# $\Omega_{cc}$ resonances with negative parity in the chiral constituent quark model

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Spectrum of the low-lying  $\Omega_{cc}$  resonances with negative parity, which are assumed to be dominated by  $sccq\bar{q}$  pentaquark components, is investigated using the chiral constituent quark model. Energies of the  $\Omega_{cc}$  resonances are obtained by considering the hyperfine interaction between quarks by exchanging a Goldstone boson. Possible  $sccq\bar{q}$  configurations with spin-parity  $1/2^-$ ,  $3/2^-$ , and  $5/2^-$  are taken into account. Numerical results show that the lowest  $\Omega_{cc}$  resonances with negative parity may lie at  $4050 \pm 100$  MeV. In addition, the transitions of the  $\Omega_{cc}$  resonance to a pseudoscalar meson and a ground baryon state are also investigated within the chiral Lagrangian approach. We expect that these  $\Omega_{cc}$  resonances could be observed in the  $D\Xi_c$  channel by future experiments.

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## I. INTRODUCTION

Recently, five narrow  $\Omega_c^0$  resonances and a doubly charmed baryon  $\Xi_{cc}^{++}$  were observed by LHCb Collaboration [1-3]. These observations are of significant importance in hadron physics since the experimental data for baryon resonances with one or more charm quarks were very poor before 2017 [4]. Accordingly, the theorists also made great efforts to describe the spectrum and decay behaviors of the observed heavy baryon resonances with charm quarks using various approaches, such as the quark model within the three- or five-quark picture [5–13], QCD sum rules [14–18], the chiral perturbation approach [19–22], the lattice OCD [23–25], etc. Furthermore, the molecular nature of these heavy baryon resonances were also studied in Refs. [26-29], where they were dynamically generated from the meson-baryon interactions in the coupled channels. Besides, it is also very interesting that the observation of doubly heavy baryons is claimed to

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On the other hand, three hidden-charm  $N_{c\bar{c}}$  pentaquark states were observed by the LHCb Collaboration in recent years [32,33], where the experimental data are in agreement with the predictions made in Refs. [34,35]. Consequently, rather than the three-quark picture, it may be more interesting to study the properties of baryon resonances near or above 4 GeV within a five-quark picture, since the energy for pulling a light quark-antiquark pair to form a pentaquark configuration as the baryon excitation may be lower than that for the traditional orbital and radial excitations of a three-quark configuration [36]. Taking the five-quark picture, the  $\Omega$  excited states with negative parity [37-44], nucleon excited states [45-51], and the newly observed  $\Omega_c^0$  resonances [11–13] are investigated explicitly. Suggestions on how to observe the  $\Omega(2012)$  state by looking at the  $\Omega_c$  weak decay process have been made in Ref. [52]. It was found that the observed small energy splitting of the  $\Omega_c^0$  resonances [13] and the masses and decay behaviors of the observed  $\Omega(2012)$  [41–44] can be well described by taking either the hadronic molecule picture or the compact pentaquark configuration, while it is true of course that one cannot rule out the three-quark components in the baryon resonances.

The observation of the  $\Xi_{cc}$  states have brought new opportunities for us to study the doubly charmed baryons,

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since this finding suggests the potential of discovering more low-lying doubly charmed baryons in the near future, and thus one needs to have solid theoretical calculations for the corresponding spectrum. In the present work, based on the chiral constituent quark model, we study the spectrum of the low-lying  $\Omega_{cc}$  resonances with negative parity. The transitions of these  $\Omega_{cc}$  states to a pseudoscalar meson and a ground baryon state (*MB*) are also studied, employing the chiral Lagrangian approach which has been explicitly developed to study the strong decays of the  $N_{s\bar{s}}$  nucleon resonances as in Ref. [49].

The present manuscript is organized as follows: in Sec. II, we briefly present our theoretical formalism which includes the Hamiltonian and wave functions for the  $\Omega_{cc}$  pentaquark system, and the chiral Lagrangian approach for strong decays of a five-quark system; we show our explicit numerical results in Sec. III; and Sec. IV contains a summary and conclusions.

#### **II. THEORETICAL FRAME**

We will briefly introduce the Hamiltonian and wave functions for the  $\Omega_{cc}$  resonances with negative parity as pentaquark states in Sec. II A, and the chiral Lagrangian approach for strong decays of the  $\Omega_{cc}$  states in Sec. II B.

#### A. Hamiltonian and wave functions

In the present work, the constituent quark model is employed to study the spectrum of  $\Omega_{cc}$  resonances, within which the Hamiltonian for a five-quark system can be written as

$$H = \sum_{i < j}^{5} H_{\text{hyp}}^{ij} + \sum_{i=1,5} m_i + H_o, \qquad (1)$$

where  $H_{hyp}^{ij}$  represents the hyperfine interaction between the *i*th and *j*th quarks in the five-quark system,  $m_i$  is the constituent mass of the *i*th quark, and  $H_o$  is the Hamiltonian concerning the orbital motions of the quarks, which should contain the kinetic term, the confinement potential of the quarks, and the flavor symmetry breaking term.

In general, the corresponding eigenvalue  $E_0$  of  $H_o + \sum_{i=1,5} m_i$  in Eq. (1) should depend on the constituent masses of quarks and the model parameters in the quark confinement model; for instance, the confinement strength C and constant  $V_0$  in the harmonic oscillator potential model [53]. In this work, we study the low-lying  $\Omega_{cc}$  resonances with negative parity as pentaquark states, which require all the quarks and antiquarks to be in their ground states. Accordingly, the eigenvalue  $E_0$  should be the same for different pentaquark configurations.

The parameter  $E_0$  has been taken to be 2127 MeV for investigations on the intrinsic sea flavor content of the

nucleon in Ref. [54], with which value the data for light sea quark asymmetry  $\overline{d} - \overline{u}$  in the proton can be well reproduced, while to fit the experimental data about  $\Omega_c^0$  resonances, we took  $E_0 = 3132$  MeV as in Ref. [13]. As discussed in details in Ref. [13], the resulted different values of  $E_0$  by fitting the experimental data should be consistent with the chiral constituent quark model if all the model parameters are taken to be the empirical values.

In this work, the value of  $E_0$  should be ~1140 MeV higher than the one we took in Ref. [13], because of the different quark content in  $\Omega_{cc}$  and  $\Omega_c$  states, while the SU(4) flavor symmetry breaking effects caused by two charm quarks in the present case will lower  $E_0$  by ~170 MeV than those caused by one charm quark as in Ref. [13]—if the Hamiltonian for symmetry breaking correction is taken to be the form similar to the one in Ref. [38]. Consequently, hereafter we will take  $E_0 =$ 4102 MeV, based on the investigations on the intrinsic sea content of the nucleon and the spectrum of low-lying  $\Omega_c^0$  resonances, and the requirements of the chiral constituent quark model. Nevertheless, we will investigate the dependency of the results on  $E_0$ .

The hyperfine interaction between quarks is taken to be mediated by goldstone boson exchange and the corresponding  $H_{hyp}^{ij}$  is taken as follows:

$$H_{\text{hyp}}^{ij} = -\vec{\sigma}_{i} \cdot \vec{\sigma}_{j} \bigg[ \sum_{a=1}^{3} V_{\pi}(r_{ij}) \lambda_{i}^{a} \lambda_{j}^{a} + \sum_{a=4}^{7} V_{K}(r_{ij}) \lambda_{i}^{a} \lambda_{j}^{a} + V_{\eta}(r_{ij}) \lambda_{i}^{8} \lambda_{j}^{8} + \sum_{a=9}^{12} V_{D}(r_{ij}) \lambda_{i}^{a} \lambda_{j}^{a} + \sum_{a=13}^{14} V_{D_{s}}(r_{ij}) \lambda_{i}^{a} \lambda_{j}^{a} + V_{\eta_{c}}(r_{ij}) \lambda_{i}^{15} \lambda_{j}^{15} \bigg],$$
(2)

where  $V_M(r_{ij})$  denotes the coupling strength for a meson M exchanged between the *i*th and *j*th quarks. In this work, the  $\pi$ , K,  $\eta$ , D,  $D_s$ , and  $\eta_c$  mesons are taken into account.

For a five-quark system with the quark flavor as  $\Omega_{cc}$  resonances—namely, the  $sccq\bar{q}$  system—a general wave function can be written as

$$\begin{split} \psi_{t,J_{z}}^{i} &= \sum_{a,b,c} \sum_{Y,y,T_{z},t_{z}} \sum_{S_{z},s_{z}} C_{[31]_{a}[211]_{a}}^{[14]} C_{[F^{i}]_{b}[S^{(i)}]_{c}}^{[31]_{a}} [F^{i}]_{b,Y,T_{z}} \\ &\times [S^{i}]_{c,S_{z}} [211;C]_{a} \langle Y,T,T_{z},y,\bar{t},t_{z}|0,0,0\rangle \\ &\times \langle S,S_{z},1/2,s_{z}|J,J_{z}\rangle \bar{\chi}_{y,t_{z}} \bar{\xi}_{s_{z}} \varphi(\{\vec{\xi}_{j}\}), \end{split}$$
(3)

where  $[F^i]_{b,Y,T_z}$ ,  $[S^i]_{c,S_z}$ , and  $[211; C]_a$  are the flavor, spin, and color wave functions of the four-quark subsystem denoted by the Young tableau, and the label *i* enumerates different pentaquark configurations. The  $\vec{\xi}_j$  is the Jacobi coordinates for a five-quark system, which is defined as

$$\vec{\xi}_j = \frac{1}{\sqrt{j+j^2}} \left( \sum_{i=1}^{j} \vec{r}_i - j\vec{r}_{j+1} \right), \quad j = 1, \dots, 4.$$
 (4)

According to the SU(2) symmetry, the spin wave function of a four-quark system may be  $[4]_S$ ,  $[31]_S$ , or  $[22]_S$ , the corresponding spin quantum numbers are 2, 1, and 0, respectively, while the coupling between spin of the four-quark subsystem and the antiquark leads to J = 1/2, 3/2, or 5/2. Given that all the quarks and antiquarks are in their ground states (namely, the orbital wave function of the four-quark system is  $[4]_X$ ) then the flavor wave function of the *sccq* subsystem can be  $[4]_F$ ,  $[31]_F^1$ ,  $[31]_F^2$ ,  $[22]_F$ , and  $[211]_F$ . Finally, the possible pentaquark configurations denoted by  $|i\rangle$  for spin-parity quantum number  $J^P = 1/2^-$  are

$$\begin{aligned} |1\rangle: \ sccq_{[4]_{X}[211]_{F}[22]_{S}[211]_{C}}\bar{q}, \\ |2\rangle: \ sccq_{[4]_{X}[31]_{F}^{1}[22]_{S}[211]_{C}}\bar{q}, \\ |3\rangle: \ sccq_{[4]_{X}[31]_{F}^{2}[22]_{S}[211]_{C}}\bar{q}, \end{aligned}$$
(5)

for  $J^P = 1/2^-$  or  $3/2^-$  are

$$\begin{aligned} |4\rangle \colon sccq_{[4]_{X}[211]_{F}[31]_{S}[211]_{C}}\bar{q}, \\ |5\rangle \colon sccq_{[4]_{X}[22]_{F}[31]_{S}[211]_{C}}\bar{q}, \\ |6\rangle \colon sccq_{[4]_{X}[31]_{F}^{-1}[31]_{S}[211]_{C}}\bar{q}, \\ |7\rangle \colon sccq_{[4]_{X}[31]_{F}^{-2}[31]_{S}[211]_{C}}\bar{q}, \\ |8\rangle \colon sccq_{[4]_{X}[4]_{F}[31]_{S}[211]_{C}}\bar{q}, \end{aligned}$$

$$(6)$$

and for  $J^{P} = 3/2^{-}$  or  $5/2^{-}$  are

$$|9\rangle: \ sccq_{[4]_{X}[31]_{F}^{1}[4]_{S}[211]_{C}}\bar{q},$$
  
$$|10\rangle: \ sccq_{[4]_{Y}[31]_{C}^{2}[4]_{S}[211]_{C}}\bar{q},$$
  
$$(7)$$

respectively.

One should note that the different spin symmetries of the four-quark subsystem result in vanishing coupling between different five-quark configurations, this is another reason



FIG. 1. Transitions of the  $sccq\bar{q}$  states to  $\bar{K}\Xi_{cc}$  ( $\Xi_{cc}^*$ ) (a) and  $D\Xi_c$  ( $\Xi_c^*$ ) (b).

for us to categorize the states in three groups by the fourquark spin wave functions.

#### B. The chiral Lagrangian approach

We consider the decays  $sccq\bar{q} \rightarrow MB$ , which mainly proceed through the process of  $q\bar{q} \rightarrow M$ , where the final baryon and meson are assumed to be composed of a threequark and a quark-antiquark pair, respectively. We name this kind of decay the annihilation transition. The  $sccq\bar{q}$  to  $\bar{K}\Xi_{cc}$  and  $D\Xi_{cc}$  transitions are shown in Fig. 1.

To compute the transitions of  $\Omega_{cc} \rightarrow \overline{K}\Xi_{cc}$  and  $\Omega_{cc} \rightarrow D\Xi_c$  shown in Fig. 1, we use the chiral Lagrangian approach. Within this approach, the quark pseudoscalar (P) and vector (V) meson couplings are

$$H_{\rm eff}^{Pqq} = \sum_{j} \bar{\psi}_{j} \gamma_{\mu}^{j} \gamma_{5}^{j} \psi_{j} \partial^{\mu} \phi_{m}, \qquad (8)$$

$$H_{\rm eff}^{Vqq} = -\sum_{j} \bar{\psi}_{j} \left( a \gamma_{\mu}^{j} + \frac{i b \sigma_{\mu\nu} k_{M}^{\nu}}{2m_{j}} \right) \phi_{m}^{\mu} \psi_{j}, \qquad (9)$$

respectively, where the summation on *j* runs over the quark in the initial hadron.  $\psi_j$  represents the quark field, and  $\phi_m$ and  $\phi_m^{\mu}$  are the pseudoscalar and vector meson fields.  $m_j$  is the constituent mass of the *j*th quark, while  $k_M^{\nu}$  denotes the four-momentum of the vector meson. *a* and *b* are the vector and tensor coupling constants, respectively.

In the nonrelativistic approximation, Eqs. (8) and (9) lead to the operators for the process involving  $q \rightarrow q'M$  transitions as follows:

$$T_d^{Pqq} = \sum_j \left( \frac{\omega_M}{E_f + M_f} \sigma_j \cdot \vec{P}_f + \frac{\omega_M}{E_i + M_i} \sigma_j \cdot \vec{P}_i - \sigma_j \cdot \vec{k}_M + \frac{\omega_M}{2\mu_q} \sigma_j \cdot \vec{p}_j \right) X_P^j \exp\{-i\vec{k}_M \cdot \vec{r}_j\},\tag{10}$$

$$T_{d,T}^{Vqq} = \sum_{j} \left\{ i \frac{b'}{2m_j} \vec{\sigma}_j \cdot (\vec{k}_M \times \vec{\epsilon}) + \frac{a}{2\mu_q} \vec{p}_j \cdot \vec{\epsilon} \right\} X_V^j \exp\{-i\vec{k}_M \cdot \vec{r}_j\},\tag{11}$$

$$T_{d,L}^{Vqq} = \sum_{j} \frac{aM_{V}}{|\vec{k}_{M}|} X_{V}^{j} \exp\{-i\vec{k}_{M} \cdot \vec{r}_{j}\},\tag{12}$$

where  $\omega_M$  and  $\vec{k}_M$  are the energy and three-momentum of the final meson.  $E_{i(f)}$ ,  $M_{i(f)}$ , and  $\vec{P}_{i(f)}$  are the energy, mass, and three-momentum of the initial (final) baryon, while  $\vec{p}_j$ ,  $\vec{r}_j$ , and  $m_j$  are the momentum, coordinate, and constituent mass of the quark which emits a meson. The  $\mu_q$  is the reduced mass of the *j*th quark before and after emitting the meson. For the vector meson emission, in Eqs. (11) and (12), the transition operators are denoted as  $T_{d,T}^{Vqq}$  and  $T_{d,L}^{Vqq}$  for the meson being transversely and longitudinally polarized, respectively. The b' in Eq. (11) is defined by b' = b - a.  $M_V$  is the mass of the vector meson, and the polarization vectors of the final vector meson are taken to be

$$\epsilon_{\mu}^{L} = \frac{1}{M_{V}} \begin{pmatrix} |k_{M}| \\ E_{V} \frac{\vec{k}_{M}}{|\vec{k}_{M}|} \end{pmatrix}, \qquad \epsilon_{\mu}^{T} = \begin{pmatrix} 0 \\ \vec{\epsilon} \end{pmatrix}, \qquad (13)$$

with

$$\vec{\epsilon}(\pm) = 1/\sqrt{2}(\mp 1, -i, 0)^T,$$
 (14)

where  $E_V$  is the energy of the final vector meson.

Finally,  $X_P^j$  and  $X_V^j$  are the operators in flavor space for a pseudoscalar and vector meson emission, which only depends on the quark-antiquark content of the emitted meson. For instance,  $X_P^j$  for a light pseudoscalar meson emission in Eq. (10) can be defined as

$$X^{j}_{\pi^{\pm}} = \mp \frac{1}{\sqrt{2}} (\lambda^{j}_{1} \mp i \lambda^{j}_{2}),$$
 (15)

$$X^j_{\pi^0} = \lambda^j_3, \tag{16}$$

$$X_{K^{\pm}}^{j} = \mp \frac{1}{\sqrt{2}} (\lambda_4^j \mp i\lambda_5^j), \qquad (17)$$

$$K^{j}_{K^{0},\bar{K}^{0}} = \mp \frac{1}{\sqrt{2}} (\lambda^{j}_{6} \mp i\lambda^{j}_{7}),$$
 (18)

$$X_{\eta}^{j} = \cos\theta\lambda_{8}^{j} - \sin\theta\sqrt{\frac{2}{3}}\mathcal{I}, \qquad (19)$$

$$X_{\eta'}^{j} = \sin\theta\lambda_{8}^{j} + \cos\theta\sqrt{\frac{2}{3}}\mathcal{I}, \qquad (20)$$

with  $\lambda_i^j$  and  $\mathcal{I}$  being the Gell-Mann matrix and unit matrix in flavor space.  $\theta$  denotes the mixing angle for the mixing between  $\eta_1$  and  $\eta_8$ , leading to the physical states  $\eta$  and  $\eta'$ :

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$$\eta = \eta_8 \cos \theta - \eta_1 \sin \theta$$
  
$$\eta' = \eta_8 \sin \theta + \eta_1 \cos \theta, \qquad (21)$$

where the empirical value for the mixing angle is  $\theta = -23^\circ$ . The flavor operators for other pseudoscalar mesons or the vector mesons can be obtained straightforwardly.

Accordingly, the transition operators for a pseudoscalar meson emission  $T_{a,T}^{Pqq}$ , a transversely polarized vector meson emission  $T_{a,T}^{Vqq}$ , and a longitudinally polarized vector meson emission can be obtained as

$$T_{a}^{Pqq} = \sum_{j} (m_{j} + m_{\bar{q}}) \mathcal{C}_{\text{XFSC}}^{j} \bar{\chi}_{z}^{\dagger} \mathcal{I}_{2} \chi_{z}^{j} X_{P}^{j} \exp\{-i\vec{k}_{M} \cdot (\vec{r}_{j} + \vec{r}_{\bar{q}})/2\},$$
(22)

$$T_{a,T}^{Vqq} = \sum_{j} \left\{ a - \frac{m_j + m_{\bar{q}}}{2m_j} b \right\} \vec{\sigma} \cdot \vec{\epsilon} X_V^j \exp\{-i\vec{k}_M \cdot (\vec{r}_j + \vec{r}_{\bar{q}})/2\},$$
(23)

$$T_{a,L}^{Vqq} = \sum_{j} \left\{ a - \frac{m_j + m_{\bar{q}}}{2m_j} b \right\} \frac{E_V \vec{\sigma} \cdot \vec{k}_M}{M_V |\vec{k}_M|} X_V^j \exp\{-i\vec{k}_M \cdot (\vec{r}_j + \vec{r}_{\bar{q}})/2\},\tag{24}$$

where  $m_j$  and  $m_{\bar{q}}$  are the constituent masses of the *j*th quark and the antiquark, respectively.  $C_{XFSC}^j$  denotes the operator to calculate the orbital, flavor, spin, and color overlap factor between the residual wave function of the pentaquark configuration after the quark-antiquark annihilation and the wave function of the final baryon.  $\bar{\chi}_z^{\dagger} \mathcal{I}_2 \chi_z^j$  is the spin operator for the quark-antiquark annihilation.

### **III. NUMERICAL RESULTS AND DISCUSSIONS**

In this section, we present our theoretical results for the mass spectrum of the low-lying  $sccq\bar{q}$  states with

 $J^P = 1/2^-$ ,  $3/2^-$ , and  $5/2^-$ , and the decay behaviors of the obtained  $\Omega_{cc}$  pentaquark states.

#### A. The mass spectrum of the low-lying *sccqq* states

With the Hamiltonian in Eq. (1) and wave function in Eq. (3), one can obtain the following nonzero  $H_{ij} = \langle i|H|j \rangle$  matrix elements:

$$\begin{split} H_{11} &= E_0 - 7.50C_D - 7.50C_{D_s}, \\ H_{12} &= H_{21} = 4.90C_D - 4.90C_{D_s}, \\ H_{13} &= H_{31} = -4.33C_K + 0.87C_D + 0.87C_{D_s} + 2.60C_{c\bar{c}}, \end{split}$$

$$H_{22} = E_0 - 1.50C_K - 2.00C_D - 2.00C_{D_s} - 1.50C_{c\bar{c}},$$
  

$$H_{23} = H_{32} = -5.66C_D + 5.66C_{D_s},$$
  

$$H_{33} = E_0 - 5.0C_K - 2.50C_D - 2.50C_{D_s} + 3.0C_{c\bar{c}},$$
 (25)

between the pentaquark configurations in Eq. (5), and

$$\begin{aligned} H_{44} &= E_0 - 2.5C_K - 4.50C_D - 4.50C_{D_s} - 1.5C_{c\bar{c}}, \\ H_{45} &= H_{54} = -6.12C_D + 6.12C_{D_s}, \\ H_{46} &= H_{64} = -3.54C_D + 3.54C_{D_s}, \\ H_{47} &= H_{74} = 5.00C_K - 2.50C_D - 2.50C_{D_s}, \\ H_{55} &= E_0 - 0.50C_K - 4.00C_D - 4.00C_{D_s} - 0.5C_{c\bar{c}}, \\ H_{56} &= H_{65} = 1.73C_K - 1.73C_{c\bar{c}}, \\ H_{57} &= H_{75} = -1.22C_D + 1.22C_{D_s}, \\ H_{58} &= H_{85} = -1.41C_K + 1.41C_D + 1.41C_{D_s} - 1.41C_{c\bar{c}}, \\ H_{66} &= E_0 + 1.50C_K - 4.0C_D - 4.0C_{D_s} + 1.50C_{c\bar{c}}, \\ H_{67} &= H_{76} = -0.71C_D + 0.71C_{D_s}, \\ H_{68} &= H_{86} = -2.45C_K + 2.45C_{c\bar{c}}, \\ H_{77} &= E_0 - 2.5C_K - 0.50C_D - 0.50C_{D_s} - 1.5C_{c\bar{c}}, \\ H_{78} &= H_{87} = -3.46C_D + 3.46C_{D_s}, \\ H_{88} &= E_0 + 0.50C_K + 1.0C_D + 1.0C_{D_s} + 0.50C_{c\bar{c}}, \end{aligned}$$

between the pentaquark configurations in Eq. (6), and

$$H_{99} = E_0 - 1.50C_K + 1.0C_D + 1.0C_{D_s} - 1.50C_{c\bar{c}},$$
  

$$H_{910} = H_{109} = 2.83C_D - 2.83C_{D_s},$$
  

$$H_{1010} = E_0 + 2.50C_K - C_D - C_{D_s} - 1.50C_{c\bar{c}},$$
 (27)

between the pentaquark configurations in Eq. (7).

In the above equations,  $C_M$  are the corresponding matrix elements of the hyperfine interaction coupling strength  $V_M(r_{ij})$  between the S-wave orbital wave functions of the quarks in the  $sccq\bar{q}$  system, namely

$$C_M = \langle \varphi(\{\vec{\xi}_j\}) | V_M(r_{ij}) | \varphi(\{\vec{\xi}_j\}) \rangle.$$
(28)

 $C_{c\bar{c}}$  is obtained from the last term in Eq. (2), and it contains the exchanges of the  $u\bar{u}$ ,  $d\bar{d}$ ,  $s\bar{s}$ , and  $c\bar{c}$  pairs. The coupling strength constants  $C_M$  are taken to be the empirical values [53] as shown in Table I.

TABLE I. The hyperfine interaction coupling strength constants (in MeV).

$C_{\pi}$	$C_K$	$C_{s\bar{s}}$	$C_D$	$C_{D_s}$	$C_{c\bar{c}}$
21.0	15.5	11.5	6.5	6.5	0



FIG. 2. Spectrum of the obtained physical states.

With the above values for the model parameters and the diagonalization of the matrices obtained by Eqs. (25)–(27), one can get the physical states which are shown in Fig. 2, while the explicit probability amplitudes are shown in Table II. For instance, Eq. (25) leads to the following energy matrix:

$$E = \begin{pmatrix} 4005 & 0 & -55.8\\ 0 & 4053 & 0\\ -55.8 & 0 & 3992 \end{pmatrix}$$
MeV. (29)

Then one can directly obtain the eigenvalues and eigenvectors of the matrix in Eq. (29). The three obtained eigenvalues are the energies of the physical states  $|i'\rangle$  with i = 1, 2, 3, respectively, and the obtained eigenvector just shows the coefficients for the decoupling of a corresponding physical state  $|i'\rangle$  to the configurations  $|i\rangle$  listed in Eq. (5).

The energies for the obtained  $\Omega_{cc}$  states in the present work are at 4050 ± 100 MeV. Similar as the results for  $\Omega_c^0$ obtained in Ref. [13], mixing between the pentaquark configurations  $|i\rangle$  caused by the goldstone boson exchange is strong, while the mass splitting for the obtained states  $|i'\rangle$ is not very large. On the other hand, the spectrum of the ten obtained states is not sensitive to the values of the coupling strength  $C_M$ .

Up to now, there were no solid experimental data for the  $\Omega_{cc}$  resonances, while theoretical investigations on the doubly heavy baryon resonances have been intensively taken using various approaches, such as the constituent quark model [9,10,55–59], QCD sum rules [17], chiral perturbation theory [20–22], the unitarized coupled channel approach [60], the lattice QCD calculations [25], etc. The corresponding obtained energies for the *P*-wave  $\Omega_{cc}$  in a three-quark picture are around 4000–4200 MeV in most of the literature, and one may note that in Ref. [60], the

	$J^{P} = 1/2^{-}$				$J^p = 1/2^- \text{ or } 3/2^-$					$J^P = 3/2^-$ or $5/2^-$	
	$\frac{ 1'\rangle}{3942}$	2'> 4053	3'> 4054	4') 3979	5'⟩ 4024	6' angle 4069	7'⟩ 4083	8') 4146	9'> 4092	10'⟩ 4128	
$ 1\rangle$	0.67	0	0.75	0	0	0	0	0	0	0	
$ 2\rangle$	0	1	0	0	0	0	0	0	0	0	
$ 3\rangle$	0.75	0	-0.67	0	0	0	0	0	0	0	
$ 4\rangle$	0	0	0	0.87	0	0	-0.50	0	0	0	
$ 5\rangle$	0	0	0	0	0.79	0.59	0	-0.16	0	0	
$ 6\rangle$	0	0	0	0	-0.58	0.64	0	-0.50	0	0	
$ 7\rangle$	0	0	0	-0.50	0	0	-0.87	0	0	0	
$ 8\rangle$	0	0	0	0	-0.19	0.49	0	0.85	0	0	
$ 9\rangle$	0	0	0	0	0	0	0	0	1	0	
$ 10\rangle$	0	0	0	0	0	0	0	0	0	1	

TABLE II. The ten physical pentaquark states obtained in the present model. Line three shows the energies for the states  $|i'\rangle$  (in MeV), and lines four through ten are the corresponding probability amplitudes.

S-wave interactions between pseudo-Nambu-Goldstone bosons ( $\pi$ , K, and  $\eta$ ) and the  $J^P = 1/2^+$  ground state doubly charmed baryons in the energy region around the corresponding thresholds are investigated, two quasistable narrow  $J^P = 1/2^- \Omega_{cc}$  are predicted to lie at the energy below 4200 MeV, and their strong decay mode is predicted to be only the  $\Omega_{cc}\pi^0$ , which is the isospin breaking channel. Therefore, the two obtained  $\Omega_{cc}$  resonances in a meson-baryon picture should be very narrow.

One may also study the spectrum of low-lying  $\Omega_{cc}$  resonances with negative parity using the chiral constituent quark model in a three-quark picture [61]. As *P*-wave states whose parity are negative, there are three possible  $\Omega_{cc}$  configurations:

$$|scc, 1\rangle = [21]_{X} [21]_{FS} [21]_{F} [21]_{S} [1^{3}]_{C},$$
  
$$|scc, 2\rangle = [21]_{X} [21]_{FS} [3]_{F} [21]_{S} [1^{3}]_{C},$$
  
$$|scc, 3\rangle = [21]_{X} [21]_{FS} [21]_{F} [3]_{S} [1^{3}]_{C},$$
  
(30)

whose spin-parity quantum number  $J^P$  may be  $1/2^-$  or  $3/2^-$  for the first two configurations, and  $1/2^-$ ,  $3/2^-$ , or  $5/2^-$  for the last one. Direct calculations employing the chiral constituent quark model as in Ref. [61] lead to the following values for the energies of the three  $\Omega_{cc}$  states:

$$E_1 = 4219 \text{ MeV}, \quad E_2 = 4246 \text{ MeV}, \quad E_3 = 4257 \text{ MeV},$$

respectively. Consequently, the energies of low-lying  $\Omega_{cc}$  states in the five-quark picture are lower than those in the three-quark picture, and this conclusion is the same as that for the  $\Omega^*$  resonances [37].

In Ref. [55], a relativistic quark model was applied to study the spectrum of doubly heavy baryons. Considering the  $\Omega_{cc}$  resonances to be dominated by three-quark components, it was obtained that the low-lying  $\Omega_{cc}$  resonances with negative parity fall in the range of 4200–4300 MeV, which are consistent with the results obtained in Ref. [10] by employing a three-quark model.

In Ref. [9], a three-quark model was employed to investigate the spectrum of the doubly heavy baryons, in which the two heavy quarks were treated as a diquark, and the resulting energies of the low-lying  $\Omega_{cc}$  were in the range of 4050–4150 MeV. Those results are about 100 MeV lower than the present rough estimation using a three-quark model, and the results in [10,55]. So one may expect that the diquark assumption for the two heavy quarks in  $\Omega_{cc}$  resonances may reduce the energies.

In any case, we can conclude that the  $\Omega_{cc}$  resonances should lie at an energy below 4200 MeV in both the compact five-quark model (present) and the meson-baryon model [60].

Finally, we show the dependency of the presently obtained spectrum on the model parameter  $E_0$ . By taking  $E_0 = 3132$  MeV as given in Ref. [13], one can get

$$E_{i'} \simeq 3075 \pm 100 \text{ MeV.}$$
 (31)

Obviously, the obtained energies are much lower than those predicted by using other approaches. Namely, the value  $E_0 = 4102$  MeV employed in our calculations should be reasonable. We also present the numerical results with  $E_0$  changed by 2%, then

$$E_{i'} \simeq 4125 \pm 100 \text{ MeV.}$$
 (32)

In fact, change of  $E_0$  should lead to almost the same change of energy of each physical state  $|i'\rangle$ . In addition, the coefficients for decompositions of the physical states  $|i'\rangle$ are not sensitive to  $E_0$ .

# B. S-wave coupling of the $sccq\bar{q}$ to pseudoscalar meson and ground baryon states

From Fig. 2 and Table II, one can find that most of the obtained physical  $sccq\bar{q}$  states are above the threshold of

	$ 1\rangle$	$ 2\rangle$	$ 3\rangle$	$ 4\rangle$	$ 5\rangle$	$ 6\rangle$	$ 7\rangle$	$ 8\rangle$	9 angle	$ 10\rangle$
$\overline{K}\Xi_{cc}$	-1	$\sqrt{6}/3$	$\sqrt{3}/3$	$\sqrt{3}$	$\sqrt{2}$	$\sqrt{6}/3$	$\sqrt{3}/3$	0		
$D\Xi_c$	0	$\sqrt{12}/3$	0	0	-2	$\sqrt{12}/3$	0	0	• • •	•••
$D\Xi_c'$	$\sqrt{6}/3$	0	$\sqrt{2}$	$-\sqrt{2}$	0	0	$\sqrt{2}$	0		
$\bar{K}\Xi_{cc}^{*}$			•••	0	0	$-\sqrt{6}/3$	$\sqrt{12}/3$	1	$\sqrt{15}/3$	$-\sqrt{30}/3$
$D\Xi_c^*$				0	0	$\sqrt{12}/3$	0	$\sqrt{2}$	$-\sqrt{30}/3$	0

TABLE III. Color-flavor-spin factors for the transitions  $J^P = 1/2^-$  (shown in the 2nd to 4th rows) and  $3/2^-$  (shown in the last two rows)  $sccq\bar{q}$  configurations  $|i\rangle$  to *MB* channels.

the SU(2) isospin breaking  $\pi\Omega_{cc}$  channel, but below the thresholds of the other pseudoscalar meson and ground state baryons channels. It is expected that the decay widths of the presently obtained  $\Omega_{cc}$  should not be very large. This is in consistent with the findings in Ref. [60]. Therefore, in the present work, we only try to estimate the *S*-wave transitions of the obtained  $1/2^-$  and  $3/2^- \Omega_{cc}$  resonances to *MB* channels.

Using the transition operator in Eq. (22), and the wave functions obtained in Sec. III A, one can calculate the transition matrix elements of the obtained  $\Omega_{cc}$  resonances to  $\overline{K}\Xi_{cc}, \overline{K}\Xi_{cc}^*, D\Xi_c$ , and  $D\Xi_c^*$  channels, respectively. It is found that all the transition amplitudes of  $\Omega_{cc} \rightarrow MB$ processes share a common factor which involves the overlap between the orbital wave functions of the pentaquark configurations and the final meson-baryon, namely,

$$\mathcal{F}(k_M^2) = \langle \phi_B(\{\vec{\xi}_i\}) | \exp\{-i\vec{k}_M \cdot (\vec{r}_j + \vec{r}_{\bar{q}})/2\} | \varphi(\{\vec{\xi}_i\}) \rangle,$$
(33)

which depends on the momentum of the final meson  $\vec{k}_M$ , and the explicit confinement potential model for the quarks in baryons. Note that in the transitions of almost all the presently obtained  $\Omega_{cc}$  resonances to the corresponding *MB* channels, the  $\Omega_{cc}$  resonances are off-shell, because of their lower energies than the mass thresholds of the *MB* channels. Thus, the momentum of the final meson  $\vec{k}_M$  in Eq. (22) cannot be pinned down in the present framework. Yet, one can estimate the partial decay widths of these obtained  $\Omega_{cc}$  resonances from the calculation of the flavor, spin, orbital, and color overlap factor for the final *MB* states and the residual three-quark-meson configurations of the  $\Omega_{cc}$  states after the annihilation of the quarkantiquark  $q\bar{q} \rightarrow M$ , namely, the transition matrix elements of the orbital-flavor-spin-color dependent operator  $C_{\text{XFSC}}^j \bar{\chi}_z^{\dagger} \mathcal{I}_2 \chi_z^j X_P^j$  in Eq. (22).

In the three-quark model, for the  $\Xi_c$  baryon, the light and strange quarks in its flavor wave function can be either symmetric or antisymmetric. We denote the former state as  $\Xi_c$ , and the latter one as  $\Xi'_c$ . Therefore, we consider the transition processes of  $\Omega_{cc}$  with spin-parity  $1/2^-$  to  $\overline{K}\Xi_{cc}$ ,  $D\Xi_c$ , and  $D\Xi'_c$  channels, and the  $\Omega^*_{cc}$  with spin-parity  $3/2^$ to  $\overline{K}\Xi^*_{cc}$  and  $D\Xi^*_c$  channels.

Then, straightforward calculations on the transition matrix elements for the operator  $C_{XFSC}^{j} \overline{z}^{\dagger} \mathcal{I}_{2} \chi_{z}^{j} X_{P}^{j}$  in the processes of  $|i\rangle$  in Eqs. (5) and (6) to the above mentioned *MB* channels lead to the results shown in Table III, where the first three rows of the numerical results are the orbital-flavor-spin-color overlap factors for the configurations  $|i\rangle$  with spin-parity quantum number  $1/2^{-}$  to *MB* channels, while the last two rows are those for  $3/2^{-}$  configurations.

Considering the probability amplitudes for the mixing of configurations  $|i\rangle$  presented in Table II, we obtain the corresponding overlap factors for the physical  $\Omega_{cc}$  resonances listed in Table IV. One may note that there are some zeros obtained for some configurations as shown in Table III, however, they become finite for the physical states. For instance, the overlap factor for the configuration  $|8\rangle$  to the channel  $\bar{K}\Xi_{cc}$  is 0, while for  $|8'\rangle \rightarrow \bar{K}\Xi_{cc}$  it is -0.635. This is because the physical state  $|8'\rangle$  decouples to the configurations  $|i\rangle$  as

$$|8'\rangle = -0.16|5\rangle - 0.50|6\rangle + 0.85|8\rangle,$$
(34)

which is shown in Table II.

TABLE IV. Color-flavor-spin overlap factors for the transitions  $J^P = 1/2^-$  (shown in the 2nd to 4th rows) and  $3/2^-$  (shown in the last two rows)  $sccq\bar{q}$  physical states  $|i'\rangle$  to *MB* channels.

	$ 1'\rangle$	$ 2'\rangle$	$ 3'\rangle$	$ 4'\rangle$	$ 5'\rangle$	6' angle	7' angle	8' angle	9' angle	$ 10'\rangle$
$\overline{K}\Xi_{cc}$	-0.237	0.816	-1.137	1.218	0.644	1.357	-1.368	-0.635		
$D\Xi_c$	0	1.155	0	0	-2.250	-0.441	0	-0.257		
$D\Xi_c'$	1.608	0	-0.335	-1.937	0	0	-0.523	0		
$\bar{K}\Xi_{cc}^{*}$				-0.577	0.284	-0.033	-1.005	1.258	1.291	-1.826
$D\Xi_c^*$	•••			0	-0.938	1.432	0	0.625	-1.826	0

Compared to the overlap factor [49,50] for the strangeness five-quark configurations  $|uuds\bar{s}\rangle$  to  $\eta p$  channels, which is about ~0.75 and which may account for the strong coupling between  $S_{11}(1535)$  and strangeness channels, one can expect that the presently obtained physical states  $|i'\rangle$  may couple strongly to the *MB* channels for which the overlap factors shown in Table IV are larger than 0.8.

It should be very interesting to compare the decay behaviors of the presently obtained  $\Omega_{cc}$  resonances with those in a three-quark model. In Ref. [9], five  $\Omega_{cc}$  resonances lying at 4208–4303 MeV were obtained, and the decay widths of these resonances to  $\overline{K}\Xi_{cc}$  or  $\overline{K}\Xi_{cc}^*$  channels were estimated explicitly. It was found that some of the obtained decay widths should be larger than 100 MeV. This is very different from the conclusion that most of the obtained  $\Omega_{cc}$  resonances using a pentaquark picture can only decay to the isospin breaking channel  $\pi\Omega_{cc}$ .

In a three-quark picture, one can also estimate the flavorspin-color overlap factor of the  $\Omega_{cc}$  resonances and the *MB* channels using Eq. (10). For instance, a straightforward calculation on the overlap factors of the three-quark states given in Eq. (30) shows that coupling for  $\Omega_{cc} \rightarrow \bar{K} \Xi_{cc}$  may be comparable to that for  $\Omega_{cc} \rightarrow D\Xi_c$ , since the obtained flavor-spin-color overlap factor for a given three-quark  $\Omega_{cc}$  resonance to the  $\bar{K}\Xi_{cc}$  channel is  $\sqrt{2}$  times of that for the  $D\Xi_c$  channel; this is determined by the flavor-spin structure of the  $\Omega_{cc}$  resonances and the effective chiral Lagrangian. However, as we can see in Table IV, the presently obtained numerical results for several states are very different from the three-quark results.

#### **IV. SUMMARY**

In the present work, we investigate the spectrum of low-lying  $\Omega_{cc}$  resonances with negative parity as pentaquark states, using the chiral constituent quark model within a five-quark picture. We obtain ten pentaquark states with spin-parity  $J^P = 1/2^-, 3/2^-, 5/2^-$ , which lie at  $4050 \pm 100$  MeV. Most of the obtained states are above the isospin breaking decay channel  $\pi\Omega_{cc}$ , but below the other meson-baryon channels. So we just try to calculate the flavor, spin, orbital, and color overlap factors for the final MB states and the residual three-quark-meson configurations of the  $\Omega_{cc}$  states after the annihilation of the quark-antiquark  $q\bar{q} \rightarrow M$ . It is found that several of the presently obtained  $\Omega_{cc}$  may couple strongly to  $D\Xi_c$  or  $\bar{K}\Xi_{cc}$  channels. One may expect that the calculations here could be compared with the future experimental measurements which are likely to be done by Belle II and/or LHCb.

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