Hidden charm hadronic molecule with strangeness $P_{cs}^*(4739)$ as a $\Sigma_c\bar{D}\bar{K}$ bound state

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Motivated by the recent discovery of the first hidden charm pentaquark state with strangeness $P_{cs}(4459)$ by the LHCb Collaboration, we study the likely existence of a three-body $\Sigma_c\bar{D}\bar{K}$ bound state, which shares the same minimal quark content as $P_{cs}(4459)$. The $\Sigma_c\bar{D}$ and DK interactions are determined by reproducing $P_c(4312)$ and $D_{s0}^*(2317)$ as $\Sigma_c\bar{D}$ and $\bar{D}\bar{K}$ molecules, respectively, while the $\Sigma_c\bar{K}$ interaction is constrained by chiral effective theory. We indeed find a three-body bound state by solving the Schrödinger equation using the Gaussian expansion method, which can be viewed as an excited hidden charm exotic state with strangeness, $P_{cs}^*(4739)$, with $I(J^P) = 1(1/2^+)$ and a binding energy of $77.8^{+25}_{-10.3}$ MeV. We further study its strong decays via triangle diagrams and show that its partial decay widths into $D\Xi_c'$ and $D_s^*\Sigma_c$ are of a few tens of MeV, with the former being dominant.

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I. INTRODUCTION

Since 2003, a large number of exotic hadronic states have been discovered, which attracted a lot of attention both theoretically and experimentally. Though their nature is largely undetermined, the hadronic molecular interpretation of these states has become rather popular because of the fact that many of them are located close to the thresholds of two conventional hadrons. For example, the well-known $X(3872)$, which is just located at the $\bar{D}D^*$ threshold, can be naturally explained as a $D^*\bar{D}$ molecule [1]. The molecular picture can also well explain its isospin-breaking decay [2–4] and the ratio between its radiative decays [5,6], for which, however, there still exist different interpretations [7–9]. Meanwhile, there are some claims that $X(3872)$ is not a pure molecule [10], but a hybrid state of charmonium and hadronic molecules [11,12]. It is interesting to note that after about 20 years of study, the nature of $X(3872)$ still remains unclear and more clarifications are needed both theoretically and experimentally.

Another intriguing exotic state is $D_{s0}^*(2317)$ [13–15], which is located 45 MeV below the DK threshold and has a decay width less than 3.8 MeV. The observed mass and width are far away from the predicted mass and width in the naive quark model, which are about 2460 MeV and hundreds of MeV respectively [16], thus $D_{s0}^*(2317)$ is difficult to be interpreted as a conventional $c\bar{s}$ meson. On the other hand, due to the strongly attractive DK interaction predicted by chiral perturbation theory and lattice QCD [17,18], $D_{s0}^*(2317)$ can be easily explained as a DK molecule [17–37]. The same molecular picture is adopted to study the pentaquark states, i.e., $P_c(4312)$, $P_c(4440)$, and $P_c(4457)$, discovered by the LHCb Collaboration [38]. These pentaquark states are explained as $\Sigma_c\bar{D}^{(*)}$ molecules due to the fact they are close to the thresholds of $\Sigma_c\bar{D}^{(*)}$ [39–54]. Actually, the $\Sigma_c\bar{D}^{(*)}$ interactions have already been investigated by several groups [55–58] before the 2015 experimental discovery [59].

The molecular explanation of some exotic states can be extended by symmetries, such as the heavy quark symmetry and the $SU(3)$ flavor symmetry. The $D_{s1}(2460)$ state, the heavy quark spin partner of $D_{s0}^*(2317)$, can also be interpreted as a D^*K molecule with heavy quark spin symmetry [22,31,36]. In this doublet molecular picture the

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TABLE I. Hadron masses needed in this work (in units of GeV).

Meson	D_{s0}^-	D_s^{*-}	K^-	$\bar{K}^0$	D^-	$\bar{D}^0$
Mass	2.3178	2.1122	0.493677	0.497611	1.86965	1.86483
Baryon	$\Xi_c'^+$	$\Xi_c'^0$	Σ_c^{++}	Σ_c^+	Σ_c^0	P_c
Mass	2.5774	2.5788	2.45397	2.4529	2.45375	4.3119

mass splitting of $D_{s0}^*(2317)$ and $D_{s1}(2460)$ can be easily understood, which supports the molecular interpretation of $D_{s0}^*(2317)$ and $D_{s1}(2460)$. Recently, a new structure was observed in the $J/\psi\Lambda$ invariant mass distribution of the $\Xi_b^- \to J/\psi\Lambda K^-$ decay by the LHCb Collaboration [60], which was predicted in Refs. [61–63]. This structure is consistent with a charmonium pentaquark state with strangeness denoted as $P_{cs}(4459)$, which could be viewed as a SU(3)-flavor symmetry partner of the P_c pentaquark states [64–68]. Motivated by this new observation and the molecular picture for $D_{s0}^*(2317)$ and the P_c states, we study the $\Sigma_c\bar{D}\bar{K}$ three-body system, whose minimum quark content is $c\bar{c}sqq$, the same as $P_{cs}(4459)$, to check whether there exist hidden charm fermionic three-body bound states, and to explore its possible decays.

It should be mentioned that hadronic few-body systems have been extensively studied by different methods in the charm sector, such as the hidden charmed $D\bar{D}K$ [69] and $D\bar{D}^*K$ [69–71], the $BB\bar{K}$ system [72], the $\pi D\bar{D}$ [73], $\rho D\bar{D}$ [74], $BD\bar{D}$ [75] and $\Xi_{cc}\bar{\Xi}_{cc}\bar{K}$ [76], the singly charmed DNN [77], $DK\bar{K}$ [78,79], and $NDK(ND\bar{K})$ [80], the doubly charmed DDK [81–84], BDD [75] and DD^*K [70] systems, the triply charmed four-body $DDDK$ [82], and the quadruply charmed $\Xi_{cc}\Xi_{cc}\bar{K}$ [76] systems. For recent reviews, see Refs. [85,86].

The decay of three-body bound states have also attracted much attention. In Ref. [83], the decay of the DDK bound state has been studied via the triangle mechanism. From the conclusion of recent works [81,82], the DDK bound state is mainly made of $DD_{s0}^*(2317)$, accordingly the decay width of DDK can be estimated through $DD_{s0}^*(2317)$ to DD_s^*/D^*D_s by exchanging K/η [83]. Using the same approach, the decay width of $D\bar{D}^{(*)}K$ was also calculated [76,87]. In addition, the $D\bar{D}^*K$ production rate in the B meson decay was studied in Ref. [88]. Although there are many studies on few-body systems in the heavy hadron sector, the $\Sigma_c\bar{D}\bar{K}$ system could generate the first hidden charm fermionic three-body bound state, which is likely to be found at the current facilities, especially considering the successful discoveries of the P_c and P_{cs} states.

The manuscript is organized as follows, In Sec. II we explain how we parametrize and determine the two-body interaction input. In Sec. III we explain how to construct the three-body wave functions and solve the $\Sigma_c\bar{D}\bar{K}$ three-body system. In Sec. IV we present our predictions of the $\Sigma_c\bar{D}\bar{K}$ bound state and study its strong decay. Finally, a short summary is given in Sec. V.

II. TWO-BODY INTERACTIONS

The study of a three-body system depends on the sub-two-body interactions. For the $\Sigma_c\bar{D}\bar{K}$ system, we need to know the $\Sigma_c\bar{D}$, $\bar{D}\bar{K}$, and $\Sigma_c\bar{K}$ interactions. The $\Sigma_c\bar{D}$ and DK interactions, as mentioned in the Introduction, are attractive enough to form bound states, namely $P_c(4312)$ and $D_{s0}^*(2317)$. Therefore we could determine the interactions by reproducing the two states. For the case of the $\Sigma_c\bar{K}$ system, there is no such information, we would resort to chiral perturbation theory and relate it to the $\bar{K}N$ interaction. For the masses of the particles used in the present study, we refer to Table I.

For the $\Sigma_c\bar{D}$ interaction, we refer to the contact-range effective field theory of Ref. [39], in which the $\Sigma_c^{(*)}\bar{D}^{(*)}$ interactions are constructed to explain $P_c(4312)$, $P_c(4440)$, and $P_c(4457)$ as part of a heavy-quark spin symmetry (HQSS) multiplet. The $\Sigma_c\bar{D}$ potential in the contact-range effective field theory reads

$$V\left(\frac{1^-}{2}, \Sigma_c\bar{D}\right) = C_a, \tag{1}$$

with C_a a coupling constant. In Ref. [39], the authors regularized the potential with a separable form factor and a cutoff Λ in momentum space and allowed the couplings to depend on the cutoff

$$\langle p|V_\Lambda|p'\rangle = C_\Lambda f\left(\frac{p}{\Lambda}\right) f\left(\frac{p'}{\Lambda}\right). \tag{2}$$

Here we propose a similar treatment in coordinate space. The contact potential of Eq. (1) in coordinate space can be obtained by Fourier transformation

$$V_{\Sigma_c\bar{D}}(\vec{r}) = C_a\delta^{(3)}(\vec{r}), \tag{3}$$

which is singular and requires regularization. For this purpose we choose a Gaussian regulator of the type

$$V_{\Sigma_c\bar{D}}(\vec{r}) = C_a\frac{e^{-(r/R_a)^2}}{\pi^{3/2}R_a^3}, \tag{4}$$

where R_c is the cutoff we use to smear the delta function. However, the previous expression is still problematic, as the prediction of a $\Sigma_c\bar{D}$ bound state and its binding energy depends on both the coupling C_a and the cutoff. This can be

solved by taking C_a cutoff dependent and therefore one is left with a renormalized potential

$$V_{\Sigma_c\bar{D}}(r;R_a) = C(R_a)e^{-(r/R_a)^2}, \tag{5}$$

with R_a the cutoff and $C(R_a)$ the running coupling constant fixed by fitting to the bound state of $\Sigma_c\bar{D}$ with a binding energy of 8.9 MeV, which corresponds to $P_c(4312)$.

The most important contribution to the DK interaction is the Weinberg-Tomozawa term between a kaon and a charmed meson [17], which in nonrelativistic normalization reads

$$V_{WT}(DK) = -\frac{C_{WT}(I)}{2f_\pi^2}, \tag{6}$$

with $f_\pi \simeq 130$ MeV and $C_{WT}(0) = 2$, $C_{WT}(1) = 0$, for the isoscalar and isovector channels, respectively. Following the same logic in treating the $\Sigma_c\bar{D}$ interaction, the isoscalar contact-range DK interaction can be Fourier transformed into coordinate space and represented by a Gaussian shape potential that was already adopted in our previous works [69,82]

$$V_{DK}(r;R_b) = C(R_b)e^{-(r/R_b)^2}, \tag{7}$$

where R_b is a cutoff, $C(R_b)$ is a running coupling constant related to R_b, which can be determined by fitting to a binding energy of 45 MeV for the DK bound state corresponding to $D_{s0}^*(2317)$.

For the $\Sigma_c\bar{K}$ interaction, we will resort to the unitarized chiral perturbation theory developed in Ref. [89] to describe the interactions between a ground-state singly charmed (bottom) baryon and a pseudo-Nambu-Goldstone boson. The leading-order chiral-effective Lagrangian reads

$$\mathcal{L} = \frac{i}{16f_0^2}\mathrm{Tr}(\bar{H}_{[6]}(x)\gamma^\mu[H_{[6]}(x),[\phi(x),(\partial_\mu\phi(x))]_-]_+), \tag{8}$$

where the $\bar{H}_{[6]}$ and ϕ collect the sextet charmed baryons and Goldstone bosons respectively (for details we refer to Ref. [89]). The Lagrangian above leads to the well-known Weinberg-Tomozawa term

$$V_{WT}(\Sigma_c\bar{K}) = \frac{C_{ij}}{4f_0^2}(\not{k}_2 + \not{k}_4), \tag{9}$$

with the coupling $C_{ij} = -3$ [90], k_2 and k_4 the momentum of the incoming and outgoing Kaons. Neglecting subleading corrections,[1] the $\Sigma_c\bar{K}$ interaction is the same as the $N\bar{K}$ interaction (see, for example, those in Refs. [91–93]). Thus

[1]In Ref. [82], we constructed a repulsive core to describe the NLO repulsive contribution to the DK interaction, and found that the subleading correction only influences the three-body binding energies by less than 1 MeV, and thus can be neglected.

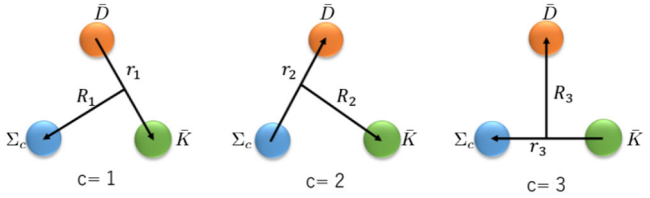

FIG. 1. Three permutations of the Jacobi coordinates for the $\Sigma_c\bar{D}\,\bar{K}$ system.

the $\Sigma_c\bar{K}$ potential is taken to be of the same form as the $N\bar{K}$ potential of Ref. [76]

$$V_{\Sigma_c\bar{K}}(r;R_c) \simeq C(R_c)e^{-(r/R_c)^2}, \tag{10}$$

while the coupling $C(R_c)$ is determined by reproducing the binding energy of $\Lambda(1405)$ as a $N\bar{K}$ bound state. A series of works indicated that the $\Lambda(1405)$ has a two-pole structure [94–96], where the upper pole mainly couples to the $N\bar{K}$ channel and the lower one mainly couples to the $\Sigma\pi$ channel. In this work, we used the nominal mass of $\Lambda(1405)$ [97] to determine the $N\bar{K}$ interaction. To estimate the uncertainties of the $\Sigma_c\bar{D}$, DK, and $\Sigma_c\bar{K}$ interactions, we vary the cutoffs R_a, R_b, and R_c from 0.5 fm to 2.0 fm. In principle, the cutoffs can be different for each sub-two-body system. However, because the uncertainties of this system mainly originate from the cutoff R_c (see Sec. IV), we choose the cutoffs R_a and R_b the same as R_c ranging from 0.5 fm to 2.0 fm in the following numerical study.

III. GAUSSIAN EXPANSION METHOD

Once all the relevant sub-two-body interactions are fixed as specified above, we employ the Gaussian expansion method [98] to solve the Schrödinger equation to study the three-body $\Sigma_c\bar{D}\,\bar{K}$ system. The corresponding Schrödinger equation is

$$\hat{H}\Psi_{JM}^{\text{total}} = E\Psi_{JM}^{\text{total}}, \tag{11}$$

with the following Hamiltonian

$$\hat{H} = \sum_{i=1}^{3}\frac{p_i^2}{2m_i} - T_{\text{c.m.}} + V_{\bar{D}\bar{K}}(r_1) + V_{\Sigma_c\bar{D}}(r_2) + V_{\bar{K}\Sigma_c}(r_3), \tag{12}$$

where $T_{\text{c.m.}}$ is the kinetic energy of the center of mass and $V(r)$'s are the potentials between the two relevant particles. The three Jacobian coordinates for the $\Sigma_c\bar{D}\,\bar{K}$ system are shown in Fig. 1.

The total wave function is a sum of the amplitudes of the three rearrangement channels ($c = 1$–3) written in Jacobian coordinates,

TABLE II. Isospin factors used in calculating the Hamiltonian matrix elements.

	$H^{c=1}_{1,0}$	$H^{c=2}_{1,1/2}$	$H^{c=3}_{1,1/2}$
$H^{c=1}_{1,0}$	1	$-\frac{1}{\sqrt{3}}$	$-\frac{1}{\sqrt{3}}$
$H^{c=2}_{1,1/2}$	$-\frac{1}{\sqrt{3}}$	1	$-\frac{1}{3}$
$H^{c=3}_{1,1/2}$	$-\frac{1}{\sqrt{3}}$	$-\frac{1}{3}$	1

$$\Psi^{\text{total}}_{JM} = \sum_{c,\alpha} C_{c,\alpha} \Psi^{c}_{JM,\alpha}(\mathbf{r}_c, \mathbf{R}_c), \tag{13}$$

where $\alpha = \{nl, NL, \Lambda, tT\}$ and $C_{c,\alpha}$ are the expansion coefficients. Here l and L are the orbital angular momentum for the coordinates r and R, t is the isospin of the two-body subsystem in each channel, Λ and T are the total orbital angular momentum and isospin, n and N are the numbers of Gaussian basis functions corresponding to coordinates r and R, respectively. The wave function of each channel has the following form

$$\Psi^{c}_{JM,\alpha}(\mathbf{r}_c, \mathbf{R}_c) = H^{c}_{T,t} \otimes [\Phi^{c}_{lL,\Lambda}]_{JM}, \tag{14}$$

where $H^{c}_{T,t}$ is the isospin wave function and $\Phi^{c}_{lL,\Lambda}$ is the orbital wave function. The total isospin wave function in each channel reads

$$\begin{aligned} H^{c=1}_{T,t_1} &= [[\eta_{\frac{1}{2}}(\bar{D})\eta_{\frac{1}{2}}(\bar{K})]_{t_1}\eta_1(\Sigma_c)]_T, \\ H^{c=2}_{T,t_2} &= [[\eta_1(\Sigma_c)\eta_{\frac{1}{2}}(\bar{D})]_{t_2}\eta_{\frac{1}{2}}(\bar{K})]_T, \\ H^{c=3}_{T,t_3} &= [[\eta_{\frac{1}{2}}(\bar{K})\eta_1(\Sigma_c)]_{t_3}\eta_{\frac{1}{2}}(\bar{D})]_T, \end{aligned} \tag{15}$$

where η is the isospin wave function of each particle. The isospin factors of this system are listed in Table II. The orbital wave function $\Phi^{c}_{lL,\Lambda}$ is given in terms of the Gaussian basis functions

$$\Phi^{c}_{lL,\Lambda}(\mathbf{r}_c, \mathbf{R}_c) = [\phi^{G}_{n_c l_c}(\mathbf{r}_c)\psi^{G}_{N_c L_c}(\mathbf{R}_c)]_\Lambda, \tag{16}$$

$$\phi^{G}_{nlm}(\mathbf{r}_c) = N_{nl} r_c^l e^{-\nu_n r_c^2} Y_{lm}(\hat{r}_c), \tag{17}$$

$$\psi^{G}_{NLM}(\mathbf{R}_c) = N_{NL} R_c^L e^{-\lambda_n R_c^2} Y_{LM}(\hat{R}_c). \tag{18}$$

Here $N_{nl}(N_{NL})$ is the normalization constant of the Gaussian basis and the parameters ν_n and λ_n are given by

$$\begin{aligned} &\nu_n = 1/r_n^2, \quad r_n = r_{\min} a^{n-1} \quad (n = 1, n_{\max}), \\ &\lambda_N = 1/R_N^2, \quad R_N = R_{\min} A^{N-1} \quad (N = 1, N_{\max}), \end{aligned} \tag{19}$$

where $\{n_{\max}, r_{\min}, a \text{ or } r_{\max}\}$ and $\{N_{\max}, R_{\min}, A \text{ or } R_{\max}\}$ are Gaussian basis parameters.

With the constraints of $D^*_{s0}(2317)$ as a DK bound state with quantum numbers $I(J^P) = 0(0^+)$ and Σ_c a $1(1/2^+)$ particle, the quantum numbers of the three-body $\Sigma_c \bar{D} \bar{K}$ system are $1(1/2^+)$ considering only S-wave interactions. More specific configurations used in the present study are listed in Table III. According to chiral perturbation theory [17], the leading-order DK interaction in the isospin-1 channel is vanishing as discussed in Sec. II, which leads to the fact that for isopin-0 and isospin-2, the $\Sigma_c \bar{D} \bar{K}$ system could not form three-body bound states, which has been confirmed numerically.

TABLE III. Quantum numbers and the numbers of Gaussian basis used in each Jacobi coordinate channel ($c = 1$–3) of the $\Sigma_c \bar{D} \bar{K}$ $I(J^P) = 1(\frac{1}{2}^+)$ system.

Channels	Coupling types	l	L	Λ	t	T	J	P	$n_{\max}$	$N_{\max}$
1	$(\bar{D}\bar{K})\Sigma_c$	0	0	0	0	1	1/2	+	10	10
2	$(\Sigma_c\bar{D})\bar{K}$	0	0	0	1/2	1	1/2	+	10	10
3	$(\bar{K}\Sigma_c)\bar{D}$	0	0	0	1/2	1	1/2	+	10	10

IV. PREDICTION OF A HIDDEN CHARM HADRONIC MOLECULAR STATE WITH STRANGENESS P^*_{cs} AND ITS STRONG DECAY

In this section, we predict the existence of a $\Sigma_c \bar{D} \bar{K}$ bound state and study its strong decays via triangle diagrams. The masses of the particles we used are found in Table I.

A. Prediction of a $\Sigma_c \bar{D} \bar{K}$ bound state as a hidden charm molecular state with strangeness

In this subsection, we study the likely existence of a $\Sigma_c \bar{D} \bar{K}$ three-body bound state formed with the regularized two-body potentials specified above. To estimate the uncertainties caused by the regulator, we vary the cutoff R_c between 0.5 fm and 2.0 fm. The $\Sigma_c \bar{D}$ and DK interactions are attractive enough to form bound states, namely $P_c(4312)$ and $D^*_{s0}(2317)$, respectively. Thus we determine the couplings C_a and C_b of these two interactions by reproducing the binding energies with respect to the corresponding thresholds. The results are summarized in Table IV, where the binding energy of the $\Sigma_c \bar{D}$ bound state is fixed at 8.9 MeV and that of DK is 45 MeV. For the $\Sigma_c \bar{K}$ subsystem, there is no direct experimental data, but one can resort to chiral perturbation theory and relate the $\Sigma_c \bar{K}$ interaction with the $N\bar{K}$ interaction via chiral

TABLE IV. Binding energies of the three-body $\Sigma_c \bar{D} \bar{K}$ system and the three two-body subsystems (in units of MeV) for three cutoffs R_c (in units of fm).

R_c	$B_2(DK)$	$B_2(\Sigma_c\bar{D})$	$B_2(\Sigma_c\bar{K})$	$B_3(\Sigma_c\bar{D}\bar{K})$
0.5	45	8.9	71.3	102.8
1.0	45	8.9	47.6	77.8
2.0	45	8.9	37.7	67.5

TABLE V. Root-mean-square radius (in units of fm) of the predicted $\Sigma_c\bar{D}\,\bar{K}$ bound states for different cutoffs R_C (in units of fm).

R_c	$r_3(DK)$	$r_3(\Sigma_c\bar{D})$	$r_3(\Sigma_c\bar{K})$
0.5	0.98	0.94	0.84
1.0	1.29	1.28	1.27
2.0	1.78	1.78	1.91

symmetry. In this case, the $\Sigma_c\bar{K}$ interaction can form a bound state, Ξ_c^*, with a binding energy ranging from 37.7 MeV to 71.3 MeV dependent on the cutoff R_c. It is interesting to note that once the cutoff is determined via the $\Lambda(1405)$, one can find a $\Sigma_c\bar{K}$ bound state, Ξ_c^*, whose binding energy is approximately twice that of $\Lambda(1405)$ as a $N\bar{K}$ bound state.[2]

For the $\Sigma_c\bar{D}\,\bar{K}$ system, we indeed find a bound state with quantum numbers $I(J^P) = 1(1/2^+)$ and a binding energy $77.8^{+25}_{-10.3}$ MeV, see Table IV. The central value is obtained with $R_c = 1.0$ fm while the uncertainties are obtained by taking $R_c = 0.5$ fm and 2.0 fm. Although the binding energy of the three-body bound state is cutoff dependent, the prediction on the existence of the $\Sigma_c\bar{D}\,\bar{K}$ bound state is robust. Since the main uncertainties come from the $\Sigma_c\bar{K}$ interaction, to show the robustness of the prediction on the existence of the $\Sigma_c\bar{D}\,\bar{K}$ bound state, we decrease the strength of the $\Sigma_c\bar{K}$ interaction to one half, one fifth, and one tenth (the lower limit to form a $\Sigma_c\bar{D}\,\bar{K}$ bound state) of the current value with a cutoff $R_c = 1.0$ fm. We find that the $\Sigma_c\bar{D}\,\bar{K}$ system still binds with a binding energy of 49.6 MeV, 47.0 MeV, and 45.0 MeV, respectively. While for the later two scenarios, the two-body $\Sigma_c\bar{K}$ system already becomes unbound, which indicated that the existence of the three-body $\Sigma_c\bar{D}\,\bar{K}$ bound state does not require all the two-body subsystems to be bound.

In Table V, we show the rms radii of the predicted $\Sigma_c\bar{D}\,\bar{K}$ bound state. The rms radius of the DK subsystem in this bound state ranges from 0.98 fm to 1.78 fm, increasing with the cutoff R_c, while those of the $\Sigma_c\bar{D}$ and $\Sigma_c\bar{K}$ subsystems range from 0.94 fm to 1.78 fm and from 0.84 fm to 1.91 fm, respectively. The rms radii of the $\Sigma_c\bar{D}\,\bar{K}$ bound state are strongly dependent on the cutoff R_c, because R_c determines the effective interaction range.

In Table VI, we present the expectation values of the Hamiltonian (potentials and kinetic energies) of the predicted $\Sigma_c\bar{D}\,\bar{K}$ bound state for different cutoffs, and give the weights of the two-body potentials with respect to the total potential. Although the absolute expectation values are strongly cutoff dependent, the relative weights of these potentials are rather stable. More specifically, the expectation value of the weight of the DK potential is from 42% to 59%, and those of the $\Sigma_c\bar{D}$ and $\Sigma_c\bar{K}$ potentials are from 0.6% to 1.5% and from 39.5% to 57.4%, respectively. This indicates that to the $\Sigma_c\bar{D}\,\bar{K}$ bound state, the DK and $\Sigma_c\bar{K}$ interaction contribute the most, which are dominant in this three-body system, while the $\Sigma_c\bar{D}$ interaction contributes the least; consistent with their interaction strengths.

TABLE VI. Expectation values of the Hamiltonian (potential and kinetic energies) (in units of MeV) of the predicted $\Sigma_c\bar{D}\,\bar{K}$ bound state for different cutoffs R_C (in units of fm). The values in brackets are the specific potential weighs with respect to the total potential.

R_c	$\langle V_{DK}\rangle$	$\langle V_{\Sigma_c\bar{D}}\rangle$	$\langle V_{\Sigma_c\bar{K}}\rangle$	$\langle T\rangle$
0.5	−167.8(42.0%)	−2.3(0.6%)	−229.2(57.4%)	296.5
1.0	−104.8(51.5%)	−2.1(1.0%)	−96.6(47.5%)	125.7
2.0	−73.4(59.0%)	−1.9(1.5%)	−49.0(39.5%)	56.7

B. Two-body strong decays of $P_{cs}^*(4757)$

In the following we explore the strong decays of P_{cs}^* via triangle diagrams. From our above study, it is clear that P_{cs}^* can not only be viewed as a $\Sigma_c\bar{D}\,\bar{K}$ bound state, but also be regarded as three kinds of quasi-two-body bound states, $\bar{D}_{s0}(2317)\Sigma_c$, $\bar{D}\Xi_c^*$, and $P_c(4312)\bar{K}$. Therefore, the decay of P_{cs}^* can also proceed through three modes as shown in Figs. 2–4. Assuming that P_{cs}^* is mainly made of $\bar{D}_{s0}(2317)\Sigma_c$ and $\bar{D}_{s0}(2317)$ is a bound state of $\bar{D}\,\bar{K}$, P_{cs}^* can decay to $\bar{D}\Xi_c'$ via the triangle diagrams shown in Fig. 2. Using the same mechanism we display the other two decay modes of P_{cs}^* as shown in Figs. 3 and 4. To give a quantitative estimate of the decays of P_{cs}^* in these processes, we employ the effective Lagrangian approach to calculate their decay widths, which has been widely used to explore strong decays of hadronic molecules, see, e.g., Refs. [83,101–103]. The relevant Lagrangians are

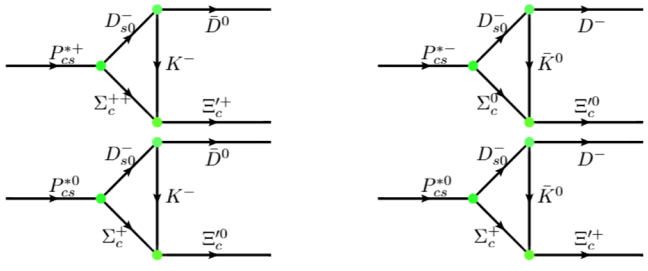

FIG. 2. Decay of P_{cs}^* to $\bar{D}\Xi_c'$ via triangle diagrams, with the hypothesis that $P_{cs}^*(4757)$ is a bound state of $\bar{D}_{s0}(2317)\Sigma_c$ and $\bar{D}_{s0}(2317)$ the bound state of $\bar{D}\,\bar{K}$.

[2]The Ξ_c^* system (with quantum numbers $I = 1/2$, $J = 1/2$) has been studied in other works. In Ref. [89], a state with a mass of 2695 MeV is predicted in a coupled channel ($\Sigma_c\bar{K} - \Omega_c K - \Xi_c'\pi - \Xi_c'\eta$) study. In Ref. [99], two states with the same quantum numbers are predicted with masses 2830 MeV and 3120 MeV respectively. In Ref. [100], the authors obtained five states with masses ranging from 2672 MeV to 4443 MeV in a coupled-channel study of cryptoexotic baryons with charm based on chiral symmetry and large-N_c QCD.

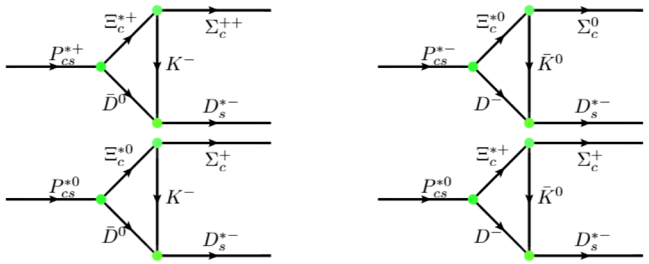

FIG. 3. Decay of P^*_{cs} to $\bar{D}^*_s\Sigma_c$ via triangle diagrams, with the hypothesis that $P^*_{cs}(4757)$ is a bound state of $\bar{D}\Xi^*_c$ and Ξ^*_c the bound state of $\Sigma_c\bar{K}$.

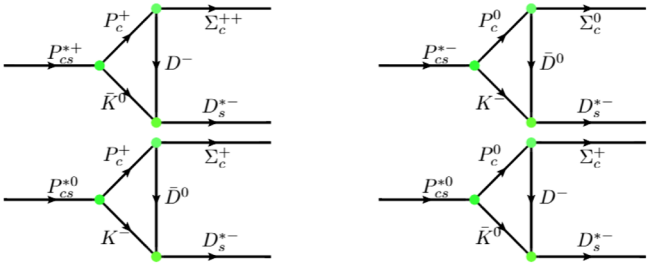

FIG. 4. Decay of P^*_{cs} to $\bar{D}^*_s\Sigma_c$ via triangle diagrams, with the hypothesis that $P^*_{cs}(4757)$ is a bound state of $P_c(4312)\bar{K}$ and $P_c(4312)$ the bound state of $\Sigma_c\bar{D}$.

$$
\begin{aligned}
\mathcal{L}_{P^*_{cs}\bar{D}_{s0}(2317)\Sigma_c} &= -ig_{P^*_{cs}\bar{D}_{s0}(2317)\Sigma_c}P^*_{cs}\bar{D}_{s0}(2317)\Sigma_c,\\
\mathcal{L}_{P^*_{cs}P_c(4312)\bar{K}} &= g_{P^*_{cs}P_c(4312)\bar{K}}P^*_{cs}P_c(4312)\bar{K},\\
\mathcal{L}_{P^*_{cs}\Xi^*_c\bar{D}} &= g_{P^*_{cs}\Xi^*_c\bar{D}}P^*_{cs}\Xi^*_c\bar{D},\\
\mathcal{L}_{\bar{D}_{s0}(2317)\bar{D}\bar{K}} &= g_{\bar{D}_{s0}(2317)\bar{D}\bar{K}}\bar{D}_{s0}(2317)\bar{D}\bar{K},\\
\mathcal{L}_{P_c(4312)\bar{D}\Sigma_c} &= -ig_{P_c(4312)\bar{D}\Sigma_c}P_c(4312)\bar{D}\Sigma_c,\\
\mathcal{L}_{\Xi^*_c\bar{K}\Sigma_c} &= -ig_{\Xi^*_c\bar{K}\Sigma_c}\Xi^*_c\bar{K}\Sigma_c,\\
\mathcal{L}_{\Sigma_c\Xi'_c\bar{K}} &= ig_{\Sigma_c\Xi'_c\bar{K}}\bar{\Sigma}_c\gamma_\mu\gamma^5\partial^\mu K\Xi'_c,\\
\mathcal{L}_{\bar{D}\bar{D}^*_s\bar{K}} &= ig_{\bar{D}\bar{D}^*_s\bar{K}}D^{*\mu}_s(\bar{D}\partial_\mu\bar{K}-\bar{K}\partial_\mu\bar{D}),
\end{aligned}
\tag{20}
$$

where the couplings of each vertex are classified into two scenarios; molecular type where a particle is assumed as a bound state of the other two particles, and scattering type where a particle can change into another particle by exchanging a light meson. For the molecular type the couplings can be estimated by the Weinberg compositeness condition [104,105], where the renormalization constant of the composite particle should be zero. To remove the ultraviolet divergence of the loop diagrams, we choose a Gaussian form factor $\exp(-p_E^2/\Lambda^2)$, where P_E is the Euclidean-Jacobi momentum and Λ characterizes the distribution of the molecular components inside the molecule. The cutoff value is often chosen to be $\Lambda = 1$ GeV [69,83,103]. With this value the coupling between P^*_{cs} and its component $\bar{D}_{s0}(2317)$ and Σ_c is found to be $g_{P^*_{cs}\bar{D}_{s0}(2317)\Sigma_c} = 3.65$. Since $\bar{D}_{s0}(2317)$ is treated as a $\bar{D}\bar{K}$ bound state in this work, the corresponding coupling can be determined as $g_{\bar{D}_{s0}(2317)\bar{D}\bar{K}} = 7.35$ GeV, whose value is a little bit smaller than that obtained in other approaches [21,24]. The other relevant couplings can also be determined in the same way as $g_{P^*_{cs}P_c(4312)\bar{K}} = 4.11$, $g_{P^*_{cs}\bar{D}\Xi^*_c} = 3.19$, $g_{P_c(4312)\bar{D}\Sigma_c} = 2.24$, and $g_{\Xi^*_c\bar{K}\Sigma_c} = 3.74$. For the scattering type vertices the couplings of $\Sigma_c\Xi'_c\bar{K}$ and $\bar{D}D^*\bar{K}$ can be determined as $g_{\Sigma_c\Xi'_c\bar{K}} = 9.01$ and $g_{\bar{D}\bar{D}^*_s\bar{K}} = 4.54$ from the couplings of $\Sigma_c\Sigma_c\pi$ and $\bar{D}\bar{D}^*\pi$, via $SU(3)$-flavor symmetry.

With the above vertices determined, we obtain the amplitudes of the corresponding triangle diagrams as

$$
\begin{aligned}
i\mathcal{M} &= g_{P^*_{cs}\bar{D}_{s0}(2317)\Sigma_c}g_{\bar{D}_{s0}(2317)\bar{D}\bar{K}}g_{\Sigma_c\Xi'_c\bar{K}}\\
&\quad\times\int\frac{d^4q}{(2\pi)^4}\bar{u}_{\Xi'_c}\gamma^\mu q_\mu\gamma_5\frac{\not{k}_1+m_{\Sigma_c}}{k_1^2-m^2_{\Sigma_c}}\frac{1}{k_2^2-m^2_{\bar{D}^*_{s0}}}\frac{1}{q^2-m^2_{\bar{K}}}u_{P^*_{cs}},\\
i\mathcal{M} &= g_{P^*_{cs}\Xi^*_c\bar{D}}g_{\Xi^*_c\bar{K}\Sigma_c}g_{\bar{D}\bar{K}\bar{D}^*_s}\\
&\quad\times\int\frac{d^4q}{(2\pi)^4}\bar{u}_{\Sigma_c}\frac{\not{k}_1+m_{\Xi^*_c}}{k_1^2-m^2_{\Xi^*_c}}\frac{2\varepsilon_{\bar{D}^*_s}\cdot q}{k_2^2-m^2_{\bar{D}}}\frac{1}{q^2-m^2_{\bar{K}}}u_{P^*_{cs}},\\
i\mathcal{M} &= g_{P^*_{cs}P_c\bar{K}}g_{P_c\bar{D}\Sigma_c}g_{\bar{D}\bar{K}\bar{D}^*_s}\\
&\quad\times\int\frac{d^4q}{(2\pi)^4}\bar{u}_{\Sigma_c}\frac{\not{k}_1+m_{P_c}}{k_1^2-m^2_{P_c}}\frac{2\varepsilon_{\bar{D}^*_s}\cdot q}{k_2^2-m^2_{\bar{K}}}\frac{1}{q^2-m^2_{\bar{D}}}u_{P^*_{cs}},
\end{aligned}
\tag{21}
$$

where k_1, k_2, and q denote the momenta of particles appearing in the triangle diagrams, $u_{P^*_{cs}}$ and $\bar{u}_{\Sigma_c}$ represent the initial and final spinors, respectively, and $\varepsilon_{\bar{D}^*_s}$ is the polarization vector of $\bar{D}^*_s$. In addition, to eliminate the ultraviolet divergence, we also add the Gaussian form factor in the above amplitudes. The partial decay width of P^*_{cs} can be finally obtained by

$$
\Gamma = \frac{1}{2J+1}\frac{1}{8\pi}\frac{|\vec{p}|}{m^2_{P^*_{cs}}}\overline{|\mathcal{M}|^2},
\tag{22}
$$

where J is the total angular momentum of the initial state P^*_{cs}, the overline indicates the sum over the polarization vectors of final states, and $|\vec{p}|$ is the momentum of either final state in the rest frame of P^*_{cs}.

Within the molecular picture studied in the present work, P^*_{cs} can decay via three possible modes, the results are presented in Table VII. If P^*_{cs} decays via $\bar{D}_{s0}(2317)\Sigma_c$ as shown in Fig. 2, we find a partial decay width of $P^{*+}_{cs}\to\bar{D}^0\Xi'^0_c$ 96.3 MeV with $\Lambda = 1$ GeV. The widths of its isospin partner $P^{*-}_{cs}\to D^-\Xi'^0_c$ and $P^{*0}_{cs}\to D^-\Xi'^+_c/\bar{D}^0\Xi'^0_c$ are 96.3 MeV and 48.2 MeV, respectively. If it decays via the second mode shown in Fig. 3, the decay widths of $P^{*-(+)}_{cs}\to\Sigma_c^{0(++)}D^{*-}_s$ and $P^{*0}_{cs}\to\Sigma^+_cD^{*-}_s$ are both 25.7 MeV. In the third mechanism shown in Fig. 4 the final states of the P^*_{cs} decay are the same as those of the second mechanism, while the decay widths are much smaller, less

TABLE VII. Partial decay widths (in units of MeV) of the predicted P_{cs}^* as three quasi-two-body bound states $\bar{D}_{s0}\Sigma_c$, $\bar{D}\Xi_c^*$, and $\bar{K}P_c$ with $\Lambda = 0.8$–1.2 GeV.

Modes	$\bar{D}_{s0}\Sigma_c \to \bar{D}\Xi_c'$	$\bar{D}\Xi_c^* \to \bar{D}_s^*\Sigma_c$	$\bar{K}P_c \to \bar{D}_s^*\Sigma_c$
P_{cs}^{*+}	$92.0 \sim 99.5$	$25.4 \sim 28.0$	$0.2 \sim 0.3$
P_{cs}^{*-}	$92.0 \sim 99.5$	$25.4 \sim 28.0$	$0.2 \sim 0.3$
P_{cs}^{*0}	$46.0 \sim 49.7$	$25.4 \sim 28.0$	$0.2 \sim 0.3$

than 1 MeV. Clearly the dominate decay mode is the first mechanism shown in Fig. 2. The final states are experimentally accessible in the $\bar{D}^0\Xi_c'^+$ or $D^-\Xi_c'^0$ channel.

V. SUMMARY AND OUTLOOK

In this work, we employed the Gaussian expansion method to study the three-body $\Sigma_c\bar{D}\,\bar{K}$ system. For the $\Sigma_c\bar{D}$ interaction, we referred to the contact-range effective field theory, and renormalized the potential with a Gaussian regulator and a cutoff R_c in coordinate space. For the DK interaction, the most important contribution is the Weinberg-Tomozawa term, which is also a contact-range potential. We made the same renormalization procedure following the case of $\Sigma_c\bar{D}$. We chose the cutoff ranging from 0.5 fm to 2.0 fm and determined the couplings of $\Sigma_c\bar{D}$ and DK potentials by reproducing $P_c(4312)$ and $D_{s0}^*(2317)$, respectively. The $\Sigma_c\bar{K}$ interaction is related to the $N\bar{K}$ one via chiral symmetry, and is also found to generate a bound state with a binding energy of 38 MeV—71 MeV depending on the chosen cutoff.

With the regularized two-body interactions, the three-body $\Sigma_c\bar{D}\,\bar{K}$ system is found to form a bound state, denoted as $P_{cs}^*(4739)$, with quantum numbers $I(J^P) = 1(1/2^+)$ and a binding energy $77.8^{+25}_{-10.3}$ MeV. The rms radii and Hamiltonian expectation values of the predicted bound states were also presented, which showed that the DK and $\Sigma_c\bar{K}$ interactions contribute most to the formation of the three-body bound state. We noted that the $\Sigma_c\bar{D}\,\bar{K}$ system can form a bound state even if the $\Sigma_c\bar{K}$ interaction is reduced to one tenth of that determined from the $N\bar{K}$ interaction via chiral symmetry.

Based on the molecular nature of the predicted three-body bound state, we studied the two-body open-charm decays of $P_{cs}^*(4739)$ with the effective Lagrangian method via triangle diagrams. We found that the $P_{cs}^*(4739)$ state can decay into $\Xi_c'\bar{D}$ and $\bar{D}_s^*\Sigma_c$ with partial decay widths of a few tens of MeV.

It is expected that the isovector $P_{cs}^*(4739)$ can also decay into the hidden-charm channel $J/\psi\Sigma$. Therefore it may be possible to search for the $P_{cs}^*(4739)$ in the Cabibbo-favored $\Lambda_b \to \pi J/\psi\Sigma$ decay. The corresponding search may be performed by the LHCb Collaboration. However, it is not easy to identify the Σ baryon at hadron-hadron colliders and thus, this poses a challenge to experiments.

The current study can be easily extended to the $\bar{\Sigma}_c DK$ system by charge conjugation symmetry. Employing the heavy-quark symmetry, the study can also be extended to the $\Sigma_c\bar{D}^*\bar{K}$ system. These predictions of the $\Sigma_c\bar{D}^{(*)}\bar{K}$ ($\bar{\Sigma}_c D^{(*)}K$) bound state provide the first hidden charm fermionic three-body molecules, which are likely to be found at the current facilities, especially considering the successful discoveries of the P_c and P_{cs} states by the LHCb Collaboration. Thus, we encourage our experimental colleagues to search for them.

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[1] E. S. Swanson, Phys. Lett. B **588**, 189 (2004).
[2] K. Abe *et al.* (Belle Collaboration), in *Proceedings of the 22nd International Symposium on Lepton-Photon Interactions at High Energy (LP 2005)* (2005) [arXiv:hep-ex/0505037].
[3] P. del Amo Sanchez *et al.* (*BABAR* Collaboration), Phys. Rev. D **82**, 011101 (2010).
[4] N. Li and S.-L. Zhu, Phys. Rev. D **86**, 074022 (2012).
[5] Y. Dong, A. Faessler, T. Gutsche, and V. E. Lyubovitskij, J. Phys. G **38**, 015001 (2011).
[6] F. Aceti, R. Molina, and E. Oset, Phys. Rev. D **86**, 113007 (2012).
[7] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. Lett. **102**, 132001 (2009).
[8] B.-Q. Li and K.-T. Chao, Phys. Rev. D **79**, 094004 (2009).
[9] V. Bhardwaj *et al.* (Belle Collaboration), Phys. Rev. Lett. **107**, 091803 (2011).
[10] S. Coito, G. Rupp, and E. van Beveren, Eur. Phys. J. C **73**, 2351 (2013).
[11] A. M. Badalian, V. D. Orlovsky, Y. A. Simonov, and B. L. G. Bakker, Phys. Rev. D **85**, 114002 (2012).
[12] M. Takizawa and S. Takeuchi, Prog. Theor. Exp. Phys. **2013**, 093D01 (2013).
[13] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. Lett. **90**, 242001 (2003).
[14] D. Besson *et al.* (CLEO Collaboration), Phys. Rev. D **68**, 032002 (2003); **75**, 119908(E) (2007).

[15] P. Krokovny *et al.* (Belle Collaboration), Phys. Rev. Lett. **91**, 262002 (2003).
[16] T. Barnes, F. E. Close, and H. J. Lipkin, Phys. Rev. D **68**, 054006 (2003).
[17] M. Altenbuchinger, L. S. Geng, and W. Weise, Phys. Rev. D **89**, 014026 (2014).
[18] D. Mohler, C. B. Lang, L. Leskovec, S. Prelovsek, and R. M. Woloshyn, Phys. Rev. Lett. **111**, 222001 (2013).
[19] E. E. Kolomeitsev and M. F. M. Lutz, Phys. Lett. B **582**, 39 (2004).
[20] J. Hofmann and M. F. M. Lutz, Nucl. Phys. **A733**, 142 (2004).
[21] D. Gamermann, E. Oset, D. Strottman, and M. J. Vicente Vacas, Phys. Rev. D **76**, 074016 (2007).
[22] D. Gamermann and E. Oset, Eur. Phys. J. A **33**, 119 (2007).
[23] F.-K. Guo, C. Hanhart, S. Krewald, and U.-G. Meissner, Phys. Lett. B **666**, 251 (2008).
[24] F.-K. Guo, P.-N. Shen, H.-C. Chiang, R.-G. Ping, and B.-S. Zou, Phys. Lett. B **641**, 278 (2006).
[25] F.-K. Guo, C. Hanhart, and U.-G. Meissner, Eur. Phys. J. A **40**, 171 (2009).
[26] A. Martinez Torres, L. R. Dai, C. Koren, D. Jido, and E. Oset, Phys. Rev. D **85**, 014027 (2012).
[27] D.-L. Yao, M.-L. Du, F.-K. Guo, and U.-G. Meißner, J. High Energy Phys. 11 (2015) 058.
[28] Z.-H. Guo, U.-G. Meißner, and D.-L. Yao, Phys. Rev. D **92**, 094008 (2015).
[29] X.-Y. Guo, Y. Heo, and M. F. M. Lutz, Phys. Rev. D **98**, 014510 (2018).
[30] M. Albaladejo, P. Fernandez-Soler, J. Nieves, and P. G. Ortega, Eur. Phys. J. C **78**, 722 (2018).
[31] L. S. Geng, N. Kaiser, J. Martin-Camalich, and W. Weise, Phys. Rev. D **82**, 054022 (2010).
[32] P. Wang and X. G. Wang, Phys. Rev. D **86**, 014030 (2012).
[33] Y.-R. Liu, X. Liu, and S.-L. Zhu, Phys. Rev. D **79**, 094026 (2009).
[34] Z.-H. Guo, L. Liu, U.-G. Meißner, J. A. Oller, and A. Rusetsky, Eur. Phys. J. C **79**, 13 (2019).
[35] L. Liu, K. Orginos, F.-K. Guo, C. Hanhart, and U.-G. Meissner, Phys. Rev. D **87**, 014508 (2013).
[36] C. B. Lang, L. Leskovec, D. Mohler, S. Prelovsek, and R. M. Woloshyn, Phys. Rev. D **90**, 034510 (2014).
[37] G. S. Bali, S. Collins, A. Cox, and A. Schäfer, Phys. Rev. D **96**, 074501 (2017).
[38] R. Aaij *et al.* (LHCb Collaboration), Phys. Rev. Lett. **122**, 222001 (2019).
[39] M.-Z. Liu, Y.-W. Pan, F.-Z. Peng, M. Sánchez Sánchez, L.-S. Geng, A. Hosaka, and M. Pavon Valderrama, Phys. Rev. Lett. **122**, 242001 (2019).
[40] R. Chen, Z.-F. Sun, X. Liu, and S.-L. Zhu, Phys. Rev. D **100**, 011502 (2019).
[41] J. He, Eur. Phys. J. C **79**, 393 (2019).
[42] H.-X. Chen, W. Chen, and S.-L. Zhu, Phys. Rev. D **100**, 051501 (2019).
[43] C. W. Xiao, J. Nieves, and E. Oset, Phys. Rev. D **100**, 014021 (2019).
[44] Z.-H. Guo and J. A. Oller, Phys. Lett. B **793**, 144 (2019).
[45] C.-J. Xiao, Y. Huang, Y.-B. Dong, L.-S. Geng, and D.-Y. Chen, Phys. Rev. D **100**, 014022 (2019).
[46] F.-K. Guo, H.-J. Jing, U.-G. Meißner, and S. Sakai, Phys. Rev. D **99**, 091501 (2019).
[47] M. I. Eides, V. Y. Petrov, and M. V. Polyakov, Mod. Phys. Lett. A **35**, 2050151 (2020).
[48] J.-B. Cheng and Y.-R. Liu, Phys. Rev. D **100**, 054002 (2019).
[49] Z.-G. Wang, Int. J. Mod. Phys. A **35**, 2050003 (2020).
[50] L. Meng, B. Wang, G.-J. Wang, and S.-L. Zhu, Phys. Rev. D **100**, 014031 (2019).
[51] X.-Z. Weng, X.-L. Chen, W.-Z. Deng, and S.-L. Zhu, Phys. Rev. D **100**, 016014 (2019).
[52] M.-L. Du, V. Baru, F.-K. Guo, C. Hanhart, U.-G. Meißner, J. A. Oller, and Q. Wang, Phys. Rev. Lett. **124**, 072001 (2020).
[53] T. J. Burns and E. S. Swanson, Phys. Rev. D **100**, 114033 (2019).
[54] M.-Z. Liu, T.-W. Wu, M. Sánchez Sánchez, M. P. Valderrama, L.-S. Geng, and J.-J. Xie, Phys. Rev. D **103**, 054004 (2021).
[55] J.-J. Wu, R. Molina, E. Oset, and B. S. Zou, Phys. Rev. Lett. **105**, 232001 (2010).
[56] J.-J. Wu, R. Molina, E. Oset, and B. S. Zou, Phys. Rev. C **84**, 015202 (2011).
[57] W. L. Wang, F. Huang, Z. Y. Zhang, and B. S. Zou, Phys. Rev. C **84**, 015203 (2011).
[58] Z.-C. Yang, Z.-F. Sun, J. He, X. Liu, and S.-L. Zhu, Chin. Phys. C **36**, 6 (2012).
[59] R. Aaij *et al.* (LHCb Collaboration), Phys. Rev. Lett. **115**, 072001 (2015).
[60] R. Aaij *et al.* (LHCb Collaboration), Sci. Bull. **66**, 1278 (2021).
[61] H.-X. Chen, L.-S. Geng, W.-H. Liang, E. Oset, E. Wang, and J.-J. Xie, Phys. Rev. C **93**, 065203 (2016).
[62] C. W. Xiao, J. Nieves, and E. Oset, Phys. Lett. B **799**, 135051 (2019).
[63] B. Wang, L. Meng, and S.-L. Zhu, Phys. Rev. D **101**, 034018 (2020).
[64] F.-Z. Peng, M.-J. Yan, M. Sánchez Sánchez, and M. P. Valderrama, Eur. Phys. J. C **81**, 666 (2021).
[65] M.-Z. Liu, Y.-W. Pan, and L.-S. Geng, Phys. Rev. D **103**, 034003 (2021).
[66] C. W. Xiao, J. J. Wu, and B. S. Zou, Phys. Rev. D **103**, 054016 (2021).
[67] Q. Wu, D.-Y. Chen, and R. Ji, Chin. Phys. Lett. **38**, 071301 (2021).
[68] J.-X. Lu, M.-Z. Liu, R.-X. Shi, and L.-S. Geng, Phys. Rev. D **104**, 034022 (2021).
[69] T.-W. Wu, M.-Z. Liu, and L.-S. Geng, Phys. Rev. D **103**, L031501 (2021).
[70] L. Ma, Q. Wang, and U.-G. Meißner, Chin. Phys. C **43**, 014102 (2019).
[71] X.-L. Ren, B. B. Malabarba, L.-S. Geng, K. P. Khemchandani, and A. Martínez Torres, Phys. Lett. B **785**, 112 (2018).
[72] T.-W. Wu and L.-S. Geng, Few-Body Syst. **62**, 89 (2021).
[73] V. Baru, A. A. Filin, C. Hanhart, Y. S. Kalashnikova, A. E. Kudryavtsev, and A. V. Nefediev, Phys. Rev. D **84**, 074029 (2011).
[74] B. Durkaya and M. Bayar, Phys. Rev. D **92**, 036006 (2015).

[75] J. M. Dias, V. R. Debastiani, L. Roca, S. Sakai, and E. Oset, Phys. Rev. D **96**, 094007 (2017).
[76] T.-W. Wu, M.-Z. Liu, L.-S. Geng, E. Hiyama, M. P. Valderrama, and W.-L. Wang, Eur. Phys. J. C **80**, 901 (2020).
[77] M. Bayar, C. W. Xiao, T. Hyodo, A. Dote, M. Oka, and E. Oset, Phys. Rev. C **86**, 044004 (2012).
[78] A. Martinez Torres, K. P. Khemchandani, M. Nielsen, and F. S. Navarra, Phys. Rev. D **87**, 034025 (2013).
[79] V. R. Debastiani, J. M. Dias, and E. Oset, Phys. Rev. D **96**, 016014 (2017).
[80] C. W. Xiao, M. Bayar, and E. Oset, Phys. Rev. D **84**, 034037 (2011).
[81] A. Martinez Torres, K. P. Khemchandani, and L.-S. Geng, Phys. Rev. D **99**, 076017 (2019).
[82] T.-W. Wu, M.-Z. Liu, L.-S. Geng, E. Hiyama, and M. P. Valderrama, Phys. Rev. D **100**, 034029 (2019).
[83] Y. Huang, M.-Z. Liu, Y.-W. Pan, L.-S. Geng, A. Martínez Torres, and K. P. Khemchandani, Phys. Rev. D **101**, 014022 (2020).
[84] J.-Y. Pang, J.-J. Wu, and L.-S. Geng, Phys. Rev. D **102**, 114515 (2020).
[85] A. Martinez Torres, K. P. Khemchandani, L. Roca, and E. Oset, Few-Body Syst. **61**, 35 (2020).
[86] T.-W. Wu, M.-Z. Liu, and L.-S. Geng, Few-Body Syst. **62**, 38 (2021).
[87] X.-L. Ren, B. B. Malabarba, K. P. Khemchandani, and A. Martinez Torres, J. High Energy Phys. 05 (2019) 103.
[88] X.-L. Ren, K. P. Khemchandani, and A. Martinez Torres, Phys. Rev. D **102**, 016005 (2020).
[89] J.-X. Lu, Y. Zhou, H.-X. Chen, J.-J. Xie, and L.-S. Geng, Phys. Rev. D **92**, 014036 (2015).
[90] Q. X. Yu, R. Pavao, V. R. Debastiani, and E. Oset, Eur. Phys. J. C **79**, 167 (2019).
[91] J. A. Oller and U. G. Meissner, Phys. Lett. B **500**, 263 (2001).
[92] J. A. Oller, Eur. Phys. J. A **28**, 63 (2006).
[93] B. Borasoy, R. Nissler, and W. Weise, Eur. Phys. J. A **25**, 79 (2005).
[94] D. Jido, J. A. Oller, E. Oset, A. Ramos, and U. G. Meissner, Nucl. Phys. **A725**, 181 (2003).
[95] Z.-H. Guo and J. A. Oller, Phys. Rev. C **87**, 035202 (2013).
[96] Z.-Y. Wang, H. A. Ahmed, and C. W. Xiao, Eur. Phys. J. C **81**, 833 (2021).
[97] P. A. Zyla *et al.* (Particle Data Group), Prog. Theor. Exp. Phys. **2020**, 083C01 (2020).
[98] E. Hiyama, Y. Kino, and M. Kamimura, Prog. Part. Nucl. Phys. **51**, 223 (2003).
[99] M. F. M. Lutz and E. E. Kolomeitsev, Nucl. Phys. **A730**, 110 (2004).
[100] J. Hofmann and M. F. M. Lutz, Nucl. Phys. **A763**, 90 (2005).
[101] Q.-F. Lü and Y.-B. Dong, Phys. Rev. D **93**, 074020 (2016).
[102] Y.-H. Lin and B.-S. Zou, Phys. Rev. D **100**, 056005 (2019).
[103] C.-J. Xiao, Y. Huang, Y.-B. Dong, L.-S. Geng, and D.-Y. Chen, Phys. Rev. D **100**, 014022 (2019).
[104] A. Faessler, T. Gutsche, V. E. Lyubovitskij, and Y.-L. Ma, Phys. Rev. D **76**, 114008 (2007).
[105] A. Faessler, T. Gutsche, V. E. Lyubovitskij, and Y.-L. Ma, Phys. Rev. D **76**, 014005 (2007).