Finite action principle and Hořava-Lifshitz gravity: Early universe, black holes, and wormholes

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In this work, we elaborate on the finite action in the framework of Hořava-Lifshitz gravity. Assuming the finite action principle, we show that the beginning of the Universe is flat and homogeneous. Depending on the version of the theory, different cosmological scenarios are possible. Furthermore, we show that the Hořava-Lifshitz gravity action selects only the regular black-hole spacetimes since the singular black holes possess infinite action. We also comment on the possibility of traversable wormholes in theories with higher-curvature invariants. The possible cosmological solutions in Hořava-Lifshitz and quadratic gravity are similar, proving that the finite action principle is not model sensitive.

I. INTRODUCTION

The path integral approach yields a powerful framework in quantum theory since it emphasises Lorentz covariance and allows for the description of nonperturbative phenomena. In the path integral, one is supposed to sum over all possible configurations of a field(s) Φ weighted by $e^{iS[\Phi]}$, where $S[\Phi]$ is the classical action of the theory. In the Wick rotated path integral, where one takes $t \to i\tau$, the field configuration is weighted by $e^{-S[\Phi]}$.

Following this notion, the finite action principle (FAP) proposes that the action should be on the fundamental entity instead of the field values. One can then ask which of the field configurations results in the finite action and which in the infinite one.

Recently, this principle has been applied to the study of black holes [1]. Since it is expected that the quantum gravity should resolve the black-hole singularity problem, one may ask which of the microscopic actions remains finite for nonsingular black holes and, conversely, interferes destructively for the singular ones. This we shall call the finite action selection principle. Only after the inclusion of higher-curvature operators, beyond the Einstein-Hilbert term, can such a selection principle be satisfied [1]. Furthermore, in asymptotic safety, the quantum corrections to the Newtonian potential eliminate the classical s ingularity [2].

On the other hand, requirement that an action of the Universe should be finite [3] is well motivated, theoretically (see also a newly proposed finite amplitudes principle [4]). This principle has a significant impact on the nature of quantum gravity and the evolution of the Universe, once the

higher-curvature terms are included [5,6]. Following this principle, unlike for the Einstein action, in Stelle gravity [7] the presence of the R^2 term implies homogeneous and isotropic conditions for the early universe if considering the off-shell action (for the on-shell action anisotropies are supposed to be washed out by inflation [4]). Furthermore, the highly symmetric state yields a vanishing Weyl tensor [8], explaining the low entropy of the early universe.

These findings suggest that by taking into account the higher curvatures one can resolve the singularities in the early universe and the black holes. Yet, an issue with the higher-curvature theory of quantum gravity is the existence of the particles with the negative mass-squared spectrum, known as "ghosts," which makes the theory nonunitary. It is the consequence of the Ostrogradsky theorem [9] and the presence of higher than second-order time derivatives in the terms beyond R in the action. However, this might be resolved by additional symmetry [10], giving up the microcausality, changing the propagator prescriptions [11,12], or taking into account infinitely many derivatives 13]], see also the discussion [14] on possible resolution in the context of asymptotic safety.

In this article, we explore yet another possibility, namely, we investigate Hořava-Lifshitz (HL) gravity [15], where the Lorentz invariance (LI) is broken at the fundamental level (see [16] for a comprehensive progress report on this subject). Kinetic terms are first order in the time derivatives, while higher spatial curvature scalars regulate the UV behavior of the gravity. Furthermore, the lower-dimensional lattice studies of causal dynamical triangulations give the same Hamiltonian as HL gravity [17–19].

In the Euclidean path integral the notion of the finite action principle is quite natural since the infinite action configurations clearly do not contribute. On the other hand,

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the Wick rotation is not well defined in quantum gravity, in general, this becomes especially difficult for the theories with higher-curvature terms. In our investigation we follow Hořava [15] and assume that the Wick rotation is well defined. This can be motivated by the lack of the higher timelike curvature invariants. In the usual higher derivative constructions the existence of massive poles in the propagator makes the Wick rotation troublesome [20]. Conversely to the latter, the new poles are massless and nontachyonic for a suitable choice of parameters [21]. Furthermore, the existence of the foliation supports that claim. For the discussion of Minkowski path integral and FAP, see [1].

In this article, we show that the finite action arguments applied to the projectable HL gravity result in a flat, homogeneous, UV-complete, and ghost-free beginning of the Universe, supporting the topological phase conjecture [22]. We also show that the finite action selection principle [1] works for HL gravity in the context of black holes (the action is finite for nonsingular black holes (BHs) and conversely for the singular). Furthermore, we have found that wormholes possess a finite action and hence contribute to the path integral of QG, therefore, they are consistent with the Einstein-Rosen (ER)=EPR hypothesis [23]. On the other hand, the stable traversable wormholes solutions are known only in the higher derivative gravities [24] (without exotic matter), so there seems to be a wormhole/nonsingular BH trade-off after taking into account the finite action principle.

II. HOŘAVA-LIFSHITZ GRAVITY

In the Hořava-Lifshitz gravity, space and time are scaled in a nonequivalent way. Diffeomorphism invariance is broken by the foliation of the four-dimensional spacetime into three-dimensional hypersurfaces of constant time, called leaves, making the theory power-counting renormalizable (see also the renormalization group studies of the subject [25–27]). The remaining symmetry respects transformations,

$$t \to \xi_0(t), \qquad x^i \to \xi^i(t, x^k), \qquad (1)$$

and is often referred to as the foliation-preserving diffeomorphism, denoted by $\text{Diff}(M, \mathcal{F})$. The diffeomorphism invariance is still present on the leaves. The four-dimensional metric may be expressed in the Arnowitt-Deser-Misner (ADM) [28] variables,

$$(N, N^i, {}^{(3)}g_{ij}), \qquad (2)$$

where $N, N^i, {}^{(3)}g_{ij}$ denote, respectively, the lapse function, shift vector, and three-dimensional induced metric on the leaves. The theory is constructed from the following quantities:

$$ {}^{(3)}g_{ij}, \qquad K_{ij}, \qquad a_i, \qquad {}^{(3)}\nabla_i, \qquad (3)$$

where ${}^{(3)}R_{ij}$ is the three-dimensional Ricci curvature tensor, ${}^{(3)}\nabla_i$ is the covariant derivative constructed from the three-dimensional metric ${}^{(3)}g_{ij}$, and $a_i := \frac{N_{,i}}{N}$. Extrinsic curvature K_{ij} is the only object, invariant under general spatial diffeomorphisms, containing exactly one time derivative of the metric tensor ${}^{(3)}g_{ij}$,

$$K_{ij} = \frac{1}{2N} \left(\frac{\partial {}^{(3)}g_{ij}}{\partial t} - {}^{(3)}\nabla_i N_j - {}^{(3)}\nabla_j N_i \right). \qquad (4)$$

Quantities (2) are tensor/vectors with respect to $\text{Diff}(M, \mathcal{F})$ possessing the following mass dimensions:

$$[{}^{(3)}R_{ij}] = 2, \qquad [K_{ij}] = 3, \qquad [a_i] = 1, \qquad [{}^{(3)}\nabla_i] = 1. \quad (5)$$

One may use (2) to construct, order by order, scalar terms appearing in the Lagrangian of the theory. Following [16,29], the action of the Hořava gravity takes the form

$$S_g = \zeta^2 \int dt dx^3 N \sqrt{{}^{(3)}g}(\mathcal{K} - V), \qquad (6)$$

where $\mathcal{K} = K_{ij}K^{ij} - \lambda K^2$ with $K = K_{ij}{}^{(3)}g^{ij}$, ${}^{(3)}g$ denotes the determinant of the three-dimensional metric, and $\zeta^2 = 1/16\pi G$. It may be expressed as the difference of the kinetic and potential part $\mathcal{L} = \mathcal{K} - V$ with $\mathcal{K} = (K_{ij}K^{ij} - \lambda K^2)$. At the sixth order, the potential part of the Lagrangian contains over 100 terms [16]. The immense number of invariants is limited by imposing further symmetries. One possible restriction for the potential comes from the projectability condition $N = N(t)$, then terms proportional to $a_i \equiv 0$ vanish. Up to the sixth order (compatible with power counting renormalizability), the potential V restricted by the projectability condition is given by

$$V = 2\Lambda\zeta^2 - {}^{(3)}R + \frac{1}{\zeta^2}(g_2 {}^{(3)}R^2 + g_3 {}^{(3)}R^{ij(3)}R_{ij})$$

$$+ \frac{1}{\zeta^4}(g_4 {}^{(3)}R^3 + g_5 {}^{(3)}R^{(3)}R^{ij(3)}R_{ij} + g_6 {}^{(3)}R^i_j {}^{(3)}R^j_k {}^{(3)}R^k_i)$$

$$+ \frac{1}{\zeta^4}(g_7 {}^{(3)}R\nabla^{2(3)}R + g_8(\nabla_i {}^{(3)}R_{jk})(\nabla^{i(3)}R^{jk})), \qquad (7)$$

where Λ is the cosmological constant, and α_{ij} are the coupling constants. For our purposes, we drop terms containing covariant derivatives ${}^{(3)}\nabla_i$. One should also mention that this "minimal theory" [30] suffers from the existence of spin 0 graviton, which is unstable in the IR, see [31–34]. Various solutions to this problem have been proposed. One can add the additional local $U(1)$ symmetry [16,35]. Then by the introduction of new fields prevents the zero-mode from propagating. On the other hand, one can

drop the projectability condition $a_i = 0$ and include the terms containing a_i in the potential term,

$$V = 2\Lambda\zeta^2 - {}^{(3)}R - \beta_0 a_i a^i + \sum_{n=3}^{6} \mathcal{L}_V^{(n)}, \qquad (8)$$

then for the spin-0 mode to be stable one requires $0 < \beta_0 < 2$ [36,37].

Note that we shall not be interested in the IR divergences stemming from boundary terms in the action and hence the famous Gibbons-Hawking-York (GHY) term in context of black holes. Those divergences have nothing to do with resolution of singularities, which is an UV issue, see also discussion in [1]. Nevertheless, it might be useful for the reader to comment on that issue. The variational principle with Dirichlet boundary condition requires variation of the action to be zero, when we fix the boundary metric [38]. However, this is not the case for the Einstein-Hilbert action, and hence the famous GHY term [39,40] has to be added, which is crucial for the finiteness of the action. On the other hand, for HL gravity the action possesses higher spatial derivatives. The variational principle requires $\delta\gamma_{ij}$, along with its derivatives, to be zero at the spatial boundary, hence the variation is well defined without the boundary term [41]. We also comment further about that issue in the context of cosmological spacetimes in the Sec. III. Furthermore, absence of the boundary term has been recently proven for the mimetic Hořava gravity, see [42,43].

III. FLATNESS, ANISOTROPIES, AND INHOMOGENEITIES IN THE EARLY UNIVERSE

A. Flatness

We begin our investigation of the early universe flatness by considering the FLRW metric given by the formula [44]

$$N \to N(t), \qquad N_i \to 0, \qquad {}^{(3)}g_{ij} \to a^2(t)\gamma_{ij}, \quad (9)$$

where γ_{ij} is a maximally symmetric constant curvature metric, with $k = +1$ for the metric on the sphere, $k = 0$ for flat spacetime, and $k = -1$ for the hyperbolic metric. We have

$${}^{(3)}R_{ij} = 2k\gamma_{ij}, \quad {}^{(3)}R = \frac{6k}{a(t)^2}, \quad \mathcal{K} = 3(1-3\lambda)\left(\frac{\dot{a}}{a}\right)^2, \quad (10)$$

and $N\sqrt{{}^{(3)}g} = Na^3(t)$. For $a(t) = t^s$ the kinetic part of the action gives us

$$N\sqrt{{}^{(3)}g}\mathcal{K} \sim t^{3s-2} \qquad (11)$$

since $N\sqrt{{}^{(3)}g}\mathcal{K} \sim t^{-1}$ leads to a logarithmic divergence at $t \to 0$, after integrating over time, we impose that the

exponent of t in the integrand should be greater than -1. Hence, for the action to remain finite as $t \to 0$, one requires $s > 1/3$. In the potential part we have exemplary terms

$$N\sqrt{{}^{(3)}g}\,{}^{(3)}R \sim kt^s, \qquad (12)$$

$$N\sqrt{{}^{(3)}g}\,{}^{(3)}R^2 \sim k^2 t^{-s}, \qquad (13)$$

$$N\sqrt{{}^{(3)}g}\,{}^{(3)}R^3 \sim k^3 t^{-3s}. \qquad (14)$$

For $k \neq 0$ equations, (11), (12), (13), (14) give rise to the following set of contradicting inequalities:

$$s > 1/3, \qquad s > -1, \qquad s < 1, \qquad s < 1/3, \quad (15)$$

this shows that for $k \neq 0$ in there is no FLRW-like the beginning of the Universe for the projectable action with potential given by (7). This means that in this version of HL gravity, the big bang with power-law time dependence of the scale factor cannot be realized (similar behavior has been observed in [4] for the LI gravity). Rejecting the cubic R^3 terms from the potential, responsible for the contradictory inequalities, yields the action to be finite. Below, we also show (on the example of Bianchi IX metric) that none of the anisotropic nonflat solutions are allowed in the action with terms cubic in Ricci curvature.

B. Anisotropies

We consider Bianchi IX metric as a representative model of nonflat anisotropic spacetimes (in this paragraph $k = 1$),

$$ds_{IX}^2 = -N^2 dt^2 + h_{ij}\omega^i\omega^j, \qquad (16)$$

where $h_{ij} = \text{diag}(M^2, Q^2, R^2)$ and M, Q, R are functions of the time only. The connection is

$$d\omega^a = \Gamma_c^a \wedge \omega^c = \Gamma_{cb}^a \omega^b \wedge \omega^c. \qquad (17)$$

The Bianchi IX forms satisfy

$$d\omega^a = \frac{1}{2}\epsilon^{abc}\omega^b \wedge \omega^c. \qquad (18)$$

Hence, $\Gamma_{bc}^a = -\frac{1}{2}\epsilon^{abc}$. The usual closed FRLW universe is obtained when $R(t) = M(t) = Q(t) = \frac{a(t)}{2}$, where $a(t)$ is the scale factor. The explicit form of the curvature invariants was calculated in [29],

$${}^{(3)}R = \frac{-1}{2M^2Q^2R^2}(M^4 + Q^4 + R^4 - (R^2 - Q^2)^2 - (R^2 - M^2)^2 - (M^2 - Q^2)^2), \qquad (19)$$

$$^{(3)}R_j^{i\,(3)}R_i^j = \frac{1}{4(MQR)^4}[3M^8 - 4M^6(Q^2 + R^2)$$
$$- 4M^2(Q^2 - R^2)^2(Q^2 + R^2)$$
$$+ 2M^4(Q^2 + R^2)^2 + (Q^2 - R^2)^2(3Q^4)$$
$$+ (Q^2 - R^2)^2(2Q^2R^2 + 3R^4)], \tag{20}$$

$$^{(3)}R_j^{i\,(3)}R_k^{j\,(3)}R_i^k = \frac{1}{8(MQR)^6}([[(M^2 - Q^2)^2 - R^4]^3$$
$$+ [(M^2 - R^2)^2 - Q^4]^3$$
$$+ [(Q^2 - R^2)^2 - M^4]^3). \tag{21}$$

The kinetic and the potential part are, respectively,

$$N\sqrt{^{(3)}g}\mathcal{K} = \frac{MQR}{N}\left[(1-\lambda)\left(\frac{\dot{M}^2}{M^2} + \frac{\dot{Q}^2}{Q^2} + \frac{\dot{R}^2}{R^2}\right)\right.$$
$$\left. - 2\lambda\left(\frac{\dot{M}\dot{Q}}{MQ} + \frac{\dot{Q}\dot{R}}{QR} + \frac{\dot{M}\dot{R}}{MR}\right)\right], \tag{22}$$

$$N\sqrt{^{(3)}g}V = -N(MQR)V. \tag{23}$$

For the Bianchi IX metric, we use the following ansatz:

$$M(t) \sim t^m, \qquad Q(t) \sim t^q, \qquad R(t) \sim t^r. \tag{24}$$

With such solutions, the kinetic term is proportional to

$$N\sqrt{^{(3)}g}\mathcal{K} \sim t^{m+q+r-2}. \tag{25}$$

This results in an inequality,

$$m + q + r > 1. \tag{26}$$

Similar reasoning is applied to all of the curvature scalars in the potential. Ricci scalar terms lead to conditions,

$$3m - q - r > -1, \quad 3q - m - r > -1, \quad 3r - m - q > -1, \tag{27}$$

$$r + q - m > 1, \quad r + m - q > -1, \quad m + q - r > -1. \tag{28}$$

Quadratic terms are numerous, and we provide explicit conditions only for the $R_{ij}R^{ij}$ terms,

$$5m - 3q - 3r > -1, \quad 3m - q - 3r > -1, \quad 3m - 3q - r > -1,$$
$$3r - m - 3q > -1, \quad r - m - q > -1, \quad q - m - r > -1, \tag{29}$$

$$3q - m - 3r > -1, \quad m + q - 3r > -1, \quad m - q - r > -1, \tag{30}$$

$$m + r - 3q > -1, \quad 5q - 3m - 3r > -1, \quad 3q - 3 - r > -1, \tag{31}$$

$$q + r - 3m > -1, \quad 3r - q - 3m > -1, \quad -3m - 3q + 5r > -1. \tag{32}$$

It is tedious to algebraically verify that the above set of conditions is not contradictory. A geometrical interpretation brings more light to the problem: each of the inequalities corresponds to half of the $\mathbb{R}^3$ space in the (q, m, r) coordinates. The common subspace restricted by a pair of such inequalities vanishes if the planes corresponding to the boundary of the half-spaces are parallel. This is easily verified by considering the vector normal to each plane. For example, the boundary plane obtained from inequality $m + q + r > 1$ is $m + q + r = 1$ and the normal vector $(1,1,1)$. If two such normal vectors are parallel (bearing in mind the correct inequality direction), then the half-spaces will be separate. The kinetic part and scalars up to quadratic order in curvature do not lead to contradictory conditions. However, including the R^3 term we have

$$N\sqrt{^{(3)}g}^{(3)}R^3 \ni \frac{N}{MQR} \sim t^{-m-q-r} \Rightarrow m + q + r < 1. \tag{33}$$

This is in clear contradiction with (26). This means that also Bianchi IX anisotropic spacetime leads to infinite action. Notice that taking the isotropic limit $m = q = r$ also leads to infinite action, as discussed in the previous paragraph. There are two ways of dealing with this—leaving the model unchanged and considering other cosmological solutions, such as oscillating universe with bounded $a(t) \in (a_{\min}, a_{\max})$, see, for example, [45]. On the other hand, this might be understood as an indication that the flatness problem is resolved via the finite action principle without the need of inflation. In particular, for flat anisotropic Bianchi I spacetime, all of the spatial invariants vanish, leaving us with the kinetic term condition

$$m + q + r > 1. \tag{34}$$

Therefore, the action is finite for both isotropic and anisotropic flat beginning of the Universe in contradistinction to the Lorentz covariant R^2 off-shell action [5], yet during the evolution of the Universe, the anisotropies might vanish dynamically [4] if spacetime is accelerating. Furthermore, this is generically true for $\Lambda > 0$, see the dynamical systems studies of the matter [46]. In our analysis we have neglected the boundary terms in the action. On nonflat FLRW spacetime they could lead only to further contradictory conditions. On the other hand, for $k = 0$ FLRW solution HL gravity those terms will play no role, as it was investigated in [4].

C. Inhomogeneities

Unlike the anisotropies, the finiteness of the action suppresses the inhomogeneities already at the second order of the spatial Ricci scalar curvature. Investigation of the inhomogeneities concerns following isotropic metric tensor,

$$ds^2 = -dt^2 + \frac{A'^2}{F^2}dr^2 + A^2(d\theta^2 + \sin^2\theta d\phi^2), \quad (35)$$

where $A = A(t, r)$, $F = F(r)$, and $A' = \partial_r A$. The homogeneous FRLW metric is retrieved when $F \to 1$. The resulting Ricci scalar and Ricci scalar squared contribution to the action are

$$\sqrt{^{(3)}g}\,^{(3)}R \sim 2AF' + \frac{A'(-1+F)^2}{F}, \quad (36)$$

$$\sqrt{^{(3)}g}\,^{(3)}R^2 \sim \frac{2AFF' + A'(F^2-1)}{A^2A'F}. \quad (37)$$

Again, we suppose that each term should be convergent as $t \to 0$. By the ansatz $A(t) \sim t^s$, inequalities stemming from $^{(3)}R$ and $^{(3)}R^2$ are contradictory. This means that $F(r) \to 1$, hence the metric of the early universe was homogeneous.

IV. BLACK HOLES AND WORMHOLES

In this section, we show that HL gravity satisfies the finite action selection principle for the microscopic action of quantum gravity [1]. We study both the solutions of HL gravity and the known off-shell BH spacetimes, due to the fact that there are no known regular BH solutions in the HL gravity. Keep in mind that a metric does not need to be a solution to the equations of motion to enter the path integral. We require singular black-hole metrics to interfere destructively, while the regular ones with finite action contribute to the probability amplitudes. We broaden this analysis by studying the wormhole solutions.

A. Singular black holes

Singularities may be categorized [47] in the three main groups: "scalar," "nonscalar," and "coordinate singularities." Scalar singularities are the ones for which (some of) the curvature invariants, like the Kretschmann scalar, become divergent, and hence, they are the object of interest in our considerations. Nonscalar singularities appear in physical quantities, such as the tidal forces. Finally, the coordinate singularities appear in the metric tensor, however, one may get rid of the divergence with a proper coordinate transformation. Yet, coordinate singularities of general relativity (GR) may become scalar singularities in the Hořava-Lifshitz gravity [48]. It is due to the fact that the spacetime diffeomorphism of GR is a broader symmetry

than the foliation-preserving diffeomorphism of HL gravity. As an example, consider the Schwarzschild metric,

$$ds^2 = -\left(1 - \frac{2m}{r}\right)dt^2 + \left(1 - \frac{2m}{r}\right)^{-1}dr^2 + r^2 d\Omega^2, \quad (38)$$

where $d\Omega^2 = r^2(d\theta^2 + \sin^2\theta d\phi^2)$. The singular points are $r = 0$ and $r = r_s = 2m$. In GR the singular point $r_s = 2m$ may be removed by the transformation

$$dt_{\text{PG}} = dt + \frac{\sqrt{2mr}}{r - 2m}dr. \quad (39)$$

The resulting Painleve-Gullstrand metric is

$$ds^2 = -dt_{\text{PG}}^2 + \left(dr - \sqrt{\frac{2m}{r}}dt_{\text{PG}}\right)^2 + r^2 d\Omega^2. \quad (40)$$

For more details see, e.g., [49]. In GR, metric tensors (38) and (40) describe the same spacetime with singularity at $r = 0$. Notice, however, that the coordinate transformation (39) does not preserve the spacetime foliation, breaking the projectability condition. Hence, in the framework of HL gravity, metric tensors (38) and (40) describe distinct spacetimes. Moreover, as we will show, Schwarzschild's metric singularity at $r = r_s$ becomes a spacetime singularity. Hence, due to the unique nature of the foliation-preserving diffeomorphism, investigating the singularities in HL gravity is a delicate matter.

We consider three representative solutions [48]: (anti-) de Sitter-Schwarzschild, which is the simplest spacetime with the black hole and the cosmological horizon, Kerr spacetime (see also the rotating HL solution [50]), and the HL solution found by Lu, Mei, and Pope (LMP) [51]. In this section, we discuss the (anti-) de-Sitter-Schwarzschild as the clearest example. The other metrics have similar features, and we explore them in the Appendix. In the following, all of the curvature scalars are three dimensional, unless stated otherwise.

1. (Anti-) de Sitter-Schwarzschild solution

The general static ADM metric with projectability condition takes the form

$$ds^2 = -dt^2 + e^{2\nu}(dr + e^{\mu-\nu}dt)^2 + r^2 d\Omega^2, \quad (41)$$

where $\mu = \mu(r)$, $\nu = \nu(r)$. (Anti-) de Sitter-Schwarzschild solutions are obtained for $\mu = \frac{1}{2}\ln(\frac{M}{r} + \frac{\Lambda}{3}r^2)$, $\nu = 0$. The resulting kinetic terms and Ricci scalar are

$$^{(3)}R = 0,$$

$$K = \left(\frac{3M + \Lambda r^3}{12r^3}\right)^{\frac{1}{2}}\left(\frac{4}{r} - \frac{3M - 2\Lambda r^3}{3M + \Lambda r^3}\right),$$

$$K_{ij}K^{ij} = \frac{3M + \Lambda r^3}{12r^3}\left[8 + \left(\frac{3M - 2\Lambda r^3}{3M + \Lambda r^3}\right)^2\right]. \quad (42)$$

The kinetic part is divergent at $r = 0$ and $r = \left(\frac{3M}{|\Lambda|}\right)^{\frac{1}{3}}$ for the negative cosmological constant. We investigate the finiteness of the function

$$S_s(r_{\text{UV}}, r_{\text{IR}}) := \int_{r_{\text{UV}}}^{r_{\text{IR}}} dr N\sqrt{g}(K_{ij}K^{ij} - \lambda K^2 + {}^{(3)}R), \quad (43)$$

which is a part of the action qualitatively describing the singularities. The r_{IR} is chosen so that the volume integral is finite, hence, we do not consider singularities stemming from the IR behavior (large distances) of the spacetime boundary and time integration. In particular, the r_{UV} is the minimal radius, in which we take $r_{\text{UV}} \to 0$. For the scalars (42) and value of $\lambda \neq 1$, the function $S_s(r_{\text{UV}}, r_{\text{IR}})$ is divergent at the expected points $r_s = r_{\text{UV}} = 0$ and $r_s = \left(\frac{3M}{|\Lambda|}\right)^{\frac{1}{3}}$. However, for $\lambda = 1$, which is the value required for low energy Einstein-Hilbert approximation, the terms divergent at $r_s = \left(\frac{3M}{|\Lambda|}\right)^{\frac{1}{3}}$ remain finite, as one could expect, since r_s corresponds to the cosmological horizon. Explicitly we have

$$S_s(r_{\text{UV}}, r_{\text{IR}}) = \frac{2}{9}\Lambda r_{\text{UV}}^3 - 8\Lambda r_{\text{UV}}^2 + \left(\frac{M}{4} - 16\Lambda\right)\ln r_{\text{UV}}$$
$$- \frac{24M}{r_{\text{UV}}} + \frac{24}{r_{\text{UV}}^2} + \text{IR terms.} \quad (44)$$

Here, only the spatial Ricci scalar is necessary for the singular solution to be suppressed in the gravitational path integral.

As mentioned previously, different gauges of the same spacetime in GR correspond to distinct spacetimes in HL gravity. Hence, we consider the (anti-) de Sitter-Schwarzschild metric in the orthogonal gauge, which is not a solution to the projectable HL theory, in contrast to the previous case,

$$ds^2 = -e^{2\Psi(r)}d\tau^2 + e^{2\Phi(r)}dr^2 + r^2 d\Omega^2, \quad (45)$$

here,

$$\Psi(r) = -\Phi(r) = \frac{1}{2}\ln\left(1 - \frac{2M}{r} + \frac{1}{3}\Lambda r^2\right). \quad (46)$$

In the orthogonal gauge, the components of the metric tensor do not depend on the time coordinate, hence the kinetic part vanishes $K_{ij} = 0$. One finds, that the Ricci scalar is constant $^{(3)}R = -2\Lambda$. However, the higher-order curvature terms (see Appendix for the general form) are divergent at the origin,

$$^{(3)}R_{ij}{}^{(3)}R^{ij} = \frac{4\Lambda^2}{3} + \frac{6M^2}{r^6},$$

$$^{(3)}R_j^i{}^{(3)}R_k^j{}^{(3)}R_i^k = -\frac{8\Lambda^3}{9} - \frac{12\Lambda M^2}{r^6} - \frac{6M^3}{r^9}, \quad (47)$$

yielding an infinite action and suppressing the singularity. The same conclusions can be drawn for Kerr spacetime and singular Lu-Mei-Pope metric, derived in the context of Hořava gravity, see Appendix.

2. Regular black holes

Due to observations of the binary black holes mergers [52] and the *Event Horizon* Telescope observations [53,54], the structure of BHs can be investigated on an unprecedented scale [55]. Furthermore, due to the expectation that the quantum gravity shall resolve the BH singularity issue, the regular black holes have been of interest recently, for discussions in various quantum gravity approaches [56–63] (see for more model independent viewpoints [64–67]). Following [1], we shall discuss the Hayward metric [65] (Dymnikova spacetime [64] is discussed in the Appendix). The Hayward metric is an example of the regular black hole solution in GR,

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega^2,$$

$$f(r) = 1 - \frac{2Mr^2}{(r^3 + 2g^3)}, \quad (48)$$

where g is an arbitrary positive parameter. The metric is nonsingular in $r \to 0$. It is not a solution to HL theory, however, we consider it as an off-shell metric present in the path integral.

The kinetic tensor vanishes $K_{ij} = 0$, while the Ricci scalar and the second-order curvature scalars are regular,

$$^{(3)}R = \frac{24g^3 GM}{(2g^3 + r^3)^2},$$

$$^{(3)}R_{ij}{}^{(3)}R^{ij} = \frac{6M^2(32g^6 + r^6)}{(2g^3 + r^3)^4}, \quad (49)$$

leading to finite action. A similar conclusion arises in the case of Dymnikova spacetime, see Appendix. These two regular solutions to GR are also regular in the off-shell HL theory.

B. Wormholes

Here, we take the first step in the direction of the investigations of the consequences of the finite action principle in the context of wormholes (WHs). The wormholes may be characterized in two classes: traversable and

nontraversable. The traversable WH, colloquially speaking, are such that one can go through it to the other side, see [68] for specific conditions. The pioneering Einstein-Rosen bridge has been found originally as a nonstatic nontraversable solution to GR. The traversable solutions are unstable, however, they might be stabilized by an exotic matter or inclusion of the higher-curvature scalar gravity [24]. This is important in the context of finite action since usually the divergences of black holes do appear in the curvature squared terms. Hence, due to the inclusion of the higher-order terms in the actions, the traversable wormholes are solutions to the equations of motions without the exotic matter. The exemplary wormhole spacetimes investigated here are the Einstein-Rosen bridge proposed in [69], the Morris-Thorne (MT) wormhole [68], the traversable exponential metric wormhole [70], and the wormhole solution discussed in the HL gravity [71]. All of them have a finite action. Here, we shall discuss the exponential metric WH. The conclusions for the other possible wormholes are similar, and we discuss them in the Appendix. For the exponential metric WH, the line element is given by

$$ds^2 = -e^{-\frac{2M}{r}}dt^2 + e^{\frac{2M}{r}}(dr^2 + r^2 d\Omega^2). \quad (50)$$

This spacetime consists of two regions: "our Universe" with $r > M$ and the "other universe" with $r < M$. Note that $r = M$ corresponds to the wormhole's throat. The spacial volume of the other universe is infinite when $r \to 0$. Such volume divergence is irrelevant to our discussion since it describes large distances in the other universe. Hence, we further consider only $r \geq M$. The resulting Ricci and Kretschmann scalars calculated in [70] and the measure are nonsingular everywhere,

$$R = -\frac{2M^2}{r^4} e^{\frac{-2M}{r}},$$

$$R_{\mu\nu\sigma\rho}R^{\mu\nu\sigma\rho} = \frac{4M^2(12r^2 - 16Mr + 7M^2)}{r^8} e^{-\frac{4M}{r}}, \quad (51)$$

resulting in the finite action for the Stelle gravity. Similarly for the HL gravity,

$$^{(3)}R = R, \qquad K^2 = K_{ij}K^{ij} = 0,$$

$$^{(3)}R_{ij}{}^{(3)}R^{ij} = \frac{2M^2(M^2 - 2Mr + 3r^2)}{r^8} e^{-\frac{4M}{r}}. \quad (52)$$

V. CONCLUSIONS AND DISCUSSION

The finite action principle is a powerful tool to study quantum gravity theories and also quantum field theories, in general. In particular, we have shown that it can be invoked to explain the flatness and homogeneity of the early universe and can possibly be a dynamic resolution of

the singularities problem of black holes in the context of Hořava-Lifshitz gravity.

The conditions stemming from the finite action principle justify the topological phase hypothesis without the need for conversion of the degrees of freedom in the early universe, which is assumed to take place in [22]. Furthermore, the anisotropic scaling of Hořava gravity admits only flat solutions for the cosmological metrics, see also the discussion on the instanton "no-boundary"-like solution [72]. Moreover, the amplitude of the cosmological perturbations are scaling as $\delta\Phi = H^{\frac{3-z}{2z}}$, hence, at $z = 3$ they are almost scale invariant [73]. Finally, the Weyl anomalies structure in HL gravity does not lead to strong nonlocal effects during the radiation domination epoch [74,75]. This stems from the fact that these anomalies are of second order in derivatives in the flat spacetimes [76–79], hence, they are harmless and allow to avoid the vanishing of conformal anomaly criteria [80,81]. In particular, it would be interesting to see whether the anisotropic Weyl anomalies can also give departure from scale invariance, as it is discussed in [22]. Yet we leave that for further investigation to be performed elsewhere. Combined with earlier results, our investigation backs up fully the topological phase conjecture hence making inflation redundant. Furthermore, it seems that this is in line with the swampland conjectures and the newly proposed finite-amplitude principle [4], making the asymptotically safe quantum gravity to pick initial conditions such that inflation ceases to be eternal [82], see also [83,84].

From the point of view of the finite action selection principle [1], they are equally good theories, resolving the black holes singularities, assuming that the ghost issue is resolved in the latter case. Yet none of the regular BH solutions have been found in the context of HL gravity [85]. Hence, it is a strong suggestion that the wormholes may appear in the UV regime of HL gravity and can serve as a "cure" for singularities [86–88].

In the case of wormholes, both traversable and nontraversable wormholes are on equal footing in the case of the finite action principle. However, this principle suggests that there is a trade-off between the resolution of black-hole singularities and the appearance of wormhole spacetimes due to higher-curvature invariants. The wormhole solutions will remain in both the LI and HL path integrals. The higher-order curvature scalars, generically present in the quantum gravity, stabilize the wormhole solutions without the need for an exotic matter.

Finally, one should mention that there are many experiments to test the Lorentz Invariance Violations (LIV) in the gravitational sector coming from gravitational waves observations [89–94], which could, in principle, validate Hořava's proposal, yet we know much more about the LIV in the matter sector (see for example [95,96]). Since these two can be related [97], one can speculate that HL gravity can be tested in the nearby future.

APPENDIX A: FURTHER BLACK HOLES AND WORMHOLES

Here, we present further examples of interesting black-hole and wormhole spacetimes in the context of the finite action principle. We find the restrictions on the LMP solutions necessary to resolve the singularity at the origin. Similarly, the spatial Ricci scalar of Kerr's spacetime yields infinite action. We further give three examples of wormholes with finite action: the Einstein-Rosen bridge, the Morris-Thorne wormhole, and a spatially symmetric and traversable wormhole solution to HL gravity.

1. Black holes

a. LPM black hole

The popular LMP [51] metric is not a solution to the vacuum HL equations. However, the second class of the LMP solutions written in the ADM frame with the project-ability condition satisfy the field equations of HL gravity coupled to anisotropic fluid with heat flow, see [48]. The LMP solutions were found in the orthogonal gauge (45) without the projectability. There are two types of solutions. Class A solutions are

$$\Phi = -\frac{1}{2}\ln(1+x^2), \qquad \Psi = \Psi(r). \qquad (A1)$$

Class B solutions consist of

$$\Phi = -\frac{1}{2}\ln\left(1+x^2-\alpha x^{\alpha_\pm}\right),$$

$$\Psi = -\beta_\pm \ln x + \frac{1}{2}\ln\left(1+x^2-\alpha x^{\alpha_\pm}\right), \qquad (A2)$$

where $x = \sqrt{|\Lambda_W|}r$, $\Lambda = \frac{3}{2}\Lambda_W$, α is an arbitrary real constant, and $\alpha_\pm$ and $\beta_\pm = 2\alpha_\pm - 1$ are parameters depending on λ. Their explicit form may be found in [48]. The LPM solutions, (A1) and (A2), have vanishing kinetic tensor $K_{ij} = 0$, while the Ricci scalar and the integral measure are given, respectively, by

$$^{(3)}R = \frac{2}{r^2}(\alpha(1+\alpha_\pm)x^{\alpha_\pm} - 3x^2), \qquad N\sqrt{g} = r^2 x^{-\beta_\pm}. \qquad (A3)$$

The S_s function (43) stands,

$$S_s(x_{\mathrm{UV}}, x_{\mathrm{IR}})$$
$$= -\frac{2}{\sqrt{\Lambda_W}}\int_{x_{\mathrm{UV}}/\sqrt{\Lambda_W}}^{x_{\mathrm{IR}}/\sqrt{\Lambda_W}} dx(\alpha(1+\alpha_\pm)x^{1-\alpha_\pm} - 3x^{3-2\alpha_\pm}), \qquad (A4)$$

where $x_{\mathrm{UV}} = \sqrt{\Lambda_W}r_{\mathrm{UV}}$ and $x_{\mathrm{IR}} = \sqrt{\Lambda_W}r_{\mathrm{IR}}$. The necessary condition for the spatial Ricci scalar to be finite is $2 > \alpha_\pm$. We proceed in the ADM gauge, which describes an independent theory in the HL gravity. Then, the class A solution is given by

$$\mu = -\infty, \qquad \nu = -\frac{1}{2}\ln\left(1-\Lambda_W r^2\right), \qquad (A5)$$

applied to (41), we get $^{(3)}R = 6\Lambda_W$. The S_s function is given by

$$S_s(r_{\mathrm{UV}}, r_{\mathrm{IR}}) = -\int_{r_{\mathrm{UV}}}^{r_{\mathrm{IR}}} \frac{6\Lambda_W r^2}{\sqrt{|1-\Lambda_W r^2|}}. \qquad (A6)$$

The exact form of $S_s(r_{\mathrm{UV}}, r_{\mathrm{IR}})$ depends on the sign of the scaled cosmological constant Λ_W, nevertheless, it is always finite when $r_{\mathrm{UV}} \to 0$. Indeed, for the negative $\Lambda_W < 0$,

$$S_s(r_{\mathrm{UV}}, r_{\mathrm{IR}}) = -\frac{3}{\sqrt{-\Lambda_W}}\mathrm{arcsinh}(\sqrt{-\Lambda_W}r_{\mathrm{UV}})$$
$$- 3r_{\mathrm{UV}}\sqrt{-\Lambda_W r_{\mathrm{UV}}^2 + 1}. \qquad (A7)$$

The positive cosmological constant splits the space in two regions. When $r > \frac{1}{\sqrt{\Lambda_W}}$, we get

$$S_s(r_{\mathrm{UV}}, r_{\mathrm{IR}}) = \frac{3}{\sqrt{\Lambda_W}}\mathrm{arctanh}\left(\frac{\sqrt{\Lambda_W}r_{\mathrm{UV}}}{\sqrt{\Lambda_W r_{\mathrm{UV}}^2 - 1}}\right)$$
$$+ 3r_{\mathrm{UV}}\sqrt{\Lambda_W r_{\mathrm{UV}}^2 - 1} \qquad (A8)$$

for a small positive cosmological constant; the above result is irrelevant for our discussion since it would describe large scales. When $r < \frac{1}{\sqrt{\Lambda_W}}$,

$$S_s(r_{\mathrm{UV}}, r_{\mathrm{IR}}) = -\frac{3}{\sqrt{\Lambda_W}}\arcsin\left(\sqrt{\Lambda_W}r_{\mathrm{UV}}\right)$$
$$+ 3r_{\mathrm{UV}}\sqrt{\Lambda_W r_{\mathrm{UV}}^2 - 1}. \qquad (A9)$$

The class B solution singularity at the origin, appearing when $2 \leq \alpha_+$, is suppressed by the finite action principle. Class A solutions are finite and contribute to the path integral, if the cosmological constant is negative or small and positive when $r_{\mathrm{UV}} \to \frac{1}{\sqrt{\Lambda_W}}$.

b. Kerr spacetime

Kerr spacetime corresponds to an axially symmetric rotating black hole with mass M and angular momentum J. It is a solution to the Einstein equations in GR, however, it has been shown order-by-order in the parameter $a = J/M$ that it is not a solution to the HL field equations [98]. Yet it can still enter the path integral as an off-shell metric. The line element in the Boyer-Lindquist coordinates is given by

$$ds^2 = -\frac{\rho^2 \Delta_r}{\Sigma^2} dt^2 + \frac{\rho^2}{\Delta_r} dr^2 + \rho^2 d\theta^2$$
$$+ \frac{\Sigma^2 \sin^2 \theta}{\rho^2} (d\phi - \xi dt^2)^2, \qquad (A10)$$

where

$$\rho^2 = r^2 + a^2 \cos^2 \theta,$$
$$\Delta_r = r^2 + a^2 - 2Mr,$$
$$\Sigma^2 = (r^2 + a^2)^2 - 2Mr,$$
$$\xi = \frac{2Mar}{\Sigma^2}. \qquad (A11)$$

We are interested in the singularity on equator plane $\cos\theta = 0$, $r = 0$, described in detail in [6]. For the explicit form of the extrinsic curvature scalars and Ricci scalar refer to [98]. Here, we only show the form of the Ricci scalar on the $\cos\theta = 0$ plane,

$$^{(3)}R = -\frac{2a^2 m^2 (a^2 + 3r^2)^2}{r^4 (r^3 + a^2 (2M + r))^2}. \qquad (A12)$$

It is singular at $r = 0$. Integrating $^{(3)}R$ with the measure $N\sqrt{g} = r^2$ results in the infinite action in the UV limit, and the Kerr spacetime does not contribute to the path integral. The vanishing four-dimensional Ricci scalar is restored in the LI limit $\lambda = 1$. It is then necessary to include the

Kretschmann scalar to resolve the singularity, as discussed in [1].

c. Dymnikova spacetime

The Dymnikova spacetime is a regular solution in GR. It is constructed with the line element (48) with

$$f(r) = 1 - \frac{2M(r)}{r}, \qquad M(r) = M(1 - e^{-\frac{r^3}{2g^3}}). \qquad (A13)$$

The corresponding curvature scalars are nonsingular,

$$^{(3)}R = \frac{6M}{g^3} e^{-\frac{r^3}{2g^3}},$$
$$^{(3)}R_{ij}{}^{(3)}R^{ij} = \frac{3M^2}{2g^6 r^6} e^{-\frac{r^3}{g^3}} (4g^6 (e^{\frac{r^3}{2g^3}} - 1)^2$$
$$- 4g^3 r^3 (e^{\frac{r^3}{2g^3}} - 1) + 9r^6), \qquad (A14)$$

and the action is finite in the limit $r_{\text{UV}} \to 0$. In particular, in this limit we have $^{(3)}R_{ij}{}^{(3)}R^{ij} \to 12M^2/g^6$.

d. Higher-order curvature scalars

Here, we give a general expression for the higher-order scalars present in the HL potential for the projectable ADM and orthogonal gauge metric tensors. The metric tensor in the projectable ADM gauge (41) yields

$$^{(3)}R_{ij}{}^{(3)}R^{ij} = \frac{2e^{-4\nu(r)} (2r^2\nu'(r)^2 + (r\nu'(r) + e^{2\nu(r)} - 1)^2)}{r^4},$$
$$^{(3)}R^i_j{}^{(3)}R^j_k{}^{(3)}R^k_i = \frac{2e^{-6\nu(r)} (4r^3\nu'(r)^3 + (r\nu'(r) + e^{2\nu(r)} - 1)^3)}{r^6}. \qquad (A15)$$

In the orthonormal gauge they are given by

$$^{(3)}R = \frac{2e^{-2\Phi(r)} (2r\Phi'(r) + e^{2\Phi(r)} - 1)}{r^2},$$
$$^{(3)}R_{ij}{}^{(3)}R^{ij} = \frac{2e^{-4\Phi(r)} (2r^2\Phi'(r)^2 + (r\Phi'(r) + e^{2\Phi(r)} - 1)^2)}{r^4},$$
$$^{(3)}R^i_j{}^{(3)}R^j_k{}^{(3)}R^k_i = \frac{2e^{-6\Phi(r)} (4r^3\Phi'(r)^3 + (r\Phi'(r) + e^{2\Phi(r)} - 1)^3)}{r^6}. \qquad (A16)$$

APPENDIX B: WORMHOLES

1. Einstein-Rosen bridge

The ER bridge smoothly glues together two copies of the Schwarzschild spacetime: black-hole and the white-hole solutions corresponding to the positive and negative coordinate u. The metric tensor of the Einstein-Rosen

wormhole proposed in [69] and discussed in, e.g., [99] is given by

$$ds^2 = \frac{-u^2}{u^2 + 4M} dt^2 + (u^2 + 4M) du^2 + \frac{1}{4}(u^2 + 4M) d\Omega^2. \qquad (B1)$$

The ER bridge is nontraversable and geodesically incomplete in $u = 0$. This fact, however, does not impact the regularity of the curvature scalars. The four-dimensional Ricci scalar is

$$R = \frac{2(64M^2 + 32Mu^2 + 4u^4 + u^2)}{(4M + u^2)^3}. \quad \text{(B2)}$$

The second-order curvature scalar $R_{\mu\nu}R^{\mu\nu}$ is

$$\frac{4(48M^2 + 8(4M + u^2)^4 + (32M - 1)(4M + u^2)^2)}{(4M + u^2)^6}, \quad \text{(B3)}$$

both of which, integrated with the measure, are non-singular,

$$\sqrt{g} = \frac{1}{4}u(4M + u^2). \quad \text{(B4)}$$

The wormhole solutions analyzed in this paper generally yield the finite action in both GR and HL. The finite action principle suggests that, in the quantum UV regime, singular black-hole spacetimes may be replaced with the regular wormhole solutions.

2. Morris-Thorne wormhole

The MT wormhole is defined in the spherically symmetric Lorentzian spacetime by the line element

$$ds^2 = -e^{2\Phi(r)}dt^2 + \frac{dr^2}{1 - \frac{b(r)}{r}} + r^2 d\Omega^2, \quad \text{(B5)}$$

where $\Phi(r)$ is known as the redshift, and there are no horizons if it is finite. Function $b(r)$ determines the wormhole's shape. We choose $\Phi(r), b(r)$ to be

$$\Phi(r) = 0, \qquad b(r) = 2M(1 - e^{r_0 - r}) + r_0 e^{r_0 - r}, \quad \text{(B6)}$$

where r_0 is the radius of the throat of the wormhole, such that $b(r_0) = r_0$. Four-dimensional curvature scalars for this spacetime have been calculated in [100]. The Ricci curvature scalar is singular at $r = 0$, however, the radial coordinate r varies between $r_0 > 0$ and infinity,

$$R = -2(2M - r_0)\frac{e^{r_0 - r}}{r^2}. \quad \text{(B7)}$$

The resulting $S_s = \int_{r_{UV}=r_0}^{r_{IR}} \sqrt{g}R$ function is divergent as $r_{UV} \to r_0$ and cannot be expressed in terms of simple functions,

$$2(2M - r_0)\int_{r_{UV}=r_0}^{r_{IR}} \sqrt{\frac{r}{r - 2M(1 - e^{r_0 - r}) + r_0 e^{r_0 - r}}}e^{r_0 - r}dr. \quad \text{(B8)}$$

However, this is only a coordinate singularity and one may get rid of it with a proper transformation. Higher-order curvature scalars for Morris-Thorne wormholes are

$$^{(3)}R = \frac{2b'(r)}{r^2},$$

$$^{(3)}R_{ij}{}^{(3)}R^{ij} = \frac{3r^2 b'(r)^2 - 2rb(r)b'(r) + 3b(r)^2}{2r^6},$$

$$^{(3)}R_j^i{}^{(3)}R_k^j{}^{(3)}R_i^k = \frac{-9r^2 b(r)b'(r)^2 + 5r^3 b'(r)^3 + 15rb(r)^2 b'(r) - 3b(r)^3}{4r^9}, \quad \text{(B9)}$$

and integrated give action that is finite.

3. HL wormhole

Static spherically traversable symmetric wormholes have been constructed in [71] in the HL theory through the modification of the Rosen-Einstein spacetime,

$$ds^2 = -N^2(\rho)dt^2 + \frac{1}{f(\rho)}d\rho^2 + (r_0 + \rho^2)^2 d\Omega^2, \quad \text{(B10)}$$

with additional $\mathbf{Z}_2$ symmetry with respect to the wormhole's throat. There are solutions with $\lambda = 1$ asymptotically

corresponding to the Minkowski vacuum. Explicitly we have

$$f = N^2 = 1 + \omega(r_0 + \rho^2)^2$$
$$- \sqrt{(r_0 + \rho^2)(\omega^2(r_0 + \rho^2)^3 + 4\omega M)}. \quad \text{(B11)}$$

Radius of the wormhole's throat is given by r_0. The parameters ω and M are connected to the coupling constants in the HL action. See [71] for their explicit form. The Ricci scalar of the HL wormhole invariants are given by

$$^{(3)}R = -\frac{1}{(\rho^2+r_0)^2}[2(-10\rho^2\sqrt{\omega(\rho^2+r_0)(4M+\omega(r_0+\omega^2)^3)}$$

$$-4r_0\sqrt{\omega(\rho^2+r_0)(4M+\omega(r_0+\omega^2)^3)} + 16\rho^6\omega + 8\rho^2 + 36\rho^4 r_0\omega + 24\rho^2 r_0^2\omega + 4r_0^3\omega + 4r_0 - 1)],$$

$$^{(3)}R_{ij}{}^{(3)}R^{ij} = \frac{1}{(\rho^2+r_0)^4}\Bigg\{2(-7\rho^2\sqrt{\omega(\rho^2+r_0)(4M+\omega(r_0+\omega^2)^3)} - 2r_0\sqrt{\omega(\rho^2+r_0)(4M+\omega(r_0+\omega^2)^3)}$$

$$+ 10\rho^6\omega + 6\rho^2 + 22\rho^4 r_0\omega + 14\rho^2 r_0^2\omega + 2r_0^3\omega + 2r_0 - 1)^2 + 4M$$

$$+ \frac{4(\rho^2+r_0)}{\omega(4M+\omega(r_0+\omega^2)^3)}[\rho^2\omega(-4(\rho^2+r_0)\sqrt{\omega(\rho^2+r_0)(4M+\omega(r_0+\omega^2)^3)} + \omega(r_0+\omega^2)^3)]$$

$$-2\sqrt{\omega(\rho^2+r_0)(4M+\omega(r_0+\omega^2)^3)}\Big[-\sqrt{\omega(\rho^2+r_0)(4M+\omega(r_0+\omega^2)^3)} + \omega(\rho^2+r_0)^2 + 1)\Big]^2\Bigg\}. \quad (B12)$$

The kinetic terms with $K_{ij} = 0$ are vanishing, while the spacial Ricci scalar and higher-curvature terms are finite. From the point of view of the finite action principle, all of the investigated wormhole spacetimes are included in the gravitational path integral.

[1] J. N. Borissova and A. Eichhorn, Universe **7**, 48 (2021).
[2] L. Bosma, B. Knorr, and F. Saueressig, Phys. Rev. Lett. **123**, 101301 (2019).
[3] J. D. Barrow and F. J. Tipler, Nature (London) **331**, 31 (1988).
[4] C. Jonas, J.-L. Lehners, and J. Quintin, Phys. Rev. D **103**, 103525 (2021).
[5] J.-L. Lehners and K. S. Stelle, Phys. Rev. D **100**, 083540 (2019).
[6] J. D. Barrow, Phys. Rev. D **101**, 023527 (2020).
[7] K. Stelle, Phys. Rev. D **16**, 953 (1977).
[8] R. Penrose, Ann. N.Y. Acad. Sci. **571**, 249 (1989).
[9] R. P. Woodard, Scholarpedia **10**, 32243 (2015).
[10] K. A. Meissner and G. Veneziano, Phys. Lett. B **267**, 33 (1991).
[11] D. Anselmi and M. Piva, J. High Energy Phys. 11 (2018) 021.
[12] J. F. Donoghue and G. Menezes, Phys. Rev. Lett. **123**, 171601 (2019).
[13] N. Barnaby and N. Kamran, J. High Energy Phys. 02 (2008) 008.
[14] A. Platania and C. Wetterich, Phys. Lett. B **811**, 135911 (2020).
[15] P. Hořava, Phys. Rev. D **79**, 084008 (2009).
[16] A. Wang, Int. J. Mod. Phys. D **26**, 1730014 (2017).
[17] J. Ambjorn, A. Gorlich, S. Jordan, J. Jurkiewicz, and R. Loll, Phys. Lett. B **690**, 413 (2010).
[18] J. Ambjørn, L. Glaser, Y. Sato, and Y. Watabiki, Phys. Lett. B **722**, 172 (2013).
[19] C. Anderson, S. J. Carlip, J. H. Cooperman, P. Horava, R. K. Kommu, and P. R. Zulkowski, Phys. Rev. D **85**, 044027 (2012).
[20] J. F. Donoghue and G. Menezes, Phys. Rev. D **100**, 105006 (2019).
[21] F. S. Bemfica and M. Gomes, Phys. Rev. D **84**, 084022 (2011).
[22] P. Agrawal, S. Gukov, G. Obied, and C. Vafa, arXiv:2009.10077.
[23] J. Maldacena and L. Susskind, Fortschr. Phys. **61**, 781 (2013).
[24] F. Duplessis and D. A. Easson, Phys. Rev. D **92**, 043516 (2015).
[25] G. D'Odorico, F. Saueressig, and M. Schutten, Phys. Rev. Lett. **113**, 171101 (2014).
[26] G. D'Odorico, J.-W. Goossens, and F. Saueressig, J. High Energy Phys. 10 (2015) 126.
[27] A. O. Barvinsky, D. Blas, M. Herrero-Valea, S. M. Sibiryakov, and C. F. Steinwachs, Phys. Rev. Lett. **119**, 211301 (2017).
[28] R. Arnowitt, S. Deser, and C. W. Misner, Gen. Relativ. Gravit. **40**, 1997 (2008).
[29] R. Maier and I. D. Soares, Phys. Rev. D **96**, 103532 (2017).
[30] P. Horava, Classical Quantum Gravity **28**, 114012 (2011).
[31] C. Charmousis, G. Niz, A. Padilla, and P. M. Saffin, J. High Energy Phys. 08 (2009) 070.
[32] M. Li and Y. Pang, J. High Energy Phys. 08 (2009) 015.
[33] D. Blas, O. Pujolas, and S. Sibiryakov, J. High Energy Phys. 10 (2009) 029.
[34] M. Henneaux, A. Kleinschmidt, and G. L. Gómez, Phys. Rev. D **81**, 064002 (2010).
[35] P. Horava and C. M. Melby-Thompson, Phys. Rev. D **82**, 064027 (2010).
[36] D. Blas, O. Pujolas, and S. Sibiryakov, J. High Energy Phys. 04 (2011) 018.
[37] S. Carloni, E. Elizalde, and P. J. Silva, Classical Quantum Gravity **28**, 195002 (2011).
[38] C. Krishnan and A. Raju, Mod. Phys. Lett. A **32**, 1750077 (2017).

[39] J. W. York, Phys. Rev. Lett. **28**, 1082 (1972).
[40] G. W. Gibbons and S. W. Hawking, Phys. Rev. D **15**, 2752 (1977).
[41] G. Lara, M. Herrero-Valea, E. Barausse, and S. M. Sibiryakov, Phys. Rev. D **103**, 104007 (2021).
[42] A. H. Chamseddine, V. Mukhanov, and T. B. Russ, Phys. Lett. B **798**, 134939 (2019).
[43] O. Malaeb and C. Saghir, arXiv:2005.02469.
[44] E. Kiritsis and G. Kofinas, Nucl. Phys. **B821**, 467 (2009).
[45] M. Fukushima, Y. Misonoh, S. Miyashita, and S. Sato, Phys. Rev. D **99**, 064004 (2019).
[46] Y. Misonoh, K.-I. Maeda, and T. Kobayashi, Phys. Rev. D **84**, 064030 (2011).
[47] G. Ellis and B. Schmidt, Gen. Relativ. Gravit. **8**, 915 (1977).
[48] R.-G. Cai and A. Wang, Phys. Lett. B **686**, 166 (2010).
[49] K. Martel and E. Poisson, Am. J. Phys. **69**, 476 (2001).
[50] A. Wang, Phys. Rev. Lett. **110**, 091101 (2013).
[51] H. Lü, J. Mei, and C. N. Pope, Phys. Rev. Lett. **103**, 091301 (2009).
[52] B. P. Abbott *et al.* (LIGO Scientific Collaboration and Virgo Collaboration), Phys. Rev. Lett. **116**, 061102 (2016).
[53] K. Akiyama *et al.* (Event Horizon Telescope Collaboration), Astrophys. J. **875**, L1 (2019).
[54] K. Akiyama *et al.* (Event Horizon Telescope Collaboration), Astrophys. J. Lett. **875**, L6 (2019).
[55] I. Dymnikova and K. Kraav, Universe **5**, 163 (2019).
[56] A. Bonanno and M. Reuter, Phys. Rev. D **62**, 043008 (2000).
[57] A. Ashtekar and M. Bojowald, Classical Quantum Gravity **23**, 391 (2006).
[58] L. Modesto, Classical Quantum Gravity **23**, 5587 (2006).
[59] A. Bonanno and M. Reuter, Phys. Rev. D **73**, 083005 (2006).
[60] K. Falls, D. F. Litim, and A. Raghuraman, Int. J. Mod. Phys. A **27**, 1250019 (2012).
[61] A. Held, R. Gold, and A. Eichhorn, J. Cosmol. Astropart. Phys. 06 (2019) 029.
[62] A. Platania, Eur. Phys. J. C **79**, 470 (2019).
[63] V. Faraoni and A. Giusti, Symmetry **12**, 1264 (2020).
[64] I. Dymnikova, Gen. Relativ. Gravit. **24**, 235 (1992).
[65] S. A. Hayward, Phys. Rev. Lett. **96**, 031103 (2006).
[66] C. Bambi and L. Modesto, Phys. Lett. B **721**, 329 (2013).
[67] V. P. Frolov, Phys. Rev. D **94**, 104056 (2016).
[68] M. S. Morris and K. S. Thorne, Am. J. Phys. **56**, 395 (1988).
[69] A. Einstein and N. Rosen, Phys. Rev. **48**, 73 (1935).
[70] P. Boonserm, T. Ngampitipan, A. Simpson, and M. Visser, Phys. Rev. D **98**, 084048 (2018).
[71] M. B. Cantcheff, N. E. Grandi, and M. Sturla, Phys. Rev. D **82**, 124034 (2010).
[72] S. F. Bramberger, A. Coates, J. a. Magueijo, S. Mukohyama, R. Namba, and Y. Watanabe, Phys. Rev. D **97**, 043512 (2018).

[73] S. Mukohyama, J. Cosmol. Astropart. Phys. 06 (2009) 001.
[74] H. Godazgar, K. A. Meissner, and H. Nicolai, J. High Energy Phys. 04 (2017) 165.
[75] K. A. Meissner and H. Nicolai, Phys. Lett. B **772**, 169 (2017).
[76] T. Griffin, P. Hořava, and C. M. Melby-Thompson, Phys. Rev. Lett. **110**, 081602 (2013).
[77] I. Arav, S. Chapman, and Y. Oz, J. High Energy Phys. 02 (2015) 078.
[78] I. Arav, Y. Oz, and A. Raviv-Moshe, J. High Energy Phys. 03 (2017) 088.
[79] I. Arav, S. Chapman, and Y. Oz, J. High Energy Phys. 06 (2016) 158.
[80] K. A. Meissner and H. Nicolai, Phys. Rev. D **96**, 041701 (2017).
[81] K. A. Meissner and H. Nicolai, Found. Phys. **48**, 1150 (2018).
[82] J. Chojnacki, J. Krajecka, J. H. Kwapisz, O. Slowik, and A. Strag, J. Cosmol. Astropart. Phys. 04 (2021) 076.
[83] T. Rudelius, J. Cosmol. Astropart. Phys. 08 (2019) 009.
[84] T. Rudelius, J. High Energy Phys. 08 (2021) 041.
[85] A. Wang (private communication).
[86] F. S. N. Lobo, Int. J. Mod. Phys. D **25**, 1630017 (2016).
[87] G. J. Olmo, D. Rubiera-Garcia, and A. Sanchez-Puente, in *14th Marcel Grossmann Meeting on Recent Developments in Theoretical and Experimental General Relativity, Astrophysics, and Relativistic Field Theories* (2017), Vol. 2, pp. 1391–1396.
[88] C. Bambi, A. Cardenas-Avendano, G. J. Olmo, and D. Rubiera-Garcia, Phys. Rev. D **93**, 064016 (2016).
[89] N. Yunes, K. Yagi, and F. Pretorius, Phys. Rev. D **94**, 084002 (2016).
[90] A. E. Gümrükçüoğlu, M. Saravani, and T. P. Sotiriou, Phys. Rev. D **97**, 024032 (2018).
[91] B. P. Abbott *et al.* (LIGO Scientific, Virgo Collaborations), Phys. Rev. Lett. **123**, 011102 (2019).
[92] O. Ramos and E. Barausse, Phys. Rev. D **99**, 024034 (2019).
[93] M. Mewes, Phys. Rev. D **99**, 104062 (2019).
[94] N. Frusciante and M. Benetti, Phys. Rev. D **103**, 104060 (2021).
[95] J. D. Tasson, Rep. Prog. Phys. **77**, 062901 (2014).
[96] N. Frusciante, M. Raveri, D. Vernieri, B. Hu, and A. Silvestri, Phys. Dark Universe **13**, 7 (2016).
[97] A. Eichhorn, A. Platania, and M. Schiffer, Phys. Rev. D **102**, 026007 (2020).
[98] H. W. Lee and Y. S. Myung, Eur. Phys. J. C **72**, 1865 (2012).
[99] M. O. Katanaev, Mod. Phys. Lett. A **29**, 1450090 (2014).
[100] B. Mattingly, A. Kar, W. Julius, M. Gorban, C. Watson, M. Ali, A. Baas, C. Elmore, B. Shakerin, E. Davis *et al.*, Universe **6**, 11 (2020).