Effect of plasma on gravitational lensing by a Schwarzschild black hole immersed in perfect fluid dark matter

Farruh Atamurotov,[1,2,3,*] Ahmadjon Abdujabbarov,[4,5,3,6,7,†] and Wen-Biao Han[4,8,9,‡]

[1]*Inha University in Tashkent, Ziyolilar 9, Tashkent 100170, Uzbekistan*
[2]*Akfa University, Kichik Halqa Yuli Street 17, Tashkent 100095, Uzbekistan*
[3]*Department of Astronomy and Astrophysics, National University of Uzbekistan, Tashkent 100174, Uzbekistan*
[4]*Shanghai Astronomical Observatory, 80 Nandan Road, Shanghai 200030, China*
[5]*Ulugh Beg Astronomical Institute, Astronomy Street 33, Tashkent 100052, Uzbekistan*
[6]*Institute of Nuclear Physics, Ulugbek Street, Tashkent 100214, Uzbekistan*
[7]*Tashkent Institute of Irrigation and Agricultural Mechanization Engineers, Kori Niyoziy 39, Tashkent 100000, Uzbekistan*
[8]*School of Fundamental Physics and Mathematical Sciences, Hangzhou Institute for Advanced Study, UCAS, Hangzhou 310024, China*
[9]*School of Astronomy and Space Science, University of Chinese Academy of Sciences, Beijing 100049, China*

(Received 19 March 2021; accepted 3 September 2021; published 1 October 2021)

We have explored the effect of both uniform and nonuniform distributed plasma medium on gravitational lensing around static black hole in the presence of perfect fluid dark matter (PFDM). We have shown that the PFDM and plasma parameters has opposite effect on the change of the value of the deflection angle of the light rays. It has been also shown that PFDM decreases the total magnification of image sources while the plasma causes opposite effect. Finally, we have analyzed the shadow cast by the black hole surrounded by PFDM and plasma. The increase of the PFDM parameter causes the decrease the size of the shadow of black hole.

DOI: 10.1103/PhysRevD.104.084015

I. INTRODUCTION

One of the first fundamental signatures of the general relativity since its discovery by Einstein in 1916 is the effect of the gravitational lensing. The effect is based on the light ray deflection near the gravitating object due to spacetime curvature. The classical gravitational lensing in general relativity has been described in reviews [1–4]. At the same time it is important to study the photon motion in curved spacetime since it is relevant for further study of gravitational lensing. Moreover, the plasma medium around compact object may interact with the electromagnetic wave and consequently affect on trajectory of photons. Particularly, in Refs. [5–29] authors have studied the photon motion in different spacetimes and some of them devoted to study the effect of plasma together with gravity on photon motion. In addition, one of the hot topic is strong gravitational lensing in a vacuum; recently, the theoretical analysis of strong lensing is studied by several authors for various space-time geometries in Refs. [30–37].

The detailed study of photon motion around black hole leads to phenomena called the shadow of black hole. The shadow cast by black hole is due to photon capture by the central object and observer will see the black spot on the bright background. In 2019 it was for the first time observed the shadow of black hole at the center of galaxy M87* [38,39]. The discovery of the shadow of black hole opens new possibilities to test the general relativity and other metric theories of gravity. Particularly the shadow of the black hole in different theories of gravity including the modification of general relativity have been studied by many authors [12,21,40–68]. In this paper, we are also interested in study of the weak gravitational lensing around black hole surrounded by the perfect fluid dark matter (PFDM) and plasma medium. Moreover, we will also study the effect of the PFDM and plasma on the observable properties of shadow in Schwarzschild spacetime.

There is some evidence for the existence of dark matter or dark energy, and according to the standard cosmological model [69] our Universe mainly contains 68% dark energy, about 27% dark matter and 5% baryonic matter [69]. Dark matter is nonbaryonic matter. Consequently one may ask the question whether dark energy and/or dark matter affect the astrophysical processes in the vicinity of compact

*atamurotov@yahoo.com
†ahmadjon@astrin.uz
‡wbhan@shao.ac.cn

objects. It is true that dark matter is invisible, so it is difficult to test it, but we can feel the influence of dark matter or dark energy from observations. From this point on, dark matter is extremely important for studying the properties of the physics of a black hole. It is well known that dark matter or dark energy also exists around a black hole, so their contribution is always very important and interesting, when we study the motion of particles around a black hole in the Universe. In the latest decay, a black hole surrounded by the quintessence of dark energy is examined in more detail [70]. The rotating black hole solution surrounded by quintessence has been obtained by [71] using Newman-Janis algorithm. Further properties of the spacetime around the black hole surrounded by quintessence have been explored in Refs. [56,72–78]. Alternative model for dark matter has been also proposed in [70] where quintessence scalar field is considered as PFDM. The solution of black hole surrounded by PFDM contains the logarithmic term $\lambda \ln(r/|\lambda|)$ appearing in the metric functions, corresponding to the nonvanishing contribution of dark matter through parameter λ. Following the idea that dark matter halo is formed by weakly interacting massive particles in Ref. [79], it has been derived a similar black hole solution involving a logarithmic term. The properties of the spacetime around the black hole surrounded by PFDM have been investigated in [80–82].

The main motivation of the study performed in this paper is to test the PFDM model for dark matter using the photon motion around compact object. Observable properties of the black hole shadow or light deflection due to gravitational lensing may help to get constraints on model and theory describing the dark matter around compact object. Consequently this will help us to explain the nature of the dark matter and the astrophysical processes related to the effects of dark matter. Particularly, in this paper we have studied the photon motion around black hole surrounded by PFDM. Performing the analysis of the effects of PFDM on light deflection we have found out that the presence of PFDM causes decrease the light deflection. This may be interpreted as negative gravitational effect of PFDM on photons since PFDM is surrounding the central object and photons. The gravitational effects on photons of PFDM and central object are opposite to each other.

The work is arranged as follows: Sec. II we consider effect of plasma on deflection angle in presence of perfect fluid dark matter for Schwarzschild black hole. In Sec. III we study magnification of image for various parameter of PFDM. Shadow of black hole study in presence of PFDM and influence of plasma in Sec. IV. Finally, in Sec. V we discuss our results that have been obtained.

II. WEAK LENSING IN THE PRESENCE OF PLASMA

The metric describing a static and spherically symmetric black hole immersed in perfect fluid dark matter in Schwarzschild coordinates (t, r, θ, φ) is given by [79]

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega^2, \tag{1}$$

with

$$f(r) = 1 - \frac{2M}{r} + \frac{\lambda}{r}\ln\frac{r}{|\lambda|}, \tag{2}$$

where M is the mass of the black hole and parameter λ is associated with the density and the pressure of dark matter [79].

We will consider a weak-field approximation defined as shown below:

$$g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta}, \tag{3}$$

where $\eta_{\alpha\beta}$ is expression for the flat geometry and $h_{\alpha\beta}$ is a perturbation of the flat spacetime geometry with the following general properties [16]:

$$\begin{aligned} &\eta_{\alpha\beta} = \mathrm{diag}(-1,1,1,1),\\ &h_{\alpha\beta} \ll 1, \qquad h_{\alpha\beta}\to 0 \qquad \text{under} \qquad x^i\to\infty,\\ &g^{\alpha\beta} = \eta^{\alpha\beta} - h^{\alpha\beta}, \qquad h^{\alpha\beta} = h_{\alpha\beta}. \end{aligned} \tag{4}$$

Here we explore the effect of the plasma on the gravitational deflection angle in the presence of the PFDM in the Schwarzschild spacetime geometry. Consider the basic expression for the deflection angle in the presence of a plasma medium as [16,26]

$$\hat{\alpha}_k = \frac{1}{2}\int_{-\infty}^{\infty}\left(h_{33} + \frac{h_{00}\omega^2 - K_e N(x^i)}{\omega^2 - \omega_e^2}\right)_{,k} dz, \tag{5}$$

where $N(x^i)$ is the concentration of charged particles in plasma, ω and ω_e are photon and plasma frequencies, respectively. $K_e = 4\pi e^2/m_e$ is constant value of plasma particle. On the other hand, one may rewrite Eq. (5) in the different form as [16]:

$$\hat{\alpha}_b = \frac{1}{2}\int_{-\infty}^{\infty}\frac{b}{r}\left(\frac{dh_{33}}{dr} + \frac{1}{1-\omega_e^2/\omega^2}\frac{dh_{00}}{dr} - \frac{K_e}{\omega^2-\omega_e^2}\frac{dN}{dr}\right)dz. \tag{6}$$

The value $\hat{\alpha}_b$ can be both negative (towards from the compact object) and positive (away from the compact object).

In the weak field regime at large distances r from the compact object surrounded by PFDM the spacetime metric can be expressed as

$$ds^2 = ds_0^2 + \left(\frac{R_s}{r} - \frac{\lambda}{r}\ln\frac{r}{|\lambda|}\right)dt^2 + \left(\frac{R_s}{r} - \frac{\lambda}{r}\ln\frac{r}{|\lambda|}\right)dr^2, \tag{7}$$

where $ds_0^2 = -dt^2 + dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$ is the line element in the flat spacetime and we have used notation $R_s = 2M$.

In order to calculate Eq. (6) one can rewrite the components $h_{\alpha\beta}$ in Cartesian coordinates in the form

$$h_{00} = \left(\frac{R_s}{r} - \frac{\lambda}{r}\ln\frac{r}{|\lambda|}\right),$$
$$h_{ik} = \left(\frac{R_s}{r} - \frac{\lambda}{r}\ln\frac{r}{|\lambda|}\right)n_i n_k,$$
$$h_{33} = \left(\frac{R_s}{r} - \frac{\lambda}{r}\ln\frac{r}{|\lambda|}\right)\cos^2\chi, \tag{8}$$

where new notation $\cos^2\chi = z^2/(b^2+z^2)$ and $r^2 = b^2 + z^2$ are introduced. One may easily calculate the derivative of h_{00} and h_{33} and get

$$\frac{dh_{33}}{dr} = -\frac{3z^2R_s}{r^4} - \frac{\lambda z^2}{r^4} + \frac{3\lambda z^2\ln(\frac{r}{|\lambda|})}{r^4},$$
$$\frac{dh_{00}}{dr} = -\frac{\lambda}{r^2} + \frac{\lambda\ln(\frac{r}{|\lambda|})}{r^2} - \frac{R_s}{r^2}. \tag{9}$$

Now one may decompose the expression for the deflection angle in the following form [26]:

$$\hat{\alpha}_b = \hat{\alpha}_1 + \hat{\alpha}_2 + \hat{\alpha}_3, \tag{10}$$

with

$$\hat{\alpha}_1 = \frac{1}{2}\int_{-\infty}^{\infty}\frac{b}{r}\frac{dh_{33}}{dr}dz,$$
$$\hat{\alpha}_2 = \frac{1}{2}\int_{-\infty}^{\infty}\frac{b}{r}\left(\frac{1}{1-\omega_e^2/\omega^2}\frac{dh_{00}}{dr}\right)dz,$$
$$\hat{\alpha}_3 = \frac{1}{2}\int_{-\infty}^{\infty}\frac{b}{r}\left(-\frac{K_e}{\omega^2-\omega_e^2}\frac{dN}{dr}\right)dz, \tag{11}$$

where the new designations $\hat{\alpha}_1$, $\hat{\alpha}_2$ and $\hat{\alpha}_3$ correspond to contribution of deflection angle due to gravity, homogeneous and inhomogeneous plasma medium, respectively. Throughout the paper we will use Eq. (10) in order to study the effects of plasma medium on the deflection angle.

A. Uniform plasma effect to the deflection angle

In this subsection, we will study the deflection angle in the presence of a uniform plasma medium using Eq. (10), which can be rewritten as

$$\hat{\alpha}_{\rm uni} = \hat{\alpha}_{\rm uni1} + \hat{\alpha}_{\rm uni2} + \hat{\alpha}_{\rm uni3} \tag{12}$$

where $\hat{\alpha}_{\rm uni1}$ and $\hat{\alpha}_{\rm uni2}$ can be interpreted as contribution to the deflection angle due to uniform plasma and $\hat{\alpha}_{\rm uni3} = 0$ due to the uniform distribution of plasma. Using Eqs. (10), (9) and (12) one may easily obtain the deflection angle formula for photons around compact object surrounded by uniform plasma and PFDM as

$$\hat{\alpha}_{\rm uni} = -\left(\frac{\lambda}{b} + \frac{\lambda\ln[\frac{1}{2\lambda}]}{b} - \frac{R_s}{b}\right) - \left(\frac{\lambda\ln[\frac{1}{2\lambda}]}{b} - \frac{R_s}{b}\right)\left(\frac{\omega^2}{\omega^2-\omega_e^2}\right). \tag{13}$$

Figure 1 shows the dependence of the deflection angle from the impact parameter of the photons for the different values of PFDM and plasma parameters. Figure 2 represents the dependence of the deflection angle form plasma and dark matter parameters for fixed values of impact parameter b. From Figs. 1 and 2 one may see that the deflection angle decreases with the increase of the PFDM parameter. On the other hand, the presence of plasma medium increase the gravitational deflection angle of photons. Note, that the deflection angle strive to zero with an increase of the impact parameter b.

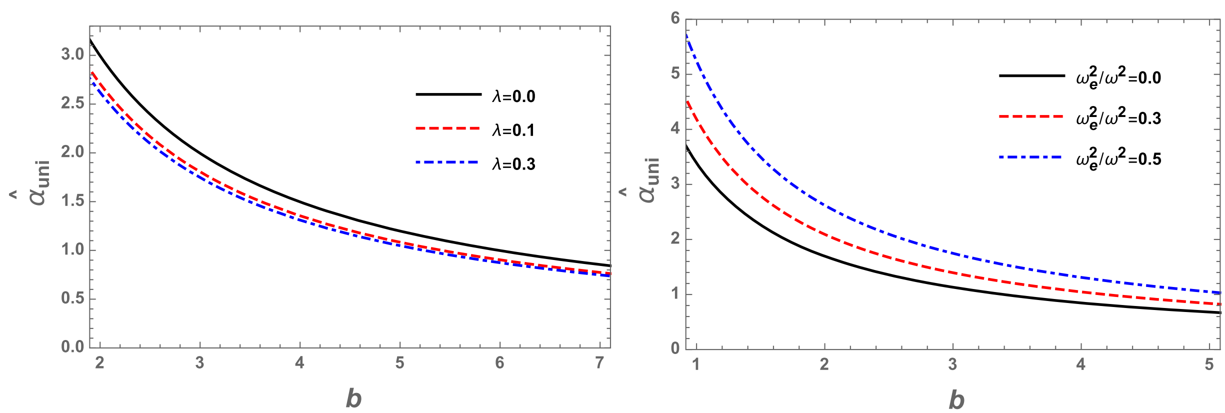

FIG. 1. The dependence of the deflection angle $\hat{\alpha}_{\rm uni}$ on the impact parameter for different values of perfect fluid dark matter parameter λ (left panel) and plasma medium (right panel).

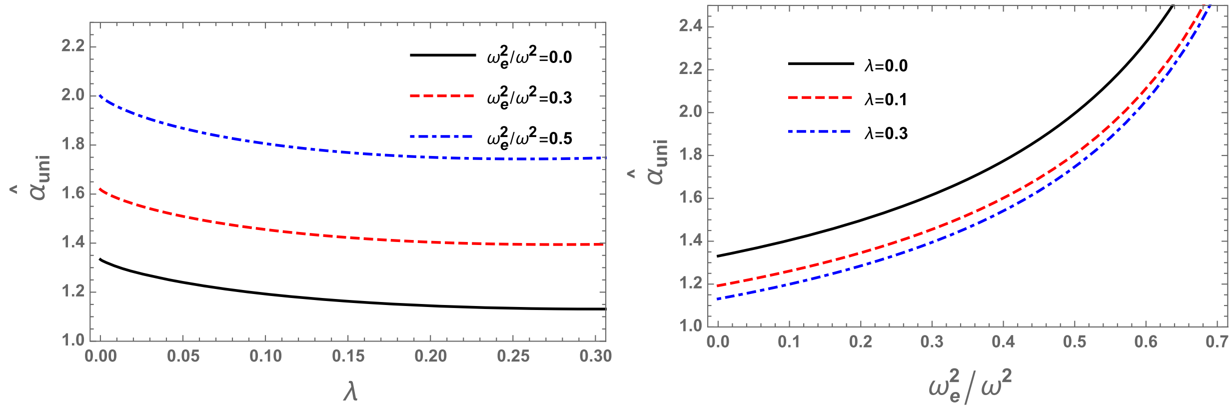

FIG. 2. The dependence of the deflection angle $\hat{\alpha}_{\rm uni}$ on the perfect fluid dark matter parameter λ (left panel) and plasma medium (right panel) for the fixed value of impact parameter $b = 3M$.

B. Nonuniform plasma effect to the deflection angle

Now we explore the deflection angle of photons around compact object in the presence of nonuniform plasma and perfect fluid dark matter. Here we use the singular isothermal sphere (SIS) medium for the nonuniform plasma distribution [16]. The plasma concentration of SIS medium has the following form [16,26,83]

$$N(r) = \frac{\rho(r)}{km_p}, \tag{14}$$

and

$$\rho(r) = \frac{\sigma_\nu^2}{2\pi r^2}, \tag{15}$$

where σ_ν is the dispersion velocity and $\rho(r)$ is the plasma density. Now Eq. (10) can be decomposed as

$$\hat{\alpha}_{\rm SIS} = \hat{\alpha}_{\rm SIS1} + \hat{\alpha}_{\rm SIS2} + \hat{\alpha}_{\rm SIS3}, \tag{16}$$

where $\hat{\alpha}_{\rm SIS1}$ and $\hat{\alpha}_{\rm SIS2}$ correspond to the contribution to the deflection angle due to pure gravity and plasma effects and $\hat{\alpha}_{\rm SIS3}$ corresponds to the contribution due to gradient of the density of the plasma medium. Using Eqs. (10), (9) and (16) one may easily obtain the deflection angle of photons around Schwarzschild black hole surrounded by nonuniform plasma in presence of PFDM in the following form:

$$\hat{\alpha}_{\rm SIS} = -\frac{\lambda w_c^2 \ln(\frac{1}{2\lambda}) R_s^2}{\pi b^2 w^2} + \frac{w_c^2 R_s^3}{\pi b^2 w^2} + \frac{w_c^2 R_s^2}{2b^2 w^2} - \frac{\lambda}{b} - \frac{2\lambda \ln(\frac{1}{2\lambda})}{b} + \frac{2R_s}{b}, \tag{17}$$

where we introduced new expression as

$$w_c^2 = \frac{\sigma_\nu^2 K_e}{2km_p R_s^2}. \tag{18}$$

Figure 3 shows the influence of dark matter parameter and nonuniform plasma medium on the deflection angle of photons around gravitation object. Figure 4 shows the dependence of the deflection angle on plasma and dark matter parameter λ for the fixed values of impact parameter and nonuniform plasma case. From Figs. 3 and

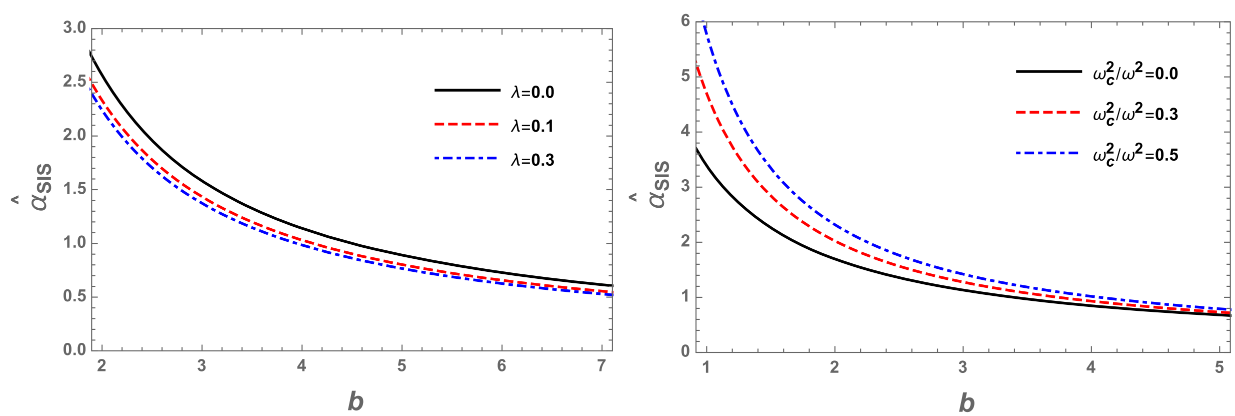

FIG. 3. The dependence of the deflection angle $\hat{\alpha}_{\rm SIS}$ on the impact parameter for the different values of PFDM parameter λ (left panel) and plasma medium (right panel).

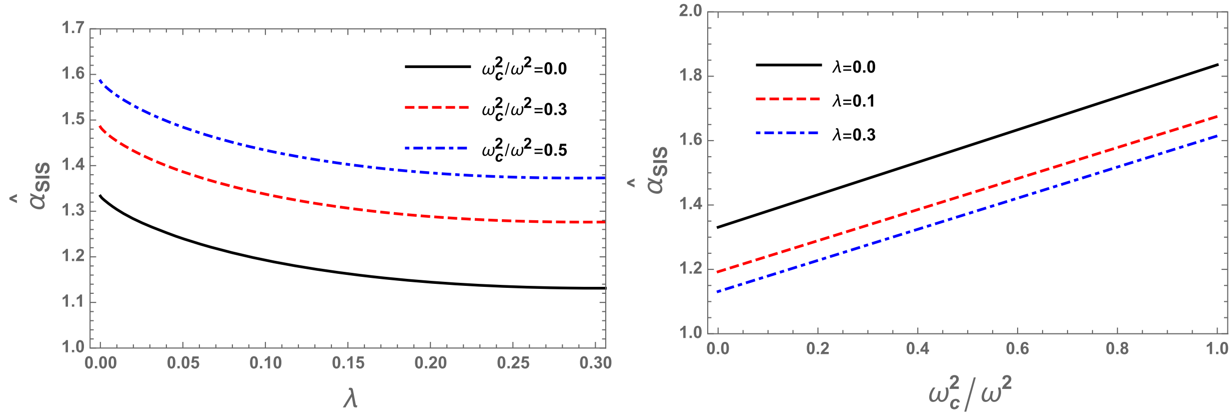

FIG. 4. The dependence of the deflection angle $\hat{\alpha}_{\mathrm{SIS}}$ on the perfect fluid dark matter parameter λ (left panel) and plasma medium (right panel) for the fixed value of impact parameter $b = 3M$.

4 one may see that the deflection angle is decreasing in the presence of PFDM parameter. Moreover, similarly to the case of uniform plasma case, due to the effect of non-uniform plasma medium the deflection angle increases.

Now we make comparison the deflection angle for the photons around Schwarzschild black hole in the presence of perfect fluid dark matter for the cases uniform and nonuniform plasma medium. Figure 5 shows the difference of the deflection angle for the cases of uniform and nonuniform plasma medium. From this dependence one may easily speculate that the deflection angle of photon beam around black hole increases. From Fig. 5 one can easily compare both cases: the deflection angle is greater in a homogeneous case than an inhomogeneous plasma medium for the fixed value of impact parameter. Moreover, one can also see that with an increase of the PFDM parameter for uniform and nonuniform cases, the orbit of the photon comes closer to the central compact object.

The decrease of the deflection angle with the increase of PFDM parameter for the fixed values of impact parameter can be interpreted as the negative effect of dark matter. Since the dark matter is distributed in the space around black hole the gravitational effect of PFDM on light rays from the outside of the photon trajectory has opposite effect with respect to the effect caused by central object. Thus gravity of the central object increases the deflection while PFDM causes to decrease the deflection angle of light rays.

III. MAGNIFICATION OF IMAGE

In this section, we study in detail the magnification of the image source in the presence of plasma using the lens equation of the form [16,26,84,85]

$$\theta D_s = \beta D_s + \hat{\alpha}_b D_{ds}, \tag{19}$$

where D_s and D_{ds} are the distances from the distant source to the observer and the lens object, respectively (see Fig. 6). $\hat{\alpha}_b$ in Eq. (19) is the deflection angle, θ and β are the angle of apparent source image and the source itself with respect to the observer-lens axis, respectively. The relation between the impact parameter b and the angle θ has the form $b = D_d\theta$, where D_d is distance from the observer to the lens object. Using these notations one can now rewrite Eq. (19) in the following form [16,19]

$$\beta = \theta - \frac{D_{ds}}{D_s}\frac{F(\theta)}{D_d}\frac{1}{\theta}, \tag{20}$$

where

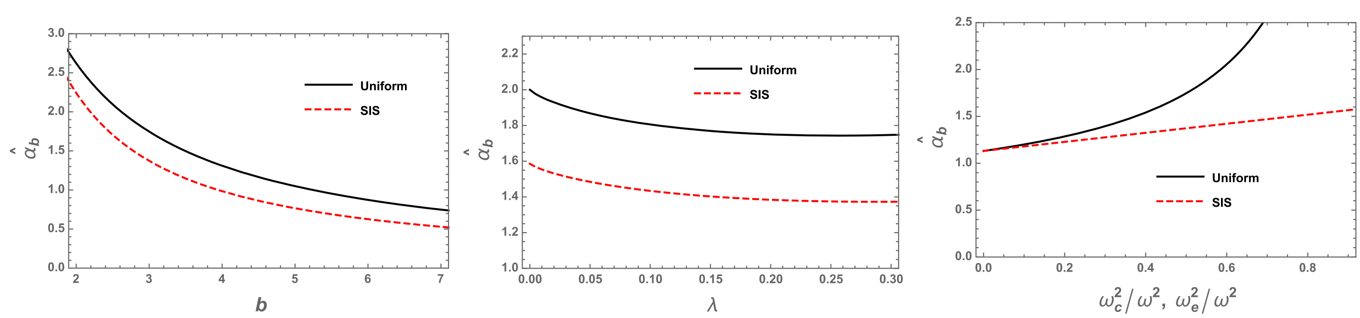

FIG. 5. The dependence of the deflection angle $\hat{\alpha}_b$ on the impact parameter (left panel), the PFDM parameter λ (middle panel) and the plasma parameters (right panel).

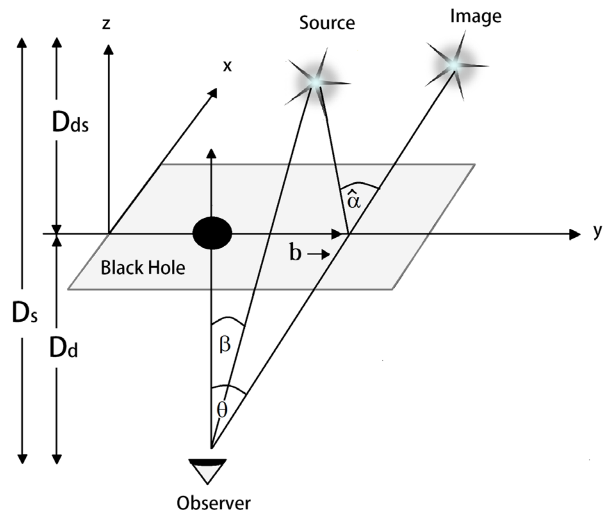

FIG. 6. Schematic view of the gravitational lensing system (adopted from Ref. [27]).

$$F(\theta) = |\alpha_b| b = |\alpha_b(\theta)| D_d \theta.$$

When the lens, observer and source lie on the single line, β vanishes. The corresponding solution of the lens equation (20) in this case is called the Einstein angle θ_E, and the image of the source will take the form so-called Einstein ring. The radius of the Einstein's ring is $R_E = D_l \theta_E$. The corresponding Einstein's angle for Schwarzschild compact object has the form

$$\theta_{\mathrm{E}} = \sqrt{2R_s \frac{D_{ds}}{D_s D_d}}. \tag{21}$$

Now we consider the magnification of the image brightness of source which has the direct relevance to the observation and lead to obtain more information about both source and lens objects. The basic equation for the calculating the magnification of image source has the following form [16]

$$\mu_\Sigma = \frac{I_{\mathrm{tot}}}{I_*} = \sum_k \left| \left(\frac{\theta_k}{\beta}\right) \left(\frac{d\theta_k}{d\beta}\right) \right|, \quad k = 1, 2, \ldots, s, \tag{22}$$

here s denotes the total number of images of the source, I_{tot} corresponds to the value of the total increased brightness due to multiple images of the source with the brightness I_*.

Below we present the analysis of the effects of plasma (both homogeneous and inhomogeneous) on magnification of image source in the presence of PFDM.

A. Effect of uniform plasma

Here we will analyze the expression for the Einstein angle (θ_0^{pl}) in the presence of plasma with uniform density. From Eqs. (13) and (20) one may easily get the following expressions for the Einstein's angle

$$(\theta_E^{\mathrm{pl}})_{\mathrm{uni}} = \theta_{\mathrm{E}} \left\{ \left[\frac{1}{2} - \frac{\lambda \ln(1/2\lambda)}{2R_s}\right] \left(1 + \frac{1}{(1 - \omega_e^2/\omega^2)}\right) - \frac{\lambda}{2R_s} \right\}^{1/2}. \tag{23}$$

Using Eq. (22) one can now easily obtain the expression for the magnification of the image source

$$\mu_{\mathrm{tot}}^{\mathrm{pl}} = \mu_+^{\mathrm{pl}} + \mu_-^{\mathrm{pl}} = \frac{x_{\mathrm{uni}}^2 + 2}{x_{\mathrm{uni}} \sqrt{x_{\mathrm{uni}}^2 + 4}}, \tag{24}$$

with

$$x_{\mathrm{uni}} = \frac{\beta}{(\theta_E^{\mathrm{pl}})_{\mathrm{uni}}} = x_0 \left\{ \left(\frac{1}{2} - \frac{\lambda \ln(1/2\lambda)}{2R_s}\right) \left(1 + \frac{1}{(1 - \omega_e^2/\omega^2)}\right) - \frac{\lambda}{2R_s} \right\}^{-\frac{1}{2}}, \tag{25}$$

where the notation $x_0 = \beta/\theta_0$ has been introduced. Finally, the magnifications of the image source read as

$$(\mu_+^{\mathrm{pl}})_{\mathrm{uni}} = \frac{1}{4} \left(\frac{x_{\mathrm{uni}}}{\sqrt{x_{\mathrm{uni}}^2 + 4}} + \frac{\sqrt{x_{\mathrm{uni}}^2 + 4}}{x_{\mathrm{uni}}} + 2 \right), \tag{26}$$

$$(\mu_-^{\mathrm{pl}})_{\mathrm{uni}} = \frac{1}{4} \left(\frac{x_{\mathrm{uni}}}{\sqrt{x_{\mathrm{uni}}^2 + 4}} + \frac{\sqrt{x_{\mathrm{uni}}^2 + 4}}{x_{\mathrm{uni}}} - 2 \right). \tag{27}$$

Figure 7 represents the dependence of the total magnification on the plasma parameter for the different values of the PFDM parameter. From this dependence one may see that total magnification of image decreases due to the effect of PFDM parameter. On the other hand, total magnification increases with the increase of plasma parameter.

B. Effect of nonuniform plasma on magnification's image

Now we turn to analysis of the total magnification of image source in the presence of nonuniform plasma medium around black hole surrounded by PFDM. The expression for the Einstein's ring takes the form

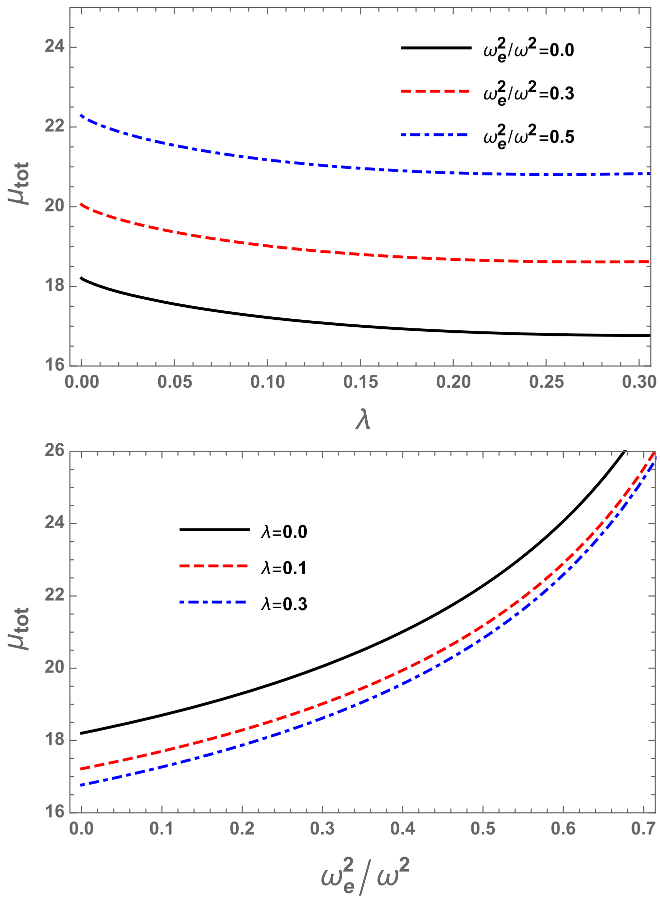

FIG. 7. The dependence of the total magnification μ_{tot} on the PFDM parameter for the different values of the plasma frequency corresponding to the fixed value of impact parameter $b = 3M$.

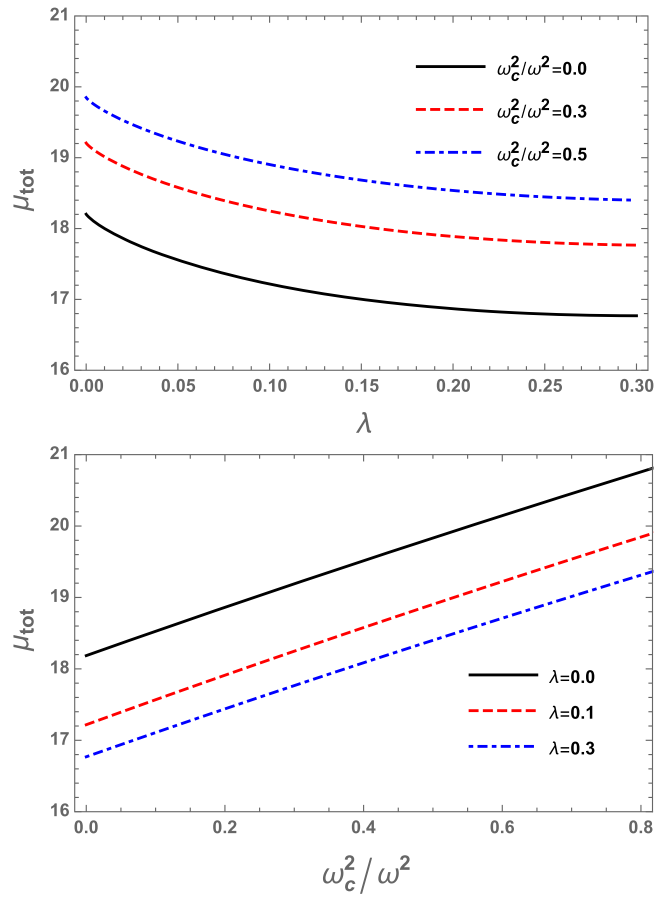

FIG. 8. The total magnification of the images as a function of the PFDM parameter. For plasma density distributed we have used SIS model with the frequency $\omega_c^2/\omega^2 = 0.5$ at $b = 3M$.

$$(\theta_E^{\text{pl}})_{\text{SIS}} = \theta_{\text{E}}\left\{1 - \frac{\lambda}{2R_s} - \frac{\lambda}{R_s}\ln(1/2\lambda) + \frac{R_s\omega_c^2}{4b\omega^2} - \frac{\lambda R_s\omega_c^2}{2b\pi\omega^2} + \frac{R_s^2\omega_c^2}{2b\pi\omega^2}\right\}^{\frac{1}{2}}. \tag{28}$$

Equation (22) allows us to express the total magnification of the image source in the following form

$$(\mu_{\text{tot}}^{\text{pl}})_{\text{SIS}} = (\mu_{+}^{\text{pl}})_{\text{SIS}} + (\mu_{-}^{\text{pl}})_{\text{SIS}} = \frac{x_{\text{SIS}}^2 + 2}{x_{\text{SIS}}\sqrt{x_{\text{SIS}}^2 + 4}}, \tag{29}$$

where

$$(\mu_{+}^{\text{pl}})_{\text{SIS}} = \frac{1}{4}\left(\frac{x_{\text{SIS}}}{\sqrt{x_{\text{SIS}}^2 + 4}} + \frac{\sqrt{x_{\text{SIS}}^2 + 4}}{x_{\text{SIS}}} + 2\right), \tag{30}$$

$$(\mu_{-}^{\text{pl}})_{\text{SIS}} = \frac{1}{4}\left(\frac{x_{\text{SIS}}}{\sqrt{x_{\text{SIS}}^2 + 4}} + \frac{\sqrt{x_{\text{SIS}}^2 + 4}}{x_{\text{SIS}}} - 2\right), \tag{31}$$

$$x_{\text{SIS}} = \frac{\beta}{(\theta_E^{\text{pl}})_{\text{SIS}}} = x_0\left\{1 - \frac{\lambda}{2R_s} - \frac{\lambda}{R_s}\ln(1/2\lambda) + \frac{R_s\omega_c^2}{4b\omega^2} - \frac{\lambda R_s\omega_c^2}{2b\pi\omega^2} + \frac{R_s^2\omega_c^2}{2b\pi\omega^2}\right\}^{-\frac{1}{2}}. \tag{32}$$

Figure 8 shows the dependence of the total magnification of images of the source from PDFM and plasma parameters. From the dependence one may see that total magnification decreases due to the effect of PFDM parameter. Moreover, the total magnification increases with the increase of the parameter of nonuniform plasma.

IV. SHADOW OF BLACK HOLE IN PLASMA

In this section we briefly explore the shadow of the black hole in the presence of uniform plasma and PDFM. In order to consider the shadow of the black hole we will consider the Hamilton-Jacobi equation for photons in the presence of the plasma [5,86]

$$\frac{1}{2}\left[g^{\mu\nu}\frac{\partial S}{\partial x^{\mu}}\frac{\partial S}{\partial x^{\nu}}-(n^2-1)\left(\frac{p_t}{\sqrt{-g_{tt}}}\right)^2\right]=0, \quad (33)$$

where S is the action for photon and refractive index of the plasma n is defined as

$$n^2=1-\frac{\omega_e^2}{\omega^2(x^i)}, \qquad \omega_e^2=\frac{4\pi e^2 N}{m_e}=K_e N, \quad (34)$$

where m_e is the electron mass and e is the electron charge. The action for photons can be expressed as

$$S=-Et+L\phi+S_r(r)+S_\theta(\theta), \quad (35)$$

where E is the energy of the photon, L is the axial angular momentum of the photon and $S_r(r)$ and $S_\theta(\theta)$ are the functions of r and θ, respectively.

After the separation of the variables one may easily obtain the following set of equations of motion for the photon around compact object surrounded by plasma and PFDM in the following form

$$\frac{dt}{d\sigma}=\frac{n^2E}{f(r)}, \quad (36)$$

$$\frac{dr}{d\sigma}=\pm\sqrt{R(r)}, \quad (37)$$

$$\frac{d\theta}{d\sigma}=\pm\frac{\sqrt{\mathcal{K}-\frac{L^2}{\sin^2\theta}}}{r^2}, \quad (38)$$

$$\frac{d\phi}{d\sigma}=\frac{L}{r^2\sin^2\theta}, \quad (39)$$

with $R(r)=[n^2E^2-\mathcal{K}f(r)/r^2]$, and $\mathcal{K}$ is so-called Carter constant [87]. One may introduce the following dimensionless impact parameters $\xi=L/E$ and $\eta=Q/E^2$. In order to describe the boundary of the unstable photon circular orbits one has to solve the equations

$$R(r)=0=dR(r)/dr. \quad (40)$$

The dimensionless impact parameter η reads

$$\eta=\frac{n^2r^2}{f(r)}. \quad (41)$$

Using the Hamiltonian-Jacobi equation (33) for photons, radial equation of motion (37), and condition (40) one may now easily find the equation determining the unstable photon circular orbits in the form

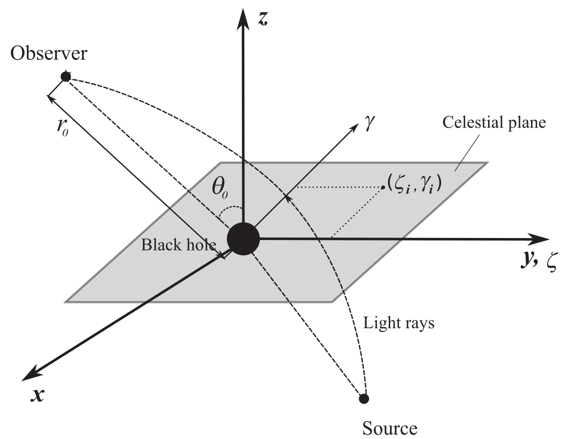

FIG. 9. Schematic view of celestial coordinate (adopted from Ref. [46]).

$$2nf(r)+2n'rf(r)-nrf'(r)=0. \quad (42)$$

The shadow of the black hole are defined in the celestial coordinates defined as (see Fig. 9) [41,46]

$$\zeta=\lim_{r_0\to\infty}\left(-r_0^2\sin\theta_0\frac{d\phi}{dr}\right), \quad (43)$$

$$\gamma=\lim_{r_0\to\infty}\left(r_0^2\frac{d\theta}{dr}\right), \quad (44)$$

where r_0 and θ_0 are the radial position of the black hole with respect to the observer and the inclination angle between the normal of observer's sky plane and observer-lens axis, respectively.

Finally, one can easily express the celestial coordinates in terms of ξ and η and obtain the parametric equations of the border of shaded area

$$\zeta=-\frac{\xi}{n\sin\theta_0}, \quad (45)$$

$$\gamma=\frac{1}{n}\sqrt{\eta-\frac{\xi^2}{\sin^2\theta_0}}. \quad (46)$$

Now using Eqs. (45) and (46) one can see that $\zeta^2+\gamma^2=\eta/n^2$. Consequently the apparent shape of the black hole shadow in the presence of PFDM and plasma is a circle with the radius $\sqrt{\eta}/n$. Figure 10 shows the shape of black hole's shadow in the presence of PFDM parameter for the fixed plasma parameter. From the dependence one may speculate that the presence of the PFDM parameter decreases the size of the shadow of black hole in the both vacuum and plasma medium cases.

The tendency of the shadow size decrease with the increase of PFDM parameter can be explained similarly as

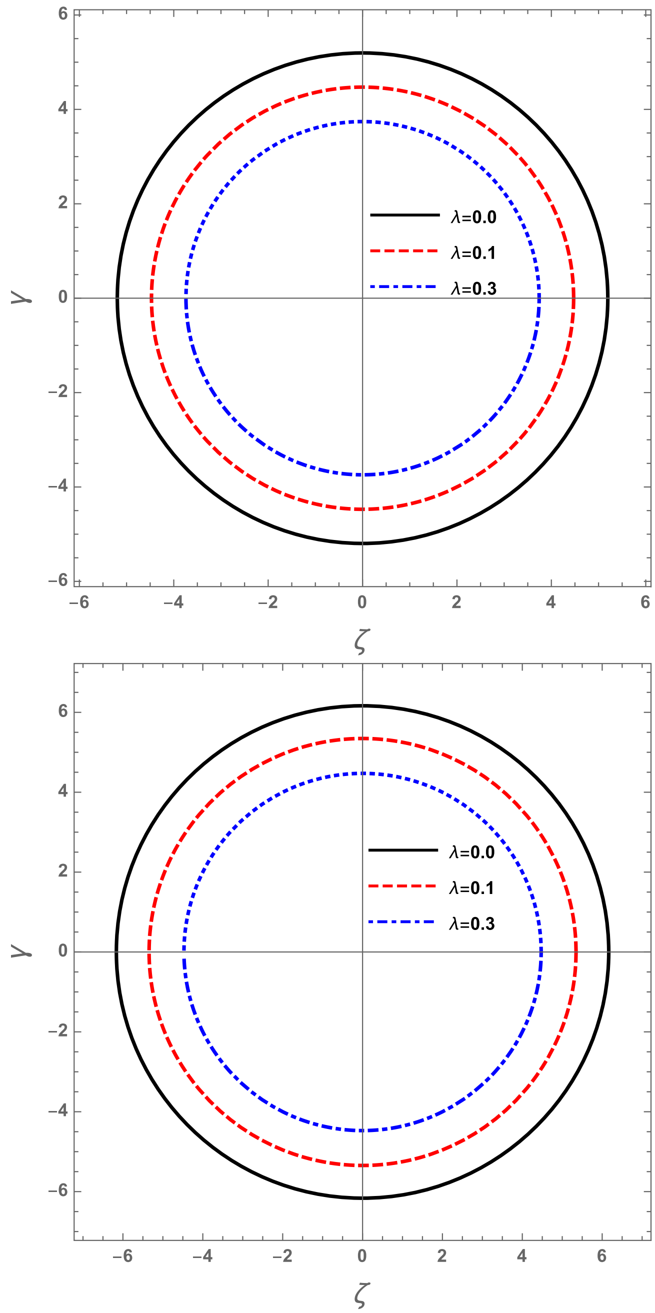

FIG. 10. The shadow of black hole immersed in perfect fluid dark matter: top panel is for the vacuum and bottom panel is for the plasma medium with $\omega_e^2/\omega^2 = 0.5$.

it has been done previously for photon deflection. The PFDM around central object decreases the gravitational effect of the central object on photons and one may observe the shrinking the size of the shadow in the presence of dark matter. The negative effect of PFDM on photon orbits may be useful to get constraints using current and future observation of the black hole shadow by event horizon telescope.

V. CONCLUSIONS

In the present work, the plasma effect on weak gravitational lensing has been studied for the Schwarzschild black hole in the presence of perfect fluid dark matter. Observable parameter such as the deflection angle is important for the gravitational lensing in general relativity. Here we have explored two cases: uniform nonuniform distributed plasma. The main results of the paper can be summarized as follows:

(i) The analysis showed that the deflection angle decreases with the increase of the PFDM parameter. At the same time the presence of plasma medium increase the gravitational deflection angle of photons.
(ii) We have obtained that the increase of parameter of nonuniform plasma medium causes the increase of deflection angle.
(iii) The comparison of the deflection angle for the photons around black hole in the presence of perfect fluid dark matter in the cases of uniform and nonuniform distributed plasma showed that the deflection angle is greater in a homogeneous case with compare to one in inhomogeneous plasma medium.
(iv) We have analyzed the total magnification of the image source due to weak gravitational lensing for the different values of the PFDM parameter. It has been shown that total magnification of image decreases due to the effect of PFDM parameter.
(v) On the other hand, total magnification increases with the increase of both uniform and nonuniform plasma parameter.
(vi) We have explored the shadow of the nonrotating black hole in PFDM in the presence of plasma. It has been shown that the presence of the PFDM parameter decreases the size of the shadow of black hole in the both vacuum and plasma medium cases.

ACKNOWLEDGMENTS

We would like to thank the referee for useful comments. F. A. acknowledges the support of INHA University in Tashkent. This research is supported by the Uzbekistan Ministry for Innovative Development, Grant No. MRB-AN-2019-29. A. A. is supported through PIFI fund of the Chinese Academy of Sciences. W. B. H. is supported by CAS Project for Young Scientists in Basic Research YSBR-006 and NSFC (National Natural Science Foundation of China) Grant No. 11773059.

[1] J. L. Synge, *Relativity: The General Theory* (North-Holland, Amsterdam, 1960).
[2] P. Schneider, J. Ehlers, and E. E. Falco, *Gravitational Lenses, XIV* (Springer-Verlag, Berlin, Heidelberg; New York, 1999), p. 560. Also Astronomy and Astrophysics Library (1999).
[3] V. Perlick, *Ray Optics, Fermat's Principle, and Applications to General Relativity*, edited by V. Perlick, Lecture Notes in Physics, Monographs Series Vol. 61 (Springer-Verlag, Berlin Heidelberg New York, 2000).
[4] V. Perlick, Living Rev. Relativity **7**, 9 (2004).
[5] A. Rogers, Mon. Not. R. Astron. Soc. **451**, 17 (2015).
[6] A. Rogers, Universe **3**, 3 (2017).
[7] X. Er and A. Rogers, Mon. Not. R. Astron. Soc. **475**, 867 (2018).
[8] A. Rogers, Mon. Not. R. Astron. Soc. **465**, 2151 (2017).
[9] A. Broderick and R. Blandford, Mon. Not. R. Astron. Soc. **342**, 1280 (2003).
[10] J. Bicak and P. Hadrava, Astron. Astrophys. **44**, 389 (1975).
[11] S. Kichenassamy and R. A. Krikorian, Phys. Rev. D **32**, 1866 (1985).
[12] V. Perlick and O. Y. Tsupko, Phys. Rev. D **95**, 104003 (2017).
[13] V. Perlick, O. Y. Tsupko, and G. S. Bisnovatyi-Kogan, Phys. Rev. D **92**, 104031 (2015).
[14] A. Abdujabbarov, B. Toshmatov, J. Schee, Z. Stuchlík, and B. Ahmedov, Int. J. Mod. Phys. D **26**, 1741011 (2017).
[15] E. F. Eiroa and C. M. Sendra, Phys. Rev. D **86**, 083009 (2012).
[16] G. S. Bisnovatyi-Kogan and O. Y. Tsupko, Mon. Not. R. Astron. Soc. **404**, 1790 (2010).
[17] G. Zaman Babar, F. Atamurotov, S. Ul Islam, and S. G. Ghosh, Phys. Rev. D **103**, 084057 (2021); F. Atamurotov, S. Shaymatov, and B. Ahmedov, Galaxies **9**, 54 (2021).
[18] O. Y. Tsupko and G. S. Bisnovatyi-Kogan, Gravitation Cosmol. **18**, 117 (2012).
[19] V. S. Morozova, B. J. Ahmedov, and A. A. Tursunov, Astrophys. Space Sci. **346**, 513 (2013).
[20] O. Y. Tsupko and G. S. Bisnovatyi-Kogan, Gravitation Cosmol. **20**, 220 (2014).
[21] G. Bisnovatyi-Kogan and O. Tsupko, Universe **3**, 57 (2017).
[22] A. Hakimov and F. Atamurotov, Astrophys. Space Sci. **361**, 112 (2016).
[23] B. Turimov, B. Ahmedov, A. Abdujabbarov, and C. Bambi, Int. J. Mod. Phys. D **28**, 2040013 (2019).
[24] C. A. Benavides, A. Cárdenas-Avendaño, and A. Larranaga, Int. J. Theor. Phys. **55**, 2219 (2016).
[25] G. V. Kraniotis, Gen. Relativ. Gravit. **46**, 1818 (2014).
[26] F. Atamurotov, A. Abdujabbarov, and J. Rayimbaev, Eur. Phys. J. C **81**, 118 (2021).
[27] G. Z. Babar, F. Atamurotov, and A. Z. Babar, Phys. Dark Universe **32**, 100798 (2021).
[28] G. Z. Babar, F. Atamurotov, S. Ul Islam, and S. G. Ghosh, Phys. Rev. D **103**, 084057 (2021).
[29] F. Atamurotov, S. Shaymatov, P. Sheoran, and S. Siwach, J. Cosmol. Astropart. Phys. 08 (2021) 045.
[30] K. S. Virbhadra and G. F. R. Ellis, Phys. Rev. D **62**, 084003 (2000).
[31] S. Rahvar and J. W. Moffat, Mon. Not. R. Astron. Soc. **482**, 4514 (2019).
[32] V. Bozza, Gen. Relativ. Gravit. **42**, 2269 (2010).
[33] V. Bozza, S. Capozziello, G. Iovane, and G. Scarpetta, Gen. Relativ. Gravit. **33**, 1535 (2001).
[34] K. S. Virbhadra, Phys. Rev. D **79**, 083004 (2009).
[35] S. U. Islam, R. Kumar, and S. G. Ghosh, J. Cosmol. Astropart. Phys. 09 (2020) 030.
[36] S. Chakraborty and S. SenGupta, J. Cosmol. Astropart. Phys. 07 (2017) 045.
[37] G. Zaman Babar, F. Atamurotov, and A. Zaman Babar, arXiv:2104.01340.
[38] K. Akiyama *et al.* (Event Horizon Telescope Collaboration), Astrophys. J. **875**, L1 (2019).
[39] K. Akiyama *et al.* (Event Horizon Telescope Collaboration), Astrophys. J. **875**, L6 (2019).
[40] R. Takahashi, Publ. Astron. Soc. Jpn. **57**, 273 (2005).
[41] K. Hioki and K.-I. Maeda, Phys. Rev. D **80**, 024042 (2009).
[42] L. Amarilla, E. F. Eiroa, and G. Giribet, Phys. Rev. D **81**, 124045 (2010).
[43] L. Amarilla and E. F. Eiroa, Phys. Rev. D **85**, 064019 (2012).
[44] L. Amarilla and E. F. Eiroa, Phys. Rev. D **87**, 044057 (2013).
[45] A. Abdujabbarov, F. Atamurotov, Y. Kucukakca, B. Ahmedov, and U. Camci, Astrophys. Space Sci. **344**, 429 (2013).
[46] F. Atamurotov, A. Abdujabbarov, and B. Ahmedov, Astrophys. Space Sci. **348**, 179 (2013).
[47] S.-W. Wei and Y.-X. Liu, J. Cosmol. Astropart. Phys. 11 (2013) 063.
[48] F. Atamurotov, A. Abdujabbarov, and B. Ahmedov, Phys. Rev. D **88**, 064004 (2013).
[49] M. Ghasemi-Nodehi, Z. Li, and C. Bambi, Eur. Phys. J. C **75**, 315 (2015).
[50] P. V. P. Cunha, C. A. R. Herdeiro, E. Radu, and H. F. Rúnarsson, Phys. Rev. Lett. **115**, 211102 (2015).
[51] A. A. Abdujabbarov, L. Rezzolla, and B. J. Ahmedov, Mon. Not. R. Astron. Soc. **454**, 2423 (2015).
[52] F. Atamurotov, B. Ahmedov, and A. Abdujabbarov, Phys. Rev. D **92**, 084005 (2015).
[53] T. Ohgami and N. Sakai, Phys. Rev. D **91**, 124020 (2015).
[54] A. Grenzebach, V. Perlick, and C. Lämmerzahl, Int. J. Mod. Phys. D **24**, 1542024 (2015).
[55] J. R. Mureika and G. U. Varieschi, Can. J. Phys. **95**, 1299 (2017).
[56] A. Abdujabbarov, B. Toshmatov, Z. Stuchlík, and B. Ahmedov, Int. J. Mod. Phys. D **26**, 1750051 (2017).
[57] A. Abdujabbarov, B. Juraev, B. Ahmedov, and Z. Stuchlík, Astrophys. Space Sci. **361**, 226 (2016).
[58] A. Abdujabbarov, M. Amir, B. Ahmedov, and S. G. Ghosh, Phys. Rev. D **93**, 104004 (2016).
[59] Y. Mizuno, Z. Younsi, C. M. Fromm, O. Porth, M. De Laurentis, H. Olivares, H. Falcke, M. Kramer, and L. Rezzolla, Nat. Astron. **2**, 585 (2018).
[60] J. Schee and Z. Stuchlík, J. Cosmol. Astropart. Phys. 06 (2015) 048.
[61] J. Schee and Z. Stuchlík, Gen. Relativ. Gravit. **41**, 1795 (2009).
[62] Z. Stuchlík and J. Schee, Classical Quantum Gravity **31**, 195013 (2014).

[63] J. Schee and Z. Stuchlík, Int. J. Mod. Phys. D **18**, 983 (2009).
[64] Z. Stuchlík and J. Schee, Classical Quantum Gravity **27**, 215017 (2010).
[65] G. Z. Babar, A. Z. Babar, and F. Atamurotov, Eur. Phys. J. C **80**, 761 (2020).
[66] F. Atamurotov, S. G. Ghosh, and B. Ahmedov, Eur. Phys. J. C **76**, 273 (2016).
[67] A. Chowdhuri and A. Bhattacharyya, arXiv:2012.12914.
[68] F. Atamurotov and U. Papnoi, arXiv:2104.14898.
[69] Planck Collaboration, Astron. Astrophys. **594**, A13 (2016).
[70] V. V. Kiselev, Classical Quantum Gravity **20**, 1187 (2003).
[71] B. Toshmatov, Z. Stuchlík, and B. Ahmedov, Eur. Phys. J. Plus **132**, 98 (2017).
[72] S. Shaymatov, B. Ahmedov, Z. Stuchlík, and A. Abdujabbarov, Int. J. Mod. Phys. D **27**, 1850088 (2018).
[73] H. Chakrabarty, A. Abdujabbarov, and C. Bambi, Eur. Phys. J. C **79**, 179 (2019).
[74] C. A. Benavides-Gallego, A. Abdujabbarov, and C. Bambi, Phys. Rev. D **101**, 044038 (2020).
[75] S. G. Ghosh, S. D. Maharaj, D. Baboolal, and T.-H. Lee, Eur. Phys. J. C **78**, 90 (2018).
[76] Y. Zhang and Y. X. Gui, Classical Quantum Gravity **23**, 6141 (2006).
[77] S. G. Ghosh, Eur. Phys. J. C **76**, 222 (2016).
[78] S. H. Hendi, A. Nemati, K. Lin, and M. Jamil, Eur. Phys. J. C **80**, 296 (2020).
[79] M.-H. Li and K.-C. Yang, Phys. Rev. D **86**, 123015 (2012).
[80] S. Haroon, M. Jamil, K. Jusufi, K. Lin, and R. B. Mann, Phys. Rev. D **99**, 044015 (2019).
[81] X. Hou, Z. Xu, and J. Wang, J. Cosmol. Astropart. Phys. 12 (2018) 040.
[82] S. Shaymatov, B. Ahmedov, and M. Jamil, Eur. Phys. J. C **81**, 588 (2021).
[83] C. A. Benavides-Gallego, A. A. Abdujabbarov, and C. Bambi, Eur. Phys. J. C **78**, 694 (2018).
[84] A. F. Zakharov and Y. V. Baryshev, Int. J. Mod. Phys. D **11**, 1067 (2002).
[85] K. K. Nandi, Y.-Z. Zhang, and A. V. Zakharov, Phys. Rev. D **74**, 024020 (2006).
[86] J. L. Synge, Mon. Not. R. Astron. Soc. **131**, 463 (1966).
[87] B. Carter, Phys. Rev. **174**, 1559 (1968).