

Charged current electroproduction of a charmed meson at an electron-ion collider

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We calculate the amplitude for exclusive electroweak production of a pseudoscalar D_s or a vector D_s^* charmed strange meson on an unpolarized nucleon, through a charged current, in leading order in α_s . We work in the framework of the collinear QCD approach where generalized gluon distributions factorize from perturbatively calculable coefficient functions. We include both $O(m_c)$ terms in the coefficient functions and $O(M_D)$ mass term contributions in the heavy meson distribution amplitudes. We show that this process may be accessed at future electron-ion colliders.

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I. INTRODUCTION

Exclusive electroproduction processes involving charged currents (i.e., through a W^\pm exchange) have not been discussed much up to now, with the notable exception of the pioneering work [1]. The reason is simple as the smallness of the weak coupling prevents exclusive cross sections from being large enough to allow sufficient counting rates at existing electron-nucleon facilities. The very high luminosity anticipated at planned high-energy electron-ion colliders [2,3] should open this physics domain to a detailed investigation of various interesting channels. In this respect, the production of a single charmed meson—which is forbidden in pure electromagnetic processes—is a specific way to study various features of hadronic physics, and in particular the effects of the heavy quarks mass in the framework of collinear QCD factorization [4]. Indeed, the well-established framework of collinear QCD factorization [5–7] for scattering amplitudes in exclusive electroproduction, in reactions mediated by a highly virtual photon, may also be applied to reactions mediated by a virtual W^\pm boson, in a similar generalized Bjorken regime [8]. This framework describes hadronic amplitudes using generalized parton distributions (GPDs) which give access to a three-dimensional analysis [9] of the internal structure of hadrons.

Since charged currents are mediated by a massive vector boson exchange which is usually highly virtual, one is tempted to apply a factorized description of the process

amplitude down to quite small values of the momentum transfer $Q^2 = -q^2$ carried by the W^\pm boson. Moreover, heavy quark production allows us to extend the range of validity of collinear factorization, the heavy quark mass playing the role of the hard scale. Indeed kinematics (detailed below) shows that the relevant scale is $O(Q^2 + m_c^2)$.

Since the scattering amplitude is proportional to the relevant Cabibbo-Kobayashi-Maskawa matrix element, the dominant production process for a charmed meson involves a $D_s^-(1968)$ or $D_s^{*-}(2112)$ charmed and strange meson. The $D_s^{*-}(2112)$ decays mostly in a $D_s^-(1968)\gamma$ pair. In this paper we shall thus restrict our study to the exclusive production of a pseudoscalar D_s^- or a D_s^{*-} vector meson through the reactions on a nucleon N (proton or neutron) target

$$e^-(k) + N(p_1) \rightarrow \nu_e(k') + D_s^-(p_D) + N'(p_2), \quad (1)$$

$$e^-(k) + N(p_1) \rightarrow \nu_e(k') + D_s^{*-}(p_D) + N'(p_2), \quad (2)$$

in the kinematical domain where collinear factorization leads to a description of the scattering amplitude as a convolution of gluon GPDs and the D_s^- or D_s^{*-} meson distribution amplitude (DA) (see Fig. 1) with the amplitude for hard subprocesses

$$W^- + g \rightarrow (\bar{c}s) + g, \quad (3)$$

calculated in the collinear kinematics taking heavy quark mass effects into account [10,11]. In order to be consistent, we shall include the order $\frac{M_D}{Q^2 + M_D^2}$ contributions related to mass terms in the distribution amplitudes of heavy mesons [see Eq. (11)].

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Our kinematical notations are as follows:

$$q = k - k', \quad Q^2 = -q^2, \quad \Delta = p_2 - p_1, \quad \Delta^2 = t, \quad s = (p_1 + k)^2.$$

We parametrize the momenta in the usual Sudakov representation as (m and M_D are the nucleon and D_s -meson masses)

$$\begin{aligned} q^\mu &= -2\xi' p^\mu + \frac{Q^2}{4\xi'} n^\mu; & \epsilon_L^\mu(q) &= \frac{1}{Q} \left[2\xi' p^\mu + \frac{Q^2}{4\xi'} n^\mu \right], & p_D^\mu &= 2(\xi - \xi') p^\mu + \frac{M_D^2 - \Delta_T^2}{4(\xi - \xi')} n^\mu - \Delta_T^\mu, \\ p_1^\mu &= (1 + \xi) p^\mu + \frac{1}{2} \frac{m^2 - \Delta_T^2/4}{1 + \xi} n^\mu - \frac{\Delta_T^\mu}{2}, & p_2^\mu &= (1 - \xi) p^\mu + \frac{1}{2} \frac{m^2 - \Delta_T^2/4}{1 - \xi} n^\mu + \frac{\Delta_T^\mu}{2}, \end{aligned} \quad (4)$$

with $p^2 = n^2 = 0$ and $p \cdot n = 1$ while m and M_D denote the nucleon and D_s (or D_s^*) charmed meson mass. We use the usual y variable to express the energy fraction carried by the W^- meson,

$$y = \frac{p_1 \cdot q}{p_1 \cdot k} = \frac{2(p_1 \cdot q)}{s - m^2}. \quad (5)$$

We define the skewness variable as

$$\xi = -\frac{(p_2 - p_1) \cdot n}{2}, \quad (6)$$

and neglecting the nucleon mass and Δ_T , its approximate value is

$$\xi \approx \frac{Q^2 + M_D^2}{4p_1 \cdot q - Q^2 - M_D^2} = \frac{Q^2 + M_D^2}{2ys - 2m^2 - Q^2 - M_D^2}, \quad (7)$$

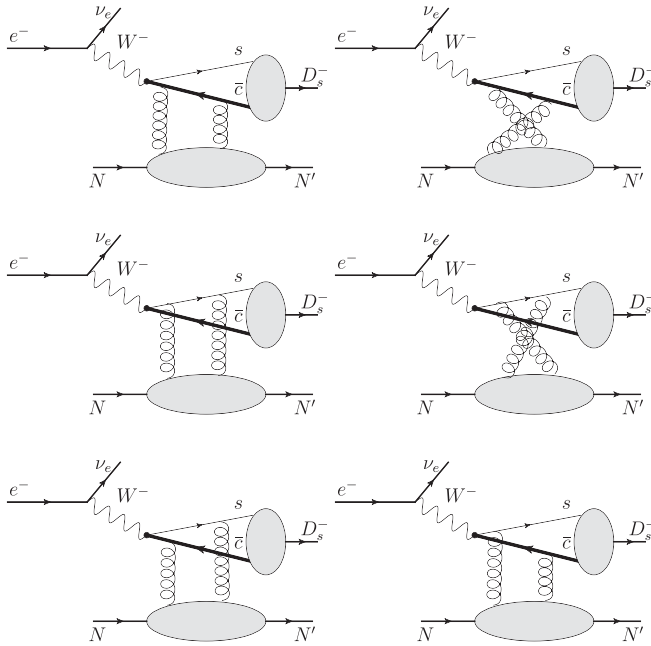


FIG. 1. Feynman diagrams for the factorized amplitude for the $e^- + N \rightarrow \nu_e + D_s^- + N'$ process involving the gluon GPDs; the thick line represents the heavy antiquark \bar{c} .

while $\xi' = -q \cdot n / 2 = \frac{\xi Q^2}{Q^2 + M_D^2}$. We show in Fig. 2 the correspondence between ξ and the energy fraction y in the two extreme scenarios of the EIC, namely the low-energy one with $s = 820 \text{ GeV}^2$ (denoted as LE) and the high-energy one with $s = 20000 \text{ GeV}^2$ (denoted as HE).

To unify the description of the scaling amplitude, we thus define a modified Bjorken variable

$$x_B^D \equiv \frac{Q^2 + M_D^2}{2p_1 \cdot q} \neq x_B \equiv \frac{Q^2}{2p_1 \cdot q}, \quad (8)$$

which allows us to express ξ in a compact form

$$\xi \approx \frac{x_B^D}{2 - x_B^D}. \quad (9)$$

If the meson mass is the relevant large scale (for instance in the limiting case where Q^2 vanishes as in the timelike Compton scattering kinematics [12])

$$\xi \approx \frac{\tau}{2 - \tau}; \quad \tau = \frac{M_D^2}{s_{WN} - m^2} = \frac{M_D^2}{ys - m^2}. \quad (10)$$

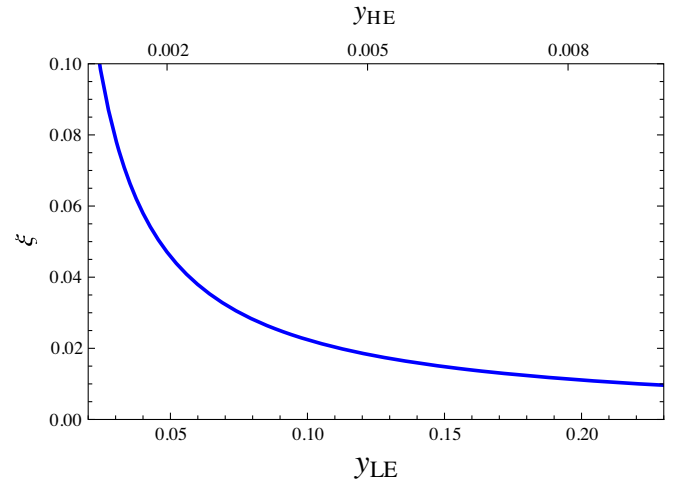


FIG. 2. The correspondence between ξ and the variable y in the low-energy (LE) and high-energy (HE) scenarios of the EIC.

II. DISTRIBUTION AMPLITUDES AND GPDS

In the collinear-factorization framework, the hadronization of the quark-antiquark pair is described by a distribution amplitude which obeys a twist expansion and evolution equations. Much work has been devoted to this subject [13]. The charmed meson distribution amplitudes are less known than the light meson ones. Here, we shall follow Ref. [14] and include some mass terms which will lead to order $\frac{M_D}{Q^2+M_D^2}$ contributions to the amplitudes; omitting the path-ordered gauge link, the relevant distribution amplitude reads for the pseudo scalar D_s^- meson.

$$\begin{aligned} \langle D_s^-(P_D) | \bar{s}_\beta(y) c_\gamma(-y) | 0 \rangle \\ = i \frac{f_{D_s}}{4} \int_0^1 dz e^{i(z-\bar{z})P_D \cdot y} [(\hat{P}_D - M_D) \gamma^5]_{\gamma\beta} \phi_{D_s}(z), \end{aligned} \quad (11)$$

where $\int_0^1 dz \phi_{D_s}(z) = 1$. As usual, we denote $\bar{z} = 1 - z$ and $\hat{p} = p_\mu \gamma^\mu$ for any vector p . Contrarily to the case of light mesons, where DAs are symmetric in a $z \rightarrow \bar{z}$ transformation, heavy meson DAs are asymmetric, the heavy quark (or antiquark) taking most of the light cone momentum of the hadron. In our case of a heavy antiquark-light quark meson, this means a DA which is strongly peaked around z_0 with $1 - z_0 = \frac{m_c}{M_D}$. In our estimates, we will thus parametrize $\phi_{D_s}(z)$ following two recent studies:

- (a) as in Ref. [14], i.e., $\phi_{D_s}(z) = 6z(1-z)(1 + C_D(1-2z))$ with $C_D \approx 1.5$, which has a maximum around $z = 0.3$.
- (b) as in Ref. [15], i.e., $\phi_{D_s}(z) = 3.12z(1-z) \times e^{2.85z(1-z) - 0.93(2z-1)}$.

As shown in Fig. 3, these two parametrizations are somewhat different, and will allow us to roughly quantify the uncertainty of our estimates with respect to a

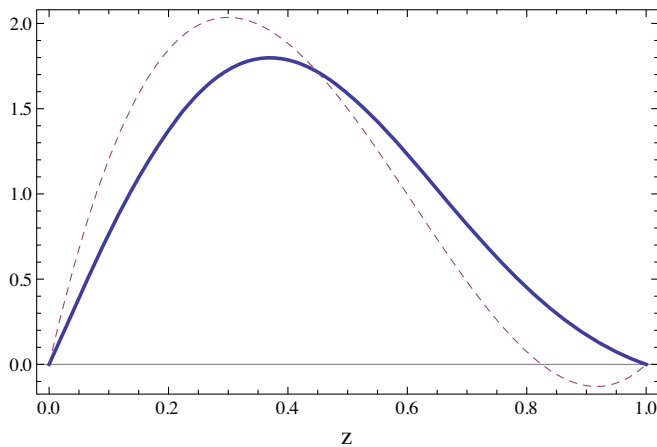


FIG. 3. The D_s meson distribution amplitudes proposed by Ref. [14] (dashed curve) and by Ref. [15] (solid curve).

reasonable choice of the D_s 's distribution amplitude. The coupling constant f_{D_s} of the pseudoscalar meson D_s has recently been calculated on the lattice as $f_{D_s} \approx 0.24$ GeV [16,17].

The DA of the vector meson D_s^* is defined as in the case of the D^* vector meson case, and for the longitudinal and transverse polarization states read:

$$\begin{aligned} \langle D_s^{*-}(P_D, \varepsilon_L) | \bar{s}_\beta(y) c_\gamma(-y) | 0 \rangle \\ = \frac{f_{D_s^*}}{4} \int_0^1 dz e^{i(z-\bar{z})P_D \cdot y} [\hat{P}_D - M_{D_s^*}]_{\gamma\beta} \phi_{D_s^*}(z), \end{aligned} \quad (12)$$

$$\begin{aligned} \langle D_s^{*-}(P_D, \varepsilon_T) | \bar{s}_\beta(y) c_\gamma(-y) | 0 \rangle \\ = \frac{f_{D_s^*}}{4} \int_0^1 dz e^{i(z-\bar{z})P_D \cdot y} [(\hat{P}_D - M_{D_s^*}) \hat{\varepsilon}_T]_{\gamma\beta} \phi_{D_s^*}(z), \end{aligned} \quad (13)$$

where $\int_0^1 dz \phi_{D_s^*}(z) = 1$. The coupling constants $f_{D_s^*}$ may be different for the transversally and longitudinally polarized mesons, but the difference is unlikely to be large. The ratio of the coupling constants of the D_s^* vector meson to the D_s pseudoscalar meson has been estimated on the lattice [16,18] as $f_{D_s^*}/f_{D_s} \approx 1.1$. Since no parametrization of the DA of the D_s^* vector meson has been proposed up to now, we shall use the same shape as for the pseudoscalar meson, $\phi_{D_s^*}(z) = \phi_{D_s}(z)$.

We define the generalized parton distributions of a parton in the nucleon target with the conventions of [19]. To get the quantitative predictions, we use the Goloskokov-Kroll (G-K) model for gluonic GPDs, based on the fits to deeply virtual meson production. Details of the model can be found in [20]. To quantify the dependence of our predictions on the poorly known gluon distributions, we present also the results based on the other simple model of GPDs described in [21].

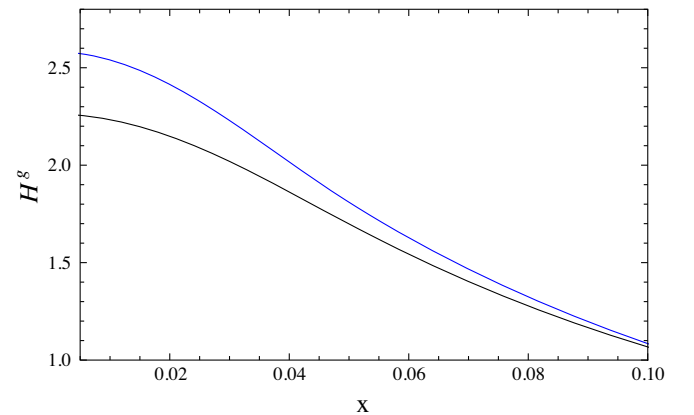


FIG. 4. The x dependence of gluon GPD $H^g(x, \xi, t, \mu_F^2)$ given by the GK (blue lines), and simple [20] (black lines) GPD models, for typical values of the parameter in an EIC experiment: $\xi = 0.05$, $Q^2 = 0.1$ GeV², $t = -0.1$ GeV² and factorization scale $\mu_F^2 = M_D^2 + Q^2$.

The difference between those two models is illustrated on Fig. 4, where the dominant gluonic GPD H^g for the kinematical configuration relevant for the described process is presented as a function of x . The values of parameters specified in the figure caption (i.e., ξ , Q^2 , t , and μ_F^2) correspond to the typical values of EIC.

III. THE D_s MESON PRODUCTION AMPLITUDE

If we neglect the strange quark content of the nucleon, there is no contribution coming from quark GPDs and the only relevant contribution comes from the diagrams of Fig. 1 with the gluon GPDs [22]. The expression for the amplitudes can be read off our previous work [11] on neutrino production. For completeness, we copy the relevant equations (with appropriate exchange of z and \bar{z}), neglecting the strange quark mass.

The six Feynman diagrams of Fig. 1 contribute to the coefficient function. The last three correspond to the first three with the substitution $x \leftrightarrow -x$, and an overall minus sign for the axial case. The transversity gluon GPDs do not contribute to the longitudinal amplitude since there is no way to flip the helicity by two units when producing a (pseudo)scalar meson. This will not be the case for the production of a vector meson D_s^* .

The symmetric and antisymmetric hard amplitudes read

$$g_{\perp}^{ij} \mathcal{M}_H^S = \left\{ \frac{Tr_a^S}{D_1 D_2} + \frac{Tr_b^S}{D_3 D_4} + \frac{Tr_c^S}{D_4 D_5} \right\} + \{x \rightarrow -x\}, \quad (14)$$

$$i\epsilon_{\perp}^{ij} \mathcal{M}_H^A = \left\{ \frac{Tr_a^A}{D_1 D_2} + \frac{Tr_b^A}{D_3 D_4} + \frac{Tr_c^A}{D_4 D_5} \right\} - \{x \rightarrow -x\}, \quad (15)$$

where the traces are

$$Tr_a^S = \frac{2z}{Q} g_T^{ij} \left[\bar{z} M_D^4 + Q^4 + Q^2 M_D^2 (2-z) - \frac{x+\xi}{2\xi} Q^2 (Q^2 + M_D^2) \right], \quad (16)$$

$$Tr_a^A = \frac{2iz\epsilon^{npij}}{Q} \left[\bar{z} M_D^4 + Q^2 M_D^2 (1+z) + \frac{x-\xi}{2\xi} Q^2 (Q^2 + M_D^2) \right], \quad (17)$$

$$Tr_b^S = \frac{2(Q^2 + M_D^2)}{Q} g_T^{ij} \left[-\frac{x+\xi}{2\xi} m_c M_D - \bar{z} \frac{x-\xi}{2\xi} Q^2 + m_c^2 + z\bar{z} M_D^2 \right], \quad (18)$$

$$Tr_b^A = \frac{2i\epsilon^{npij}}{Q} \left[-\bar{z} \frac{x-\xi}{2\xi} Q^4 + Q^2 M_D^2 (2-z) \left(-\bar{z} - \frac{x-\xi}{2\xi} \right) + M_D^4 \left(\bar{z}^2 - \bar{z} - 1 - \frac{x+\xi}{2\xi} \right) + (M_D^2 - m_c^2)(M_D^2 - Q^2) + M_D(M_D + m_c)(Q^2 + M_D^2) \frac{x+\xi}{2\xi} \right], \quad (19)$$

$$Tr_c^S = -\frac{Q^2 + M_D^2}{\xi Q} g_T^{ij} \left[(Q^2 + M_D^2) \frac{x^2 - \xi^2}{2\xi} + 2\bar{z} M_D^2 (\xi\bar{z} + x) - M_D(m_c + M_D)(x - \xi + 2\xi\bar{z}) \right], \quad (20)$$

$$Tr_c^A = \frac{2i\epsilon^{npij}}{Q} \left[(\bar{z}^2 - 1) Q^2 M_D^2 + \left(z M_D^2 - (Q^2 + M_D^2) \frac{x+\xi}{2\xi} \right) \left((2-z) M_D^2 + (Q^2 + M_D^2) \frac{x-\xi}{2\xi} \right) + M_D(m_c + M_D) \left[(Q^2 + M_D^2) \frac{x+\xi}{2\xi} + z(Q^2 - M_D^2) \right] \right], \quad (21)$$

and the denominators read (with $\alpha = \frac{2\xi M_D^2}{M_D^2 + Q^2}$)

$$D_1 = z[-\bar{z} M_D^2 - Q^2 + i\epsilon], \quad (22)$$

$$D_2 = z \left[z M_D^2 + \frac{x-\xi}{2\xi} (Q^2 + M_D^2) + i\epsilon \right] = z \frac{Q^2 + M_D^2}{2\xi} (x - \xi + \alpha z + i\epsilon), \quad (23)$$

$$D_3 = -\bar{z} Q^2 - z\bar{z} M_D^2 - m_c^2 + i\epsilon = -\bar{z} (Q^2 + M_D^2) + \bar{z}^2 M_D^2 - m_c^2 + i\epsilon, \quad (24)$$

$$D_4 = \bar{z}^2 M_D^2 - m_c^2 + \frac{\bar{z}(x - \xi)}{2\xi} (Q^2 + M_D^2) + i\epsilon, \quad (25)$$

$$D_5 = z \left[z M_D^2 - \frac{x + \xi}{2\xi} (Q^2 + M_D^2) + i\epsilon \right] = z \frac{Q^2 + M_D^2}{2\xi} (-x - \xi + \alpha z + i\epsilon). \quad (26)$$

Inserting these results inside Eqs. (14)–(15) with the help of Eq. (11), we get the following form of the gluonic contribution to the amplitude

$$T_L^g = \frac{iC_g}{2} \int_{-1}^1 dx \frac{-1}{(x + \xi - i\epsilon)(x - \xi + i\epsilon)} \int_0^1 dz f_{D_s} \phi_{D_s}(z) \cdot \left[\bar{N}(p_2) \left[H^g \hat{n} + E^g \frac{i\sigma^{n\Delta}}{2m} \right] N(p_1) \mathcal{M}_H^S + \bar{N}(p_2) \left[\tilde{H}^g \hat{n} \gamma^5 + \tilde{E}^g \frac{\gamma^5 n \cdot \Delta}{2m} \right] N(p_1) \mathcal{M}_H^A \right] \quad (27)$$

$$\equiv \frac{-iC_g}{2Q} \bar{N}(p_2) \left[\mathcal{H}^g \hat{n} + \mathcal{E}^g \frac{i\sigma^{n\Delta}}{2m} + \tilde{\mathcal{H}}^g \hat{n} \gamma^5 + \tilde{\mathcal{E}}^g \frac{\gamma^5 n \cdot \Delta}{2m} \right] N(p_1), \quad (28)$$

where $C_g = T_f \frac{\pi}{3} \alpha_s V_{sc}$ with $T_f = \frac{1}{2}$ and the factor $\frac{-1}{(x + \xi - i\epsilon)(x - \xi + i\epsilon)}$ comes from the conversion of the strength tensor to the gluon field. The last line defines the gluonic form factors \mathcal{H}^g , $\tilde{\mathcal{H}}^g$, \mathcal{E}^g , $\tilde{\mathcal{E}}^g$, which depend on ξ , t , and the factorization scale $\mu_F^2 = M_D^2 + Q^2$. Note that there is no singularity in the integral over z if the DA vanishes like $z\bar{z}$ at the limits of integration.

We plot on Fig. 5 the ξ dependence of the real and imaginary parts of the dominant form factor $\mathcal{H}(\xi)$ for $\Delta_T = 0$ and $Q^2 = 0.1 \text{ GeV}^2$. This form factor is energy independent, and its dependence on y occurs through the relation between y and ξ in Eq. (10).

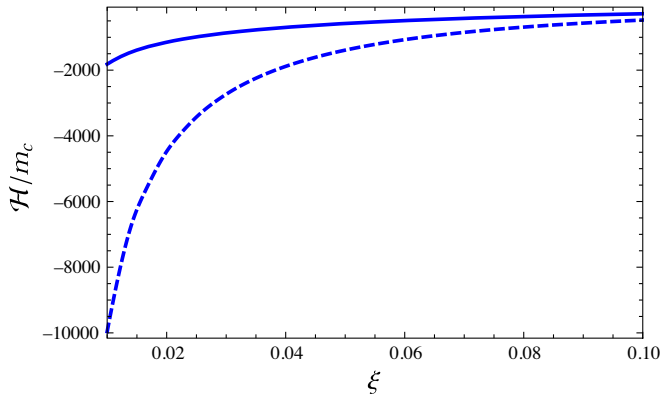


FIG. 5. The ξ dependence of the dominant form factor $\mathcal{H}(\xi)$ divided by the charm quark mass m_c for $\Delta_T = 0$ and $Q^2 = 0.1 \text{ GeV}^2$. Solid (dashed) lines refer to the real (imaginary) parts.

IV. THE D_s^* MESON PRODUCTION AMPLITUDE

Let us now consider the exclusive production of the D_s^{*-} (2112) vector meson through the reaction

$$e^-(k) + p(p_1, \lambda) \rightarrow \nu_e(k') + D_s^{*-}(p_D, \epsilon_D) + p'(p_2, \lambda'). \quad (29)$$

In [23], we showed that neutrino production of D^* vector mesons may help to measure the gluon transversity GPDs, the phenomenology of which is presently restricted to angular asymmetries in DVCS [24], which turns out to be quite difficult to access experimentally. We do not follow this task here since the reconstruction of the decay products of the D_s^* —which must be carried on to isolate the transversity gluon GPD contribution—is likely to be a too hard challenge. The amplitude for charged current D_s^{*-} production may be read off from the neutrino-production study [23]. There are three nonzero ($W^- \rightarrow D_s^{*-}$) helicity amplitudes:

- a longitudinal (W^-) to longitudinal (D_s^{*-}) amplitude \mathcal{M}_{00} ;
- a left (W^-) to left (D_s^{*-}) \mathcal{M}_{LL} ;
- a left (W^-) to right (D_s^{*-}) \mathcal{M}_{LR} , which is proportional to transversity gluon GPDs.

Apart from trivial changes in the masses and coupling constants, the amplitude for the longitudinally polarized vector meson D_s^* production is calculated in the same way as the one for the pseudoscalar D_s production. The additional γ_5 matrix in the definition of the pseudoscalar DA does not alter the magnitude of the coefficient function

acting on the gluon GPD, as previously shown in the massless-quark case of π vs ρ_L production [25]; this result is also true for the massive charm quark case studies here. The main difference in the production rates will thus come from the higher values of the skewness ξ at fixed values of y and beam energy, which results in smaller values of the gluon GPDs.

The amplitude for the transversely polarized D_s^* production has two components; one which depends on the

usual gluon GPDs and contributes to σ_{--} and another one which depends on the gluon transversity GPD; the magnitude of which is unknown up to now. This latter contribution vanishes in the angular integrated cross section and can only be separated through an analysis of the angular distribution of decay products of the vector meson D_s^* , which is a very hard challenge. The amplitude \mathcal{M}_{LL} which contributes to the azimuthal angle integrated cross section is expressed in terms of form factors \mathcal{H}_T^g , \mathcal{E}_T^g , $\tilde{\mathcal{H}}_T^g$, $\tilde{\mathcal{E}}_T^g$ as

$$\mathcal{M}_{LL} = \frac{iC_g}{2} \int_{-1}^1 dx \frac{-1}{(x+\xi-i\epsilon)(x-\xi+i\epsilon)} \int_0^1 dz f_T \phi_{D_s^*}(z) \cdot \left[\bar{N}(p_2) \left[H^g \hat{n} + E^g \frac{i\sigma^{n\Delta}}{2m} \right] N(p_1) G_T + \bar{N}(p_2) \left[\tilde{H}^g \hat{n} \gamma^5 + \tilde{E}^g \frac{\gamma^5 n \cdot \Delta}{2m} \right] N(p_1) \tilde{G}_T \right] \quad (30)$$

$$\equiv \frac{-iC_g}{2Q} \bar{N}(p_2) \left[\mathcal{H}_T^g \hat{n} + \mathcal{E}_T^g \frac{i\sigma^{n\Delta}}{2m} + \tilde{\mathcal{H}}_T^g \hat{n} \gamma^5 + \tilde{\mathcal{E}}_T^g \frac{\gamma^5 n \cdot \Delta}{2m} \right] N(p_1), \quad (31)$$

with $T_f = \frac{1}{2}$ and $C_g = T_f \frac{\pi}{3} \alpha_s V_{sc}$ for D_s^* production, and where G_T and \tilde{G}_T factors read [23]

$$G_T = \frac{-8M_D z \epsilon_D^* \cdot \epsilon_W (\kappa(x-3\xi) + M_D^2 z)}{D_1(x, \xi) D_2(x, \xi)} + \frac{8iM_D z \kappa(x+\xi) \epsilon^{pne_D^* \epsilon_W}}{D_1(x, \xi) D_2(x, \xi)} - 8 \frac{\kappa(\bar{z}M_D + m_c)(x-\xi) \epsilon_D^* \cdot \epsilon_W}{D_3(x, \xi) D_4(x, \xi)} + \{x \rightarrow -x\}, \quad (32)$$

$$\tilde{G}_T = \frac{8iM_D z p \cdot \epsilon_W p \cdot \epsilon_D^* (M_D^2 (\xi - x - 2\xi\bar{z}) + 4\kappa\xi(x-\xi))}{\kappa D_1(x, \xi) D_2(x, \xi)} + \frac{-8M_D z \epsilon^{np\epsilon_D^* \epsilon_W} (M_D^2 z + \kappa(x-3\xi))}{D_1(x, \xi) D_2(x, \xi)} - \{x \rightarrow -x\}, \quad (33)$$

with $\kappa = \frac{M_D^2 + Q^2}{4\xi}$.

V. OBSERVABLES

The (initial electron-spin averaged) differential cross section for the production of a pseudoscalar charmed meson is written, after azimuthal integration, as [26]

$$\frac{d\sigma(e^- N \rightarrow \nu N' D_s^-)}{dy dQ^2 dt} = \pi \bar{\Gamma} \left\{ \frac{1 + \sqrt{1 - \epsilon^2}}{2} \sigma_{--} + \epsilon \sigma_{00} \right\}, \quad (34)$$

with $y = \frac{p \cdot q}{p \cdot k}$, $Q^2 = x_B y (s - m^2)$, $\epsilon \approx \frac{1-y}{1-y+y^2/2}$, and

$$\bar{\Gamma} = \frac{G_F^2}{(2\pi)^4} \frac{1}{32y} \frac{1}{\sqrt{1 + 4x_B^2 m^2 / Q^2}} \frac{1}{(s - m^2)^2} \frac{Q^2}{1 - \epsilon}, \quad (35)$$

where the ‘‘cross sections’’ $\sigma_{lm} = \epsilon_l^{\mu} W_{\mu\nu} \epsilon_m^{\nu}$ are the product of amplitudes for the process $W^-(\epsilon_l) N \rightarrow D_s^- N'$, averaged (summed) over the initial (final) hadron polarizations.

For pseudoscalar D_s^- -meson production, σ_{--} vanishes while the longitudinal cross section σ_{00} is straightforwardly obtained by squaring the sum of the amplitudes T_L^g ; at zeroth order in Δ_T , it reads

$$\sigma_{00}|_{D_s^-} = \frac{1}{Q^2} \left\{ [|C_g \mathcal{H}^g|^2 + |C_g \tilde{\mathcal{H}}^g|^2] (1 - \xi^2) + \frac{\xi^4}{1 - \xi^2} [|C_g \tilde{\mathcal{E}}^g|^2 + |C_g \mathcal{E}^g|^2] - 2\xi^2 \text{Re}[C_g \mathcal{H}^g][C_g \mathcal{E}^{g*}] - 2\xi^2 \text{Re}[C_g \tilde{\mathcal{H}}^g][C_g \tilde{\mathcal{E}}^{g*}] \right\}. \quad (36)$$

For transversely polarized vector D_s^{*-} meson production the cross sections are given by Eq. (36) after appropriate replacement of form factors in Eq. (36) by those defined in Eq. (31)

$$\sigma_{00}|_{D_s^-} = \frac{1}{Q^2} \left\{ [|C_g \mathcal{H}_L^g|^2 + |C_g \tilde{\mathcal{H}}_L^g|^2](1 - \xi^2) + \frac{\xi^4}{1 - \xi^2} [|C_g \tilde{\mathcal{E}}_L^g|^2 + |C_g \mathcal{E}_L^g|^2] - 2\xi^2 \text{Re}[C_g \mathcal{H}_L^g][C_g \mathcal{E}_L^{g*}] - 2\xi^2 \text{Re}[C_g \tilde{\mathcal{H}}_L^g][C_g \tilde{\mathcal{E}}_L^{g*}] \right\}, \quad (37)$$

$$\sigma_{--}|_{D_s^-} = \frac{1}{Q^2} \left\{ [|C_g \mathcal{H}_T^g|^2 + |C_g \tilde{\mathcal{H}}_T^g|^2](1 - \xi^2) + \frac{\xi^4}{1 - \xi^2} [|C_g \tilde{\mathcal{E}}_T^g|^2 + |C_g \mathcal{E}_T^g|^2] - 2\xi^2 \text{Re}[C_g \mathcal{H}_T^g][C_g \mathcal{E}_T^{g*}] - 2\xi^2 \text{Re}[C_g \tilde{\mathcal{H}}_T^g][C_g \tilde{\mathcal{E}}_T^{g*}] \right\}. \quad (38)$$

Let us now present our estimates for the D_s^- and D_s^{*-} production cross sections. Since the gluon axial GPDs are quite smaller than the vector ones, due to the known smallness of the ratio of the relevant helicity dependent vs spin-independent gluon parton distribution functions $\frac{\Delta g(x)}{g(x)}$ [27], we neglect their contributions in our following numerical analysis. Contributions of GPD E and \tilde{E} , in the considered region of $\xi < 0.1$, are additionally suppressed

by factors ξ^2 or ξ^4 , so we also omit them. We remind the reader that the relevant factorization scale used in the calculation is given by $\mu_F^2 = m_c^2 + Q^2$, which for $Q^2 = 0.1\text{--}0.3 \text{ GeV}^2$ gives $\mu_F^2 \approx 1.8\text{--}2 \text{ GeV}^2$, which is very close to the scale at which our GPD models are defined (2 GeV^2), making evolution effects negligible.

We show on Fig. 6 and on Fig. 7 the Q^2 and y dependence of the cross section for the pseudoscalar D_s^-

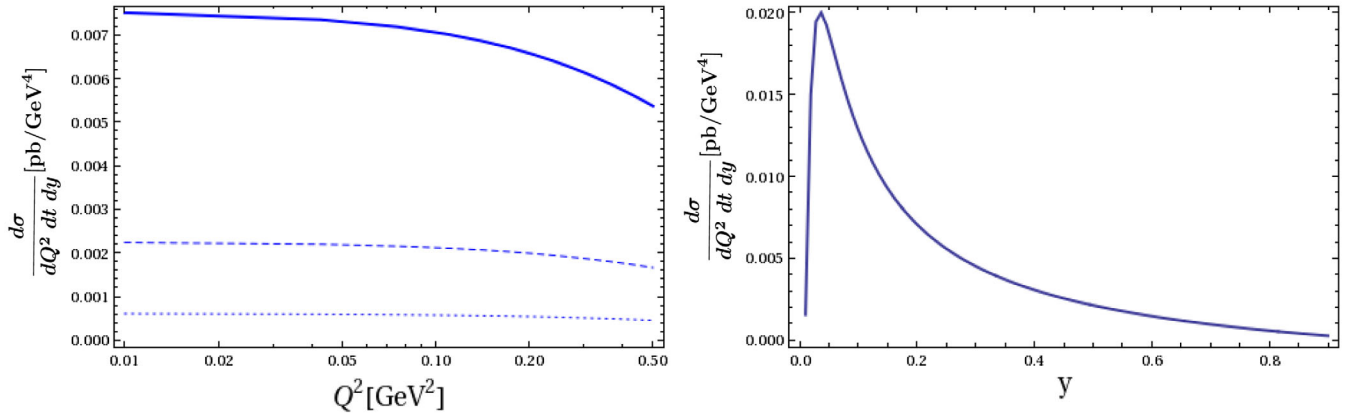


FIG. 6. Left panel: The Q^2 dependence of the cross section $\frac{d\sigma(e^-N \rightarrow \nu ND_s^-)}{dy dQ^2 dt}$ (in pb GeV $^{-4}$) for $\Delta_T = 0$ and $s = 820 \text{ GeV}^2$ and $y = 0.2$ (solid curve), $y = 0.5$ (dashed curve) and $y = 0.8$ (dotted curve). Right panel: The y dependence of the cross section $\frac{d\sigma(e^-N \rightarrow \nu ND_s^-)}{dy dQ^2 dt}$ (in pb GeV $^{-4}$) for $Q^2 = 0.1 \text{ GeV}^2$, $\Delta_T = 0$, and $s = 820 \text{ GeV}^2$.

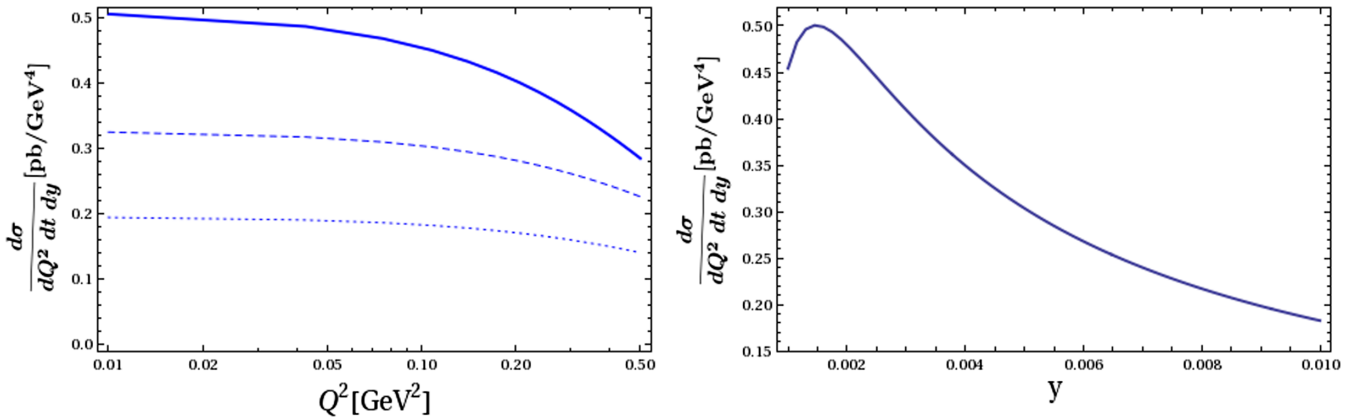


FIG. 7. Left panel: The Q^2 dependence of the cross section $\frac{d\sigma(e^-N \rightarrow \nu ND_s^-)}{dy dQ^2 dt}$ (in pb GeV $^{-4}$) for $\Delta_T = 0$ and $s = 20000 \text{ GeV}^2$ and $y = 10^{-3}$ (solid curve), $y = 5 \times 10^{-3}$ (dashed curve) and $y = 10^{-2}$ (dotted curve). Right panel: The y dependence of the cross section $\frac{d\sigma(e^-N \rightarrow \nu ND_s^-)}{dy dQ^2 dt}$ (in pb GeV $^{-4}$) for $Q^2 = 0.1 \text{ GeV}^2$, $\Delta_T = 0$, and $s = 20000 \text{ GeV}^2$.

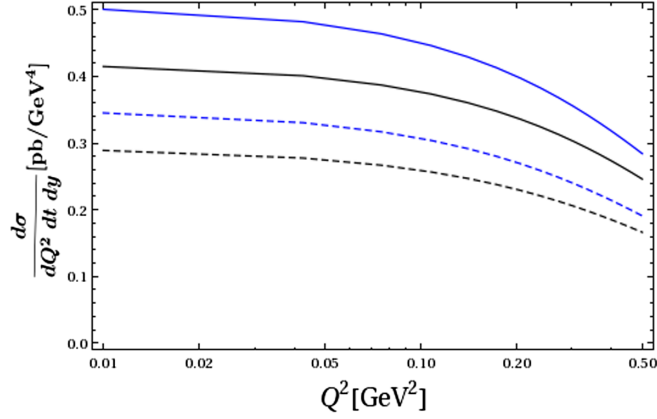


FIG. 8. The Q^2 dependence of the cross section $\frac{d\sigma(e^-N \rightarrow \nu ND_s^-)}{dy dQ^2 dt}$ (in pb GeV^{-4}) for $\Delta_T = 0$, $s = 20000 \text{ GeV}^2$, and $y = 10^{-4}$ with GK (blue lines), and simple [21] (black lines) GPD models, and with DAs from [14] (solid lines) and [15] (dashed lines).

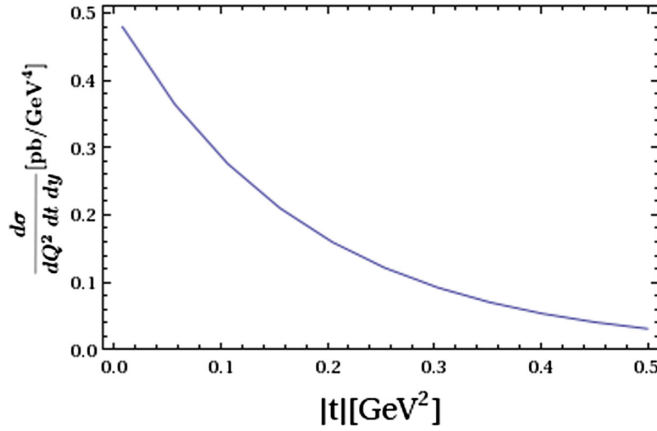


FIG. 9. The t -dependence of the cross section $\frac{d\sigma(e^-N \rightarrow \nu ND_s^-)}{dy dQ^2 dt}$ (in pb GeV^{-4}) for $y = 2 \times 10^{-3}$, $s = 20000 \text{ GeV}^2$, and $Q^2 = 0.1 \text{ GeV}^2$.

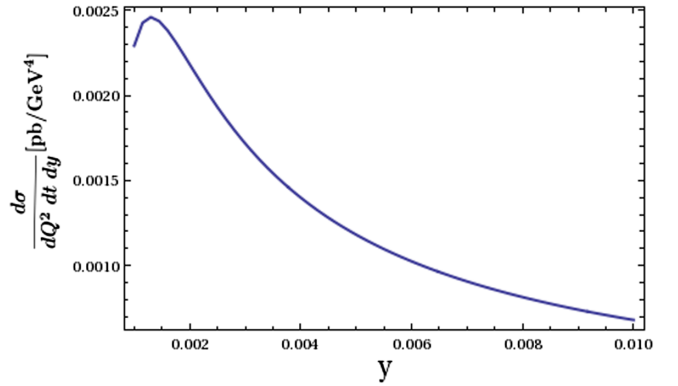
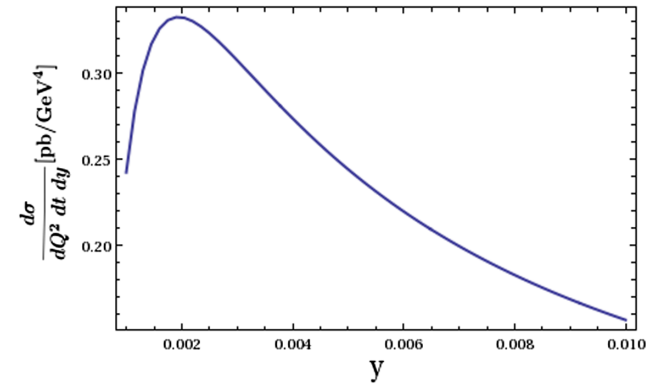


FIG. 10. Left panel: The y dependence of the longitudinal cross section $\frac{d\sigma(e^-N \rightarrow \nu ND_s^*)}{dy dQ^2 dt}$ (in pb GeV^{-4}) for $\Delta_T = 0$, $s = 20000 \text{ GeV}^2$, and $Q^2 = 0.1 \text{ GeV}^2$. Right panel: idem for the production cross section of a transversely polarized D_s^* .

production in the low $s = 820 \text{ GeV}^2$ and high $s = 20000 \text{ GeV}^2$ energy modes of the EIC. As it may have been anticipated the Q^2 dependence is quite modest at small $Q^2 \ll M_D^2$. The y dependence is quite strong resulting in the dominance of the moderate skewness region. The dependence of our results with respect to the choice of the gluon GPDs and heavy meson DAs is illustrated in Fig. 8.

The possibility to perform the three-dimensional tomography of nucleons is strongly related (by Fourier transform) to the t -dependence of GPDs and hence to the cross sections. At the present state of our knowledge we are unable to predict unambiguously t -dependence of GPDs directly from QCD, so we must rely on predictions depending on a GPD model confronted with available experimental data. In Fig. 9 we present the cross section for the pseudoscalar D_s^- production as a function of $-t$ for the high-energy EIC mode with $s = 20000 \text{ GeV}^2$, $Q^2 = 0.1 \text{ GeV}^2$ and $y = 0.002$, as predicted by the GK model [20] for GPDs. Let us stress that the t -dependence of gluon GPDs is quite unrestricted and will remain unrestricted in the ξ range probed here until EIC data come out. Consequently, the measurement of the t -dependence of the cross section of the present process will contain much new interesting information on the transverse size of the gluonic cloud in the nucleon.

We show in Fig. 10 both σ_{00} and σ_{--} for D_s^* production, which could be separated by a Rosenbluth-type separation, although this separation is more difficult to perform in a charged-current event than in the usual photon exchange process where the incoming energy is easier to measure in order to determine the value of ε . One could also separate σ_{00} from σ_{--} by an angular analysis of the decay products of the D_s^* meson, which allows us to quantify the relative production of transversely- vs longitudinally-polarized vector mesons. This however does not seem an easy job with the dominant decay channel ($D_s^* \rightarrow D_s \gamma$) of the D_s^* vector meson. Since the D_s^* vector meson is heavier than the

corresponding pseudoscalar meson, the skewness parameter ξ is somewhat larger (see Eq. (6) and consequently the gluon GPD is smaller and the longitudinal D_s^* production cross section is smaller than the D_s one. The transverse cross section shown in the right panel of Fig. 10 is much smaller than the longitudinal one. At small Q^2 , this can be traced back to the Q/M_D additional factor present in the Dirac trace for the transversely polarized charmed mesons.

The overall conclusion is that the cross section is large enough for the vector charmed meson D_s^{*-} to be produced through the exclusive reaction studied here, at a sizeable rate in future high-luminosity electron-ion colliders, and that it will dominantly be produced with a longitudinal polarization.

In the case of a beam of polarized electrons with definite helicities only the left-handed electrons are able to emit a W^- boson, and the beam asymmetry for both the pseudoscalar and vector charmed-meson production will be maximal

$$\mathcal{A} = \frac{d\sigma(\lambda_e = -) - d\sigma(\lambda_e = +)}{d\sigma(\lambda_e = -) + d\sigma(\lambda_e = +)} = 1, \quad (39)$$

which is a very clear signature of the charged current process. This may be helpful to analyze the background from neutral current (i.e., quasireal photon exchange) events with missing or misidentified mesons in the final state.

VI. CONCLUSION

Collinear QCD factorization has allowed us to calculate charged current exclusive electroproduction of D_s^- and D_s^{*-} mesons in terms of GPDs. Our study complements the previous calculations [1] which were dedicated to the production of pseudoscalar and vector light mesons.

The inclusive production of D mesons was also recently discussed in [28] in the context of high-multiplicity collisions at EIC.

Our study also applies to the production of a D_s^+ or a D_s^{*+} by a positron beam, with the obvious replacements of W^- by W^+ and left-handed polarizations by right-handed ones.

We have demonstrated that the production cross sections for exclusive D_s charmed strange mesons, although small, are in the reach of future high-luminosity electron-ion colliders making them another potential source of information for future programs aiming at the extraction of GPDs [29]. The rate for the longitudinally polarized D_s^* vector meson is of the same order of magnitude as the one for the pseudoscalar D_s meson. Both are in fact of the same order of magnitudes as the rates for light mesons at a Q^2 value of the order a few GeV^2 [1]. A detailed feasibility study, taking care of the difficult reconstruction of the D_s and D_s^* mesons through their decay products, is needed to decide whether the reaction we study here is fully observable.

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