Two- and three-gluon glueballs of $C = +$

Hua-Xing Chen[1,*] Wei Chen[2,†] and Shi-Lin Zhu[3,‡]

[1]School of Physics, Southeast University, Nanjing 210094, China
[2]School of Physics, Sun Yat-Sen University, Guangzhou 510275, China
[3]School of Physics and Center of High Energy Physics, Peking University, Beijing 100871, China

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We study two- and three-gluon glueballs of $C = +$ using the method of QCD sum rules. We systematically construct their interpolating currents, and find that all the spin-1 currents of $C = +$ vanish. This suggests that the "ground-state" spin-1 glueballs of $C = +$ do not exist within the relativistic framework. We calculate masses of the two-gluon glueballs with $J^{PC} = 0^{\pm+}/2^{\pm+}$ and the three-gluon glueballs with $J^{PC} = 0^{\pm+}/2^{\pm+}$. We propose searching for the $J^{PC} = 0^{-+}/2^{-\pm}/3^{\pm-}$ three-gluon glueballs in their three-meson decay channels in the future BESIII, GlueX, LHC, and PANDA experiments.

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I. INTRODUCTION

Glueballs, composed of valence gluons, are important for the understanding of nonperturbative QCD [1–3]. There have been tremendous theoretical studies on them in the past fifty years using various models and methods, such as the MIT bag model [4], the flux-tube model [5], the Coulomb Gauge model [6,7], Regge trajectories [8], holographic QCD [9], lattice QCD [10–14], and QCD sum rules [15–30], etc. However, experimental efforts in searching for glueballs are confronted with the difficulty of identifying them unambiguously, and there is currently no definite experimental evidence for their existence.

Recently the D0 and TOTEM Collaborations studied pp and $p\bar{p}$ [31] cross sections, which are found to be different with a significance of 3.4σ [32]. Together with their previous result [33], this significance can be increased to 5.2σ–5.7σ. The above difference leads to the evidence of a t-channel exchanged odderon [34–38], that is predominantly a three-gluon glueball of $C = -$. We refer to Refs. [39–47] and a review of [48] for more discussions. Due to these studies, interest in glueballs have recently been revived. Since the above odderon evidence is still indirect, it is crucial and important to directly study the glueball itself.

The lowest-lying two-, three-, and four-gluon glueballs have been systematically investigated in Ref. [49], where the authors constructed their corresponding nonrelativistic low-dimension operators. These operators have been successfully used in lattice QCD calculations. In this paper we systematically study two- and three-gluon glueballs of $C = +$. We shall construct their corresponding relativistic glueball currents, and calculate the masses of these glueballs using the method of QCD sum rules. The same approach has been applied in Ref. [50] to study three-gluon glueballs of $C = -$, so a rather complete QCD sum rule study will be done on the lowest-lying glueballs composed of two- or three-valence gluons. These studies can largely improve our understanding of the gluon degree of freedom as well as the nonperturbative behaviors of the strong interaction at the low-energy region.

This paper is organized as follows. We systematically construct relativistic two- and three-gluon glueball currents of $C = +$ in Sec. II. We apply them to perform QCD sum rule analyses in Sec. III, and perform numerical analyses in Sec. IV. The obtained results are summarized and discussed in Sec. V, and are compared with lattice QCD results [11–14].

II. RELATIVISTIC GLUEBALL CURRENTS

In this section we systematically construct relativistic glueball currents, including the two-gluon glueball currents and the $C = +$ three-gluon glueball currents. We shall do this separately in the following subsections. Note that the two-gluon glueball currents cannot reach $C = -$ [51], and the $C = -$ three-gluon glueball currents have been systematically constructed in Ref. [50].

A. Couplings of tensor currents

In the present study we shall use some special tensor currents to study glueballs with nonzero spins $J \neq 0$. These currents have $2 \times J$ Lorentz indices with certain symmetries,

*hxchen@seu.edu.cn
†chenwei29@mail.sysu.edu.cn
‡zhusl@pku.edu.cn

and they couple to both positive- and negative-parity glueballs. In this subsection we briefly explain how we deal with them.

We assume $J_{\alpha\beta}$ to be a tensor current with two antisymmetric Lorentz indices μ and ν. Taking the current $J_{\alpha\beta} = \bar{c}\sigma_{\alpha\beta}c$ as an example, it can be separated into ($\alpha, \beta = 0, 1, 2, 3$ and $i, j = 1, 2, 3$)

$$J_{\alpha\beta} = \bar{c}\sigma_{\alpha\beta}c \to \begin{cases} \bar{c}\sigma_{ij}c, & P = + \\ \bar{c}\sigma_{0i}c, & P = -. \end{cases} \tag{1}$$

Accordingly, it couples to both positive- and negative-parity charmonia through

$$\langle 0|J_{\alpha\beta}|h_c(\epsilon, p)\rangle = i f^T_{h_c}\epsilon_{\alpha\beta\mu\nu}\epsilon^\mu p^\nu, \tag{2}$$

$$\langle 0|J_{\alpha\beta}|J/\psi(\epsilon, p)\rangle = i f^T_{J/\psi}(p_\alpha\epsilon_\beta - p_\beta\epsilon_\alpha), \tag{3}$$

where $f^T_{h_c}$ and $f^T_{J/\psi}$ are relevant decay constants. Given the Lorentz structures of J/ψ and h_c are totally different, they can be clearly separated from each other. For example, we can isolate h_c at the hadron level by investigating the two-point correlation function containing

$$\begin{aligned} &\langle 0|J_{\alpha\beta}|h_c\rangle\langle h_c|J^\dagger_{\alpha'\beta'}|0\rangle \\ &= (f^T_{h_c})^2\epsilon_{\alpha\beta\mu\nu}\epsilon^\mu p^\nu\epsilon_{\alpha'\beta'\mu'\nu'}\epsilon^{*\mu'}p^{\nu'} \\ &= -(f^T_{h_c})^2 p^2(g_{\alpha\alpha'}g_{\beta\beta'} - g_{\alpha\beta'}g_{\beta\alpha'}) + \cdots, \end{aligned} \tag{4}$$

since the correlation function of J/ψ does not contain the above coefficient. It is not so easy to isolate J/ψ from $J_{\alpha\beta}$ at the hadron level. Instead, we can investigate its partner current

$$\tilde{J}_{\alpha\beta} = \epsilon_{\alpha\beta\gamma\delta} \times J^{\gamma\delta}, \tag{5}$$

which couples to J/ψ and h_c (just in the opposite way)

$$\langle 0|\tilde{J}_{\alpha\beta}|J/\psi(\epsilon, p)\rangle = i\tilde{f}^T_{J/\psi}\epsilon_{\alpha\beta\mu\nu}\epsilon^\mu p^\nu, \tag{6}$$

$$\langle 0|\tilde{J}_{\alpha\beta}|h_c(\epsilon, p)\rangle = i\tilde{f}^T_{h_c}(p_\alpha\epsilon_\beta - p_\beta\epsilon_\alpha). \tag{7}$$

Accordingly, we can use the two currents $J_{\alpha\beta}$ and $\tilde{J}_{\alpha\beta}$ to study and separate J/ψ and h_c.

We apply the above process to generally investigate the current $J^{\alpha_1\cdots\alpha_N,\beta_1\cdots\beta_N}$, which has $2N = 2J$ Lorentz indices with certain symmetries, e.g., the spin-2 current $J^{\alpha_1\alpha_2,\beta_1\beta_2}$ has four Lorentz indices, satisfying

$$J^{\alpha_1\alpha_2,\beta_1\beta_2} = -J^{\beta_1\alpha_2,\alpha_1\beta_2} = -J^{\alpha_1\beta_2,\beta_1\alpha_2} = J^{\alpha_2\alpha_1,\beta_2\beta_1}. \tag{8}$$

Its coupling can be written as

$$\langle 0|J^{\alpha_1\cdots\alpha_N,\beta_1\cdots\beta_N}|X\rangle = i f_X \mathcal{S}[\epsilon^{\alpha_i\beta_i\mu_i\nu_i}p_{\nu_i}]^N\epsilon_{\mu_1\cdots\mu_N}, \tag{9}$$

where X is the corresponding state having the same parity as $J^{i_1\cdots i_N,j_1\cdots j_N}$ ($i_1\cdots j_N = 1, 2, 3$); $\mathcal{S}$ denotes symmetrization and subtracting trace terms in the two sets $\{\alpha_1\cdots\alpha_N\}$ and $\{\beta_1\cdots\beta_N\}$ simultaneously, with

$$[\cdots]^N = \epsilon^{\alpha_1\beta_1\mu_1\nu_1}p_{\nu_1}\cdots\epsilon^{\alpha_N\beta_N\mu_N\nu_N}p_{\nu_N}. \tag{10}$$

Note that the current $J^{\alpha_1\cdots\alpha_N,\beta_1\cdots\beta_N}$ can also couple to the other state X' having opposite parity to X, but this state X' cannot be easily isolated at the hadron level, so we do not consider it in the present study.

B. Two-gluon glueball currents

In this subsection we use the gluon field strength tensor $G^a_{\mu\nu}$ to construct two-gluon glueball currents, with a the color index and μ, ν the Lorentz indices. We also need $\tilde{G}^a_{\mu\nu} = G^{a,\rho\sigma} \times \epsilon_{\mu\nu\rho\sigma}/2$ to denote the dual gluon-field strength tensor, and f^{abc} to denote the totally antisymmetric $SU(3)_C$ structure constant. In the present study we only consider local glueball currents without explicit derivatives, although $G^a_{\mu\nu}$ and $\tilde{G}^a_{\mu\nu}$ contain covariant derivatives within them.

In Ref. [49] the authors use the chromoelectric and chromomagnetic fields ($i, j = 1, 2, 3$),

$$E_i = G_{i0} \quad \text{and} \quad B_i = -\frac{1}{2}\epsilon_{ijk}G^{jk}, \tag{11}$$

to write down all the nonrelativistic low-dimension two-gluon glueball operators,

$$\begin{aligned} &0^{++} && \vec{E}^2_a \pm \vec{B}^2_a, \\ &0^{-+} && \vec{E}_a \cdot \vec{B}_a, \\ &1^{-+} && \vec{E}_a \times \vec{B}_a, \\ &2^{++} && \mathcal{S}'[E^i_aE^j_a \pm B^i_aB^j_a], \\ &2^{-+}\mathcal{S}' && [E^i_aB^j_a - B^i_aE^j_a], \end{aligned} \tag{12}$$

where $\mathcal{S}'$ denotes symmetrization and subtracting trace terms in the set $\{ij\}$.

We construct their corresponding relativistic currents in order to perform QCD sum rule analyses,

$$J_0 = g_s^2 G_a^{\mu\nu}G^a_{\mu\nu}, \tag{13}$$

$$\tilde{J}_0 = g_s^2 G_a^{\mu\nu}\tilde{G}^a_{\mu\nu}, \tag{14}$$

$$J_1^{\alpha\beta} = g_s^2 G_a^{\alpha\mu}\tilde{G}^{a,\beta}_\mu - \{\alpha \leftrightarrow \beta\}, \tag{15}$$

$$J_2^{\alpha_1\alpha_2,\beta_1\beta_2} = \mathcal{S}[g_s^2 G_a^{\alpha_1\beta_1}G^{a,\alpha_2\beta_2}], \tag{16}$$

$$\tilde{J}_2^{\alpha_1\alpha_2,\beta_1\beta_2} = \mathcal{S}[g_s^2 G_a^{\alpha_1\beta_1}\tilde{G}^{a,\alpha_2\beta_2}]. \tag{17}$$

We shall explicitly prove in Appendix that the third current $J_1^{\alpha\beta}$ vanishes, suggesting that the "ground-state" two-gluon glueball of $J^{PC}=1^{-+}$ does not exist within the relativistic framework.

The former two currents J_0 of $J^{PC}=0^{++}$ and $\tilde{J}_0$ of $J^{PC}=0^{-+}$ couple to the $J^{PC}=0^{++}$ and 0^{-+} two-gluon glueballs $|\mathrm{GG};J^{PC}\rangle$, respectively,

$$\langle 0|J_0|\mathrm{GG};0^{++}\rangle = f_{0^{++}}, \quad (18)$$

$$\langle 0|\tilde{J}_0|\mathrm{GG};0^{-+}\rangle = f_{0^{-+}}, \quad (19)$$

where $f_{0^{++}}$ and $f_{0^{-+}}$ are decay constants. In addition, the current J_0 has a partner,

$$J_0' = g_s^2 \tilde{G}_a^{\mu\nu}\tilde{G}^a_{\mu\nu}, \quad (20)$$

whose sum rule result is the same as that of J_0.

The latter two currents $J_2^{\alpha_1\alpha_2,\beta_1\beta_2}$ and $\tilde{J}_2^{\alpha_1\alpha_2,\beta_1\beta_2}$ couple to the $J^{PC}=2^{++}$ and 2^{-+} glueballs through

$$\langle 0|J_2^{\cdots}|\mathrm{GG};2^{++}\rangle = i f_{2^{++}} \mathcal{S}[\epsilon^{\alpha_i\beta_i\mu_i\nu_i} p_{\nu_i}]^2 \epsilon_{\mu_1\mu_2}, \quad (21)$$

$$\langle 0|\tilde{J}_2^{\cdots}|\mathrm{GG};2^{-+}\rangle = i f_{2^{-+}} \mathcal{S}[\epsilon^{\alpha_i\beta_i\mu_i\nu_i} p_{\nu_i}]^2 \epsilon_{\mu_1\mu_2}. \quad (22)$$

The current $J_2^{\alpha_1\alpha_2,\beta_1\beta_2}$ also has a partner,

$$J_2'^{\alpha_1\alpha_2,\beta_1\beta_2} = \mathcal{S}[g_s^2 \tilde{G}_a^{\alpha_1\beta_1}\tilde{G}^{a,\alpha_2\beta_2}]. \quad (23)$$

whose sum rule result is the same as that of $J_2^{\alpha_1\alpha_2,\beta_1\beta_2}$.

C. Three-gluon glueball currents of $C=+$

In this subsection we use $G^a_{\mu\nu}$ and $\tilde{G}^a_{\mu\nu}$ to construct three-gluon glueball currents of $C=+$. Some of their corresponding nonrelativistic operators have been constructed in Ref. [49]

$$\begin{aligned}
&0^{++} && f^{abc}(\vec{E}_a\times\vec{E}_b)\cdot\vec{B}_c,\\
&0^{-+} && f^{abc}(\vec{E}_a\times\vec{E}_b)\cdot\vec{E}_c,\\
&1^{++} && f^{abc}(\vec{B}_a\cdot\vec{E}_b)\vec{E}_c,\\
&1^{-+} && f^{abc}(\vec{B}_a\cdot\vec{E}_b)\vec{B}_c,\\
&2^{++} && f^{abc}\mathcal{S}'[(\vec{B}_a\times\vec{B}_b)^i B_c^j]+\cdots,\\
&2^{-+} && f^{abc}\mathcal{S}'[(\vec{E}_a\times\vec{E}_b)^i E_c^j]+\cdots. \quad (24)
\end{aligned}$$

We further construct their corresponding relativistic currents as follows:

$$\eta_0 = f^{abc} g_s^3 G_a^{\mu\nu} G_{b,\nu\rho} G^\rho_{c,\mu}, \quad (25)$$

$$\tilde{\eta}_0 = f^{abc} g_s^3 \tilde{G}_a^{\mu\nu} \tilde{G}_{b,\nu\rho} \tilde{G}^\rho_{c,\mu}, \quad (26)$$

$$\eta_1^{\alpha\beta} = f^{abc} g_s^3 \tilde{G}_a^{\mu\nu} G_{b,\mu\nu} \tilde{G}_c^{\alpha\beta}, \quad (27)$$

$$\tilde{\eta}_1^{\alpha\beta} = f^{abc} g_s^3 \tilde{G}_a^{\mu\nu} G_{b,\mu\nu} G_c^{\alpha\beta}, \quad (28)$$

$$\eta_2^{\alpha_1\alpha_2,\beta_1\beta_2} = f^{abc}\mathcal{S}[g_s^3 G_a^{\alpha_1\beta_1} G_b^{\alpha_2\mu} G^{\beta_2}_{c,\mu} - \{\alpha_2\leftrightarrow\beta_2\}], \quad (29)$$

$$\tilde{\eta}_2^{\alpha_1\alpha_2,\beta_1\beta_2} = f^{abc}\mathcal{S}[g_s^3 \tilde{G}_a^{\alpha_1\beta_1} \tilde{G}_b^{\alpha_2\mu} \tilde{G}^{\beta_2}_{c,\mu} - \{\alpha_2\leftrightarrow\beta_2\}]. \quad (30)$$

We shall explicitly prove in Appendix that the third and fourth currents $\eta_1^{\alpha\beta}$ and $\tilde{\eta}_1^{\alpha\beta}$ both vanish, suggesting that the "ground-state" three-gluon glueballs of $J^{PC}=1^{++}$ and 1^{-+} do not exist within the relativistic framework.

The former two currents η_0 of $J^{PC}=0^{++}$ and $\tilde{\eta}_0$ of $J^{PC}=0^{-+}$ couple to the $J^{PC}=0^{++}$ and 0^{-+} three-gluon glueballs $|\mathrm{GGG};J^{PC}\rangle$, respectively,

$$\langle 0|\eta_0|\mathrm{GGG};0^{++}\rangle = f'_{0^{++}}, \quad (31)$$

$$\langle 0|\tilde{\eta}_0|\mathrm{GGG};0^{-+}\rangle = f'_{0^{-+}}. \quad (32)$$

The latter two currents $\eta_2^{\alpha_1\alpha_2,\beta_1\beta_2}$ and $\tilde{\eta}_2^{\alpha_1\alpha_2,\beta_1\beta_2}$ couple to the $J^{PC}=2^{++}$ and 2^{-+} glueballs through

$$\langle 0|\eta_2^{\cdots}|\mathrm{GGG};2^{++}\rangle = i f'_{2^{++}} \mathcal{S}[\epsilon^{\alpha_i\beta_i\mu_i\nu_i} p_{\nu_i}]^2 \epsilon_{\mu_1\mu_2}, \quad (33)$$

$$\langle 0|\tilde{\eta}_2^{\cdots}|\mathrm{GGG};2^{-+}\rangle = i f'_{2^{-+}} \mathcal{S}[\epsilon^{\alpha_i\beta_i\mu_i\nu_i} p_{\nu_i}]^2 \epsilon_{\mu_1\mu_2}. \quad (34)$$

III. QCD SUM RULE ANALYSES

In this section we use the two-gluon glueball currents J_0, $\tilde{J}_0$, $J_2^{\alpha_1\alpha_2,\beta_1\beta_2}$, and $\tilde{J}_2^{\alpha_1\alpha_2,\beta_1\beta_2}$ as well as the three-gluon glueball currents η_0, $\tilde{\eta}_0$, $\eta_2^{\alpha_1\alpha_2,\beta_1\beta_2}$, and $\tilde{\eta}_2^{\alpha_1\alpha_2,\beta_1\beta_2}$ to perform QCD sum rule analyses. This method has been widely applied in the field of hadron phenomenology [52,53] to study various exotic hadrons [54–56]; all the above spin-2 currents have four Lorentz indices with certain symmetries, so that they couple to both positive- and negative-parity glueballs simultaneously. We refer to Ref. [50] for detailed discussions.

We take the current $\tilde{J}_0$ defined in Eq. (14) as an example, and calculate its two-point correlation function

$$\Pi(q^2) \equiv i\int d^4x e^{iqx}\langle 0|\mathbf{T}[\tilde{J}_0(x)\tilde{J}_0^\dagger(0)]|0\rangle, \quad (35)$$

separately at hadron and quark-gluon levels.

At the hadron level we express Eq. (35) using the dispersion relation as

$$\Pi(q^2) = \int_0^\infty \frac{\rho(s)}{s-q^2-i\varepsilon} ds, \quad (36)$$

with $\rho(s) = \mathrm{Im}\Pi(s)/\pi$ the spectral density. It is parametrized using one pole dominance for the ground state X as well as the continuum contribution,

$$\rho(s) \equiv \sum_n \delta(s - M_n^2)\langle 0|\tilde{J}_0|n\rangle\langle n|\tilde{J}_0^\dagger|0\rangle = f_X^2\delta(s - M_X^2) + \text{continuum}. \quad (37)$$

At the quark-gluon level we insert Eq. (14) into Eq. (35), and calculate it using the method of operator product expansion (OPE). After performing the Borel transformation to Eq. (36) at both hadron and quark-gluon levels, we approximate the continuum using the spectral density above a threshold value s_0, and obtain

$$\Pi(s_0, M_B^2) \equiv f_X^2 e^{-M_X^2/M_B^2} = \int_0^{s_0} e^{-s/M_B^2}\rho(s)ds. \quad (38)$$

This equation can be used to further calculate the mass of X through

$$M_X^2(s_0, M_B) = \frac{\int_0^{s_0} e^{-s/M_B^2} s\rho(s)ds}{\int_0^{s_0} e^{-s/M_B^2}\rho(s)ds}. \quad (39)$$

Since the gluon field strength tensor $G^a_{\mu\nu}$ is defined as

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_s f^{abc} A_{b,\mu} A_{c,\nu}, \quad (40)$$

it can be naturally separated into two parts. As shown in Fig. 1, we depict the former two terms using the single-gluon line, and the latter one term using the double-gluon line with a red vertex [see also diagram Fig. 2($c-3$)]. Here A^a_μ is the gluon field, whose propagator is [57]

$$\langle 0|\mathbf{T}[A^a_\mu(x)A^b_\nu(y)]|0\rangle = \frac{\delta^{ab} g_{\mu\nu}}{4\pi^2(x-y)^2} + \frac{g_s \ln(-(x-y)^2)}{8\pi^2} f^{abc} G_{c,\mu\nu}(0) - \frac{g_s g_{\mu\nu} x^\alpha y^\beta}{8\pi^2(x-y)^2} f^{abc} G_{c,\alpha\beta}(0). \quad (41)$$

We work in the fixed-point gauge so that

$$A^a_\mu(x) \approx -\frac{1}{2} x^\nu G^a_{\mu\nu}(0). \quad (42)$$

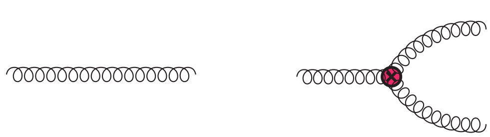

FIG. 1. The gluon field strength tensor $G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_s f^{abc} A_{b,\mu} A_{c,\nu}$, naturally separated into two parts (a) and (b).

In the present study we consider the Feynman diagrams depicted in Fig. 2 (for three-gluon glueballs), and calculate OPEs up to dimension eight ($D = 8$) condensates. We take into account the perturbative term, the two-gluon condensate $\langle g_s^2 GG\rangle$, the three-gluon condensate $\langle g_s^3 G^3\rangle$, and the $D = 8$ condensate $\langle g_s^2 GG\rangle^2$,

$$\Pi_{|GG;0^{++}\rangle}(s_0, M_B^2) = \int_0^{s_0} (32\alpha_s^2 s^2 + 60\alpha_s^2\langle g_s^2 GG\rangle) e^{-s/M_B^2} ds + 24\pi\alpha_s\langle g_s^3 G^3\rangle, \quad (43)$$

$$\Pi_{|GG;2^{++}\rangle}(s_0, M_B^2) = \int_0^{s_0} \left(\frac{2\alpha_s^2}{15} s^2 - \frac{5\alpha_s^2\langle g_s^2 GG\rangle}{24}\right) e^{-s/M_B^2} ds + \frac{\pi\alpha_s\langle g_s^3 G^3\rangle}{3}, \quad (44)$$

$$\Pi_{|GG;0^{-+}\rangle}(s_0, M_B^2) = \int_0^{s_0} 32\alpha_s^2 s^2 e^{-s/M_B^2} ds - 40\pi\alpha_s\langle g_s^3 G^3\rangle, \quad (45)$$

$$\Pi_{|GG;2^{-+}\rangle}(s_0, M_B^2) = \int_0^{s_0} \left(\frac{2\alpha_s^2}{5} s^2 + \frac{\alpha_s^2\langle g_s^2 GG\rangle}{12}\right) e^{-s/M_B^2} ds - \frac{\pi\alpha_s\langle g_s^3 G^3\rangle}{2}, \quad (46)$$

$$\Pi_{|GGG;0^{++}\rangle}(s_0, M_B^2) = \int_0^{s_0} \left(\frac{3\alpha_s^3}{10\pi} s^4 + \frac{135\alpha_s^3\langle g_s^2 GG\rangle}{32\pi} s^2 - \frac{81\alpha_s^2\langle g_s^3 G^3\rangle}{2} s\right) e^{-s/M_B^2} ds, \quad (47)$$

$$\Pi_{|GGG;2^{++}\rangle}(s_0, M_B^2) = \int_0^{s_0} \left(\frac{2\alpha_s^3}{315\pi} s^4 + \frac{\alpha_s^2\langle g_s^2 GG\rangle}{15} s^2 + \frac{53\alpha_s^3\langle g_s^2 GG\rangle}{320\pi} s^2 + \frac{\alpha_s^2\langle g_s^3 G^3\rangle}{3} s\right) e^{-s/M_B^2} ds, \quad (48)$$

$$\Pi_{|GGG;0^{-+}\rangle}(s_0, M_B^2) = \int_0^{s_0} \left(\frac{3\alpha_s^3}{10\pi} s^4 + \frac{135\alpha_s^3\langle g_s^2 GG\rangle}{32\pi} s^2 + \frac{27\alpha_s^2\langle g_s^3 G^3\rangle}{2} s\right) e^{-s/M_B^2} ds, \quad (49)$$

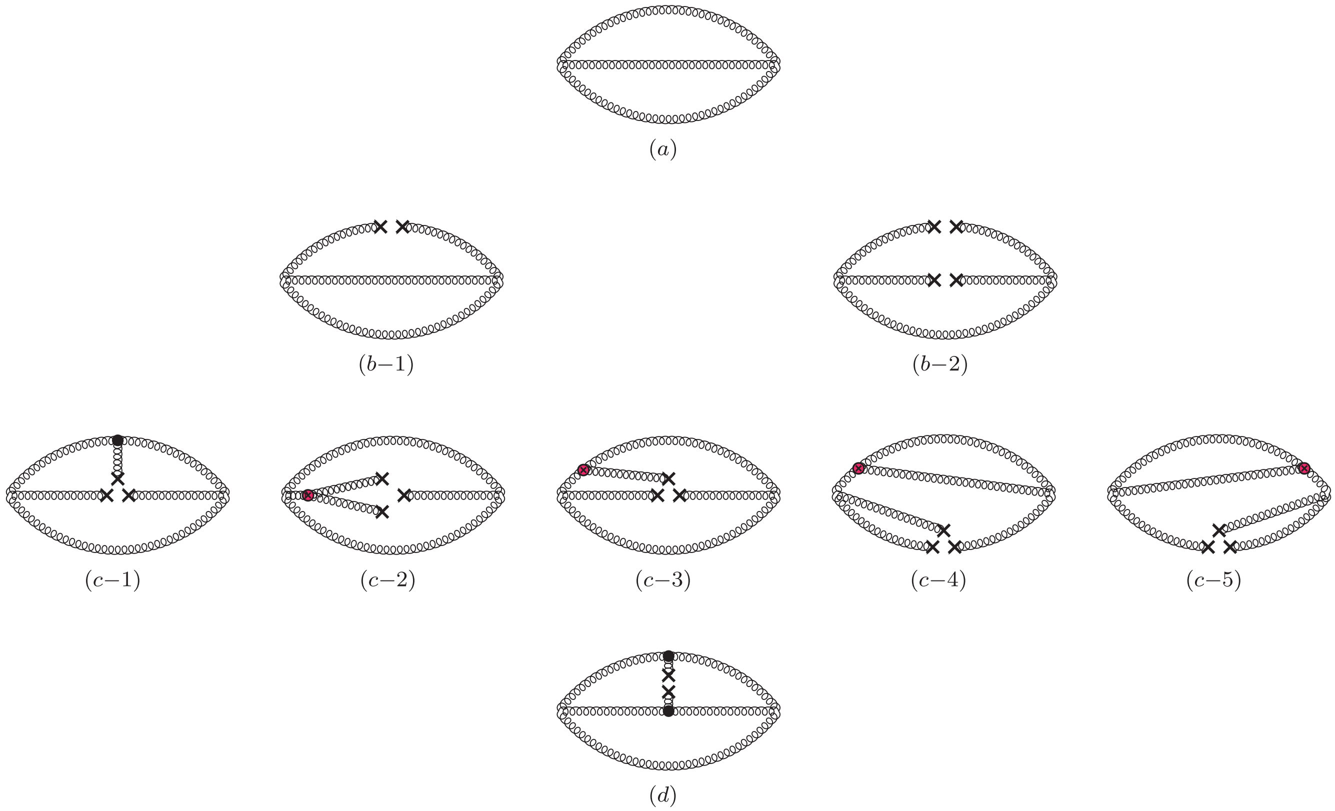

FIG. 2. Feynman diagrams for three-gluon glueball currents, including the perturbative term, the two-gluon condensate $\langle g_s^2 GG\rangle$, the three-gluon condensate $\langle g_s^3 G^3\rangle$, and the $D = 8$ condensate $\langle g_s^2 GG\rangle^2$. Diagrams (a) and $(b-i)$ are proportional to $\alpha_s^3 \times g_s^0$, diagrams $(c-i)$ are proportional to $\alpha_s^3 \times g_s^1$, and diagram (d) is proportional to $\alpha_s^3 \times g_s^2$.

$$\Pi_{|GGG;2^{-+}\rangle}(s_0, M_B^2) = \int_0^{s_0}\left(\frac{2\alpha_s^3}{315\pi}s^4 - \frac{\alpha_s^2\langle g_s^2 GG\rangle}{15}s^2 + \frac{57\alpha_s^3\langle g_s^2 GG\rangle}{320\pi}s^2 + \frac{5\alpha_s^2\langle g_s^3 G^3\rangle}{4}s\right)e^{-s/M_B^2}ds. \quad (50)$$

In the calculations we have considered all the diagrams proportional to $\alpha_s^n \times g_s^0$ and $\alpha_s^n \times g_s^1$ ($n = 2$ for two-gluon glueballs and $n = 3$ for three-gluon glueballs); however, there are so many diagrams proportional to $\alpha_s^n \times g_s^2$, so we have only taken into account one of them. Specifically, we find that all the $D = 8$ terms proportional to $\langle g_s^2 GG\rangle^2$ vanish, so the convergence of the above OPE series are quite good.

In Ref. [22] the authors studied $J^{PC} = 0^{++}$ three-gluon glueballs using the current η_0 defined in Eq. (25), where they calculated the Feynman diagrams depicted in Figs. 2 $(a, b-i, c-1, c-2)$. In Ref. [24] the authors studied $J^{PC} = 0^{-+}$ three-gluon glueballs using the current $\tilde{\eta}_0$ defined in Eq. (26), where they calculated the diagrams depicted in Figs. 2 $(a, b-i, c-i)$. Their calculations are done (mainly) by hand. In the present study we use the software *Mathematica* with the package *FeynCalc*, and we can obtain exactly the same results for these diagrams. In Refs. [16,21,23] the authors studied $J^{PC} = 0^{++}$ and 0^{-+} two-gluon glueballs using the currents J_0 and $\tilde{J}_0$ defined in Eqs. (13) and (14), where they calculated more diagrams than those calculated in the present study. However, such calculations are too complicated to be applied to three-gluon glueballs, and we still calculate similar diagrams as those depicted in Fig. 2 for two-gluon glueballs to make the present study unified as a whole.

For completeness, we also investigate the following three-gluon glueball currents of $C = -$,

$$\xi_1^{\alpha\beta} = d^{abc} g_s^3 G_a^{\mu\nu} G_{b,\mu\nu} G_c^{\alpha\beta}, \quad (51)$$

$$\tilde{\xi}_1^{\alpha\beta} = d^{abc} g_s^3 G_a^{\mu\nu} G_{b,\mu\nu} \tilde{G}_c^{\alpha\beta}, \quad (52)$$

$$\xi_2^{\alpha_1\alpha_2,\beta_1\beta_2} = d^{abc}\mathcal{S}[g_s^3 \tilde{G}_a^{\alpha_1\beta_1} G_b^{\alpha_2\mu} \tilde{G}_{c,\mu}^{\beta_2} - \{\alpha_2 \leftrightarrow \beta_2\}], \quad (53)$$

$$\tilde{\xi}_2^{\alpha_1\alpha_2,\beta_1\beta_2} = d^{abc}\mathcal{S}[g_s^3 G_a^{\alpha_1\beta_1} \tilde{G}_b^{\alpha_2\mu} \tilde{G}_{c,\mu}^{\beta_2} - \{\alpha_2 \leftrightarrow \beta_2\}], \quad (54)$$

$$\xi_3^{\cdots} = d^{abc}\mathcal{S}[g_s^3 G_a^{\alpha_1\beta_1} G_b^{\alpha_2\beta_2} G_c^{\alpha_3\beta_3}], \quad (55)$$

$$\tilde{\xi}_3^{\cdots} = d^{abc}\mathcal{S}[g_s^3 \tilde{G}_a^{\alpha_1\beta_1} \tilde{G}_b^{\alpha_2\beta_2} \tilde{G}_c^{\alpha_3\beta_3}], \quad (56)$$

where d^{abc} is the totally symmetric $SU(3)_C$ structure constant. Their sum rule equations are

$$\Pi_{|\mathrm{GGG};1^{+-}\rangle}(s_0,M_B^2) = \int_0^{s_0}\left(\frac{4\alpha_s^3}{81\pi}s^4+\frac{10\alpha_s^2\langle g_s^2GG\rangle}{9}s^2 + \frac{35\alpha_s^3\langle g_s^2GG\rangle}{36\pi}s^2+\frac{5\alpha_s^2\langle g_s^3G^3\rangle}{27}s\right)e^{-s/M_B^2}ds, \tag{57}$$

$$\Pi_{|\mathrm{GGG};2^{+-}\rangle}(s_0,M_B^2) = \int_0^{s_0}\left(\frac{\alpha_s^3}{324\pi}s^4-\frac{5\alpha_s^2\langle g_s^2GG\rangle}{108}s^2 + \frac{15\alpha_s^3\langle g_s^2GG\rangle}{128\pi}s^2+\frac{65\alpha_s^2\langle g_s^3G^3\rangle}{216}s\right)e^{-s/M_B^2}ds, \tag{58}$$

$$\Pi_{|\mathrm{GGG};3^{+-}\rangle}(s_0,M_B^2) = \int_0^{s_0}\left(\frac{5\alpha_s^3}{2016\pi}s^4+\frac{\alpha_s^2\langle g_s^2GG\rangle}{16}s^2 - \frac{59\alpha_s^3\langle g_s^2GG\rangle}{512\pi}s^2-\frac{\alpha_s^2\langle g_s^3G^3\rangle}{2}s\right)e^{-s/M_B^2}ds, \tag{59}$$

$$\Pi_{|\mathrm{GGG};1^{--}\rangle}(s_0,M_B^2) = \int_0^{s_0}\left(\frac{4\alpha_s^3}{81\pi}s^4-\frac{10\alpha_s^2\langle g_s^2GG\rangle}{9}s^2 + \frac{25\alpha_s^3\langle g_s^2GG\rangle}{36\pi}s^2+\frac{35\alpha_s^2\langle g_s^3G^3\rangle}{27}s\right)e^{-s/M_B^2}ds, \tag{60}$$

$$\Pi_{|\mathrm{GGG};2^{--}\rangle}(s_0,M_B^2) = \int_0^{s_0}\left(\frac{\alpha_s^3}{324\pi}s^4+\frac{5\alpha_s^2\langle g_s^2GG\rangle}{108}s^2 + \frac{15\alpha_s^3\langle g_s^2GG\rangle}{128\pi}s^2+\frac{5\alpha_s^2\langle g_s^3G^3\rangle}{24}s\right)e^{-s/M_B^2}ds, \tag{61}$$

$$\Pi_{|\mathrm{GGG};3^{--}\rangle}(s_0,M_B^2) = \int_0^{s_0}\left(\frac{5\alpha_s^3}{2016\pi}s^4-\frac{\alpha_s^2\langle g_s^2GG\rangle}{16}s^2 - \frac{49\alpha_s^3\langle g_s^2GG\rangle}{1536\pi}s^2-\frac{11\alpha_s^2\langle g_s^3G^3\rangle}{432}s\right)e^{-s/M_B^2}ds. \tag{62}$$

The above three-gluon glueball currents of $C=-$ have been systematically studied in Ref. [50], but there we did not calculate the Feynman diagrams depicted in Figs. 2 ($c-3$, $c-4$, $c-5$). Similar to Eqs. (43)–(50), we find all the $D=8$ terms proportional to $\langle g_s^2GG\rangle^2$ vanish, so the convergence of the above OPE series are also quite good.

We shall use the above sum rule equations to perform numerical analyses in the next section.

IV. NUMERICAL ANALYSES

In this section we perform numerical analyses using the sum rules given in Eqs. (43)–(50) and (57)–(62). The glueball mass M_X depends significantly on the gluon condensates $\langle g_s^2GG\rangle$ and $\langle g_s^3G^3\rangle$, both of which are still not precisely known. In the present study we use the following values for these parameters [58,59]

$$\langle\alpha_sGG\rangle=(6.35\pm0.35)\times10^{-2}\ \mathrm{GeV}^4,$$
$$\langle g_s^3G^3\rangle=\langle\alpha_sGG\rangle\times(8.2\pm1.0)\ \mathrm{GeV}^2. \tag{63}$$

Additionally, we use the following value for the strong coupling constant at the QCD scale $\Lambda_{\mathrm{QCD}}=300$ MeV [60],

$$\alpha_s(Q^2)=\frac{4\pi}{11\ln(Q^2/\Lambda_{\mathrm{QCD}}^2)}. \tag{64}$$

We still take the current $\tilde{J}_0$ as an example, and use Eq. (39) to calculate the mass of $|\mathrm{GG};0^{-+}\rangle$. It depends on two free parameters, the Borel mass M_B and the threshold value s_0. We use two criteria to determine the Borel window. The first criterion is to ensure the convergence of OPE by requiring: a) the $\alpha_s^2\times g_s^2$ term $\alpha_s^2\langle g_s^2GG\rangle$ to be less than 5%, and b) the $D=6$ term $\alpha_s\langle g_s^3G^3\rangle$ to be less than 10%

$$\mathrm{CVG}_A\equiv\left|\frac{\Pi^{g_s^{n=6}}(s_0,M_B^2)}{\Pi(s_0,M_B^2)}\right|\le5\%, \tag{65}$$

$$\mathrm{CVG}_B\equiv\left|\frac{\Pi^{\mathrm{D}=6}(s_0,M_B^2)}{\Pi(s_0,M_B^2)}\right|\le10\%. \tag{66}$$

As shown in Fig. 3 using the dashed curves, we determine the lower limit of M_B to be $M_B^2\ge3.28\ \mathrm{GeV}^2$ when setting $s_0=9.0\ \mathrm{GeV}^2$.

The above condition is the cornerstone of a reliable sum rule analysis, where we have taken into account two terms

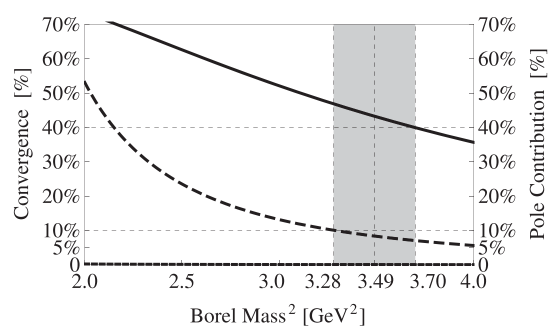

FIG. 3. CVG_A [short-dashed curve, defined in Eq. (65)], CVG_B [long-dashed curve, defined in Eq. (66)], and PC [solid curve, defined in Eq. (69)] as functions of the Borel mass M_B. The current $\tilde{J}_0$ is used here when setting $s_0=9.0\ \mathrm{GeV}^2$.

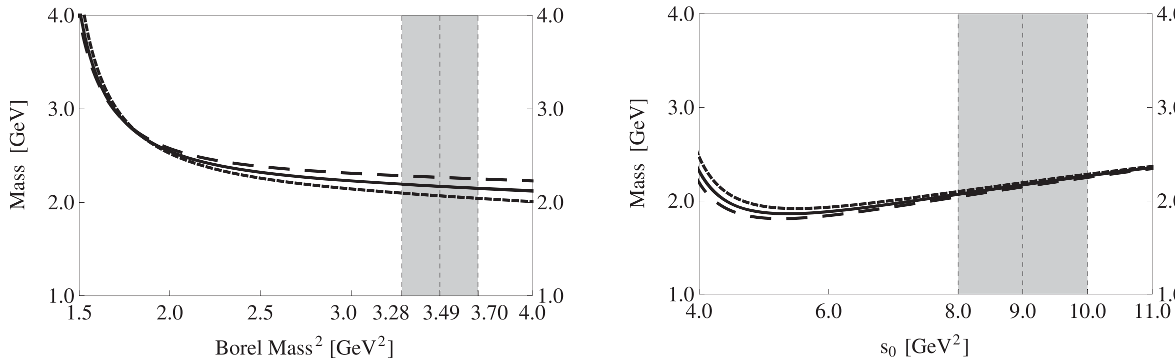

FIG. 4. Mass of the two-gluon glueball $|\mathrm{GG};0^{-+}\rangle$ as a function of the Borel mass M_B (left) and the threshold value s_0 (right), calculated using the current $\tilde{J}_0$. In the left panel the short-dashed/solid/long-dashed curves are obtained by setting $s_0 = 8.0/9.0/10.0\ \mathrm{GeV}^2$, respectively. In the right panel the short-dashed/solid/long-dashed curves are obtained by setting $M_B^2 = 3.28/3.49/3.70\ \mathrm{GeV}^2$, respectively.

because the OPE is expanded in two directions; the dimension of condensates and the coupling constant g_s. Equations (65) and (66) are for two-gluon glueball currents, and the conditions for three-gluon glueball currents are

$$\mathrm{CVG}'_A \equiv \left| \frac{\Pi^{g_s^{n=8}}(s_0, M_B^2)}{\Pi(s_0, M_B^2)} \right| \leq 5\%, \tag{67}$$

$$\mathrm{CVG}'_B \equiv \left| \frac{\Pi^{\mathrm{D}=6}(s_0, M_B^2)}{\Pi(s_0, M_B^2)} \right| \leq 10\%. \tag{68}$$

The second criterion is to insure the one-pole-dominance assumption by requiring the pole contribution (PC) to be larger than 40%,

$$\mathrm{PC} \equiv \left| \frac{\Pi(s_0, M_B^2)}{\Pi(\infty, M_B^2)} \right| \geq 40\%. \tag{69}$$

As shown in Fig. 3 (using the solid curve), we determine the upper limit of M_B to be $M_B^2 \leq 3.70\ \mathrm{GeV}^2$ when setting $s_0 = 9.0\ \mathrm{GeV}^2$.

Altogether we determine the Borel window to be $3.28\ \mathrm{GeV}^2 \leq M_B^2 \leq 3.70\ \mathrm{GeV}^2$ for the fixed threshold value $s_0 = 9.0\ \mathrm{GeV}^2$. Then we redo the same procedures by changing s_0, and find that there exists nonvanishing Borel windows as long as $s_0 \geq s_0^{\mathrm{min}} = 8.2\ \mathrm{GeV}^2$. Accordingly, we choose s_0 to be slightly larger, and determine our working regions to be $8.0\ \mathrm{GeV}^2 \leq s_0 \leq 10.0\ \mathrm{GeV}^2$ and $3.28\ \mathrm{GeV}^2 \leq M_B^2 \leq 3.70\ \mathrm{GeV}^2$, where we calculate the mass of $|\mathrm{GG};0^{-+}\rangle$ to be

$$M_{|\mathrm{GG};0^{-+}\rangle} = 2.17 \pm 0.11\ \mathrm{GeV}. \tag{70}$$

Its central value corresponds to $M_B^2 = 3.49\ \mathrm{GeV}^2$ and $s_0 = 9.0\ \mathrm{GeV}^2$, its uncertainty comes from the threshold value s_0, the Borel mass is M_B, and the gluon condensates are listed in Eqs. (63).

We show $M_{|\mathrm{GG};0^{-+}\rangle}$ in the left panel of Fig. 4 as a function of the Borel mass M_B, and find it quite stable inside the Borel window $3.28\ \mathrm{GeV}^2 \leq M_B^2 \leq 3.70\ \mathrm{GeV}^2$. We also show it in the right panel of Fig. 4 as a function of the threshold value s_0. We find there exists a mass minimum around $s_0 \sim 5\ \mathrm{GeV}^2$, and the s_0 dependence is weak and acceptable inside the working region $8.0\ \mathrm{GeV}^2 \leq s_0 \leq 10.0\ \mathrm{GeV}^2$.

Similarly, we use the sum rules given in Eqs. (43)–(50) and (57)–(62) to perform numerical analyses, and calculate masses of two- and three-gluon glueballs systematically. The obtained results are summarized in Table I, where we choose threshold values s_0 for two-gluon glueballs to be around 9–10 GeV^2, and those for three-gluon glueballs to be around 33–38 GeV^2.

V. SUMMARY AND DISCUSSIONS

In this paper we study two- and three-gluon glueballs of $C = +$ using the method of QCD sum rules, including

(a) the two-gluon glueballs with the quantum numbers $J^{PC} = 0^{\pm+}$, 1^{-+}, and $2^{\pm+}$;
(b) the three-gluon glueballs with the quantum numbers $J^{PC} = 0^{\pm+}$, $1^{\pm+}$, and $2^{\pm+}$.

We systematically construct their interpolating currents, and find that all the spin-1 currents of $C = +$ vanish, suggesting that the "ground-state" spin-1 glueballs of $C = +$ do not exist within the relativistic framework. This behavior is consistent with lattice QCD calculations [11–14].

We use spin-0 and spin-2 glueball currents to perform QCD sum rule analyses, and calculate the masses of their corresponding spin-0 and spin-2 glueballs. All these spin-2 currents have four Lorentz indices with certain symmetries, so that they couple to both positive- and negative-parity glueballs, which need to be further separated at the hadron level. We refer to Ref. [50] for detailed discussions.

We summarize the obtained results in Table II, which are compared with the lattice QCD results obtained using

TABLE I. QCD sum rule results of two- and three-gluon glueballs.

Glueball	Current	s_0^{min} [GeV^2]	Working Regions s_0 [GeV^2]	Working Regions M_B^2 [GeV^2]	Pole [%]	Mass [GeV]
$\lvert GG;0^{++}\rangle$	J_0	7.8	9.0 ± 1.0	3.70–4.19	40–48	$1.78^{+0.14}_{-0.17}$
$\lvert GG;2^{++}\rangle$	$J_2^{\alpha_1\alpha_2,\beta_1\beta_2}$	8.5	10.0 ± 1.0	3.99–4.60	40–50	$1.86^{+0.14}_{-0.17}$
$\lvert GG;0^{-+}\rangle$	$\tilde{J}_0$	8.2	9.0 ± 1.0	3.28–3.70	40–47	$2.17^{+0.11}_{-0.11}$
$\lvert GG;2^{-+}\rangle$	$\tilde{J}_2^{\alpha_1\alpha_2,\beta_1\beta_2}$	8.1	10.0 ± 1.0	3.27–4.20	40–55	$2.24^{+0.11}_{-0.11}$
$\lvert GGG;0^{++}\rangle$	η_0	31.6	33.0 ± 3.0	7.25–7.61	40–44	$4.46^{+0.17}_{-0.19}$
$\lvert GGG;2^{++}\rangle$	$\eta_2^{\alpha_1\alpha_2,\beta_1\beta_2}$	16.0	35.0 ± 3.0	4.77–9.04	40–90	$4.18^{+0.19}_{-0.42}$
$\lvert GGG;0^{-+}\rangle$	$\tilde{\eta}_0$	17.0	33.0 ± 3.0	4.48–8.13	40–88	$4.13^{+0.18}_{-0.36}$
$\lvert GGG;2^{-+}\rangle$	$\tilde{\eta}_2^{\alpha_1\alpha_2,\beta_1\beta_2}$	33.1	35.0 ± 3.0	8.10–8.53	40–44	$4.29^{+0.20}_{-0.22}$
$\lvert GGG;1^{+-}\rangle$	$\xi_1^{\alpha\beta}$	9.0	34.0 ± 4.0	3.16–9.09	40–99	$4.01^{+0.26}_{-0.95}$
$\lvert GGG;2^{+-}\rangle$	$\xi_2^{\alpha_1\alpha_2,\beta_1\beta_2}$	32.7	35.0 ± 4.0	7.53–8.09	40–46	$4.42^{+0.24}_{-0.29}$
$\lvert GGG;3^{+-}\rangle$	$\xi_3^{\alpha_1\alpha_2\alpha_3,\beta_1\beta_2\beta_3}$	30.2	33.0 ± 4.0	7.69–8.40	40–47	$4.30^{+0.23}_{-0.26}$
$\lvert GGG;1^{--}\rangle$	$\tilde{\xi}_1^{\alpha\beta}$	31.2	34.0 ± 4.0	5.81–6.77	40–51	$4.91^{+0.20}_{-0.18}$
$\lvert GGG;2^{--}\rangle$	$\tilde{\xi}_2^{\alpha_1\alpha_2,\beta_1\beta_2}$	19.7	36.0 ± 4.0	5.80–9.47	40–81	$4.25^{+0.22}_{-0.33}$
$\lvert GGG;3^{--}\rangle$	$\tilde{\xi}_3^{\alpha_1\alpha_2\alpha_3,\beta_1\beta_2\beta_3}$	35.8	38.0 ± 4.0	6.15–7.22	40–49	$5.59^{+0.33}_{-0.22}$

TABLE II. Masses of two- and three-gluon glueballs, in units of GeV. Our QCD sum rule results are listed in the second column. Lattice QCD results are listed in the third–sixth columns, taken from Refs. [11–13] (quenched) and Ref. [14] (unquenched).

Glueball	QCD sum rules	Ref. [11]	Ref. [12]	Ref. [13]	Ref. [14]
$\lvert GG;0^{++}\rangle$	$1.78^{+0.14}_{-0.17}$	$1.71\pm0.05\pm0.08$	$1.73\pm0.05\pm0.08$	$1.48\pm0.03\pm0.07$	1.80 ± 0.06
$\lvert GG;2^{++}\rangle$	$1.86^{+0.14}_{-0.17}$	$2.39\pm0.03\pm0.12$	$2.40\pm0.03\pm0.12$	$2.15\pm0.03\pm0.10$	2.62 ± 0.05
$\lvert GG;0^{-+}\rangle$	$2.17^{+0.11}_{-0.11}$	$2.56\pm0.04\pm0.12$	$2.59\pm0.04\pm0.13$	$2.25\pm0.06\pm0.10$	$\cdots$
$\lvert GG;2^{-+}\rangle$	$2.24^{+0.11}_{-0.11}$	$3.04\pm0.04\pm0.15$	$3.10\pm0.03\pm0.15$	$2.78\pm0.05\pm0.13$	3.46 ± 0.32
$\lvert GGG;0^{++}\rangle$	$4.46^{+0.17}_{-0.19}$	$\cdots$	$2.67\pm0.18\pm0.13$	$2.76\pm0.03\pm0.12$	3.76 ± 0.24
$\lvert GGG;2^{++}\rangle$	$4.18^{+0.19}_{-0.42}$	$\cdots$	$\cdots$	$2.88\pm0.10\pm0.13$	$\cdots$
$\lvert GGG;0^{-+}\rangle$	$4.13^{+0.18}_{-0.36}$	$\cdots$	$3.64\pm0.06\pm0.18$	$3.37\pm0.15\pm0.15$	4.49 ± 0.59
$\lvert GGG;2^{-+}\rangle$	$4.29^{+0.20}_{-0.22}$	$\cdots$	$\cdots$	$3.48\pm0.14\pm0.16$	$\cdots$
$\lvert GGG;1^{+-}\rangle$	$4.01^{+0.26}_{-0.95}$	$2.98\pm0.03\pm0.14$	$2.94\pm0.03\pm0.14$	$2.67\pm0.07\pm0.12$	3.27 ± 0.34
$\lvert GGG;2^{+-}\rangle$	$4.42^{+0.24}_{-0.29}$	$4.23\pm0.05\pm0.20$	$4.14\pm0.05\pm0.20$	$\cdots$	$\cdots$
$\lvert GGG;3^{+-}\rangle$	$4.30^{+0.23}_{-0.26}$	$3.60\pm0.04\pm0.17$	$3.55\pm0.04\pm0.17$	$3.27\pm0.09\pm0.15$	3.85 ± 0.35
$\lvert GGG;1^{--}\rangle$	$4.91^{+0.20}_{-0.18}$	$3.83\pm0.04\pm0.19$	$3.85\pm0.05\pm0.19$	$3.24\pm0.33\pm0.15$	$\cdots$
$\lvert GGG;2^{--}\rangle$	$4.25^{+0.22}_{-0.33}$	$4.01\pm0.05\pm0.20$	$3.93\pm0.04\pm0.19$	$3.66\pm0.13\pm0.17$	4.59 ± 0.74
$\lvert GGG;3^{--}\rangle$	$5.59^{+0.33}_{-0.22}$	$4.20\pm0.05\pm0.20$	$4.13\pm0.09\pm0.20$	$4.33\pm0.26\pm0.20$	$\cdots$–

nonrelativistic-glueball operators [11–14]. For completeness, we also reanalyze the results of $C=-$ three-gluon glueballs (also called odderons), which have been previously studied in Ref. [50]. Thus, a rather complete QCD sum rule study has been done on the lowest-lying glueballs composed of two or three valence gluons. We find that our QCD sum rule results are generally consistent with unquenched lattice QCD results [14].

To end this paper, we briefly discuss possible decay patterns of two- and three-gluon glueballs. The two-gluon glueballs can decay after exciting two quark-antiquark pairs, and recombine into two mesons; however, it is rather difficult to differentiate them from standard $q\bar{q}$ states. The three-gluon glueballs can decay after exciting three quark-antiquark pairs, and recombine into three mesons. Their possible decay patterns are

$$
\begin{aligned}
0^{-+} &\to \quad VVP, VVV \quad (S\text{-wave}),\\
0^{++} &\to \quad VPP, VVP, VVV \quad (P\text{-wave}),\\
1^{--} &\to \quad VPP, VVP, VVV \quad (S\text{-wave}),\\
1^{+-} &\to PPP, VPP, VVP, VVV \quad (P\text{-wave}),\\
2^{-\pm} &\to \quad VVP, VVV \quad (S\text{-wave}),\\
2^{+\pm} &\to \quad VPP, VVP, VVV \quad (P\text{-wave}),\\
3^{--} &\to \quad VVV \quad (S\text{-wave}),\\
3^{+-} &\to \quad VVP, VVV \quad (P\text{-wave}),
\end{aligned}
$$

where P and V denote light vector and pseudoscalar mesons, respectively. Considering their limited decay patterns, the $J^{PC} = 0^{-+}/2^{-\pm}/3^{\pm-}$ three-gluon glueballs may have relatively smaller widths, and we propose to search for them in their VVV and VVP decay channels in future BESIII, GlueX, LHC, and PANDA experiments.

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APPENDIX: SPIN-1 CURRENTS OF $C = +$

In this Appendix we prove that the three spin-1 currents $J_1^{\alpha\beta}$, $\eta_1^{\alpha\beta}$, and $\tilde{\eta}_1^{\alpha\beta}$ all vanish. Their definitions are given in Eqs. (15), (27), and (28), respectively. For simplicity, we shall not differentiate the superscript and subscript in the following calculations.

Firstly, we investigate the current $J_1^{\alpha\beta}$. Due to the Lorentz invariant, we simply assume $\alpha = 0$ and $\beta = 1\ldots3$. Additionally, we need the Lorentz indices $\mu = 0/i$, $\rho = 0/k$, and $\sigma = 0/l$, with $i/k/l = 1\ldots3$. We obtain

$$
\begin{aligned}
2J_1^{0\beta} &= 2G_a^{0\mu}\tilde{G}_a^{\beta\mu} - \{0 \leftrightarrow \beta\},\\
&= G_a^{0\mu}G_a^{\rho\sigma}\epsilon^{\beta\mu\rho\sigma} - G_a^{\beta\mu}G_a^{\rho\sigma}\epsilon^{0\mu\rho\sigma}\\
&= G_a^{0i}G_a^{k0}\epsilon^{\beta ik0} + G_a^{0i}G_a^{0l}\epsilon^{\beta i0l} - G_a^{\beta i}G_a^{kl}\epsilon^{0ikl}. \quad \text{(A1)}
\end{aligned}
$$

After interchanging $i \leftrightarrow k$, the first term turns out to be zero,

$$
G_a^{0i}G_a^{k0}\epsilon^{\beta ik0} = G_a^{0k}G_a^{i0}\epsilon^{\beta ki0} = G_a^{0i}G_a^{k0}\epsilon^{\beta ki0} \to 0, \quad \text{(A2)}
$$

as does the second term. The third term is nonzero when $\beta = k$ or $\beta = l$; however, for the case $\beta = k$, we can interchange $i \leftrightarrow l$ and obtain (not sum over β here),

$$
G_a^{\beta i}G_a^{\beta l}\epsilon^{0i\beta l} = G_a^{\beta l}G_a^{\beta i}\epsilon^{0l\beta i} = G_a^{\beta i}G_a^{\beta l}\epsilon^{0l\beta i} \to 0, \quad \text{(A3)}
$$

similar to the case $\beta = l$. Therefore, the third term is also zero, and the current $J_1^{\alpha\beta}$ vanishes.

Secondly, we investigate the current $\eta_1^{\alpha\beta}$,

$$
\begin{aligned}
2\eta_1^{\alpha\beta} &= 2f_{abc}\tilde{G}_a^{\mu\nu}G_b^{\mu\nu}G_c^{\alpha\beta}\\
&= f_{abc}\epsilon^{\mu\nu\rho\sigma}G_a^{\rho\sigma}G_b^{\mu\nu}G_c^{\alpha\beta}\\
&= f_{abc}\epsilon^{\mu\nu\rho\sigma}G_a^{\mu\nu}G_b^{\rho\sigma}G_c^{\alpha\beta}\\
&= -f_{abc}\epsilon^{\mu\nu\rho\sigma}G_b^{\mu\nu}G_a^{\rho\sigma}G_c^{\alpha\beta}\\
&\to 0. \quad \text{(A4)}
\end{aligned}
$$

In the above expressions, we have interchanged $\mu\nu \leftrightarrow \rho\sigma$ and $a \leftrightarrow b$. Similarly, we can prove the current $\tilde{\eta}_1^{\alpha\beta}$ to be zero.

One can construct more spin-1 three-gluon glueball currents of $C = +$, such as

$$
\eta_1'^{\alpha\beta} = f_{abc}G_a^{\alpha\mu}G_b^{\mu\nu}G_c^{\nu\beta} - \{\alpha \leftrightarrow \beta\}, \quad \text{(A5)}
$$

$$
\tilde{\eta}_1'^{\alpha\beta} = f_{abc}G_a^{\alpha\mu}G_b^{\mu\nu}\tilde{G}_c^{\nu\beta} - \{\alpha \leftrightarrow \beta\}. \quad \text{(A6)}
$$

It is straightforward to prove the former current $\eta_1'^{\alpha\beta}$ to be zero,

$$
\begin{aligned}
\eta_1'^{\alpha\beta} &= f_{abc}G_a^{\alpha\mu}G_b^{\mu\nu}G_c^{\nu\beta} - \{\alpha \leftrightarrow \beta\}\\
&= f_{abc}G_a^{\alpha\nu}G_b^{\nu\mu}G_c^{\mu\beta} - \{\alpha \leftrightarrow \beta\}\\
&= -f_{abc}G_a^{\beta\nu}G_b^{\nu\mu}G_c^{\mu\alpha} + \{\alpha \leftrightarrow \beta\}\\
&= f_{abc}G_c^{\beta\nu}G_b^{\nu\mu}G_a^{\mu\alpha} - \{\alpha \leftrightarrow \beta\}\\
&= -f_{abc}G_c^{\nu\beta}G_b^{\mu\nu}G_a^{\alpha\mu} - \{\alpha \leftrightarrow \beta\}\\
&\to 0. \quad \text{(A7)}
\end{aligned}
$$

It is a bit tricky but one can still prove the latter current $\tilde{\eta}_1'^{\alpha\beta}$ to be zero, after explicitly writing out all its Lorentz indices. We have done this using the software *Mathematica*.

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