Study of *CP* violation and *CPT* violation in $K^*(892) \to K^0_{S,L}\pi$ decays at BESIII

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The decays $K^*(892) \to K^0_{S,L}\pi$ can be used to study *CP* violation and *CPT* violation. The $K^*(892)$ (hereinafter referred to as K^*) meson can be produced via J/ψ decays at BESIII. In this paper, we study *CP* violation and $K^0_S - K^0_L$ asymmetry in the J/ψ decays involving the K^* meson in the final states and obtain the following results: $\mathcal{A}_{CP}(J/\psi \to K^* f_{J/\psi} \to K^0_S \pi f_{J/\psi}) = (3.64 \pm 0.04) \times 10^{-3}$, $\mathcal{A}_{CP}(J/\psi \to K^* f_{J/\psi} \to K^0_L \pi f_{J/\psi}) = (-3.32 \pm 0.04) \times 10^{-3}$, and $R(J/\psi \to K^* f_{J/\psi} \to K^0_{S,L}\pi f_{J/\psi}) = (3.51 \pm 0.03) \times 10^{-3}$, $R(J/\psi \to \bar{K}^* \bar{f}_{J/\psi} \to K^0_{S,L}\pi \bar{f}_{J/\psi}) = (-3.45 \pm 0.03) \times 10^{-3}$. Based on two cases, the samples of 10^{10} and 10^{12} J/ψ events, we calculate the expected numbers of the observed signal events on the *CP* violation and the $K^0_S - K^0_L$ asymmetry in the J/ψ decays with K^* meson in the final states, and we find that the BESIII experiment may be able to unambiguously observe *CP* violation and $K^0_S - K^0_L$ asymmetry for each of these two cases. We study the possibility to constrain the *CPT* violation parameter z and discuss the sensitivity for the measurement of z in J/ψ decays with K^* meson in the final states at BESIII. The sensitivity of the measurement of z depends on the measured precision of the parameters $m_L - m_S$, Γ_L, Γ_S, p, and q and the consistence between the values of t_0 and t_1 and the event selection criteria in experiment.

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CP violation and *CPT* violation play important roles in deepening the understanding of Nature and studying physics beyond the Standard Model [1–5]. The decays with neutral K meson in the final states can be used to study *CP* violation [6–11] and *CPT* violation [12]. The *CP* violations in the decays $D \to K^0_S\pi$ and $\tau \to \pi K^0_S \bar{\nu}_\tau$ have been reported by Belle [13–15], *BABAR* [16,17], CLEO [18,19], and FOCUS [20] collaborations. A 2.8σ discrepancy is observed between the latest *BABAR* measurement and the Standard Model prediction of the *CP* asymmetry in the $\tau \to \pi K^0_S \bar{\nu}_\tau$ decay [21–23]. Such a discrepancy has motivated many studies of possible origins of the direct *CP* asymmetry in $\tau \to \pi K^0_S \bar{\nu}_\tau$ decay [24–34]. However, the measurements of the *CP*-violating effect in $\tau \to \pi K^0_S \bar{\nu}_\tau$ decay still have large uncertainties and no clear conclusion can be drawn at the present stage, so more precise data and more decays are needed in both experiment and theory. Moreover, it is crucial to study the *CP* violation and *CPT* violation in various reactions, to see the correlations between different processes and probe the sources of *CP* violation and *CPT* violation [35–37].

In this paper, we consider the possible *CP* and *CPT* asymmetric observations in $K^* \to K^0_{S,L}\pi$ decays, in which the possible *CP* violation and *CPT* violation are due to $K^0 - \bar{K}^0$ oscillation within the Standard Model. Because *CP* is conserved in the strong decays of the K^* meson into $K\pi$, we can obtain the following properties:

$$\mathcal{B}(K^{*0} \to K^0\pi^0) = \mathcal{B}(\bar{K}^{*0} \to \bar{K}^0\pi^0), \tag{1}$$

$$\mathcal{B}(K^{*0} \to K^+\pi^-) = \mathcal{B}(\bar{K}^{*0} \to K^-\pi^+), \tag{2}$$

$$\mathcal{B}(K^{*+} \to K^0\pi^+) = \mathcal{B}(K^{*-} \to \bar{K}^0\pi^-), \tag{3}$$

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$$\mathcal{B}(K^{*+} \to K^+\pi^0) = \mathcal{B}(K^{*-} \to K^-\pi^0). \tag{4}$$

In the $K^0 - \bar{K}^0$ system, the mass eigenstates can be written [38] as

$$|K_L^0\rangle = p\sqrt{1+z}|K^0\rangle - q\sqrt{1-z}|\bar{K}^0\rangle, \tag{5}$$

$$|K_S^0\rangle = p\sqrt{1-z}|K^0\rangle + q\sqrt{1+z}|\bar{K}^0\rangle, \tag{6}$$

and the corresponding mass eigenbras read [39,40]

$$\langle K_L^0| = \frac{q\sqrt{1+z}\langle K^0| - p\sqrt{1-z}\langle \bar{K}^0|}{2pq}, \tag{7}$$

$$\langle K_S^0| = \frac{q\sqrt{1-z}\langle K^0| + p\sqrt{1+z}\langle \bar{K}^0|}{2pq}, \tag{8}$$

where p, q, and z are complex mixing parameters. If CPT invariance held, we would have $z = 0$; if CP and CPT invariance held, we would have $p = q = \sqrt{2}/2$ and $z = 0$. The mass and width eigenstates, $K_{S,L}^0$, may also be described with the popular notations

$$|K_L^0\rangle = \frac{1+\epsilon-\delta}{\sqrt{2(1+|\epsilon-\delta|^2)}}|K^0\rangle - \frac{1-\epsilon+\delta}{\sqrt{2(1+|\epsilon-\delta|^2)}}|\bar{K}^0\rangle, \tag{9}$$

$$|K_S^0\rangle = \frac{1+\epsilon+\delta}{\sqrt{2(1+|\epsilon+\delta|^2)}}|K^0\rangle + \frac{1-\epsilon-\delta}{\sqrt{2(1+|\epsilon+\delta|^2)}}|\bar{K}^0\rangle, \tag{10}$$

where the complex parameter ϵ signifies deviation of the mass eigenstates from the CP eigenstates, and δ is the CPT violating complex parameter. The parameters p, q, and z can be expressed in terms of ϵ and δ [neglecting terms of $\epsilon\delta$ and $\mathcal{O}(\delta)$]

$$p = \frac{1+\epsilon}{\sqrt{2(1+|\epsilon|^2)}}, \quad q = \frac{1-\epsilon}{\sqrt{2(1+|\epsilon|^2)}}, \quad z = -2\delta. \tag{11}$$

The time-evolved states of the $K^0 - \bar{K}^0$ system can be expressed by the mass eigenstates

$$|K_{\rm phys}^0(t)\rangle = \frac{\sqrt{1+z}}{2p}e^{-im_Lt-\frac{1}{2}\Gamma_Lt}|K_L^0\rangle + \frac{\sqrt{1-z}}{2p}e^{-im_St-\frac{1}{2}\Gamma_St}|K_S^0\rangle, \tag{12}$$

$$|\bar{K}_{\rm phys}^0(t)\rangle = -\frac{\sqrt{1-z}}{2q}e^{-im_Lt-\frac{1}{2}\Gamma_Lt}|K_L^0\rangle + \frac{\sqrt{1+z}}{2q}e^{-im_St-\frac{1}{2}\Gamma_St}|K_S^0\rangle. \tag{13}$$

With Eqs. (12) and (13), the time-dependent amplitudes of the cascade decays $K^* \to K^0\pi \to f_{K^0}\pi$ can be written as

$$A(K^* \to K^0(t)\pi \to f_{K^0}(t)\pi) = A(K^* \to K^0\pi) \cdot A(K_{\rm phys}^0(t) \to f_{K^0}), \tag{14}$$

$$A(\bar{K}^* \to \bar{K}^0(t)\pi \to f_{K^0}(t)\pi) = A(\bar{K}^* \to \bar{K}^0\pi) \cdot A(\bar{K}_{\rm phys}^0(t) \to f_{K^0}), \tag{15}$$

where K^* denotes the K^{*0} (or K^{*+}) meson, $\bar{K}^*$ denotes the charge conjugate state of K^*, and f_{K^0} denotes the final state from the decay of the K^0 or $\bar{K}^0$ meson. $A(K_{\rm phys}^0(t) \to f_{K^0})$ and $A(\bar{K}_{\rm phys}^0(t) \to f_{K^0})$ denote the amplitude of the $K_{\rm phys}^0(t) \to f_{K^0}$ and $\bar{K}_{\rm phys}^0(t) \to f_{K^0}$ decays, respectively, and they have the following forms:

$$A(K_{\rm phys}^0(t) \to f_{K^0}) = \frac{\sqrt{1+z}}{2p}e^{-im_Lt-\frac{1}{2}\Gamma_Lt}A(K_L^0 \to f_{K^0}) + \frac{\sqrt{1-z}}{2p}e^{-im_St-\frac{1}{2}\Gamma_St}A(K_S^0 \to f_{K^0}), \tag{16}$$

$$A(\bar{K}_{\rm phys}^0(t) \to f_{K^0}) = -\frac{\sqrt{1-z}}{2q}e^{-im_Lt-\frac{1}{2}\Gamma_Lt}A(K_L^0 \to f_{K^0}) + \frac{\sqrt{1+z}}{2q}e^{-im_St-\frac{1}{2}\Gamma_St}A(K_S^0 \to f_{K^0}). \tag{17}$$

Making use of Eqs. (14)–(17) and performing integration over phase space, we can obtain

$$\begin{aligned}\mathcal{B}(K^* \to K^0(t)\pi \to f_{K^0}(t)\pi) = \frac{\mathcal{B}(K^* \to K^0\pi)}{4|p|^2} \cdot [|\sqrt{1+z}|^2 \cdot e^{-\Gamma_Lt} \cdot \Gamma(K_L^0 \to f_{K^0}) + |\sqrt{1-z}|^2 \cdot e^{-\Gamma_St} \cdot \Gamma(K_S^0 \to f_{K^0}) \\ + \sqrt{1+z} \cdot (\sqrt{1-z})^* \cdot e^{-i\Delta mt-\Gamma t} \cdot A^*(K_S^0 \to f_{K^0}) \cdot A(K_L^0 \to f_{K^0}) \\ + \sqrt{1-z} \cdot (\sqrt{1+z})^* \cdot e^{i\Delta mt-\Gamma t} \cdot A(K_S^0 \to f_{K^0}) \cdot A^*(K_L^0 \to f_{K^0})],\end{aligned} \tag{18}$$

$$\mathcal{B}(\bar{K}^* \to \bar{K}^0(t)\pi \to f_{K^0}(t)\pi) = \frac{\mathcal{B}(\bar{K}^* \to \bar{K}^0\pi)}{4|q|^2} \cdot [|\sqrt{1-z}|^2 \cdot e^{-\Gamma_L t} \cdot \Gamma(K_L^0 \to f_{K^0}) + |\sqrt{1+z}|^2 \cdot e^{-\Gamma_S t} \cdot \Gamma(K_S^0 \to f_{K^0}) \\ -\sqrt{1-z} \cdot (\sqrt{1+z})^* \cdot e^{-i\Delta m t - \Gamma t} \cdot A^*(K_S^0 \to f_{K^0}) \cdot A(K_L^0 \to f_{K^0}) \\ - \sqrt{1+z} \cdot (\sqrt{1-z})^* \cdot e^{i\Delta m t - \Gamma t} \cdot A(K_S^0 \to f_{K^0}) \cdot A^*(K_L^0 \to f_{K^0})], \quad (19)$$

where Δm denotes the difference in masses of K_L^0 and K_S^0, and Γ denotes the average in widths of K_L^0 and K_S^0

$$\Delta m = m_L - m_S, \qquad \Gamma = \frac{\Gamma_L + \Gamma_S}{2}. \quad (20)$$

The first, second, and the last two terms in the bracket in Eqs. (18) and (19) are related to the effects of the K_L^0 decay and the K_S^0 decay and their interference, respectively.

In BESIII, the K_S^0 state is reconstructed via its decay into the final states $\pi^+\pi^-$ and a time difference between the K^* decay and the K_S^0 decay in the $K^* \to K_S^0\pi$ decay [22]. By taking into account these experimental features, the branching ratio for the $K^* \to K_S^0\pi$ decay can be defined as

$$\mathcal{B}(K^* \to K_S^0\pi) = \frac{\int_{t_0}^{t_1} \mathcal{B}(K^* \to K^0(t)\pi \to \pi^+\pi^-(t)\pi)dt}{(e^{-\Gamma_S t_0} - e^{-\Gamma_S t_1}) \cdot \mathcal{B}(K_S^0 \to \pi^+\pi^-)}, \quad (21)$$

where $t_0 = 0.1\tau_S$ and $t_1 = 2\tau_S \sim 20\tau_S$ with τ_S is the K_S^0 lifetime, we adopt $t_1 = 10\tau_S$ in our calculation. Combining Eqs. (18) and (21), we can obtain

$$\mathcal{B}(K^* \to K_S^0\pi) = \frac{\mathcal{B}(K^* \to K^0\pi)}{4|p|^2} \cdot \left[|\sqrt{1-z}|^2 + |\sqrt{1+z}|^2 \cdot \frac{e^{-\Gamma_L t_0} - e^{-\Gamma_L t_1}}{e^{-\Gamma_S t_0} - e^{-\Gamma_S t_1}} \cdot \frac{\mathcal{B}(K_L^0 \to \pi^+\pi^-)}{\mathcal{B}(K_S^0 \to \pi^+\pi^-)} \\ + 2\mathrm{Re}\left(\sqrt{1+z} \cdot (\sqrt{1-z})^* \cdot \frac{e^{-i\Delta m t_0 - \Gamma t_0} - e^{-i\Delta m t_1 - \Gamma t_1}}{e^{-\Gamma_S t_0} - e^{-\Gamma_S t_1}} \cdot \frac{\Gamma_S}{\Gamma + i\Delta m} \cdot \frac{A(K_L^0 \to \pi^+\pi^-)}{A(K_S^0 \to \pi^+\pi^-)}\right)\right]. \quad (22)$$

Using Eqs. (5) and (6) and assuming that the direct *CP* violation in the $K^0 \to \pi^+\pi^-$ decay can be neglected, we derive

$$\frac{A(K_L^0 \to \pi^+\pi^-)}{A(K_S^0 \to \pi^+\pi^-)} = \frac{p\sqrt{1+z} - q\sqrt{1-z}}{p\sqrt{1-z} + q\sqrt{1+z}}. \quad (23)$$

By combining Eq. (23) with Eq. (22) and introducing the following substitution

$$t_{K_S^0 - K_L^0} = \frac{e^{-i\Delta m t_0 - \Gamma t_0} - e^{-i\Delta m t_1 - \Gamma t_1}}{e^{-\Gamma_S t_0} - e^{-\Gamma_S t_1}} \cdot \frac{\Gamma_S}{\Gamma + i\Delta m}, \quad (24)$$

we can obtain

$$\mathcal{B}(K^* \to K_S^0\pi) = \frac{\mathcal{B}(K^* \to K^0\pi)}{4|p|^2} \cdot \left[|\sqrt{1-z}|^2 + |\sqrt{1+z}|^2 \cdot \frac{e^{-\Gamma_L t_0} - e^{-\Gamma_L t_1}}{e^{-\Gamma_S t_0} - e^{-\Gamma_S t_1}} \cdot \frac{\mathcal{B}(K_L^0 \to \pi^+\pi^-)}{\mathcal{B}(K_S^0 \to \pi^+\pi^-)} \\ + 2\mathrm{Re}\left(\sqrt{1+z} \cdot (\sqrt{1-z})^* \cdot t_{K_S^0 - K_L^0} \cdot \frac{p\sqrt{1+z} - q\sqrt{1-z}}{p\sqrt{1-z} + q\sqrt{1+z}}\right)\right]. \quad (25)$$

Similarly, we can derive the branching ratio for the $\bar{K}^* \to K_S^0\pi$ decay

$$\mathcal{B}(\bar{K}^* \to K_S^0\pi) = \frac{\mathcal{B}(\bar{K}^* \to \bar{K}^0\pi)}{4|q|^2} \cdot \left[|\sqrt{1+z}|^2 + |\sqrt{1-z}|^2 \cdot \frac{e^{-\Gamma_L t_0} - e^{-\Gamma_L t_1}}{e^{-\Gamma_S t_0} - e^{-\Gamma_S t_1}} \cdot \frac{\mathcal{B}(K_L^0 \to \pi^+\pi^-)}{\mathcal{B}(K_S^0 \to \pi^+\pi^-)} \\ - 2\mathrm{Re}\left(\sqrt{1-z} \cdot (\sqrt{1+z})^* \cdot t_{K_S^0 - K_L^0} \cdot \frac{p\sqrt{1+z} - q\sqrt{1-z}}{p\sqrt{1-z} + q\sqrt{1+z}}\right)\right]. \quad (26)$$

With the values of the parameters, which are listed in Table I, we can obtain

TABLE I. The values of the input parameters used in this paper [38,41].

$\mathrm{Re}(\epsilon) = (1.66 \pm 0.02) \times 10^{-3}$	$\mathrm{Im}(\epsilon) = (1.57 \pm 0.02) \times 10^{-3}$
$\lvert\epsilon\rvert = (2.228 \pm 0.011) \times 10^{-3}$	$\Delta m = (3.481 \pm 0.007) \times 10^{-15}$ GeV
$\Gamma_L = (1.287 \pm 0.005) \times 10^{-17}$ GeV	$\Gamma_S = (7.351 \pm 0.003) \times 10^{-15}$ GeV
$\mathcal{B}(K_L^0 \to \pi^+\pi^-) = (1.967 \pm 0.010) \times 10^{-3}$	$\mathcal{B}(K_S^0 \to \pi^+\pi^-) = (69.2 \pm 0.05) \times 10^{-2}$

$$\frac{e^{-\Gamma_L t_0} - e^{-\Gamma_L t_1}}{e^{-\Gamma_S t_0} - e^{-\Gamma_S t_1}} \cdot \frac{\mathcal{B}(K_L^0 \to \pi^+\pi^-)}{\mathcal{B}(K_S^0 \to \pi^+\pi^-)} = (5.40 \pm 0.03) \times 10^{-5}. \tag{27}$$

Combining Eqs. (25)–(27), we can obtain

$$\mathcal{B}(K^* \to K_S^0\pi) = \frac{\mathcal{B}(K^* \to K^0\pi)}{4|p|^2} \cdot \left[|\sqrt{1-z}|^2 + 2\mathrm{Re}\left(\sqrt{1+z} \cdot (\sqrt{1-z})^* \cdot t_{K_S^0-K_L^0} \cdot \frac{p\sqrt{1+z} - q\sqrt{1-z}}{p\sqrt{1-z} + q\sqrt{1+z}}\right)\right], \tag{28}$$

$$\mathcal{B}(\bar{K}^* \to K_S^0\pi) = \frac{\mathcal{B}(\bar{K}^* \to \bar{K}^0\pi)}{4|q|^2} \cdot \left[|\sqrt{1+z}|^2 - 2\mathrm{Re}\left(\sqrt{1-z} \cdot (\sqrt{1+z})^* \cdot t_{K_S^0-K_L^0} \cdot \frac{p\sqrt{1+z} - q\sqrt{1-z}}{p\sqrt{1-z} + q\sqrt{1+z}}\right)\right]. \tag{29}$$

In BESIII, the K_L^0 state is defined via a large time difference between the K^* decay and the K_L^0 decay and mostly decay outside the detector in the $K^* \to K_L^0\pi$ decay [42–44]. Based on these experimental features, the branching ratio for the $K^* \to K_L^0\pi$ decay can be defined as

$$\mathcal{B}(K^* \to K_L^0\pi) = \frac{\int_{t_2}^{+\infty} \mathcal{B}(K^* \to K^0(t)\pi \to f_{K_L^0}(t)\pi)dt}{e^{-\Gamma_L t_2} \cdot \mathcal{B}(K_L^0 \to f_{K_L^0})}, \tag{30}$$

where $f_{K_L^0}$ denotes the final state of the K_L^0 decay, $t_2 \geq 100\tau_S$. Combining Eqs. (18) and (30), we can obtain

$$\mathcal{B}(K^* \to K_L^0\pi) = \frac{\mathcal{B}(K^* \to K^0\pi)}{4|p|^2} \cdot \left[|\sqrt{1+z}|^2 + |\sqrt{1-z}|^2 \cdot e^{-(\Gamma_S-\Gamma_L)t_2} \cdot \frac{\mathcal{B}(K_S^0 \to f_{K_L^0})}{\mathcal{B}(K_L^0 \to f_{K_L^0})}\right.$$
$$\left. + 2\mathrm{Re}\left(\sqrt{1-z} \cdot (\sqrt{1+z})^* \cdot \frac{e^{i\Delta m t_2 - \frac{\Gamma_S-\Gamma_L}{2}t_2}}{\Gamma - i\Delta m} \cdot \frac{A(K_S^0 \to f_{K_L^0})}{A(K_L^0 \to f_{K_L^0})}\right)\right]. \tag{31}$$

Using the values of the parameters in Table I, we can obtain

$$e^{-(\Gamma_S-\Gamma_L)t_2} \leq 4.4 \times 10^{-44}, \quad e^{-\frac{\Gamma_S-\Gamma_L}{2}t_2} \leq 2.1 \times 10^{-22}, \tag{32}$$

so the second and third terms in the brackets in Eq. (31), which correspond respectively to the effects of the K_S^0 decay and the interference between the K_S^0 decay and the K_L^0 decay, can be neglected, then we obtain

$$\mathcal{B}(K^* \to K_L^0\pi) = \mathcal{B}(K^* \to K^0\pi)\frac{|\sqrt{1+z}|^2}{4|p|^2}. \tag{33}$$

Similarly, we can derive the branching ratio for the $\bar{K}^* \to K_L^0\pi$ decay

$$\mathcal{B}(\bar{K}^* \to K_L^0\pi) = \mathcal{B}(\bar{K}^* \to \bar{K}^0\pi)\frac{|\sqrt{1-z}|^2}{4|q|^2}. \tag{34}$$

By combining Eqs. (1)–(4), (28)–(29), and (33)–(34), and assuming $z = 0$, we can derive the following observations of CP asymmetry:

$$\mathcal{A}_{CP}(K^* \to K_S^0\pi) = \frac{\mathcal{B}(K^* \to K_S^0\pi) - \mathcal{B}(\bar{K}^* \to K_S^0\pi)}{\mathcal{B}(K^* \to K_S^0\pi) + \mathcal{B}(\bar{K}^* \to K_S^0\pi)}$$
$$= \frac{|q|^2 - |p|^2}{|q|^2 + |p|^2} + 2\mathrm{Re}\left(t_{K_S^0-K_L^0} \cdot \frac{p-q}{p+q}\right) \tag{35}$$

and

$$\mathcal{A}_{CP}(K^* \to K_L^0\pi) = \frac{\mathcal{B}(K^* \to K_L^0\pi) - \mathcal{B}(\bar{K}^* \to K_L^0\pi)}{\mathcal{B}(K^* \to K_L^0\pi) + \mathcal{B}(\bar{K}^* \to K_L^0\pi)} = \frac{|q|^2 - |p|^2}{|q|^2 + |p|^2}, \quad (36)$$

where K^* denotes the K^{*0} (or K^{*+}) meson and $\bar{K}^*$ denotes the charge conjugate state of K^*.

The K^* meson can be produced through the J/ψ decays at BESIII. The *CP* asymmetry observables can be defined in the J/ψ decays which involve the K^* meson in the final states

$$\mathcal{A}_{CP}(J/\psi \to K^* f_{J/\psi} \to K_{S,L}^0\pi f_{J/\psi}) = \frac{\mathcal{B}(J/\psi \to K^* f_{J/\psi} \to K_{S,L}^0\pi f_{J/\psi}) - \mathcal{B}(J/\psi \to \bar{K}^* \bar{f}_{J/\psi} \to K_{S,L}^0\pi \bar{f}_{J/\psi})}{\mathcal{B}(J/\psi \to K^* f_{J/\psi} \to K_{S,L}^0\pi f_{J/\psi}) + \mathcal{B}(J/\psi \to \bar{K}^* \bar{f}_{J/\psi} \to K_{S,L}^0\pi \bar{f}_{J/\psi})}, \quad (37)$$

where $f_{J/\psi}$ denotes the final state (K^* excepted) in the J/ψ decays, and $\bar{f}_{J/\psi}$ is the charge conjugate state of $f_{J/\psi}$. For example, the *CP* asymmetry observables can be defined in $J/\psi \to \gamma K^{*0}\bar{K}^{*0}$ and $J/\psi \to K^\pm K^{*\mp}$ decays

$$\mathcal{A}_{CP}(J/\psi \to \gamma K^{*0}\bar{K}^{*0} \to \gamma K_{S,L}^0\pi^0 K^-\pi^+) = \frac{\mathcal{B}(J/\psi \to \gamma K^{*0}\bar{K}^{*0} \to \gamma K_{S,L}^0\pi^0 K^-\pi^+) - \mathcal{B}(J/\psi \to \gamma K^{*0}\bar{K}^{*0} \to \gamma K^+\pi^- K_{S,L}^0\pi^0)}{\mathcal{B}(J/\psi \to \gamma K^{*0}\bar{K}^{*0} \to \gamma K_{S,L}^0\pi^0 K^-\pi^+) + \mathcal{B}(J/\psi \to \gamma K^{*0}\bar{K}^{*0} \to \gamma K^+\pi^- K_{S,L}^0\pi^0)}, \quad (38)$$

$$\mathcal{A}_{CP}(J/\psi \to K^\pm K^{*\mp} \to K^\pm K_{S,L}^0\pi^\mp) = \frac{\mathcal{B}(J/\psi \to K^- K^{*+} \to K^- K_{S,L}^0\pi^+) - \mathcal{B}(J/\psi \to K^+ K^{*-} \to K^+ K_{S,L}^0\pi^-)}{\mathcal{B}(J/\psi \to K^- K^{*+} \to K^- K_{S,L}^0\pi^+) + \mathcal{B}(J/\psi \to K^+ K^{*-} \to K^+ K_{S,L}^0\pi^-)}. \quad (39)$$

According to Eqs. (2) and (4) and (35)–(37), we can derive

$$\mathcal{A}_{CP}(J/\psi \to K^* f_{J/\psi} \to K_{S,L}^0\pi f_{J/\psi}) = \mathcal{A}_{CP}(K^* \to K_{S,L}^0\pi). \quad (40)$$

By using Eqs. (37) and (40), the number of the observed signal events on the *CP* violation in J/ψ decays can be derived,

$$\begin{aligned} N_{CP}^{K_{S,L}^0} &= |\mathcal{A}_{CP}(K^* \to K_{S,L}^0\pi)| \cdot N_{J/\psi} \cdot \varepsilon_{K_{S,L}^0} \cdot \mathcal{B}(J/\psi \to K^* f_{J/\psi} + \text{c.c.} \to K_{S,L}^0\pi f_{J/\psi} + \text{c.c.} \to f_{K_{S,L}^0}\pi f_{J/\psi} + \text{c.c.}) \\ &\approx 2|\mathcal{A}_{CP}(K^* \to K_{S,L}^0\pi)| \cdot N_{J/\psi} \cdot \varepsilon_{K_{S,L}^0} \cdot \mathcal{B}(J/\psi \to K^* f_{J/\psi} \to K_{S,L}^0\pi f_{J/\psi} \to f_{K_{S,L}^0}\pi f_{J/\psi}), \end{aligned} \quad (41)$$

where $N_{J/\psi}$ is the number of J/ψ events accumulated at BESIII. During several run periods from 2009 to 2019, a total data sample of 10^{10} J/ψ events was collected with the BESIII detector [42,45,46]. Moreover, accelerators at the tau-charm energy region with luminosity 100 times higher than BEPCII are being proposed [47–49]; the detectors in these new facilities will be able to collect 10^{12} J/ψ events in one year's running time. In this paper, we will perform the calculation under two cases: $N_{J/\psi} = 10^{10}$ and $N_{J/\psi} = 10^{12}$. $\varepsilon_{K_{S,L}^0}$ in Eq. (41) are the selection efficiencies of J/ψ decays at BESIII. $f_{K_{S,L}^0}$ in Eq. (41) denote the final states in the $K_{S,L}^0$ decays. Here, it is worth noting that the K_L^0 meson could not be reconstructed by its decays because of its large life [42–44]. Instead, all particles except for the K_L^0 are reconstructed and the presence of a K_L^0 can be inferred from the missing four momentum in the decays that contain a K_L^0 in the final states. So we do not consider the branching ratios of K_L^0 decays in the calculation.

The branching ratios of the $J/\psi \to K^* f_{J/\psi} \to K_{S,L}^0\pi f_{J/\psi}$ decays can be expressed as the products of the branching ratios of the $J/\psi \to K^* f_{J/\psi}$ decays and the $K^* \to K_{S,L}^0\pi$ decays. The branching ratios of the $J/\psi \to K^* f_{J/\psi}$ decays can be taken directly from the Particle Data Group (PDG) [38] and Ref. [50]. For the branching ratios of the $K^* \to K_{S,L}^0\pi$ decays, we apply the PDG result $\mathcal{B}(K^* \to K\pi) \approx 100\%$ and isospin relation to obtain

$$\mathcal{B}(K^{*0} \to K^0\pi^0) = \frac{1}{3}, \qquad \mathcal{B}(K^{*0} \to K^+\pi^-) = \frac{2}{3}, \quad (42)$$

$$\mathcal{B}(\bar{K}^{*0} \to \bar{K}^0\pi^0) = \frac{1}{3}, \qquad \mathcal{B}(\bar{K}^{*0} \to K^-\pi^+) = \frac{2}{3}, \quad (43)$$

$$\mathcal{B}(K^{*+} \to K^0\pi^+) = \frac{2}{3}, \qquad \mathcal{B}(K^{*+} \to K^+\pi^0) = \frac{1}{3}, \quad (44)$$

$$\mathcal{B}(K^{*-} \to \bar{K}^0\pi^-) = \frac{2}{3}, \qquad \mathcal{B}(K^{*-} \to K^-\pi^0) = \frac{1}{3}. \quad (45)$$

Then, from Eqs. (42)–(45) and under the assumptions of CP conservation and CPT conservation, we have

$$\mathcal{B}(K^{*0} \to K_S^0\pi^0) = \frac{1}{6}, \qquad \mathcal{B}(K^{*0} \to K_L^0\pi^0) = \frac{1}{6}, \tag{46}$$

$$\mathcal{B}(\bar{K}^{*0} \to K_S^0\pi^0) = \frac{1}{6}, \qquad \mathcal{B}(\bar{K}^{*0} \to K_L^0\pi^0) = \frac{1}{6}, \tag{47}$$

$$\mathcal{B}(K^{*+} \to K_S^0\pi^+) = \frac{1}{3}, \qquad \mathcal{B}(K^{*+} \to K_L^0\pi^+) = \frac{1}{3}, \tag{48}$$

$$\mathcal{B}(K^{*-} \to K_S^0\pi^-) = \frac{1}{3}, \qquad \mathcal{B}(K^{*-} \to K_L^0\pi^-) = \frac{1}{3}. \tag{49}$$

According to Eqs. (42)–(49) and the values for the branching ratios of the $J/\psi \to K^* f_{J/\psi}$ and $K_S^0 \to f_{K_S^0}$ decays which are taken directly from PDG [38] and Ref. [50], we collect the branching ratios of $J/\psi \to K^* f_{J/\psi} \to K_{S,L}^0 \pi f_{J/\psi} \to f_{K_{S,L}^0} \pi f_{J/\psi}$ decays, which are shown in Table II. Here, we only consider the decay channels with a branching ratio larger than 1.0×10^{-5}. With the values of the parameters in Table I and combining Eqs. (11), (24), (35), and (36), we can obtain

$$\mathcal{A}_{CP}(K^* \to K_S^0\pi) = (3.64 \pm 0.04) \times 10^{-3}, \tag{50}$$

$$\mathcal{A}_{CP}(K^* \to K_L^0\pi) = (-3.32 \pm 0.04) \times 10^{-3}. \tag{51}$$

According to Table II and combining Eqs. (41), (50), and (51), we derive the numerical results of $N_{CP}^{K_{S,L}^0}$, which are shown in Table III, based on the samples of 10^{10} and 10^{12} J/ψ events. Obviously, the CP violations in J/ψ decays with the K^* meson in the final states can be unambiguously observed at BESIII.

Here, we note that the interferences between the K^* meson and other K^* mesons, such as $K^*(1680)$, $K_2^*(1430)$, and $K_2^*(1980)$, have non-negligible effects on the branching ratios of the $J/\psi \to K^{*\pm}K^{*\mp}$, $J/\psi \to K^{*\pm}K^{\mp}$, $J/\psi \to K^{*\pm}K^{\mp}\pi^0$, $J/\psi \to K^{*0}K^-\pi^+$, and $J/\psi \to \bar{K}^{*0}K^+\pi^-$ decays [50–54]. Fortunately, the CP asymmetries in all the $K^{*\prime} \to K_{S,L}^0\pi$ decays can be derived

$$\frac{\mathcal{B}(K^{*\prime} \to K_{S,L}^0\pi) - \mathcal{B}(\bar{K}^{*\prime} \to K_{S,L}^0\pi)}{\mathcal{B}(K^{*\prime} \to K_{S,L}^0\pi) + \mathcal{B}(\bar{K}^{*\prime} \to K_{S,L}^0\pi)} = \frac{\mathcal{B}(K^{*} \to K_{S,L}^0\pi) - \mathcal{B}(\bar{K}^{*} \to K_{S,L}^0\pi)}{\mathcal{B}(K^{*} \to K_{S,L}^0\pi) + \mathcal{B}(\bar{K}^{*} \to K_{S,L}^0\pi)}, \tag{52}$$

where $K^{*\prime}$ denotes all the K^* mesons except the $K^*(892)$ meson. Therefore the interferences between the K^* meson and other K^* mesons have no effect on the CP violations in $K^* \to K_{S,L}^0\pi$ decays. Here, one can consider the possibility of summing over all the decays involving K^* resonance in order to obtain a statistically significant signal of CP violation. From Eqs. (35)–(37), and (40), we can see that the resonance structures in the $K^{*\pm}K^{\mp}$ invariant mass spectrum of the $J/\psi \to \eta' K^{*\pm}K^{\mp}$ decays also have no effect on the CP violations in these decays [55,56].

Now, we proceed to study the $K_S^0 - K_L^0$ asymmetries in the $K^* \to K_{S,L}^0\pi$ decays, which are defined as [57–59]

$$R(K^* \to K_{S,L}^0\pi) = \frac{\mathcal{B}(K^* \to K_S^0\pi) - \mathcal{B}(K^* \to K_L^0\pi)}{\mathcal{B}(K^* \to K_S^0\pi) + \mathcal{B}(K^* \to K_L^0\pi)}, \tag{53}$$

TABLE II. The branching fractions of $J/\psi \to K^* f_{J/\psi} \to K_{S,L}^0 \pi f_{J/\psi} \to f_{K_{S,L}^0} \pi f_{J/\psi}$ decays.

Branching ratio for the decay channel	Numerical result
$\mathcal{B}(J/\psi \to K^{*0}\bar{K}^{*0} \to K_S^0\pi^0K^-\pi^+ \to \pi^+\pi^-\pi^0K^-\pi^+)$	$(1.77 \pm 0.46) \times 10^{-5}$
$\mathcal{B}(J/\psi \to K^{*0}\bar{K}^{*0} \to K_L^0\pi^0K^-\pi^+)$	$(2.56 \pm 0.67) \times 10^{-5}$
$\mathcal{B}(J/\psi \to K^{*+}K^{*-} \to K_S^0\pi^+K^-\pi^0 \to \pi^+\pi^-\pi^+K^-\pi^0)$	$(7.69^{-3.08}_{+1.69}) \times 10^{-5}$
$\mathcal{B}(J/\psi \to K^{*+}K^{*-} \to K_L^0\pi^+K^-\pi^0)$	$(1.11^{-0.44}_{+0.24}) \times 10^{-4}$
$\mathcal{B}(J/\psi \to \eta K^{*0}\bar{K}^{*0} \to \eta K_S^0\pi^0K^-\pi^+ \to \gamma\gamma\pi^+\pi^-\pi^0K^-\pi^+)$	$(3.48 \pm 0.79) \times 10^{-5}$
$\mathcal{B}(J/\psi \to \eta K^{*0}\bar{K}^{*0} \to \gamma\gamma K_L^0\pi^0K^-\pi^+)$	$(5.04 \pm 1.14) \times 10^{-5}$
$\mathcal{B}(J/\psi \to \eta' K^{*+}K^- \to \eta' K_S^0\pi^+K^- \to \pi^+\pi^-\eta K_S^0\pi^+K^- \to \pi^+\pi^-\gamma\gamma\pi^+\pi^-\pi^+K^-)$	$(2.86 \pm 0.25) \times 10^{-5}$
$\mathcal{B}(J/\psi \to \eta' K^{*+}K^- \to \pi^+\pi^-\eta K_L^0\pi^+K^- \to \pi^+\pi^-\gamma\gamma K_L^0\pi^+K^-)$	$(4.13 \pm 0.37) \times 10^{-5}$
$\mathcal{B}(J/\psi \to K^{*+}K^- \to K_S^0\pi^+K^- \to \pi^+\pi^-\pi^+K^-)$	$(6.92^{-1.15}_{+0.92}) \times 10^{-4}$
$\mathcal{B}(J/\psi \to K^{*+}K^- \to K_L^0\pi^+K^-)$	$(1.00^{-0.17}_{+0.13}) \times 10^{-3}$
$\mathcal{B}(J/\psi \to K^{*0}K^-\pi^+ \to K_S^0\pi^0K^-\pi^+ \to \pi^+\pi^-\pi^0K^-\pi^+)$	$(4.44 \pm 0.92) \times 10^{-4}$
$\mathcal{B}(J/\psi \to K^{*0}K^-\pi^+ \to K_L^0\pi^0K^-\pi^+)$	$(6.42 \pm 1.33) \times 10^{-4}$
$\mathcal{B}(J/\psi \to K^{*+}K^-\pi^0 \to K_S^0\pi^+K^-\pi^0 \to \pi^+\pi^-\pi^+K^-\pi^0)$	$(3.46 \pm 1.11) \times 10^{-4}$
$\mathcal{B}(J/\psi \to K^{*+}K^-\pi^0 \to K_L^0\pi^+K^-\pi^0)$	$(5.00 \pm 1.57) \times 10^{-4}$
$\mathcal{B}(J/\psi \to \omega K^{*+}K^- \to \omega K_S^0\pi^+K^- \to \pi^+\pi^-\pi^0\pi^+\pi^-\pi^+K^-)$	$(6.28 \pm 0.93) \times 10^{-4}$
$\mathcal{B}(J/\psi \to \omega K^{*+}K^- \to \pi^+\pi^-\pi^0K_L^0\pi^+K^-)$	$(9.07 \pm 1.34) \times 10^{-4}$

TABLE III. The expected numbers of the observed signal events on the *CP* violation in J/ψ decays.

Decay channel	$N_{CP}^{K^0_{S,L}}$
$J/\psi \to K^{*0}\bar{K}^{*0} \to K^0_S\pi^0K^-\pi^+ \to \pi^+\pi^-\pi^0K^-\pi^+$	$N_{J/\psi}=10^{10}$: $(1.3\pm0.3)\times10^3\times\varepsilon_{K^0_S}$
	$N_{J/\psi}=10^{12}$: $(1.3\pm0.3)\times10^5\times\varepsilon_{K^0_S}$
$J/\psi \to K^{*0}\bar{K}^{*0} \to K^0_L\pi^0K^-\pi^+$	$N_{J/\psi}=10^{10}$: $(1.7\pm0.4)\times10^3\times\varepsilon_{K^0_L}$
	$N_{J/\psi}=10^{12}$: $(1.7\pm0.4)\times10^5\times\varepsilon_{K^0_L}$
$J/\psi \to K^{*+}K^{*-} \to K^0_S\pi^+K^-\pi^0 \to \pi^+\pi^-\pi^+K^-\pi^0$	$N_{J/\psi}=10^{10}$: $(5.6^{-2.2}_{+1.2})\times10^3\times\varepsilon_{K^0_S}$
	$N_{J/\psi}=10^{12}$: $(5.6^{-2.2}_{+1.2})\times10^5\times\varepsilon_{K^0_S}$
$J/\psi \to K^{*+}K^{*-} \to K^0_L\pi^+K^-\pi^0$	$N_{J/\psi}=10^{10}$: $(7.4^{-3.0}_{+1.6})\times10^3\times\varepsilon_{K^0_L}$
	$N_{J/\psi}=10^{12}$: $(7.4^{-3.0}_{+1.6})\times10^5\times\varepsilon_{K^0_L}$
$J/\psi \to \eta K^{*0}\bar{K}^{*0} \to \eta K^0_S\pi^0K^-\pi^+ \to \gamma\gamma\pi^+\pi^-\pi^0K^-\pi^+$	$N_{J/\psi}=10^{10}$: $(2.5\pm0.6)\times10^3\times\varepsilon_{K^0_S}$
	$N_{J/\psi}=10^{12}$: $(2.5\pm0.6)\times10^5\times\varepsilon_{K^0_S}$
$J/\psi \to \eta K^{*0}\bar{K}^{*0} \to \gamma\gamma K^0_L\pi^0K^-\pi^+$	$N_{J/\psi}=10^{10}$: $(3.3\pm0.8)\times10^3\times\varepsilon_{K^0_L}$
	$N_{J/\psi}=10^{12}$: $(3.3\pm0.8)\times10^5\times\varepsilon_{K^0_L}$
$J/\psi \to \eta' K^{*+}K^- \to \pi^+\pi^-\eta K^0_S\pi^+K^- \to \pi^+\pi^-\gamma\gamma\pi^+\pi^-\pi^+K^-$	$N_{J/\psi}=10^{10}$: $(2.1\pm0.2)\times10^3\times\varepsilon_{K^0_S}$
	$N_{J/\psi}=10^{12}$: $(2.1\pm0.2)\times10^5\times\varepsilon_{K^0_S}$
$J/\psi \to \eta' K^{*+}K^- \to \pi^+\pi^-\eta K^0_L\pi^+K^- \to \pi^+\pi^-\gamma\gamma K^0_L\pi^+K^-$	$N_{J/\psi}=10^{10}$: $(2.7\pm0.2)\times10^3\times\varepsilon_{K^0_L}$
	$N_{J/\psi}=10^{12}$: $(2.7\pm0.2)\times10^5\times\varepsilon_{K^0_L}$
$J/\psi \to K^{*+}K^- \to K^0_S\pi^+K^- \to \pi^+\pi^-\pi^+K^-$	$N_{J/\psi}=10^{10}$: $(5.0^{-0.8}_{+0.7})\times10^4\times\varepsilon_{K^0_S}$
	$N_{J/\psi}=10^{12}$: $(5.0^{-0.8}_{+0.7})\times10^6\times\varepsilon_{K^0_S}$
$J/\psi \to K^{*+}K^- \to K^0_L\pi^+K^-$	$N_{J/\psi}=10^{10}$: $(6.6^{-1.1}_{+0.9})\times10^4\times\varepsilon_{K^0_L}$
	$N_{J/\psi}=10^{12}$: $(6.6^{-1.1}_{+0.9})\times10^6\times\varepsilon_{K^0_L}$
$J/\psi \to K^{*0}K^-\pi^+ \to K^0_S\pi^0K^-\pi^+ \to \pi^+\pi^-\pi^0K^-\pi^+$	$N_{J/\psi}=10^{10}$: $(3.2\pm0.7)\times10^4\times\varepsilon_{K^0_S}$
	$N_{J/\psi}=10^{12}$: $(3.2\pm0.7)\times10^6\times\varepsilon_{K^0_S}$
$J/\psi \to K^{*0}K^-\pi^+ \to K^0_L\pi^0K^-\pi^+$	$N_{J/\psi}=10^{10}$: $(4.3\pm0.9)\times10^4\times\varepsilon_{K^0_L}$
	$N_{J/\psi}=10^{12}$: $(4.3\pm0.9)\times10^6\times\varepsilon_{K^0_L}$
$J/\psi \to K^{*+}K^-\pi^0 \to K^0_S\pi^+K^-\pi^0 \to \pi^+\pi^-\pi^+K^-\pi^0$	$N_{J/\psi}=10^{10}$: $(2.5\pm0.8)\times10^4\times\varepsilon_{K^0_S}$
	$N_{J/\psi}=10^{12}$: $(2.5\pm0.8)\times10^6\times\varepsilon_{K^0_S}$
$J/\psi \to K^{*+}K^-\pi^0 \to K^0_L\pi^+K^-\pi^0$	$N_{J/\psi}=10^{10}$: $(3.3\pm1.0)\times10^4\times\varepsilon_{K^0_L}$
	$N_{J/\psi}=10^{12}$: $(3.3\pm1.0)\times10^6\times\varepsilon_{K^0_L}$
$J/\psi \to \omega K^{*+}K^- \to \omega K^0_S\pi^+K^- \to \pi^+\pi^-\pi^0\pi^+\pi^-\pi^+K^-$	$N_{J/\psi}=10^{10}$: $(4.6\pm0.7)\times10^4\times\varepsilon_{K^0_S}$
	$N_{J/\psi}=10^{12}$: $(4.6\pm0.7)\times10^6\times\varepsilon_{K^0_S}$
$J/\psi \to \omega K^{*+}K^- \to \pi^+\pi^-\pi^0K^0_L\pi^+K^-$	$N_{J/\psi}=10^{10}$: $(6.0\pm0.9)\times10^4\times\varepsilon_{K^0_L}$
	$N_{J/\psi}=10^{12}$: $(6.0\pm0.9)\times10^6\times\varepsilon_{K^0_L}$

$$R(\bar{K}^*\to K^0_{S,L}\pi)=\frac{\mathcal{B}(\bar{K}^*\to K^0_S\pi)-\mathcal{B}(\bar{K}^*\to K^0_L\pi)}{\mathcal{B}(\bar{K}^*\to K^0_S\pi)+\mathcal{B}(\bar{K}^*\to K^0_L\pi)}. \quad (54)$$

Substituting Eqs. (25), (26), (33), and (34) into Eqs. (53) and (54) and assuming $z=0$, we can obtain

$$R(K^*\to K^0_{S,L}\pi)=\frac{1}{2}\frac{e^{-\Gamma_L t_0}-e^{-\Gamma_L t_1}}{e^{-\Gamma_S t_0}-e^{-\Gamma_S t_1}}\cdot\frac{\mathcal{B}(K^0_L\to\pi^+\pi^-)}{\mathcal{B}(K^0_S\to\pi^+\pi^-)}+\mathrm{Re}\left(t_{K^0_S-K^0_L}\cdot\frac{p-q}{p+q}\right), \quad (55)$$

$$R(\bar{K}^*\to K^0_{S,L}\pi)=\frac{1}{2}\frac{e^{-\Gamma_L t_0}-e^{-\Gamma_L t_1}}{e^{-\Gamma_S t_0}-e^{-\Gamma_S t_1}}\cdot\frac{\mathcal{B}(K^0_L\to\pi^+\pi^-)}{\mathcal{B}(K^0_S\to\pi^+\pi^-)}-\mathrm{Re}\left(t_{K^0_S-K^0_L}\cdot\frac{p-q}{p+q}\right). \quad (56)$$

In the $J/\psi\to K^*f_{J/\psi}\to K^0_{S,L}\pi f_{J/\psi}$ and $J/\psi\to\bar{K}^*\bar{f}_{J/\psi}\to K^0_{S,L}\pi\bar{f}_{J/\psi}$ decays, the $K^0_S-K^0_L$ asymmetry can de defined as

$$R(J/\psi \to K^* f_{J/\psi} \to K^0_{S,L}\pi f_{J/\psi}) = \frac{\mathcal{B}(J/\psi \to K^* f_{J/\psi} \to K^0_S\pi f_{J/\psi}) - \mathcal{B}(J/\psi \to K^* f_{J/\psi} \to K^0_L\pi f_{J/\psi})}{\mathcal{B}(J/\psi \to K^* f_{J/\psi} \to K^0_S\pi f_{J/\psi}) + \mathcal{B}(J/\psi \to K^* f_{J/\psi} \to K^0_L\pi f_{J/\psi})}, \quad (57)$$

$$R(J/\psi \to \bar{K}^* \bar{f}_{J/\psi} \to K^0_{S,L}\pi \bar{f}_{J/\psi}) = \frac{\mathcal{B}(J/\psi \to \bar{K}^* \bar{f}_{J/\psi} \to K^0_S\pi \bar{f}_{J/\psi}) - \mathcal{B}(J/\psi \to \bar{K}^* \bar{f}_{J/\psi} \to K^0_L\pi \bar{f}_{J/\psi})}{\mathcal{B}(J/\psi \to \bar{K}^* \bar{f}_{J/\psi} \to K^0_S\pi \bar{f}_{J/\psi}) + \mathcal{B}(J/\psi \to \bar{K}^* \bar{f}_{J/\psi} \to K^0_L\pi \bar{f}_{J/\psi})}. \quad (58)$$

Combining Eqs. (53) and (54) with Eqs. (57) and (58), we can obtain the following relations:

$$R(J/\psi \to K^* f_{J/\psi} \to K^0_{S,L}\pi f_{J/\psi}) = R(K^* \to K^0_{S,L}\pi), \quad (59)$$

$$R(J/\psi \to \bar{K}^* \bar{f}_{J/\psi} \to K^0_{S,L}\pi \bar{f}_{J/\psi}) = R(\bar{K}^* \to K^0_{S,L}\pi). \quad (60)$$

With Eqs. (57)–(60), we can derive the numbers of the observed signal events on the $K^0_S - K^0_L$ asymmetry in J/ψ decays

$$N^{K^*}_{K^0_S-K^0_L} = |R(K^* \to K^0_{S,L}\pi)| \cdot N_{J/\psi} \cdot \varepsilon_{K^0_L} \cdot \mathcal{B}(J/\psi \to K^* f_{J/\psi} \to K^0_S\pi f_{J/\psi} + K^0_L\pi f_{J/\psi}), \quad (61)$$

$$N^{\bar{K}^*}_{K^0_S-K^0_L} = |R(\bar{K}^* \to K^0_{S,L}\pi)| \cdot N_{J/\psi} \cdot \varepsilon_{K^0_L} \cdot \mathcal{B}(J/\psi \to \bar{K}^* \bar{f}_{J/\psi} \to K^0_S\pi \bar{f}_{J/\psi} + K^0_L\pi \bar{f}_{J/\psi}). \quad (62)$$

Using the values of the parameters in Table I and combining Eqs. (11), (24), (55), and (56), we can obtain

$$R(K^* \to K^0_{S,L}\pi) = (3.51 \pm 0.03) \times 10^{-3}, \quad (63)$$

$$R(\bar{K}^* \to K^0_{S,L}\pi) = (-3.45 \pm 0.03) \times 10^{-3}. \quad (64)$$

The branching ratios for the $J/\psi \to K^* f_{J/\psi} \to K^0_{S,L}\pi f_{J/\psi}$ and $J/\psi \to \bar{K}^* \bar{f}_{J/\psi} \to K^0_{S,L}\pi \bar{f}_{J/\psi}$ decays can be obtained directly from PDG [38] and Table II. With the values of the branching ratios for these decays and combining Eqs. (61)–(64), we calculate the numerical results of $N^{K^*}_{K^0_S-K^0_L}$ and $N^{\bar{K}^*}_{K^0_S-K^0_L}$, which are listed in Table IV, with a 10^{10} J/ψ event sample and a 10^{12} J/ψ event sample, respectively. Here, we also note that the detection efficiency $\varepsilon_{K^0_L}$ is at the level of 10^{-3} at BESIII [43,60], so the $K^0_S - K^0_L$ asymmetry can be observed in the decays where $N^{K^*}_{K^0_S-K^0_L}$ (or $N^{\bar{K}^*}_{K^0_S-K^0_L}$) is larger than $10^4 \times \varepsilon_{K^0_L}$.

The $K^* \to K^0_{S,L}\pi$ decays can also be used to study the CPT violation. We define the following observables, which are related to the CPT violation parameter z:

$$\mathcal{A}^m_{CPT}(K^* \to K^0_{S,L}\pi) = \frac{\mathcal{A}^-_{K^0_S} - \mathcal{A}^-_{K^0_L}}{\mathcal{A}^+_{K^0_S} + \mathcal{A}^+_{K^0_L}}, \quad (65)$$

$$\mathcal{A}^p_{CPT}(K^* \to K^0_{S,L}\pi) = \frac{\mathcal{A}^+_{K^0_S} - \mathcal{A}^+_{K^0_L}}{\mathcal{A}^+_{K^0_S} + \mathcal{A}^+_{K^0_L}}, \quad (66)$$

where

$$\mathcal{A}^\pm_{K^0_{S,L}} = \mathcal{B}(K^* \to K^0_{S,L}\pi) \pm \mathcal{B}(\bar{K}^* \to K^0_{S,L}\pi). \quad (67)$$

Substituting Eqs. (25), (26), (33), and (34) into Eqs. (65) and (66), we can easily find

$$\mathcal{A}^m_{CPT}(K^* \to K^0_{S,L}\pi) = -\mathrm{Re}(z) + \mathrm{Re}\left(t_{K^0_S-K^0_L} \cdot \left(\frac{p-q}{p+q} + \frac{z}{2}\right)\right), \quad (68)$$

$$\mathcal{A}^p_{CPT}(\bar{K}^* \to K^0_{S,L}\pi) = -\mathrm{Re}(z) \cdot \frac{|q|^2 - |p|^2}{|q|^2 + |p|^2} + \frac{1}{2}\frac{e^{-\Gamma_L t_0} - e^{-\Gamma_L t_1}}{e^{-\Gamma_S t_0} - e^{-\Gamma_S t_1}} \cdot \frac{\mathcal{B}(K^0_L \to \pi^+\pi^-)}{\mathcal{B}(K^0_S \to \pi^+\pi^-)} + \mathrm{Re}\left(\left(\frac{|q|^2 - |p|^2}{|q|^2 + |p|^2} + i\mathrm{Im}(z)\right) t_{K^0_S-K^0_L} \cdot \left(\frac{p-q}{p+q} + \frac{z}{2}\right)\right). \quad (69)$$

Obviously, $\mathcal{A}^{m,p}_{CPT}(K^* \to K^0_{S,L}\pi)$ contains the terms $t_{K^0_S-K^0_L} \cdot \frac{p-q}{p+q}$ and $(|q|^2 - |p|^2)/(|q|^2 + |p|^2)$, which are independent from the CPT violation parameter z, so the precise calculations of $t_{K^0_S-K^0_L} \cdot \frac{p-q}{p+q}$ and $(|q|^2 - |p|^2)/(|q|^2 + |p|^2)$, which are the functions of the parameters $m_L - m_S, \Gamma_L, \Gamma_S, p, q, t_0$, and t_1, are crucial to constrain the CPT violation parameter z in the $K^* \to K^0_{S,L}\pi$ decays.

TABLE IV. The expected numbers of the observed signal events on the $K_S^0 - K_L^0$ asymmetry in J/ψ decays.

Decay channel	$N^{K^*}_{K_S^0-K_L^0}(N^{\bar{K}^*}_{K_S^0-K_L^0})$
$J/\psi \to K^{*0}\bar{K}^{*0} \to K_{S,L}^0\pi^0K^-\pi^+$	$N_{J/\psi} = 10^{10}$: $(1.8 \pm 0.5) \times 10^3 \times \varepsilon_{K_L^0}$
	$N_{J/\psi} = 10^{12}$: $(1.8 \pm 0.5) \times 10^5 \times \varepsilon_{K_L^0}$
$J/\psi \to K^{*0}\bar{K}^{*0} \to K^+\pi^-K_{S,L}^0\pi^0$	$N_{J/\psi} = 10^{10}$: $(1.8 \pm 0.5) \times 10^3 \times \varepsilon_{K_L^0}$
	$N_{J/\psi} = 10^{12}$: $(1.8 \pm 0.5) \times 10^5 \times \varepsilon_{K_L^0}$
$J/\psi \to K^{*+}K^{*-} \to K_{S,L}^0\pi^+K^-\pi^0$	$N_{J/\psi} = 10^{10}$: $(7.8^{-3.1}_{+1.7}) \times 10^3 \times \varepsilon_{K_L^0}$
	$N_{J/\psi} = 10^{12}$: $(7.8^{-3.1}_{+1.7}) \times 10^5 \times \varepsilon_{K_L^0}$
$J/\psi \to K^{*+}K^{*-} \to K^+\pi^0K_{S,L}^0\pi^-$	$N_{J/\psi} = 10^{10}$: $(7.7^{-3.1}_{+1.7}) \times 10^3 \times \varepsilon_{K_L^0}$
	$N_{J/\psi} = 10^{12}$: $(7.7^{-3.1}_{+1.7}) \times 10^5 \times \varepsilon_{K_L^0}$
$J/\psi \to \eta K^{*0}\bar{K}^{*0} \to \gamma\gamma K_{S,L}^0\pi^0K^-\pi^+$	$N_{J/\psi} = 10^{10}$: $(3.5 \pm 0.8) \times 10^3 \times \varepsilon_{K_L^0}$
	$N_{J/\psi} = 10^{12}$: $(3.5 \pm 0.8) \times 10^5 \times \varepsilon_{K_L^0}$
$J/\psi \to \eta K^{*0}\bar{K}^{*0} \to \gamma\gamma K^+\pi^-K_{S,L}^0\pi^0$	$N_{J/\psi} = 10^{10}$: $(3.5 \pm 0.8) \times 10^3 \times \varepsilon_{K_L^0}$
	$N_{J/\psi} = 10^{12}$: $(3.5 \pm 0.8) \times 10^5 \times \varepsilon_{K_L^0}$
$J/\psi \to \eta' K^{*+}K^- \to \pi^+\pi^-\eta K_{S,L}^0\pi^+K^- \to \pi^+\pi^-\gamma\gamma K_{S,L}^0\pi^+K^-$	$N_{J/\psi} = 10^{10}$: $(2.9 \pm 0.3) \times 10^3 \times \varepsilon_{K_L^0}$
	$N_{J/\psi} = 10^{12}$: $(2.9 \pm 0.3) \times 10^5 \times \varepsilon_{K_L^0}$
$J/\psi \to \eta' K^{*-}K^+ \to \pi^+\pi^-\eta K_{S,L}^0\pi^-K^+ \to \pi^+\pi^-\gamma\gamma K_{S,L}^0\pi^-K^+$	$N_{J/\psi} = 10^{10}$: $(2.9 \pm 0.3) \times 10^3 \times \varepsilon_{K_L^0}$
	$N_{J/\psi} = 10^{12}$: $(2.9 \pm 0.3) \times 10^5 \times \varepsilon_{K_L^0}$
$J/\psi \to K^{*+}K^- \to K_{S,L}^0\pi^+K^-$	$N_{J/\psi} = 10^{10}$: $(7.0^{-1.2}_{+0.9}) \times 10^4 \times \varepsilon_{K_L^0}$
	$N_{J/\psi} = 10^{12}$: $(7.0^{-1.2}_{+0.9}) \times 10^6 \times \varepsilon_{K_L^0}$
$J/\psi \to K^{*-}K^+ \to K_{S,L}^0\pi^-K^+$	$N_{J/\psi} = 10^{10}$: $(6.9^{-1.2}_{+0.9}) \times 10^4 \times \varepsilon_{K_L^0}$
	$N_{J/\psi} = 10^{12}$: $(6.9^{-1.2}_{+0.9}) \times 10^6 \times \varepsilon_{K_L^0}$
$J/\psi \to K^{*0}K^-\pi^+ \to K_{S,L}^0\pi^0K^-\pi^+$	$N_{J/\psi} = 10^{10}$: $(4.5 \pm 0.9) \times 10^4 \times \varepsilon_{K_L^0}$
	$N_{J/\psi} = 10^{12}$: $(4.5 \pm 0.9) \times 10^6 \times \varepsilon_{K_L^0}$
$J/\psi \to \bar{K}^{*0}K^+\pi^- \to K_{S,L}^0\pi^0K^+\pi^-$	$N_{J/\psi} = 10^{10}$: $(4.4 \pm 0.9) \times 10^4 \times \varepsilon_{K_L^0}$
	$N_{J/\psi} = 10^{12}$: $(4.4 \pm 0.9) \times 10^6 \times \varepsilon_{K_L^0}$
$J/\psi \to K^{*+}K^-\pi^0 \to K_{S,L}^0\pi^+K^-\pi^0$	$N_{J/\psi} = 10^{10}$: $(3.5 \pm 1.1) \times 10^4 \times \varepsilon_{K_L^0}$
	$N_{J/\psi} = 10^{12}$: $(3.5 \pm 1.1) \times 10^6 \times \varepsilon_{K_L^0}$
$J/\psi \to K^{*-}K^+\pi^0 \to K_{S,L}^0\pi^-K^+\pi^0$	$N_{J/\psi} = 10^{10}$: $(3.5 \pm 1.1) \times 10^4 \times \varepsilon_{K_L^0}$
	$N_{J/\psi} = 10^{12}$: $(3.5 \pm 1.1) \times 10^6 \times \varepsilon_{K_L^0}$
$J/\psi \to \omega K^{*+}K^- \to \pi^+\pi^-\pi^0K_{S,L}^0\pi^+K^-$	$N_{J/\psi} = 10^{10}$: $(6.4 \pm 0.9) \times 10^4 \times \varepsilon_{K_L^0}$
	$N_{J/\psi} = 10^{12}$: $(6.4 \pm 0.9) \times 10^6 \times \varepsilon_{K_L^0}$
$J/\psi \to \omega K^{*-}K^+ \to \pi^+\pi^-\pi^0K_{S,L}^0\pi^-K^+$	$N_{J/\psi} = 10^{10}$: $(6.3 \pm 0.9) \times 10^4 \times \varepsilon_{K_L^0}$
	$N_{J/\psi} = 10^{12}$: $(6.3 \pm 0.9) \times 10^6 \times \varepsilon_{K_L^0}$

In the J/ψ decays involving the K^* meson in the final states, we define the CPT asymmetry observables

$$\mathcal{A}^m_{CPT}(J/\psi \to K^* f_{J/\psi}) = \frac{\mathcal{A}^-_{J/\psi K_S^0} - \mathcal{A}^-_{J/\psi K_L^0}}{\mathcal{A}^+_{J/\psi K_S^0} + \mathcal{A}^+_{J/\psi K_L^0}}, \quad (70)$$

$$\mathcal{A}^p_{CPT}(J/\psi \to K^* f_{J/\psi}) = \frac{\mathcal{A}^+_{J/\psi K_S^0} - \mathcal{A}^+_{J/\psi K_L^0}}{\mathcal{A}^+_{J/\psi K_S^0} + \mathcal{A}^+_{J/\psi K_L^0}}, \quad (71)$$

where

$$\mathcal{A}^\pm_{J/\psi K_S^0} = \mathcal{B}(J/\psi \to K^* f_{J/\psi} \to K_S^0\pi f_{J/\psi}) \pm \mathcal{B}(J/\psi \to \bar{K}^* \bar{f}_{J/\psi} \to K_S^0\pi \bar{f}_{J/\psi}), \quad (72)$$

$$\mathcal{A}^\pm_{J/\psi K_L^0} = \mathcal{B}(J/\psi \to K^* f_{J/\psi} \to K_L^0\pi f_{J/\psi}) \pm \mathcal{B}(J/\psi \to \bar{K}^* \bar{f}_{J/\psi} \to K_L^0\pi \bar{f}_{J/\psi}). \quad (73)$$

Obviously, according to Eqs. (65)–(67) and (70)–(73), we can obtain

$$\mathcal{A}^m_{CPT}(J/\psi \to K^* f_{J/\psi}) = \mathcal{A}^m_{CPT}(K^* \to K_{S,L}^0\pi), \quad (74)$$

$$\mathcal{A}^{p}_{CPT}(J/\psi \to K^* f_{J/\psi}) = \mathcal{A}^{p}_{CPT}(K^* \to K^0_{S,L}\pi). \quad (75)$$

With the values of the parameters in Table I and combining Eqs. (11), (24), (68), and (69), we can obtain

$$\mathrm{Re}\left(t_{K^0_S-K^0_L}\cdot\left(\frac{p-q}{p+q}\right)\right) = (3.48\pm 0.03)\times 10^{-3}, \quad (76)$$

$$\frac{1}{2}\frac{e^{-\Gamma_L t_0}-e^{-\Gamma_L t_1}}{e^{-\Gamma_S t_0}-e^{-\Gamma_S t_1}}\cdot\frac{\mathcal{B}(K^0_L\to\pi^+\pi^-)}{\mathcal{B}(K^0_S\to\pi^+\pi^-)} + \frac{|q|^2-|p|^2}{|q|^2+|p|^2}\cdot\mathrm{Re}\left(t_{K^0_S-K^0_L}\cdot\frac{p-q}{p+q}\right) = (1.54\pm 0.03)\times 10^{-5}, \quad (77)$$

which accuracy can reach 10^{-4} and 10^{-6}, respectively. Combining Eqs. (68) and (69) with Eqs. (74)–(77), we can obtain the numerical results of $\mathcal{A}^{m,p}_{CPT}(J/\psi\to K^* f_{J/\psi})$ under the assumption of $z=0$

$$\mathcal{A}^{m}_{CPT}(J/\psi\to K^* f_{J/\psi})_{z=0} = (3.48\pm 0.03)\times 10^{-3}, \quad (78)$$

$$\mathcal{A}^{p}_{CPT}(J/\psi\to K^* f_{J/\psi})_{z=0} = (1.54\pm 0.03)\times 10^{-5}. \quad (79)$$

From Eqs. (70) and (71) and taking into account only the statistical errors, the errors of $\mathcal{A}^{m,p}_{CPT}(J/\psi\to K^* f_{J/\psi})$ can be derived

$$\Delta(\mathcal{A}^{m,p}_{CPT}(J/\psi\to K^* f_{J/\psi})) \approx \frac{1}{\sqrt{N_{J/\psi}\cdot(\mathcal{A}^{+}_{J/\psi K^0_S}+\mathcal{A}^{+}_{J/\psi K^0_L})\cdot\varepsilon_{K^0_L}}}. \quad (80)$$

From PDG [38] and Table II, the branching ratios for $J/\psi\to K^* f_{J/\psi}\to K^0_{S,L}\pi f_{J/\psi}$ can reach the level of 10^{-3}, so the errors $\Delta(\mathcal{A}^{m,p}_{CPT}(J/\psi\to K^* f_{J/\psi}))$ can arrive at the level of 10^{-4}, if we assume the selection efficiency is 10^{-3} and the total number of J/ψ events is 10^{12}. According to these results, Eqs. (68) and (76), we can obtain that the sensitivity for the measurement of the CPT violation parameter z is expected to be at the level of 10^{-3} with $\mathcal{A}^{m}_{CPT}(J/\psi\to K^* f_{J/\psi})$ at BESIII. Because there exists a suppression effect on the CPT violation parameter z in Eq. (69), the observable $\mathcal{A}^{p}_{CPT}(J/\psi\to K^* f_{J/\psi})$ is insensitive to the measurement of z with a 10^{12} J/ψ event sample. Currently, the best result for $\mathrm{Re}(z)$ is $-(5.2\pm 5.0)\times 10^{-4}$ which is obtained from a combined fit, including KLOE [4,61] and CPLEAR [62], by the Particle Data Group [38], so the measurement of the CPT violation parameter z is expected to be competitive with the current best result with a 10^{12} J/ψ event sample at BESIII. Here, we note that the sensitivity of the measurement of the CPT violation parameter z depends on measured precision of the parameters m_L-m_S, Γ_L, Γ_S, p, and q and the consistency between the values of t_0 and t_1 and the event selection criteria in experiment.

In addition, we can define the following observable:

$$\mathcal{A}_{CPT}(K^*\to K^0_L\pi) = \frac{4|p|^2\cdot\mathcal{B}(K^*\to K^0_L\pi)-4|q|^2\cdot\mathcal{B}(\bar{K}^*\to K^0_L\pi)}{4|p|^2\cdot\mathcal{B}(K^*\to K^0_L\pi)+4|q|^2\cdot\mathcal{B}(\bar{K}^*\to K^0_L\pi)}. \quad (81)$$

Substituting Eqs. (33) and (34) into Eq. (81), we can find

$$\mathcal{A}_{CPT}(K^*\to K^0_L\pi) = \frac{\mathrm{Re}(z)}{1+\frac{|z|^2}{4}}, \quad (82)$$

which is sensitive to the CPT violation parameter $\mathrm{Re}(z)$. However, the sensitivity of the measurement of the CPT violation parameter $\mathrm{Re}(z)$ with the observable $\mathcal{A}_{CPT}(K^*\to K^0_L\pi)$ depends on the measured precisions of the parameters p and q.

In conclusion, we discuss the CP asymmetry, $K^0_S-K^0_L$ asymmetry, and CPT asymmetry in $K^*\to K^0_{S,L}\pi$ decays at BESIII. The CP asymmetries in the $K^*\to K^0_{S,L}\pi$ decays are dominated by $K^0-\bar{K}^0$ mixing. We calculate the numerical results of the CP asymmetries in the J/ψ decays involving the K^* meson in the final states

$$\mathcal{A}_{CP}(J/\psi\to K^* f_{J/\psi}\to K^0_S\pi f_{J/\psi}) = (3.64\pm 0.04)\times 10^{-3}, \quad (83)$$

$$\mathcal{A}_{CP}(J/\psi\to K^* f_{J/\psi}\to K^0_L\pi f_{J/\psi}) = (-3.32\pm 0.04)\times 10^{-3}. \quad (84)$$

We estimate the expected numbers of the observed signal events on the CP violations in J/ψ decays with a K^* meson in the final states based on a 10^{10} J/ψ event sample and 10^{12} J/ψ event sample in BESIII experiment, respectively. We find that the BESIII experiment may be able to make a significant measurement of the CP violation in these decays with 10^{10} J/ψ events, which has been accumulated in four runs in 2009, 2012, 2018, and 2019 [42,47].

The $K^0_S-K^0_L$ asymmetries in J/ψ decays with a K^* meson in the final states are also studied. The numerical results of the $K^0_S-K^0_L$ asymmetries can be obtained as

$$R(J/\psi\to K^* f_{J/\psi}\to K^0_{S,L}\pi f_{J/\psi}) = (3.51\pm 0.03)\times 10^{-3}, \quad (85)$$

$$R(J/\psi\to \bar{K}^* \bar{f}_{J/\psi}\to K^0_{S,L}\pi \bar{f}_{J/\psi}) = (-3.45\pm 0.03)\times 10^{-3}. \quad (86)$$

Together with these results and the branching ratios for the J/ψ decays, we calculate the expected numbers of the observed signal events on the $K_S^0 - K_L^0$ asymmetries in the case of a 10^{10} J/ψ event sample and 10^{12} J/ψ event sample in BESIII experiment, respectively. If we assume that the detection efficiency $\varepsilon_{K_L^0}$ is at the level of 10^{-3} at BESIII, the $K_S^0 - K_L^0$ asymmetry can be observed in the decays when the expected number of the observed signal events on the $K_S^0 - K_L^0$ asymmetry is larger than $10^4 \times \varepsilon_{K_L^0}$.

We investigate the possibility to constrain the *CPT* violation parameter z in J/ψ decays with a K^* meson in the final states at BESIII. We discuss the sensitivity for the measurement of the *CPT* violation parameter z under the assumption that the selection efficiency is 10^{-3} and the total number of J/ψ events is 10^{12}. We find that the sensitivity for the measurement of the *CPT* violation parameter z is expected to be at the level of 10^{-3} with 10^{12} J/ψ event sample at BESIII. The sensitivity of the measurement of the *CPT* violation parameter z depends on measured precision of the parameters $m_L - m_S$, Γ_L, Γ_S, p, and q and the consistency between the values of t_0 and t_1 and the event selection criteria in experiment.

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