scale structure (LSS) surveys [11,12]] support this parametrization for the scalar fluctuations with $10^9 A_S \approx 2.1$ and $n_S \approx 0.965$ [5,9,10].

In the last decade, the bound on the amplitude of PGWs (parametrized typically with the tensor-to-scalar ratio, r) has not yet seen significant improvement, where only an upper limit $r_{0.002} < 0.056$ at 95% C.L. has been provided in the latest data released by the *Planck* Collaboration [9] combining *Planck* and BICEP2/Keck array (BK15) data [13]. Detecting those PGWs would give a direct measurement of the energy scale during inflation, as well as a clear distinguishable signature of the quantum origin of primordial fluctuations. In the upcoming decade, a new generation of CMB experiments (e.g., BICEP3 [14], CLASS [15], SPT-3G [16], Advanced ACTPol [17], LBIRD [18], and CMB-S4 [19]) are expected to strongly improve the sensitivity on the B-mode polarization in the CMB, possibly revealing the first evidence of inflationary tensor modes with amplitudes $r \sim 0.01$–0.001. That range is precisely expected in many well-motivated models, such as Starobinsky inflation, which is considered the benchmark of future CMB experiments. However, while a measure of a nonvanishing r would be of key importance for inflationary theories, it will not allow us to understand the inflationary mechanism in detail, but rather only its energy scale. Therefore, it is time to investigate, given future CMB experiments, what would be the freedom in a generic inflationary framework that is left in case of the optimistic scenario of a nonvanishing tensor-to-scalar ratio measure.

In particular, there is a general class of models called α attractors that have gained popularity because of their agreement with observational constraints and the universality of their predictions for the inflationary observables [20–24]. Recently, α attractors have also been used in the context of dark energy to explain the late-time cosmic acceleration [25–29]. This set of models has also been embedded in a more general multifield inflationary scenario and in $\mathcal{N} = 1$ supergravity. In the context of supergravity, the α attractor can be represented by a potential of the form

$$\frac{V(\varphi)}{V_0} = (\tanh(\beta\varphi/2))^{2n}, \tag{3}$$

where $\beta^2 = 2/3\alpha$ and n is an arbitrary value. It is important to note that the "attractor behavior" of this potential comes from the fact that the observable predictions are the same up to leading order regardless of the value of n, while they differ only in subleading corrections. Assuming slow-roll inflation and the α-attractor form of the inflationary potential, the observational predictions for the inflationary observables can be written as

$$r = \frac{12\alpha}{N^2}, \tag{4a}$$

$$n_S = 1 - \frac{2}{N} = 1 - \sqrt{\frac{r}{3\alpha}}, \tag{4b}$$

$$\alpha_S = -\frac{2}{N^2} = -\frac{r}{6\alpha}, \tag{4c}$$

where N is the number of e-folds for inflation to last. These definitions in terms of the parameter α encompass several inflationary models and clearly reduce to the well-known Starobinsky inflation for $\alpha = 1$ [30–32]. Moreover, for a broad class of potentials V, as long as $\alpha \ll O(1)$, the scalar spectral index n_S, its running α_S, and the tensor-to-scalar ratio r converge to the functional form of Eqs. (4) regardless of the kinetic terms of the theory [33]. It has also been shown that this statement holds true in some multifield inflation regimes [33], where the conditions that guarantee the universality of the observational predictions for the inflationary parameters are derived by imposing constraints on the potential. The universality of the observational constraints is one of the most important features of single-field α-attractor models.

II. CONSTRAINTS FROM CURRENT CMB AND LSS DATA

Current CMB [1–5] and LSS [11] data are unable to constrain the tensor-to-scalar ratio if r is sampled independently from the scalar index n_S. However, by imposing the α-attractor model *a priori*, we force a specific functional relation between n_S and r that allows us to translate the subpercent constraints on n_S from current data into a constraint on r in the context of α attractors.

In fact, by imposing an inflationary model we are selecting a subset of the parameter space allowed by the data when r and n_S are considered independently in cosmological parameter estimations. This is particularly evident if one considers the relation between r and n_s in the α-attractor model given by Eq. (4b).

Current data cannot break the $r - \alpha$ degeneracy, and therefore, sampling it in our analysis would give no insight on the α-attractor models. For this reason, instead of sampling r and α independently, we use the ratio r/α as

TABLE I. Range of uniform prior distributions imposed on the sampled parameters during the analysis.

Parameter	Prior range
$\Omega_b h^2$	[0.005, 0.1]
$\Omega_c h^2$	[0.001, 0.99]
H_0	[20, 100]
τ	[0.01, 0.8]
$r_{0.002}/\alpha \cdot 10^3$	[0.5, 8]
$\log(10^{10} A_s)$	[1.61, 3.91]
$\sum m_\nu$	[0, 1]
$N_{\rm eff}$	[2, 5]

a parameter for our Markov chain Monte Carlo (MCMC) analysis.

Along with the ratio r/α, we consider as independent parameters the other five standard ΛCDM ones: the baryon density $\omega_b = \Omega_b h^2$ and cold dark matter density $\omega_c = \Omega_c h^2$, the Hubble constant H_0, the optical depth τ, and the amplitude of scalar perturbations A_s. Along with these parameters we also include the running of the spectral index, α_S. As α attractors satisfy the usual inflationary consistency relation, we fix the index of tensor modes to $n_T = -r/8$. The uniform prior distributions imposed on these parameters are reported in Table I.

The predictions of the theoretical observational probes are calculated using the latest version of the cosmological Boltzmann integrator code CAMB [34,35]. To compare our theoretical predictions with data, we use the full 2018 *Planck* (P18) temperature and polarization data sets which also include multipoles $\ell < 30$ [36]. We combine the *Planck* likelihood with the BICEP/Keck 2015B-mode (BK15) data [13] and the combination of galaxy clustering and weak lensing data from the first year of the Dark Energy Survey (DES Y1) [37]. The posterior distributions of the cosmological parameters have been explored using the publicly available version of the Bayesian analysis tool COBAYA [38]. In particular, the posteriors have been sampled using the MCMC algorithm developed for CosmoMC [39,40] and tailored for parameter spaces with a speed hierarchy.

The 1D posteriors of r/α, n_S, and α_S resulting from our Bayesian statistical analysis employing *Planck* 2018 data in combination with DES and BK15 data are reported in Fig. 1. Given the subpercent constraints on the scalar index, we found a sub-percentage constraint on the tensor amplitude, i.e., $r/\alpha = 0.00387^{+0.00078}_{-0.00094}$ for *Planck* 2018 data alone. We also found no differences between the results using *Planck* 2018 data alone or combined with the BICEP/Keck 2015 data ($r/\alpha = 0.00400^{+0.00076}_{-0.00095}$) as the constraint on the scalar index is the same for the two datasets (if not for a statistically insignificant shift in the posterior mean). When LSS data (i.e., DES) are included in the analysis, a shift in the spectral index n_s with respect to CMB data is found. DES data prefer a slightly higher value for n_s, shifting the running of the spectral index α_S and the tensor-to-scalar ratio r accordingly. However, the results remain consistent with that from P18 and P18 + BK15 within 1σ (see also Fig. 1).

Incidentally, we also obtain a constraint on the running of the scalar index α_S [related to r/α by Eq. (4c)] away from zero at 4 standard deviations, i.e., $\alpha_S = -6.4^{+1.6}_{-1.3} \times 10^{-4}$. It is worth noting that (as for r) this is due to the specific correlation which arises in α-attractor inflation between the parameters of the scalar and tensor spectra. This result, however, shows that future measurements of $r_{0.002}$ and $n_{\rm nrun}$ could potentially rule out the α-attractor model: they are key parameters in studying the viability of an inflationary model and should be considered in the future analysis of CMB and LSS data.

We conclude this section with the following two considerations:

(1) Given $n_S \approx 0.965$ and $r \to 0$, the best-fit model for CMB and LSS data is the case of $\alpha = 1$, which corresponds to Starobinsky inflation, and this cannot be distinguished from a generic model with $\alpha \neq 1$ unless future experiments provide a measure of either polarization B modes or α_S.

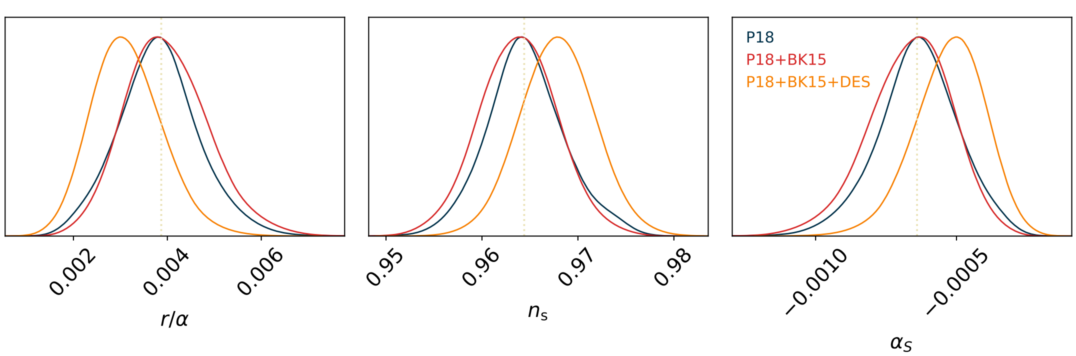

FIG. 1. Posterior distribution for *Planck* 2018 data alone and combined with the BICEP/Keck 2015B-mode data and with large-scale structure DES data. The dotted lines denote the expected values for Starobinsky inflation ($\alpha = 1$ and $N \approx 60$).

(2) Current data are consistent with $r \to 0$. Nevertheless, not all values of α are allowed. Instead, they set an upper limit on the value of the tensor-to-scalar ratio and this knowledge can be used to obtain an upper limit for α:

$$\alpha \lesssim \frac{r_{\rm lim}}{r_0} \equiv \alpha_{\rm lim}, \tag{5}$$

where $r_{\rm lim}$ is the experimental threshold for a given experimental configuration and r_0 is the mean of the r/α posterior. Since $r_{\rm lim} \approx 0.1$ for P18 data and $r_{\rm lim} \approx 0.06$ for P18 + BK15 data combined [5], we correspondingly find $\alpha_{\rm lim} \approx 25$ and $\alpha_{\rm lim} \approx 15$ for P18 and P18 + BK15, respectively. These upper limits would be the same that one would obtain by running an MCMC analysis with r and α considered independently (see the Appendix).

III. FORECAST FOR FUTURE CMB-S4 OBSERVATIONS

While current data are unable to constrain the value of α given the current experimental sensitivity, future CMB experiments are expected to strongly improve the sensitivity on the B-mode polarization signal of the CMB, possibly discovering evidence of a primordial tensor mode with an amplitude in the range of $r \sim 0.01$–0.001 [41–44]. In particular, this is the range predicted by many well-motivated inflationary models such as Starobinsky inflation, considered the benchmark for future CMB observations [41–44]. In this section, we study the optimistic scenario of a future detection in the CMB anisotropies of a nonvanishing tensor amplitude and we forecast the constraints achievable with a CMB-S4-like experiment on the parameters of the α-attractor model.

We consider as a baseline model a minimal extended ΛCDM cosmology with the inclusion of a nonvanishing tensor-to-scalar ratio r, and α. This extended model constitutes our simulated data sets. The value of r is chosen correspondingly to the best-fit value obtained with a Starobinsky model using only *Planck* 2018 data i.e., $r = 0.00387$, while we fix $\alpha = 1$. The value of the scalar index and its running are also fixed to $n_s = 0.964$ and $\alpha_S = 0.0006$. The remaining ΛCDM parameters values are $\omega_b = 0.0221$, $\omega_c = 0.12$, $H_0 = 67.3$, $\tau = 0.06$, and $\ln(10^{10} A_s) = 3.05$. As the new generation of CMB experiments also expects to shed some light on the neutrino sector, we also explore the number of effective degrees of freedom of relativistic species $N_{\rm eff}$ and the sum of the neutrino masses $\sum m_\nu$ in the forecast.

Both simulated data and theoretical models are computed with the latest version of the Boltzmann code CAMB [34,35]. To extract constraints on cosmological parameters, we make use of the MCMC code CosmoMC [39,40] which compares theory with a simulated data set using a given likelihood.

As in Refs. [45–48], we build our forecasts for future CMB experiments following a well-established and common method. Using the set of fiducial parameters described above, we compute the angular power spectra of temperature C_ℓ^{TT}, E and B polarization $C_\ell^{EE,BB}$, and cross temperature-polarization C_ℓ^{TE} anisotropies. We produce a fiducial realization of future data by adding to the theoretical power spectra an exponential noise of the form [49]

$$N_\ell = w^{-1} \exp(\ell(\ell+1)\theta^2/8\ln 2), \tag{6}$$

where θ is the FWHM angular resolution and w^{-1} is the experimental sensitivity expressed in μK arcmin. The polarization noise is derived equivalently assuming $w_p^{-1} = 2w^{-1}$ since one detector measures two polarization states. The simulated spectra, realized accordingly to the previous discussion, are compared with theoretical ones using a "CMB-like" likelihood as in Refs. [49,50].

For this paper, we have constructed synthetic realizations of CMB data for only one experimental configuration, namely, CMB-S4 (see, e.g., Ref. [51]). The CMB-S4 data set is constructed using $\theta = 3'$ and $w = 1\mu$K arcmin, and it operates over the range of multipoles $5 \leq \ell \leq 3000$, with a sky coverage of 40%. Furthermore, CMB-S4 is expected to reach a target sensitivity on the tensor-to-scalar ratio of $\Delta r \sim 0.0006$, the goal of which is to provide a 95% upper limit of $r < 0.001$. Therefore, the value chosen for our fiducial model is well within the scope of an experiment like CMB-S4. However, the corresponding sensitivity on the value of the running of the scalar index α_S would be only $\Delta\alpha_S = 0.002$, which would clearly not be enough for a joint detection of r and α_S assuming Starobinsky inflation. Thus, it may not be possible to distinguish between a generic α-attractor model with $r \sim 0.004$ and Starobinsky inflation despite a future detection of a nonvanishing tensor amplitude.

In α attractors, however, the uncertainties about the correct shape of the inflationary potential, defining the values of r, n_s, and α_S, are parametrized with the α parameter. Therefore, a measure of the value of α would also give us insights about the correct shape of the inflationary potential and correspondingly the correct theory of inflation. If a non vanishing tensor amplitude is ever detected, a CMB-S4-like experiment will be able to provide such insights.

Current data is only able to constrain the upper bound $0 \leq \alpha \lesssim 15$ [52] correspondingly to P18 + BK15 upper limit on the tensor amplitude $r < 0.056$ at 95% C.L. [9]. To correctly explore the available parameter space for α, we therefore employ a logarithmic prior on its value $-6 \leq \log_{10}\alpha \leq 1$ while the other parameters are sampled using the priors shown in Table I. We refer to this model as αCDM. From our CMB-S4 forecasts we obtain a 15%

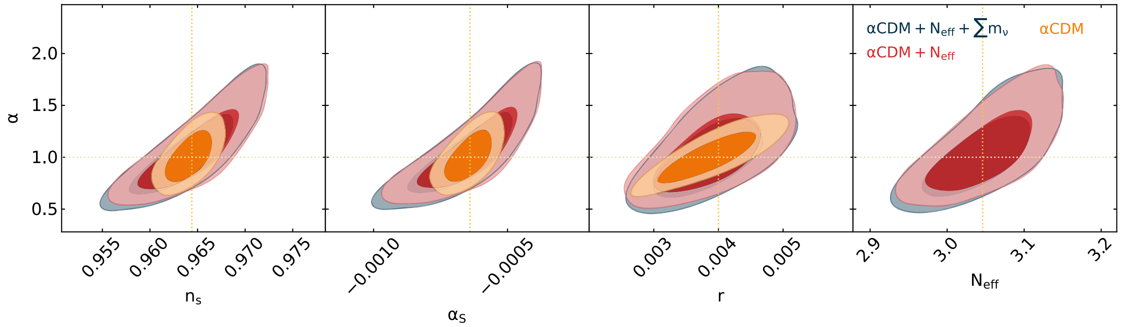

FIG. 2. CMB-S4 forecasted 2D contours at 68% and 95% for α-attractor inflationary parameters in αCDM, $\alpha\mathrm{CDM} + N_{\mathrm{eff}}$, and $\alpha\mathrm{CDM} + N_{\mathrm{eff}} + \sum m_\nu$.

bound on the parameter $\alpha = 1.01^{+0.14}_{-0.18}$, clearly showing the ability of future CMB experiments to bound single-field slow-roll inflationary models. Models with $\alpha \geq 2$ and $\alpha \leq 0.5$ would be potentially excluded at more than 2 standard deviations in the optimistic scenario of a PGW detection with an amplitude in the range of the Starobinsky model. We then extend this baseline model by including the number of relativistic neutrino species N_{eff}, ($\alpha\mathrm{CDM} + N_{\mathrm{eff}}$). When N_{eff} is varied, we find a 5% reduction of the accuracy with which α is measured, i.e., $\alpha = 1.07^{+0.18}_{-0.23}$, while the bound on the tensor-to-scalar ratio is basically the same in the two cases, i.e., $\sigma(r) = 0.00050$. Conversely, we find an increase in the error budget of the scalar index and running, passing from $\sigma(n_s) = 0.0016$ and $\sigma(\alpha_S) = 0.00006$ (αCDM) to $\sigma(n_s) = 0.0035$ and $\sigma(\alpha_S) = 0.0001$ ($\alpha\mathrm{CDM} + N_{\mathrm{eff}}$) a worsening of a factor around two in both cases. It is worth stressing that primordial gravitational waves may also contribute to the number of relativistic species, being themselves relativistic degrees of freedom [48,53,54]. This contribution can be calculated analytically to be

$$N_{\mathrm{eff,GW}} \sim \frac{rA_s}{n_T}(A^{n_T} - B^{n_T}), \tag{7}$$

where A and B are two real numbers and $A, B \gg 1$. This contribution is clearly extremely small for red spectra ($n_T \leq 0$) but may be important in inflationary theories where blue spectra ($n_T > 0$) can be produced (see, e.g., Refs. [55–61]). Consequently, the only interaction between PGWs and neutrinos considered in this work is the one arising from neutrino anisotropic stress after neutrino decoupling at $T \lesssim 1$ MeV [62]. These constraints are virtually unmodified when we further extend our baseline model, allowing the whole neutrino sector to vary, i.e., $N_{\mathrm{eff}} + \sum m_\nu$. The 2D contours for both of our forecasts are reported in Fig. 2. A strong correlation now arises between α and the other inflationary parameters conversely to what we found with the *Planck* data. This is due to the power of CMB-S4 to resolve the B-mode spectrum, consequently breaking the degeneracy between r and n_s. Nevertheless, the situation is unchanged for the scalar running. The strong bound we find on the scalar running is in fact due to imposing the α model *a priori*. Even a Stage IV experiment would not have the required accuracy to measure the tiny scalar running predicted by α-attractor inflation. When r, n_s, and α_S are independently varied [i.e., neglecting the consistency relation in Eqs. (4)] the running is fixed only with an error $\sigma(\alpha_S) = 0.0029$ at 68% C.L., an order of magnitude higher than when the α model is imposed *a priori* and in good agreement with the expected sensitivity for the CMB-S4 experiment [19].

We note that, as well as for current data, one can forecast the corresponding upper limit on α from Eq. (5) in the pessimist scenario where CMB-S4 does not detect a tensor-to-scalar ratio above the target sensitivity. Assuming $r_{\mathrm{lim}} = 0.001$ [43], one finds $\alpha_{\mathrm{lim}} \approx 0.26$ which would exclude Starobinsky inflation at 10 standard deviations. With respect to *Planck* data, CMBS4 will provide an improvement on the

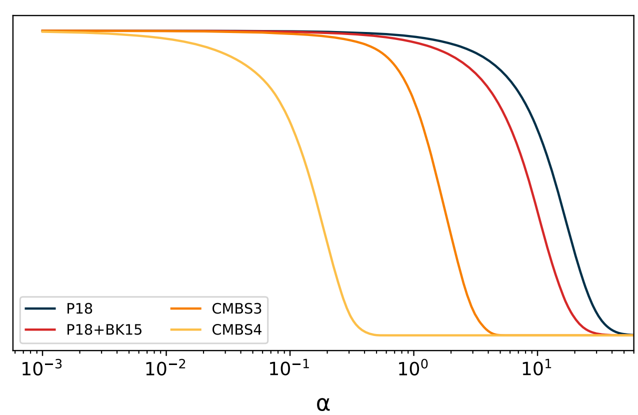

FIG. 3. 1D posterior for the parameter α for several experimental configurations. These posterior distributions are obtained with the method described in the Appendix. The CMB Stage-III (CMBS3) constraint is obtained assuming a target sensitivity of $r_{\mathrm{lim}} = 0.01$ corresponding to $\alpha_{\mathrm{lim}} \approx 3$. This sensitivity would be achievable by a Stage III experiment such as SPT-3G [16] or BICEP3 [14].

measure of $\alpha_{\rm lim}$ of 2 orders of magnitude even in the pessimistic case of not detecting any B-mode polarization signal. The constraints on r/α will be instead improved only by a factor of 4, leading to $r/\alpha = 0.00386 \pm 0.00035$ when neutrino parameters are fixed to their ΛCDM values. In Fig. 3 we show a comparison of the upper bounds on α achievable by the experimental configurations considered in this work.

IV. CONCLUSIONS

We have carried out a Bayesian analysis with current CMB and LSS data to constrain inflationary observables (the scalar spectral index n_s, its running α_S, and the tensor-to-scalar ratio r). With the current constraining power on n_s and by imposing the α-attractor model *a priori* in our analysis, the possible values of the ratio r/α are narrowed to a band of around 0.004. However, current data do not have enough sensitivity to break the degeneracy between r and α and consequently to constrain any deviation from the Starobinsky inflationary model due to the fact that the predicted tensor-to-scalar ratio is much smaller than the current upper limit of LSS and CMB data. Consequently, we focused our attention on forecasting the constraints achievable by a future CMB-S4 experiment assuming a tensor-to-scalar ratio corresponding to the value obtained from *Planck* data by imposing the α-attractor model *a priori*; see Sec. II.

The forecast was performed using a Bayesian statistical approach, where α was sampled from a logarithmic prior distribution. Future CMB-S4 experiments will then be able to constrain α as long as the value of r is above the target sensitivity expected from such an experiment, i.e., $r > 0.001$ [43,51]. Conversely, in the pessimist scenario where future CMB-S4 data does not measure a tensor amplitude above the target sensitivity, the situation will be similar to the current one with the value of α constrained only to an upper limit. We forecasted the corresponding limit on α to be $\alpha_{\rm lim} \approx 0.26$, an improvement of 2 orders of magnitude with respect to *Planck* data alone.

In conclusion, a future CMB-S4 experiment will have enough sensitivity to significantly constrain single-field slow-roll inflationary models. In the case of an optimistic detection of a nonvanishing tensor amplitude, it would be able to shed light on both the energy scale and the shape of the inflationary potential, while in the pessimistic scenario of a nondetection of tensor modes it would still be able to place a tight upper limit on the value of α and exclude Starobinsky inflation at 10σ. We emphasize that when the running of the spectral index α_S is free to vary it is always different from zero, as is expected from the inflationary consistency relation of the α-attractor model. However, we showed that the value expected for the scalar running given the current constraints on the scalar index is so small that it will not be detectable by a future CMB-S4 experiment (with an expected sensitivity of $\Delta\alpha_S \sim 0.003$), but it may be reachable when information from future weak-lensing and galaxy clustering measurements are included [63–65]. The combination of future weak-lensing surveys and CMB-S4 would possibly reach a target sensitivity of $\Delta\alpha_S \sim 0.001$, a factor of 3 better than CMB-S4 alone.[1] This is enough to constrain α_S at a level compatible with the value expected from α-attractor models. Note that a measure of α_S would constitute a smoking gun for inflation as well as a measure of a nonzero tensor amplitude. Therefore, future LSS surveys and CMB experiments will either give us a measure of both r and α_S in the most optimistic scenario, or they will be able to significantly reduce the available parameter space for single-field slow-roll inflation in the most pessimistic one.

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APPENDIX: FROM A TWO-SIGMA BOUND TO AN UPPER LIMIT

In this appendix we briefly describe the procedure used to convert the two-sigma bound on r/α into an upper limit on α assuming an experimental threshold $r_{\rm lim}$. Let us start by noting that an upper limit on the tensor-to-scalar ratio $r < r_{\rm lim}$ at 95% C.L. can be represented by a half-normal distribution with a standard deviation σ given by the following equation:

$$\int_{-r_{\rm lim}}^{r_{\rm lim}} \mathcal{N}(x|0,\sigma)dx = 0.95. \tag{A1}$$

Solving for σ and applying an inverse transform sampling technique, we can extract samples from the half-normal distribution. Then, for each sample of the half-Gaussian of r/α, we can use Eq. (5) to calculate a sample of the distribution of α. This procedure allows to reconstruct the posterior of α starting from the bound on

[1] This is derived assuming an improvement of a factor $\sigma(\alpha_S)_{\rm Planck}/\sigma(\alpha_S)_{\rm CMBS4} \sim 3$ of the forecasted constraints on α_S with respect to the combination of weak-lensing and *Planck* data from Table 21 of Ref. [66].

r/α and it is equivalent to performing a full MCMC analysis with an experimental configuration that can reveal tensor modes with amplitude $r > r_{\rm lim}$ at 95% C.L. As shown in Fig. 3, the results on α agree almost perfectly with the approximate results obtained considering a delta distribution for r_0 and $r_{\rm lim}$. Thus, we conclude that the uncertainties in the measure of r/α can be negligible in deriving an upper limit for the values of α.

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