

Gravitational particle production of superheavy massive particles in quintessential inflation: A numerical analysis

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We compute numerically the reheating temperature due to the gravitational production of conformally-coupled superheavy particles during the phase transition from the end of inflation to the beginning of kination in two different quintessential inflation (QI) scenarios, namely Lorentzian quintessential inflation and α -attractors in the context of quintessential inflation (α -QI). Once these superheavy particles have been created, they must decay into lighter ones to form a relativistic plasma, whose energy density will eventually dominate the one of the inflaton field in order to reheat our Universe after its inflation period with a very high temperature—in both cases greater than 10^7 GeV—contrary to the usual belief that heavy masses suppress the particle production and, thus, lead to an inefficient reheating temperature. Finally, we will show that the overproduction of gravitational waves during this phase transition, when one deals with our models, does not disturb the big bang nucleosynthesis success.

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I. INTRODUCTION

Today, the inflationary paradigm is the most accepted implementation to the big bang (BB) theory in order to solve a number of shortcomings associated with the standard BB cosmology, such as the horizon, the flatness or the primordial monopole problems [1,2], and to explain correctly the early Universe at background level. More remarkable is the fact that inflation is also able to explain the origin of inhomogeneities in the Universe as quantum fluctuations [3–7], leading to theoretical results that match well with the recent observational data provided by the Planck team [8].

Once having a viable theory explaining the early Universe, one can extend it in order to deal with the whole evolution of the Universe, thus, unifying its early- and late-time accelerated expansions (see [9–11] for a review of the current dark energy models). One of the most attractive scenarios able to do it is the so-called quintessential inflation (QI), introduced for the first time by Peebles and Vilenkin in their seminal paper [12] (see [13] for a review of the Peebles-Vilenkin model), where the idea behind their proposal comes through the introduction of a single scalar field, also named inflaton, that at early times is the responsible for inflation while at late times it allows the current cosmic acceleration via quintessence.

Due to the simplicity of this proposal and since the behavior of the slow-roll regime is the one of an attractor, the dynamics of the model are simply obtained with the initial value of the scalar field and its derivative at some moment during this regime. Therefore, the models of QI [14–16], which generally only depend on two parameters, caught the attention of some researchers who wanted to confront QI with the observational data [17–35], thus becoming a popular topic in some reduced circles.

By dealing with QI in this way, it is well known that all the scenarios containing a period of inflation need a reheating mechanism to match the hot BB Universe [1] because the particles existing before the beginning of this period were completely diluted at the end of inflation resulting in a very cold universe. Here, we will choose as a reheating mechanism the so-called *gravitational particle production* [36–43] of superheavy particles conformally coupled with gravity, which was applied to standard inflation (potentials with a deep well) in [44–48]. However, the gravitational reheating in QI is normally applied to very light fields [12,17,49,50] and only in a few papers, which deal with toy discontinuous models as the Peebles-Vilenkin one, is it applied to massive particles [22,24,26,51,52] (see also [53] where the authors also deals with the gravitational production of dark matter). In fact, regarding smooth QI potentials, particle creation and, in particular, gravitational particle production are sometimes associated to an overbarrier problem (see for instance Sec. VII of [54]), which has to be analytically studied

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using the complex Wentzel-Kramers-Brillouin approximation, that is, the Stokes phenomenon [47], whose real application is limited to the creation of particles by parabolic potentials [54].

For this reason, in the present work we deal numerically with smooth QI potentials such as the ones provided by Lorentzian quintessential inflation (LQI) [55–57] and α -attractors in quintessential inflation (α -QI) [58]. Then, in order to get the reheating temperature, we will calculate numerically the energy density of the superheavy particles produced during the phase transition from the end of inflation to the beginning of kination (the period where all the energy of the field becomes kinetic [59]). To do this, we will use the well-known Hamiltonian diagonalization method (see [60] for a review), showing that the time-dependent β -Bogoliubov coefficient encodes the polarization effects associated to the creation and annihilation of the so-called quasiparticles [60] and also the real particles created during the phase transition. However, through a toy model inspired in the Peebles-Vilenkin one, and presenting a discontinuity of the first derivative of the potential at the beginning of kination, we will show that these polarization effects disappear when the Universe evolves adiabatically, which happens soon after the beginning of kination. Thus, in order to calculate the energy density of the produced particles (the real particles), one can safely use the value of the β -Bogoliubov coefficient after the beginning of kination. This is the key point of our investigation, and we have numerically checked that this also happens for our QI smooth potentials.

Another important point is the overproduction of gravitational waves (GWs) in QI, which are also produced during the phase transition from the end of inflation to the beginning of kination. This overproduction in many QI models may disturb the success of the big bang nucleosynthesis (BBN), but, as we will show, the reheating via gravitational particle production of superheavy particles in the QI scenarios studied in this work prevents the incompatibilities of the BBN with the overproduction of GWs. The reason why this happens is because the gravitational production of superheavy particles (contrary to the standard belief that in analytic calculations only ultraviolet modes are taken into account) is very efficient for long-wavelength modes, leading to a high reheating temperature able to overcome all the constraints ensuring the BBN success.

Finally, a few words about the viable values of the reheating temperature are in order. A lower bound for the

reheating temperature comes from the fact that the radiation-dominated era occurs before the BBN epoch, which takes place in the 1 MeV regime [61], and thus, the reheating temperature should naturally be greater than 1 MeV. On the contrary, the upper bound of this temperature is dependent on the theory we are concerned with. In fact, in some supergravity theories such as α -attractors containing particles with only gravitational interactions, the late-time decay of these relics may jeopardize the success of the standard BBN [62]. To solve this problem one has to consider a sufficiently low reheating temperature (of the order of 10^9 GeV or less) [63].

The units used in the manuscript are $\hbar = c = 1$ and the reduced Planck's mass is denoted by $M_{pl} \equiv \frac{1}{\sqrt{8\pi G}} \cong 2.44 \times 10^{18}$ GeV.

II. THE DIAGONALIZATION METHOD

This short section is a review of our previous work [64] (see also the pioneering works [37–39,65] for a more detailed vision of the topic). The idea of the method goes as follows; given a quantum scalar field of superheavy particles conformally coupled to gravity, namely χ , the Klein-Gordon (KG) equation in the Fourier space, which is satisfied by the modes in the flat Friedmann-Lemaître-Robertson-Walker (FLRW) spacetime, is given by [66]

$$\chi_k''(\tau) + \omega_k^2(\tau)\chi_k(\tau) = 0, \quad (1)$$

where the prime denotes the derivative with respect to the conformal time τ , and $\omega_k(\tau) = \sqrt{k^2 + m_\chi^2 a^2(\tau)}$ is the time-dependent frequency being m_χ the mass of the quantum field χ .

As usual, the modes that define the vacuum state at a given initial time τ_i are the ones that minimize the energy density, so they must satisfy the conditions

$$\begin{aligned} \chi_k(\tau_i) &= \frac{1}{\sqrt{2\omega_k(\tau_i)}} e^{-i \int^{\tau_i} \omega_k(\tilde{\eta}) d\tilde{\eta}}, \\ \chi_k'(\tau_i) &= -i\omega_k(\tau_i)\chi_k(\tau_i), \end{aligned} \quad (2)$$

and thus, the vacuum expectation value of the energy density will be given by [67]

$$\langle \rho(\tau) \rangle \equiv \langle 0 | \hat{\rho}(\tau) | 0 \rangle = \frac{1}{4\pi^2 a^4(\tau)} \int_0^\infty k^2 dk (|\chi_k'(\tau)|^2 + \omega_k^2(\tau) |\chi_k(\tau)|^2 - \omega_k(\tau)), \quad (3)$$

where, in order to obtain a finite energy density [60], we have subtracted the energy density of the zero-point oscillations of the vacuum $\frac{1}{(2\pi)^3 a^4(\tau)} \int d^3 k \frac{1}{2} \omega_k(\tau)$.

Following the method developed in [65] (see also Sec. 9.2 of [60]), we will write the modes as follows:

$$\chi_k(\tau) = \alpha_k(\tau) \frac{e^{-i \int^\tau \omega_k(\bar{\tau}) d\bar{\tau}}}{\sqrt{2\omega_k(\tau)}} + \beta_k(\tau) \frac{e^{i \int^\tau \omega_k(\bar{\tau}) d\bar{\tau}}}{\sqrt{2\omega_k(\tau)}}, \quad (4)$$

where $\alpha_k(\tau)$ and $\beta_k(\tau)$ are the time-dependent Bogoliubov coefficients. Now, imposing that the modes satisfy the condition

$$\chi'_k(\tau) = -i\omega_k(\tau) \left(\alpha_k(\tau) \frac{e^{-i \int^\tau \omega_k(\bar{\tau}) d\bar{\tau}}}{\sqrt{2\omega_k(\tau)}} - \beta_k(\tau) \frac{e^{i \int^\tau \omega_k(\bar{\tau}) d\bar{\tau}}}{\sqrt{2\omega_k(\tau)}} \right), \quad (5)$$

one can show that the Bogoliubov coefficients must satisfy the system

$$\begin{cases} \alpha'_k(\tau) &= \frac{\omega'_k(\tau)}{2\omega_k(\tau)} e^{2i \int^\tau \omega_k(\bar{\tau}) d\bar{\tau}} \beta_k(\tau) \\ \beta'_k(\tau) &= \frac{\omega'_k(\tau)}{2\omega_k(\tau)} e^{-2i \int^\tau \omega_k(\bar{\tau}) d\bar{\tau}} \alpha_k(\tau), \end{cases} \quad (6)$$

in order for Eq. (4) to be a solution of Eq. (1).

Finally, inserting (4) into the expression for the vacuum energy density (3), and taking into account that the Bogoliubov coefficients satisfy the equation $|\alpha_k(\tau)|^2 - |\beta_k(\tau)|^2 = 1$, one finds that

$$\langle \rho(\tau) \rangle = \frac{1}{2\pi^2 a^4(\tau)} \int_0^\infty k^2 \omega_k(\tau) |\beta_k(\tau)|^2 dk, \quad (7)$$

where it is important to notice that $|\beta_k(\tau)|^2$ encodes the vacuum polarization effects and also the production of particles, which only happens when the adiabatic evolution breaks. In fact, the quantity

$$\langle N(\tau) \rangle = \frac{1}{2\pi^2 a^3(\tau)} \int_0^\infty k^2 |\beta_k(\tau)|^2 dk \quad (8)$$

was named, in the Russian literature, as the number density of quasiparticles [60], which, as we will see in next section dealing with a toy model, is very different from the number density of the produced particles because it also contains the vacuum polarization effects, that is, the creation and annihilation of pairs.

III. PARTICLE CREATION OF SUPERHEAVY PARTICLES CONFORMALLY COUPLED TO GRAVITY

This section is devoted to the numerical calculation of the energy density of the gravitationally produced particles in two different QI scenarios with smooth potentials, namely:

- (1) Lorentzian quintessential inflation;

Based on the well-known Lorentzian distribution, the authors of [55,56] considered following the ansatz

$$\epsilon(N) = \frac{\xi}{\pi} \frac{\Gamma/2}{N^2 + \Gamma^2/4}, \quad (9)$$

where ϵ is the main slow-roll parameter, N denotes the number of e -folds, ξ is the amplitude of the Lorentzian distribution, and Γ is its width. From this ansatz, one can find the exact corresponding potential of the scalar field, namely

$$V(\varphi) = \lambda M_{pl}^4 \exp \left[-\frac{2\xi}{\pi} \arctan(\sinh(\gamma\varphi/M_{pl})) \right] \cdot \left(1 - \frac{2\gamma^2 \xi^2}{3\pi^3} \frac{1}{\cosh(\gamma\varphi/M_{pl})} \right), \quad (10)$$

where λ is a dimensionless parameter and the parameter γ is defined by

$$\gamma \equiv \sqrt{\frac{\pi}{\Gamma\xi}}.$$

Here, to simplify the structure of the potential although without modifying its properties, we set $\xi = \gamma$ and disregard the term $(1 - \frac{2\gamma^2 \xi^2}{3\pi^3} \frac{1}{\cosh(\gamma\varphi/M_{pl})})$, which has no influence on the dynamics. Hence, we obtain the simplified version

$$V(\varphi) = \lambda M_{pl}^4 \exp \left[-\frac{2\gamma}{\pi} \arctan(\sinh(\gamma\varphi/M_{pl})) \right], \quad (11)$$

plotted in Fig. 1, and where in order to match with the current observational data one has to choose $\lambda \sim 10^{-69}$ and $\gamma \cong 122$ (see [57] for details).

Summing up, in this scenario kination starts when $H \sim H_{\text{kin}} \cong 4 \times 10^{-8} M_{pl}$ with $\varphi_{\text{kin}} \cong -0.03 M_{pl}$,

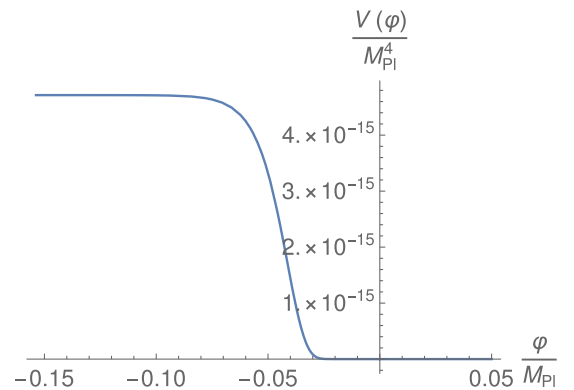


FIG. 1. Plot of the Lorentzian quintessential inflation potential.

inflation ends when $\varphi_{END} \cong -0.078M_{pl}$ and the pivot scale leaves the Hubble radius when $\varphi_* \cong -0.154M_{pl}$. Finally, at very late times the effective equation of state (EOS) parameter goes to -1 , which leads to an eternal acceleration.

(2) Exponential α -attractor in quintessential inflation;

The corresponding potential is obtained, combined with a standard exponential potential, from the following Lagrangian motivated by supergravity and corresponding to a nontrivial Kähler manifold (see for instance [27] and the references therein),

$$\mathcal{L} = \frac{1}{2} \frac{\dot{\phi}^2}{\left(1 - \frac{\phi^2}{6\alpha}\right)^2} M_{pl}^2 - \lambda M_{pl}^4 e^{-\kappa\phi}, \quad (12)$$

where ϕ is a dimensionless scalar field, and κ and λ are positive dimensionless constants.

In order that the kinetic term has the canonical form, one can redefine the scalar field as follows:

$$\phi = \sqrt{6\alpha} \tanh\left(\frac{\varphi}{\sqrt{6\alpha}M_{pl}}\right), \quad (13)$$

obtaining the following potential plotted in Fig. 2,

$$V(\varphi) = \lambda M_{pl}^4 e^{-n \tanh\left(\frac{\varphi}{\sqrt{6\alpha}M_{pl}}\right)}, \quad (14)$$

where we have introduced the notation $n \equiv \kappa\sqrt{6\alpha}$, and by taking $\alpha \sim 10^{-2}$ one has to choose $n \sim 10^2$ and $\lambda \sim 10^{-66}$ in order to match with the observational data (see [58] for details).

Finally, it is interesting to note that for this case kination starts later than in LQI; more precisely, when $\varphi_{kin} \cong -0.5M_{pl}$ with $H_{kin} \sim 4 \times 10^{-7}M_{pl}$. However, the end of inflation and the horizon crossing occurs earlier than in LQI, $\varphi_{END} \cong -0.89M_{pl}$ and $\varphi_* \cong -1.7M_{pl}$, respectively.

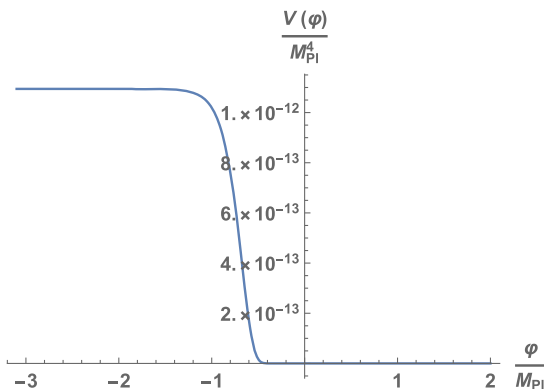


FIG. 2. Plot of the exponential α -attractor potential.

A. A toy model

Before dealing with our models, and in order to better understand the gravitational particle production, we warm up by reviewing a toy potential inspired in the Peebles-Vilenkin model [64],

$$V(\varphi) = \begin{cases} \frac{1}{2}m^2(\varphi^2 - M_{pl}^2 + M^2) & \text{for } \varphi \leq -M_{pl} \\ \frac{1}{2}m^2 \frac{M^6}{(\varphi + M_{pl})^4 + M^4} & \text{for } \varphi \geq -M_{pl}, \end{cases} \quad (15)$$

where $m \sim 5 \times 10^{-6}M_{pl}$ is the mass of the inflaton field and $M \cong 20$ GeV is a very small mass needed to match the theoretical results provided by the model with the current cosmic acceleration [13].

The toy model contains a discontinuity of the first derivative of the potential at the beginning of kination, i.e., when $\varphi = 0$. Then, according to the conservation equation, the second temporal derivative of the scalar field is discontinuous at the beginning of kination, as well as the second temporal derivative of the Hubble parameter. Consequently, the third derivative of the frequency $\omega_k(\tau)$ (which depends on the scale factor) is discontinuous at the beginning of kination, namely τ_{kin} , which is the moment when particles are gravitationally created because it is when the adiabatic evolution is broken.

On the other hand, before performing analytic calculations with this toy model, the following remark, which helps us to understand the more realistic ones, is needed; we have to make some assumptions to be sure that the polarization effects do not affect the evolution of the inflaton field during the slow-roll period, that is, during the slow-roll regime we will demand that the polarization effects will be subdominant and do not affect the dynamics of the inflaton field. Analytically, a condition that ensures that polarization effects are subdominant is $H/m_\chi \ll 1$, which implies that $\omega'_k/\omega_k^2 \ll 1$, i.e., an adiabatic evolution, and where once again m_χ is the mass of superheavy field χ . Taking into account that a classical picture of the Universe (quantum gravitational effects could be neglected) appears at grand unified theory (GUT) scales with $H_{GUT} \cong 5 \times 10^{-5}M_{pl} \cong 10^{14}$ GeV, we have to choose superheavy massive fields with a mass greater or equal than $m_\chi \sim 5 \times 10^{-4}M_{pl} \cong 10^{15}$ GeV, which is a mass of the same order as those of the vector mesons responsible for transforming quarks into leptons in simple theories with $SU(5)$ symmetry [68].

Therefore, once we have chosen the mass of the χ field, in order to obtain the value of the β -Bogoliubov coefficient we come back to Eq. (6) and, in the first approximation, we take $\alpha_k(\tau) = 1$, getting

$$\beta_k(\tau) = \int^\tau \frac{\omega'_k(\eta)}{2\omega_k(\eta)} e^{-2i \int^\eta \omega_k(\bar{\eta}) d\bar{\eta}} d\eta, \quad (16)$$

which, after integration by parts, yields before the beginning of kination

$$\beta_k(\tau) = \left(-\frac{\omega'_k(\tau)}{4i\omega_k^2(\tau)} + \frac{1}{8\omega_k(\tau)} \left(\frac{\omega'_k(\tau)}{\omega_k^2(\tau)} \right)' + \frac{1}{16i\omega_k(\tau)} \left(\frac{1}{\omega_k(\tau)} \left(\frac{\omega'_k(\tau)}{\omega_k^2(\tau)} \right)' \right)' + \dots \right) e^{-2i \int^\tau \omega_k(\bar{\eta}) d\bar{\eta}}. \quad (17)$$

However, after the beginning of kination the β -Bogoliubov coefficient must be given by

$$\beta_k(\tau) = \left(-\frac{\omega'_k(\tau)}{4i\omega_k^2(\tau)} + \frac{1}{8\omega_k(\tau)} \left(\frac{\omega'_k(\tau)}{\omega_k^2(\tau)} \right)' + \frac{1}{16i\omega_k(\tau)} \left(\frac{1}{\omega_k(\tau)} \left(\frac{\omega'_k(\tau)}{\omega_k^2(\tau)} \right)' \right)' + \dots \right) e^{-2i \int^\tau \omega_k(\bar{\eta}) d\bar{\eta}} + C, \quad (18)$$

where the constant C has to be chosen in order that the β -Bogoliubov coefficient becomes continuous at τ_{kin} because Eq. (6) is a first-order differential equation, and mathematically it is necessary to demand the solution to be continuous. Thus, after some cumbersome calculations (see [64] for details) one has

$$C = \left(\frac{m_\chi^2 m^3 a_{\text{kin}}^5}{16i\omega_k^5(\tau_{\text{kin}})} + \dots \right) e^{-2i \int^{\tau_{\text{kin}}} \omega_k(\bar{\eta}) d\bar{\eta}}, \quad (19)$$

where we have introduced the definition $a_{\text{kin}} \equiv a(\tau_{\text{kin}})$.

The terms of the β -Bogoliubov coefficient different from C lead to subleading geometric quantities in the energy density. Effectively, the term $-\frac{\omega'_k(\tau)}{4i\omega_k^2(\tau)}$ leads to the following contribution to the energy density, $\frac{m_\chi^2 H^2}{96\pi}$, which is negligible compared with $H^2 M_{pl}^2$. The same happens with $\frac{1}{8\omega_k(\tau)} (\omega'_k(\tau)/\omega_k^2(\tau))'$ leading to a term of order H^4 , which satisfies $H^4 \ll H^2 M_{pl}^2$. The product of the first and second term generates in the right-hand side of the modified semiclassical Friedmann equation a term of the order $H^3 m_\chi$, which is also subleading compared with $H^2 M_{pl}^2$. Finally, the third term of (18) leads in the right-hand side of the semiclassical Friedmann equation to the subleading term $\frac{H^6}{m_\chi^2}$.

Fortunately, this does not happen with C , whose leading term gives the main contribution of the vacuum-energy density due to the gravitational particle production. In fact, the time-dependent terms, which as we have already shown are always subleading, are vacuum polarization effects, and they rapidly disappear in the adiabatic regime, that is, soon after the beginning of kination $|\beta_k(\tau)|^2$ approaches $|C|^2$, obtaining

$$\langle \rho(\tau) \rangle \cong \begin{cases} 0 & \text{when } \tau < \tau_{\text{kin}} \\ 10^{-5} \left(\frac{m}{m_\chi} \right)^2 m^4 \left(\frac{a_{\text{kin}}}{a(\tau)} \right)^3 & \text{when } \tau \geq \tau_{\text{kin}}, \end{cases} \quad (20)$$

which at the beginning of kination is subdominant with respect to the energy density of the inflaton—but it will eventually dominate because the one of the inflatons decreases during kination as $a^{-6}(\tau)$.

Finally, in order to understand these results better it is useful to recall, that the authors of the diagonalization

method assume that during the whole evolution of the Universe, quanta named quasiparticles are created and annihilated due to the interaction of the quantum field with gravity [60], i.e., this is a vacuum-polarization effect where pairs are created and annihilated. And, following this interpretation, the number density of the created quasiparticles at time τ is given by $\langle N(\tau) \rangle = \frac{1}{2\pi^2 a^3(\tau)} \int_0^\infty k^2 |\beta_k(\tau)|^2 dk$. However, one has to be very careful with this interpretation and especially keep in mind that, as we have already pointed out in our toy model (15), real particles are only created when the adiabatic regime breaks. Effectively, before the beginning of kination, i.e., before the break of the adiabatic evolution, the main term of the β -Bogoliubov coefficient is given by $-\frac{\omega'_k(\tau)}{4i\omega_k^2(\tau)}$, whose contribution to the energy density is $\frac{m_\chi^2 H^2}{96\pi}$, and to the number density of the quasiparticles $\frac{m_\chi H^2}{512\pi}$, and thus, at any time τ before the beginning of kination $\langle \rho(\tau) \rangle \neq m_\chi \langle N(\tau) \rangle$, meaning that the quasiparticles do not evolve as real massive particles. On the contrary, during kination the polarization effects disappear and the leading term of $\langle N(\tau) \rangle$ is given by $10^{-5} \left(\frac{m}{m_\chi} \right)^3 m^3 \left(\frac{a_{\text{kin}}}{a(\tau)} \right)^3$, having $\langle \rho(\tau) \rangle = m_\chi \langle N(\tau) \rangle$ and a decay of $a^{-3}(\tau)$, which justifies the interpretation of massive particle production.

B. Gravitational particle creation in Lorentzian quintessential inflation

Now we are ready to calculate the particle production for our QI models. Coming back to the LQI potential (11), we have first of all integrated numerically the conservation equation for the inflaton field, namely

$$\ddot{\varphi} + 3H\dot{\varphi} + V_\varphi = 0, \quad (21)$$

where $H = \frac{1}{\sqrt{3}M_{pl}} \sqrt{\frac{\dot{\varphi}^2}{2} + V(\varphi)}$, with initial conditions at the horizon crossing, i.e., when the pivot scales leaves the Hubble radius. Recall that in that moment the system is in the slow-roll phase and (since this regime is an attractor) one only has to take initial conditions in the basin of attraction of the slow-roll solution, for example, $\varphi_* = -0.154M_{pl}$ and $\dot{\varphi}_* = -\frac{V_\varphi(\varphi_*)}{3H_*}$ (where the “star” denotes that the quantities are evaluated at the horizon crossing).

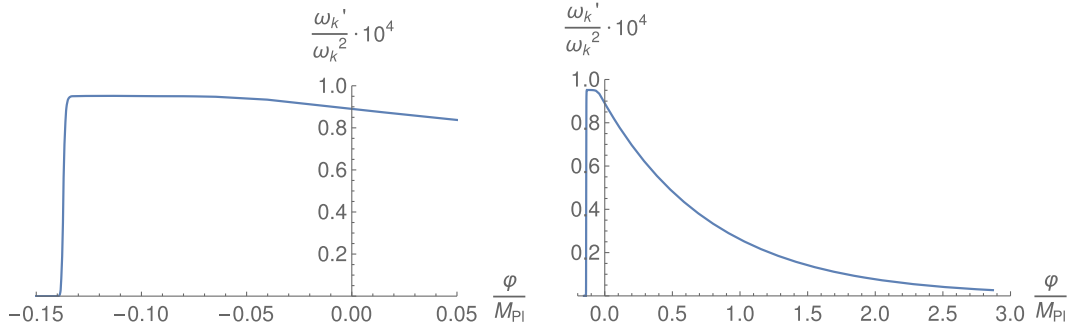


FIG. 3. Plot of the adiabatic evolution for a heavy field with mass $m_\chi \cong 10^{15}$ GeV, when the background is given by the Lorentzian quintessential inflation potential. The value $k = a_{\text{kin}} H_{\text{kin}}$ has been used for the quantities a_{kin} , H_{kin} of this model. On the left, with the same scale as the one used in Fig. 1 and on the right with a wider range of values of the scalar field in order to appreciate the extent of the nonadiabatic region.

Once we have obtained the evolution of the background, and in particular the evolution of the Hubble rate, we compute the evolution of the scale factor, which is given by

$$a(t) = a_* e^{\int_{t_*}^t H(s) ds}, \quad (22)$$

where we have chosen as the value of the scale factor at the horizon crossing $a_* = 1$.

From the evolution of the scale factor, we can see in Fig. 3 that a spike appears in the plot of the quantity ω'_k / ω_k^2 during the phase transition from the end of inflation to the beginning of kination, that is, at that moment when the

adiabatic evolution is broken and particles are gravitationally produced.

Then, from the knowledge acquired studying our toy model (15), we have numerically solved the equation (6), with initial conditions $\alpha_k(\tau_*) = 1$ and $\beta_k(\tau_*) = 0$ at the horizon crossing (there were neither particles nor polarization effects at that moment because during the slow-roll regime the derivatives of the Hubble rate are negligible compared with the powers of H , i.e., the system is in the adiabatic regime).

In order to get rid of complex exponentials we have transformed equation (6) into a second-order differential equation, namely

$$\begin{cases} \alpha_k''(\tau) = \alpha_k'(\tau) \left(\frac{\omega_k''(\tau)}{\omega_k'(\tau)} - \frac{\omega_k'(\tau)}{\omega_k(\tau)} + 2i\omega_k(\tau) \right) + \left(\frac{\omega_k'(\tau)}{2\omega_k(\tau)} \right)^2 \alpha_k(\tau) \\ \beta_k''(\tau) = \beta_k'(\tau) \left(\frac{\omega_k''(\tau)}{\omega_k'(\tau)} - \frac{\omega_k'(\tau)}{\omega_k(\tau)} - 2i\omega_k(\tau) \right) + \left(\frac{\omega_k'(\tau)}{2\omega_k(\tau)} \right)^2 \beta_k(\tau) \end{cases}. \quad (23)$$

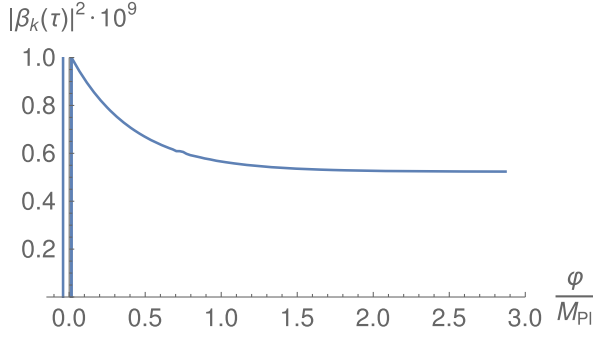
Given that $\alpha_k(\tau_*) = 1$ and $\beta_k(\tau_*) = 0$ leads to $\alpha_k'(\tau_*) = 0$, we are interested in solving the equation for $\alpha_k(\tau)$, which can be split into the real and imaginary form in the following way,

$$\begin{cases} \alpha_{k,\text{Re}}''(\tau) = \alpha_{k,\text{Re}}'(\tau) \left(\frac{\omega_k''(\tau)}{\omega_k'(\tau)} - \frac{\omega_k'(\tau)}{\omega_k(\tau)} \right) - 2\omega_k(\tau) \alpha_{k,\text{Im}}'(\tau) + \left(\frac{\omega_k'(\tau)}{2\omega_k(\tau)} \right)^2 \alpha_{k,\text{Re}}(\tau) \\ \alpha_{k,\text{Im}}''(\tau) = \alpha_{k,\text{Im}}'(\tau) \left(\frac{\omega_k''(\tau)}{\omega_k'(\tau)} - \frac{\omega_k'(\tau)}{\omega_k(\tau)} \right) + 2\omega_k(\tau) \alpha_{k,\text{Re}}'(\tau) + \left(\frac{\omega_k'(\tau)}{2\omega_k(\tau)} \right)^2 \alpha_{k,\text{Im}}(\tau) \end{cases}, \quad (24)$$

and then $|\beta_k(\tau)|^2 = |\alpha_k(\tau)|^2 - 1$ because of the well-known conservation property of the Wronskian. For the value $k = a_{\text{kin}} H_{\text{kin}}$, we obtain in Fig. 4 that $|\beta_k(\tau)|^2$ soon stabilizes to a nonzero value after the beginning of kination, containing only particle-production effects. We have numerically verified that this happens for the range $0.05 \lesssim \frac{k}{a_{\text{kin}} H_{\text{kin}}} \lesssim 7 \times 10^4$, which leads to values of $|\beta_k|^2$ of the order of 10^{-10} and 10^{-11} .

Then, introducing these values of the β -Bogoliubov coefficient in energy density [Eq. (7)], we have obtained a vacuum energy density of the order of 10^{44} GeV⁴. So, the energy density of the produced particles evolves as

$$\langle \rho(\tau) \rangle = \langle \bar{\rho} \rangle \left(\frac{\bar{a}}{a(\tau)} \right)^3, \quad (25)$$

FIG. 4. Evolution of $|\beta_k(\tau)|^2$.

where $\langle \bar{\rho} \rangle \cong 10^{44} \text{ GeV}^4 \cong 3 \times 10^{-30} M_{pl}^4$ and \bar{a} are, respectively, the energy density of the produced particles and the value of the scale factor at the end of the nonadiabatic phase ($\varphi \sim M_{pl}$, which coincides with the stabilization of the β -Bogoliubov coefficient). Finally, the energy density of the background at this moment is given by $\bar{\rho}_\varphi = 3\bar{H}^2 M_{pl}^2 \cong 2 \times 10^{57} \text{ GeV}^4 \cong 7 \times 10^{-17} M_{pl}^4$, showing that the energy density of the produced particles is subleading close to the beginning of kination, but will eventually be dominant because, during the kination regime, the energy density of the inflaton field decreases as a^{-6} .

Remark 3.1 Note that the values of the wave number k , that leads to significant values of the β -Bogoliubov coefficient, are not in the ultraviolet regime. On the contrary, analytic calculations only deal with the ultraviolet spectrum leading to an insignificant value of the β -Bogoliubov coefficient for heavy masses. For this reason, without taking into account the long wavelengths, it is usual to assume that the heavy masses suppress the particle production, thus leading to an inefficient reheating temperature. However, as we will see, this is not the case when one performs the numerical calculations considering all the spectrum of values of the wave number.

Remark 3.2 In the case of the α -attractors in the context of quintessential inflation, we have numerically obtained

that the corresponding energy densities are of the order $\langle \bar{\rho} \rangle \sim 3 \times 10^{-27} M_{pl}^4$ and $\bar{\rho}_\varphi \sim 6 \times 10^{-17} M_{pl}^4$ (see Fig. 5).

We would like to finish this subsection showing more details about the function $\frac{\omega'_k}{\omega_k^2} = \frac{m_\chi^2 a^3 H}{\omega_k^3}$. We have calculated its temporal derivative, obtaining

$$\frac{d}{dt} \left(\frac{\omega'_k}{\omega_k^2} \right) = \frac{3H^2 a^3 m_\chi^2}{\omega_k^3} \left(\frac{1 - w_{\text{eff}}}{2} - \frac{m_\chi^2 a^2}{\omega_k^2} \right), \quad (26)$$

where w_{eff} denotes the effective EOS parameter, i.e., the ratio of the pressure to the energy density. We will easily see that during kination, i.e., when $w_{\text{eff}} = 1$, this is a decreasing function, and is only an increasing function for the modes satisfying

$$k > \sqrt{\frac{1 + w_{\text{eff}}}{1 - w_{\text{eff}}}} m_\chi a. \quad (27)$$

Then, for the relevant models in LQI that contribute to the energy density of produced particles ($0.05 \lesssim \frac{k}{a_{\text{kin}} H_{\text{kin}}} \lesssim 7 \times 10^4$), the condition (27) is fulfilled during all the inflationary regime (see Fig. 3 and recall that LQI inflation ends when $\varphi_{\text{END}} \cong -0.078$), and thus one obtains the spike given in Fig. 3.

In addition, for the relevant modes, at the horizon crossing the quantity $\frac{\omega'_k}{\omega_k^2}$ is extremely small and, since it is an increasing function during inflation, the quantity $\frac{\omega'_k}{\omega_k^2}$ is negligible before the horizon crossing; that is, during this period there is no particle production and the polarization effects are negligible—for this reason we can ensure that before the horizon crossing the relevant modes of the field χ are in the vacuum. After the horizon crossing, polarization effects appear and later, during the phase transition, particles are created as we will see in Fig. 4. Finally, the β -Bogoliubov coefficient stabilizes [at the same time that the nonadiabatic regime finishes (right plot of Fig. 3)]

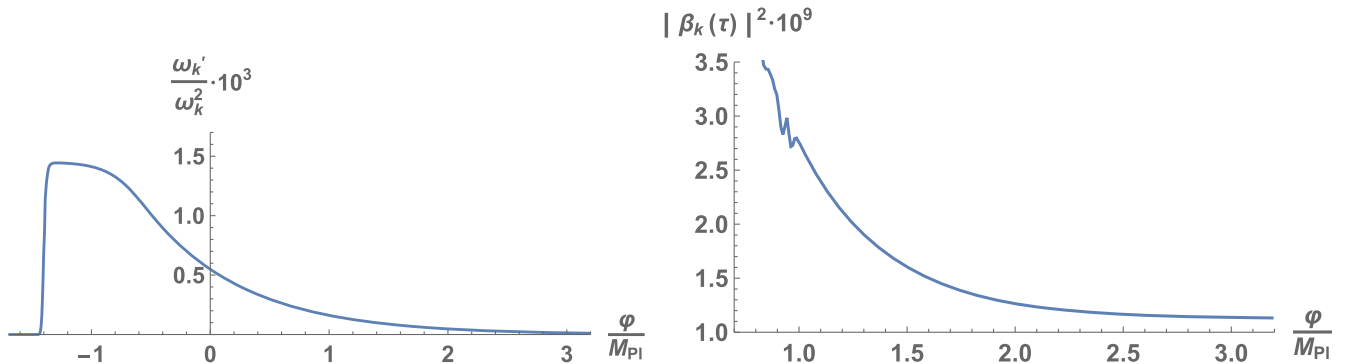


FIG. 5. Plot of the adiabatic evolution (left) and the β -Bogoliubov coefficient (right) for a heavy field with mass $m_\chi \cong 10^{15} \text{ GeV}$, when the background is given by the exponential α -attractor potential. Here we have used the value $k = a_{\text{kin}} H_{\text{kin}}$, since we have observed that particle creation takes place for modes in the range $1 \lesssim \frac{k}{a_{\text{kin}} H_{\text{kin}}} \lesssim 10^3$.

during kination, that is, the polarization effects are negligible and it only encodes the particle creation.

Note that exactly the same happens in the α -QI scenario.

IV. THE REHEATING PROCESS

After the production of the heavy massive particles with masses around 10^{15} GeV, they have to decay into lighter ones which, after the thermalization process, form a relativistic plasma that depicts our hot Universe. Two different situations may arise, as follows:

- (1) The decay, which occurs at time τ_{dec} , is before the end of the kination regime, when the energy density of the inflaton becomes equal to the one of the χ -field.
- (2) The decay is after the end of the kination regime.

A. Decay before the end of kination

Let us begin the discussion with the LQI potential. In this case, the energy density of the background, i.e., the one of the inflaton field, and the one of the relativistic plasma, when the decay is finished, when $\Gamma \sim H_{\text{dec}} = \bar{H}(\frac{\bar{a}}{a_{\text{dec}}})^3 \cong 10^{10}(\frac{\bar{a}}{a_{\text{dec}}})^3 \text{ GeV} \sim 5 \times 10^{-9}(\frac{\bar{a}}{a_{\text{dec}}})^3 M_{pl}$, will be

$$\rho_{\varphi,\text{dec}} = 3\Gamma^2 M_{pl}^2 \quad \text{and} \quad \langle \rho_{\text{dec}} \rangle \cong 3 \times 10^{-30} \left(\frac{\bar{a}}{a_{\text{dec}}} \right)^3 M_{pl}^4 \sim 10^{-21} \Gamma M_{pl}^3. \quad (28)$$

Imposing that the end of the decay precedes the end of kination means $\langle \rho_{\text{dec}} \rangle \leq \rho_{\varphi,\text{dec}}$, and one gets $\Gamma \geq 10^{-21} M_{pl}$.

In addition, since the decay is after the beginning of the kination, one has $\Gamma \leq H_{\text{kin}} \cong 4 \times 10^{-8} M_{pl}$. So, we have the following bound for the decay rate,

$$10^{-21} M_{pl} \leq \Gamma \leq 4 \times 10^{-8} M_{pl}. \quad (29)$$

Finally, the reheating temperature, i.e., the temperature of the Universe when the relativistic plasma in thermal equilibrium starts to dominate (which happens when $\rho_{\varphi,\text{reh}} = \langle \rho_{\text{reh}} \rangle$) can be calculated as follows: Since after the decay the evolution of the respective energy densities is given by

$$\begin{aligned} \rho_{\varphi,\text{reh}} &= \rho_{\varphi,\text{dec}} (a_{\text{dec}}/a_{\text{reh}})^6, \\ \langle \rho_{\text{reh}} \rangle &= \langle \rho_{\text{dec}} \rangle (a_{\text{dec}}/a_{\text{reh}})^4, \end{aligned} \quad (30)$$

we will have $\frac{\langle \rho_{\text{dec}} \rangle}{\rho_{\varphi,\text{dec}}} = (a_{\text{dec}}/a_{\text{reh}})^2$, and thus, the reheating temperature will be

$$\begin{aligned} T_{\text{reh}} &= \left(\frac{30}{\pi^2 g_{\text{reh}}} \right)^{1/4} \langle \rho_{\text{reh}} \rangle^{1/4} = \left(\frac{30}{\pi^2 g_{\text{reh}}} \right)^{1/4} \langle \rho_{\text{dec}} \rangle^{1/4} \sqrt{\frac{\langle \rho_{\text{dec}} \rangle}{\rho_{\varphi,\text{dec}}}} \\ &\sim 4 \times 10^{-17} \left(\frac{M_{pl}}{\Gamma} \right)^{1/4} M_{pl}, \end{aligned} \quad (31)$$

where $g_{\text{reh}} = 106.75$ is the effective number of degrees of freedom for the Standard Model. So, taking into account the bound (29), the reheating temperature ranges between $7 \times 10^3 \text{ GeV}$ and $2 \times 10^7 \text{ GeV}$.

B. Decay after the end of kination

In the case that the decay of the χ -field is after the end of kination, one has to impose $\Gamma \leq H(\tau_{\text{end}}) \equiv H_{\text{end}}$, where we have denoted by τ_{end} the time at which kination ends. Taking this into account, one has

$$H_{\text{end}}^2 = \frac{2\rho_{\varphi,\text{end}}}{3M_{pl}^2} \quad \text{and} \quad \rho_{\varphi,\text{end}} = \bar{\rho}_{\varphi} \left(\frac{\bar{a}}{a_{\text{end}}} \right)^6 = \frac{\langle \bar{\rho} \rangle^2}{\bar{\rho}_{\varphi}}, \quad (32)$$

where we have used that the kination ends when $\langle \rho(\tau_{\text{end}}) \rangle = \rho_{\varphi,\text{end}}$, meaning $(\bar{a}/a_{\text{end}})^3 = \frac{\langle \bar{\rho} \rangle}{\bar{\rho}_{\varphi}}$. So, the condition $\Gamma \leq H_{\text{end}}$ leads to the bound

$$\Gamma \leq 10^{-23} M_{pl}. \quad (33)$$

On the other hand, assuming once again instantaneous thermalization, the reheating temperature (i.e., the temperature of the universe when the thermalized plasma starts to dominate) will be obtained when all the superheavy particles decay, i.e., when $H \sim \Gamma$, obtaining

$$T_{\text{reh}} = \left(\frac{30}{\pi^2 g_{\text{reh}}} \right)^{1/4} \langle \rho_{\text{dec}} \rangle^{1/4} = \left(\frac{90}{\pi^2 g_{\text{reh}}} \right)^{1/4} \sqrt{\Gamma M_{pl}}, \quad (34)$$

where we have used that, after the end of the kination regime, the energy density of the produced particles dominates the one of the inflaton field.

Consequently, since the BBN epoch occurs at the 1 MeV regime and taking once again $g_{\text{reh}} = 106.75$, one can find that, in that case, the reheating temperature is bounded by

$$1 \text{ MeV} \leq T_{\text{reh}} \leq 5 \times 10^6 \text{ GeV}. \quad (35)$$

Remark 4.1 Since for α -attractors in QI the number of produced particles is higher than in LQI, then the reheating temperature also increases. In fact, following step by step the same calculations that we have done above, we have obtained a very high reheating temperature when the decay is before the end of kination, between 10^7 GeV and 10^9 GeV . And, when the decay is after the end of kination, the reheating temperature belongs in the range between 1 MeV and 10^9 GeV .

V. GRAVITATIONAL WAVES AND THE BBN SUCCESS

A. The overproduction of gravitational waves

In the seminal paper [12], the authors pointed out that in quintessential inflation a reheating due to the gravitational production of light particles is incompatible with the overproduction of gravitational waves. However, as we will show in this subsection, for our LQI model, (and also in α -QI, because in this scenario, as we have already explained, the production of particles is greater than in LQI), when the reheating is due to the gravitational creation of superheavy particles, then the overproduction of GWs does not disturb the BBN success.

To prove our statement, first of all we recall that the energy density of the produced GWs during the phase transition from the end of inflation to the beginning of kination is given by [69]

$$\langle \rho_{\text{GW}}(\tau) \rangle \cong \frac{H_{\text{kin}}^4}{2\pi^3} \left(\frac{a_{\text{kin}}}{a(\tau)} \right)^4 \cong 10^{-2} H_{\text{kin}}^4 \left(\frac{a_{\text{kin}}}{a(\tau)} \right)^4. \quad (36)$$

Then, taking into account that the success of the BBN demands that the ratio of the energy density of GWs to the one of the produced particles at the reheating time satisfies [70]

$$\frac{\langle \rho_{\text{GW,reh}} \rangle}{\langle \rho_{\text{reh}} \rangle} \leq 10^{-2}, \quad (37)$$

we will see that the constraint (37) is overcome when the decay of the superheavy particles is before the end of kination. Effectively, if the decay occurs before the end of kination one has

$$\frac{\langle \rho_{\text{GW,reh}} \rangle}{\langle \rho_{\text{reh}} \rangle} = \frac{\langle \rho_{\text{GW,dec}} \rangle}{\langle \rho_{\text{dec}} \rangle}, \quad (38)$$

because the energy density of light relativistic particles evolves as the one of the GWs. Next, noting that during kination the energy density of the background scales as a^{-6} , it yields that

$$\left(\frac{a_{\text{kin}}}{a_{\text{dec}}} \right)^4 = \left(\frac{\rho_{\phi,\text{dec}}}{\rho_{\phi,\text{kin}}} \right)^{2/3}, \quad (39)$$

and thus, using our previous results [see Eq. (28)] and recalling that the value of the Hubble rate at the beginning of kination is $H_{\text{kin}} \cong 4 \times 10^{-8} M_{\text{pl}}$, we get

$$\frac{\langle \rho_{\text{GW,reh}} \rangle}{\langle \rho_{\text{reh}} \rangle} \cong 10^{-1} \left(\frac{\Gamma}{M_{\text{pl}}} \right)^{1/3}. \quad (40)$$

Therefore, the bound (37) is overcome when

$$\Gamma \leq 7 \times 10^{-4} M_{\text{pl}}, \quad (41)$$

which is completely compatible with the bound (29).

On the other hand, when the decay is produced after the end of kination, and assuming once again instantaneous thermalization, the reheating time will coincide with the decay one. Then, since $\langle \rho_{\text{dec}} \rangle = 3\Gamma^2 M_{\text{pl}}^2$ and

$$\begin{aligned} H_{\text{dec}} &= H_{\text{end}} \left(\frac{a_{\text{end}}}{a_{\text{dec}}} \right)^{3/2} \\ \Rightarrow \left(\frac{a_{\text{end}}}{a_{\text{dec}}} \right)^{3/2} &= \sqrt{\frac{3}{2}} \frac{\Gamma M_{\text{pl}} \sqrt{\bar{\rho}_{\phi}}}{\langle \bar{\rho} \rangle}, \end{aligned} \quad (42)$$

where we have used that $H_{\text{end}} = \sqrt{\frac{2}{3}} \frac{\langle \bar{\rho} \rangle}{M_{\text{pl}} \sqrt{\bar{\rho}_{\phi}}}$.

Thus, we will have

$$\begin{aligned} \langle \rho_{\text{GW,dec}} \rangle &= \langle \rho_{\text{GW,end}} \rangle \left(\frac{a_{\text{end}}}{a_{\text{dec}}} \right)^4 \\ &= \langle \rho_{\text{GW,end}} \rangle \left(\sqrt{\frac{3}{2}} \frac{\Gamma M_{\text{pl}} \sqrt{\bar{\rho}_{\phi}}}{\langle \bar{\rho} \rangle} \right)^{8/3}, \end{aligned} \quad (43)$$

and, using that

$$\left(\frac{a_{\text{end}}}{a_{\text{dec}}} \right)^4 = \frac{\rho_{\phi,\text{end}}}{\rho_{\phi,\text{kin}}} \quad \text{and} \quad \rho_{\phi,\text{end}} = \frac{\langle \bar{\rho} \rangle^2}{\bar{\rho}_{\phi}}, \quad (44)$$

we get

$$\begin{aligned} \langle \rho_{\text{GW,dec}} \rangle &= \left(\frac{2}{16} \right)^{1/3} \left(\frac{H_{\text{kin}}}{M_{\text{pl}}} \right)^2 \left(\frac{\bar{\rho}_{\phi}}{\langle \bar{\rho} \rangle^2} \right)^{1/3} (\Gamma M_{\text{pl}})^{8/3} \\ &\cong 10^{-2} \Gamma^{8/3} M_{\text{pl}}^{4/3}, \end{aligned} \quad (45)$$

and thus

$$\frac{\langle \rho_{\text{GW,reh}} \rangle}{\langle \rho_{\text{reh}} \rangle} = \frac{\langle \rho_{\text{GW,dec}} \rangle}{\langle \rho_{\text{dec}} \rangle} \cong 3 \times 10^{-3} \left(\frac{\Gamma}{M_{\text{pl}}} \right)^{2/3} \leq 10^{-17}, \quad (46)$$

where we have used the bound (33). So, the constraint (37) is clearly overcome.

B. BBN constraints from the logarithmic spectrum of GWs

As we have already explained, GWs are produced in the postinflationary period, and its logarithmic spectrum of GWs, namely Ω_{GW} defined as $\Omega_{\text{GW}} \equiv \frac{1}{\rho_c} \frac{d\rho_{\text{GW}}(k)}{d \ln k}$ [where $\rho_{\text{GW}}(k)$ is the energy density spectrum of the produced GWs; $\rho_c = 3H_0^2 M_{\text{pl}}^2$, where H_0 is the present value of the Hubble rate, is the so-called critical density] scales as k^2 during kination [71], producing a spike in the spectrum of GWs at high frequencies. Then, so that GWs do not destabilize the BBN, the following bound must be imposed (see Sec. 7.1 of [72]),

$$I \equiv h_0^2 \int_{k_{BBN}}^{k_{\text{end}}} \Omega_{\text{GW}}(k) d \ln k \leq 10^{-5}, \quad (47)$$

where $h_0 \cong 0.678$ parametrizes the experimental uncertainty to determine the current value of the Hubble constant and k_{BBN} , k_{end} are the momenta associated to the horizon scale at the BBN and at the end of inflation, respectively. As has been shown in [73], the main contribution of the integral (47) comes from the modes that leave the Hubble radius before the end of the inflationary epoch and finally reenter during kination, that means, for $k_{\text{end}} \leq k \leq k_{\text{kin}}$, where $k_{\text{end}} = a_{\text{end}} H_{\text{end}}$ and $k_{\text{kin}} = a_{\text{kin}} H_{\text{kin}}$. For these modes one can calculate the logarithmic spectrum of GWs as in [43] (see also [71,74–76])

$$\Omega_{\text{GW}}(k) = \tilde{\epsilon} \Omega_\gamma h_{\text{GW}}^2 \left(\frac{k}{k_{\text{end}}} \right) \ln^2 \left(\frac{k}{k_{\text{kin}}} \right), \quad (48)$$

where $h_{\text{GW}}^2 = \frac{1}{8\pi} \left(\frac{H_{\text{kin}}}{M_{pl}} \right)^2$ is the amplitude of the GWs; $\Omega_\gamma \cong 2.6 \times 10^{-5} h_0^{-2}$ is the present density fraction of radiation, and the quantity $\tilde{\epsilon}$, which is approximately equal to 0.05 for the Standard Model of particle physics, takes into account the variation of massless degrees of freedom between decoupling and thermalization (see [71,73] for more details). As has been derived in [73], the specific form of the expression above comes from the behavior of the Hankel functions for small arguments. Now, plugging Eq. (48) into Eq. (47) and disregarding the subleading logarithmic terms, one finds

$$\begin{aligned} 2\tilde{\epsilon} h_0^2 \Omega_\gamma h_{\text{GW}}^2 \left(\frac{k_{\text{kin}}}{k_{\text{end}}} \right) &\leq 10^{-5} \Rightarrow 10^{-2} \left(\frac{H_{\text{kin}}}{M_{pl}} \right)^2 \left(\frac{k_{\text{kin}}}{k_{\text{end}}} \right) \leq 1 \\ \Rightarrow 10^{-17} \left(\frac{k_{\text{kin}}}{k_{\text{end}}} \right) &\leq 1, \end{aligned} \quad (49)$$

because in our LQI model at the beginning of kination the Hubble rate is $H_{\text{kin}} \sim 4 \times 10^{-8} M_{pl}$, and the same happens for our α -QI model.

So, we continue with the LQI, but taking into account that all the reasoning is also valid for the other model. Then, to calculate the ratio $k_{\text{kin}}/k_{\text{end}}$, we will have to study the following two different situations:

(1) When the decay occurs before the end of kination.

In this case the reheating time coincides with the end of kination, and thus, a simple calculation leads to

$$\frac{k_{\text{kin}}}{k_{\text{end}}} = \frac{k_{\text{kin}}}{k_{\text{reh}}}. \quad (50)$$

Using the formulas

$$\begin{aligned} \rho_{\phi, \text{reh}} &= \rho_{\phi, \text{kin}} \left(\frac{a_{\text{kin}}}{a_{\text{reh}}} \right)^6 \quad \text{and} \\ \langle \rho_{\text{reh}} \rangle &= \langle \rho_{\text{dec}} \rangle \left(\frac{a_{\text{dec}}}{a_{\text{reh}}} \right)^4, \end{aligned} \quad (51)$$

we get

$$\begin{aligned} \frac{a_{\text{kin}}}{a_{\text{reh}}} &= \left(\frac{\langle \rho_{\text{dec}} \rangle}{\rho_{\phi, \text{kin}}} \right)^{1/6} \left(\frac{a_{\text{dec}}}{a_{\text{reh}}} \right)^{2/3} \\ &= \left(\frac{\langle \rho_{\text{dec}} \rangle}{\rho_{\phi, \text{kin}}} \right)^{1/6} \left(\frac{\langle \rho_{\text{dec}} \rangle}{\rho_{\phi, \text{dec}}} \right)^{1/3}, \end{aligned} \quad (52)$$

where we have used the relation $\left(\frac{a_{\text{dec}}}{a_{\text{reh}}} \right)^2 = \frac{\langle \rho_{\text{dec}} \rangle}{\rho_{\phi, \text{dec}}}$. Then, taking into account that $H_{\text{reh}} = \Gamma \left(\frac{a_{\text{dec}}}{a_{\text{reh}}} \right)^3$, we obtain

$$\begin{aligned} \frac{k_{\text{kin}}}{k_{\text{end}}} &= \frac{k_{\text{kin}}}{k_{\text{reh}}} = \frac{H_{\text{kin}} a_{\text{kin}}}{H_{\text{reh}} a_{\text{reh}}} \\ &= \frac{H_{\text{kin}} \rho_{\phi, \text{dec}}}{\Gamma \langle \rho_{\text{dec}} \rangle} \left(\frac{\rho_{\phi, \text{dec}}}{\rho_{\phi, \text{kin}}} \right)^{1/6}. \end{aligned} \quad (53)$$

Finally, from our previous results (28)

$$\begin{aligned} \rho_{\phi, \text{dec}} &= 3\Gamma^2 M_{pl}^2, \quad \langle \rho_{\text{dec}} \rangle \sim 10^{-21} \Gamma M_{pl}^3 \quad \text{and} \\ \rho_{\phi, \text{kin}} &= 3H_{\text{kin}}^2 M_{pl}^2, \end{aligned} \quad (54)$$

we arrive at

$$\frac{k_{\text{kin}}}{k_{\text{end}}} \sim 10^{15} \left(\frac{\Gamma}{M_{pl}} \right)^{1/3}, \quad (55)$$

and consequently the constraint (49) becomes

$$\left(\frac{\Gamma}{M_{pl}} \right)^{1/3} \leq 10^2, \quad (56)$$

which is obviously overcome for all the viable values of Γ , i.e., for all values between $10^{-21} M_{pl}$ and $10^{-8} M_{pl}$.

(2) When the decay occurs after the end of kination.

Now, since during kination we have $H_{\text{end}} = H_{\text{kin}} \left(\frac{a_{\text{kin}}}{a_{\text{end}}} \right)^3$, one gets

$$\frac{k_{\text{kin}}}{k_{\text{end}}} = \left(\frac{H_{\text{kin}}}{H_{\text{end}}} \right)^{2/3}, \quad (57)$$

and taking into account that

$$H_{\text{end}} = \sqrt{\frac{2\langle\bar{\rho}\rangle^2}{3M_{pl}^2\bar{\rho}_\phi}} \sim 3 \times 10^{-22} M_{pl} \quad \text{and}$$

$$H_{\text{kin}} \sim 4 \times 10^{-8} M_{pl}, \quad (58)$$

the bound (49) is completely overcome.

VI. CONCLUSIONS

In the present work we numerically studied the gravitational particle production of superheavy particles conformally coupled to gravity for two classes of QI scenarios; namely LQI and α -QI. To calculate the energy density of the produced particles we have used the well-known diagonalization method, where the key point is the calculation and interpretation of the time-dependent β -Bogoliubov coefficient.

In fact, this coefficient encodes all the polarization effects (creation and annihilation of pairs named, in the Russian literature, quasiparticles) and the produced superheavy particles—the real ones which after decaying into lighter ones form a relativistic plasma which reheats the universe during the phase transition from the end of inflation to the beginning of kination. However, and this is the main observation of the present work, the polarization effects disappear soon after the end of the phase transition (during the kination regime), i.e., when the evolution of the Universe gets adiabatic again. So, we have numerically checked that effectively the value of the β -Bogoliubov coefficient stabilizes during the kination, which allows us to compute it numerically, and thus, to calculate numerically the energy density of the particles created during this phase transition.

Once these superheavy particles have been created, they must decay into lighter ones to form a relativistic plasma which eventually becomes dominant and matches with the hot big bang Universe. Then two different situations arise; namely, when the decay occurs before the end of the kination regime and when the decay occurs after the end of the kination regime. Thus, since we have numerically computed the energy density of the superheavy particles, for both situations we have been able to calculate the reheating temperature of the Universe, which depends on the decay rate of these superheavy particles and whose maximum value is quite big—more or less around 10^7 GeV in LQI and around 10^9 GeV in α -QI—which demystifies the belief, never checked numerically, that heavy masses suppress the particle production thus leading to an abnormally low reheating temperature. In fact, the main contribution of particle production is to a long-wavelength regime, which is impossible to quantify analytically because only ultraviolet effects can be calculated with analytic methods, and this is the reason why in many papers the production of superheavy particles is simply disregarded.

Finally, we have checked that the overproduction of GWs does not disturb the BBN, because all the bounds preserving its success are clearly overcome.

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- [1] A. Guth, *Phys. Rev. D* **23**, 347 (1981).
 - [2] A. Linde, *Phys. Lett.* **108B**, 389 (1982).
 - [3] G. Chibisov and V. Mukhanov, *Mon. Not. R. Astron. Soc.* **200**, 535 (1982).
 - [4] A. A. Starobinsky, *Phys. Lett.* **117B**, 175 (1982).
 - [5] A. H. Guth and S.-Y. Pi, *Phys. Rev. Lett.* **49**, 1110 (1982).
 - [6] J. M. Bardeen, P. J. Steinhardt, and M. S. Turner, *Phys. Rev. D* **28**, 679 (1983).
 - [7] A. D. Linde, *Phys. Lett.* **116B**, 335 (1982).
 - [8] P. A. R. Ade *et al.* (Planck Collaboration), *Astron. Astrophys.* **594**, A20 (2016).
 - [9] K. Bamba, S. Capozziello, S. Nojiri, and S. D. Odintsov, *Astrophys. Space Sci.* **342**, 155 (2012).
 - [10] M. Li, X-D Li, S. Wang, and Y. Wang, *Commun. Theor. Phys.* **56**, 525 (2011).
 - [11] E. J. Copeland, M. Sami, and S. Tsujikawa, *Int. J. Mod. Phys. D* **15**, 1753 (2006).
 - [12] P. J. E. Peebles and A. Vilenkin, *Phys. Rev. D* **59**, 063505 (1999).
 - [13] J. Haro, J. Amorós, and S. Pan, *Eur. Phys. J. C* **79**, 505 (2019).
 - [14] J. de Haro, J. Amorós, and S. Pan, *Phys. Rev. D* **93**, 084018 (2016).
 - [15] J. de Haro and E. Elizalde, *Gen. Relativ. Gravit.* **48**, 77 (2016).
 - [16] J. de Haro, *Gen. Relativ. Gravit.* **49**, 6 (2017).
 - [17] K. Dimopoulos, *Nucl. Phys. B, Proc. Suppl.* **95**, 70 (2001).
 - [18] K. Dimopoulos and J. W. F. Valle, *Astropart. Phys.* **18**, 287 (2002).
 - [19] M. Giovannini, *Phys. Rev. D* **67**, 123512 (2003).
 - [20] M. W. Hossain, R. Myrzakulov, M. Sami, and E. N. Saridakis, *Phys. Rev. D* **89**, 123513 (2014).
 - [21] M. W. Hossain, R. Myrzakulov, M. Sami, and E. N. Saridakis, *Int. J. Mod. Phys. D* **24**, 1530014 (2015).

- [22] J. de Haro, J. Amorós, and S. Pan, *Phys. Rev. D* **94**, 064060 (2016).
- [23] C. Q. Geng, C. C. Lee, M. Sami, E. N. Saridakis, and A. A. Starobinsky, *J. Cosmol. Astropart. Phys.* **06** (2017) 011.
- [24] L. A. Saló and J. de Haro, *Eur. Phys. J. C* **77**, 798 (2017).
- [25] J. Haro and S. Pan, *Int. J. Mod. Phys. D* **27**, 1850052 (2018).
- [26] J. Haro, W. Yang, and S. Pan, *J. Cosmol. Astropart. Phys.* **01** (2019) 023.
- [27] K. Dimopoulos and C. Owen, *J. Cosmol. Astropart. Phys.* **06** (2017) 027.
- [28] K. Dimopoulos, *Phys. Rev. D* **68**, 123506 (2003).
- [29] J. C. B. Sanchez and K. Dimopoulos, *Phys. Lett. B* **642**, 294 (2006); **647**, 526(E) (2007).
- [30] J. C. B. Sanchez and K. Dimopoulos, *J. Cosmol. Astropart. Phys.* **10** (2007) 002.
- [31] J. C. B. Sanchez and K. Dimopoulos, *J. Cosmol. Astropart. Phys.* **11** (2007) 007.
- [32] K. Dimopoulos and T. Markkanen, *J. Cosmol. Astropart. Phys.* **06** (2018) 021.
- [33] K. Dimopoulos and L. Donaldson-Wood, *Phys. Lett. B* **796**, 26 (2019).
- [34] K. Dimopoulos, M. Karčiauskas, and C. Owen, *Phys. Rev. D* **100**, 083530 (2019).
- [35] K. Dimopoulos and S. S. López, *Phys. Rev. D* **103**, 043533 (2021).
- [36] L. Parker, *Phys. Rev. Lett.* **21**, 562 (1968); *Phys. Rev.* **183**, 1057 (1969); *Phys. Rev. D* **3**, 346 (1971).
- [37] V. M. Folov, S. G. Mamayev, and V. M. Mostepanenko, *Phys. Lett.* **55A**, 7 (1976).
- [38] A. A. Grib, B. A. Levitskii, and V. M. Mostepanenko, *Teor. Mat. Fiz.* **19**, 59 (1974).
- [39] A. A. Grib, S. G. Mamayev, and V. M. Mostepanenko, *Gen. Relativ. Gravit.* **7**, 535 (1976); *Sov. Phys. J.* **17**, 1700 (1974).
- [40] L. H. Ford, *Phys. Rev. D* **35**, 2955 (1987).
- [41] Y. B. Zeldovich and A. A. Starobinsky, *JETP Lett.* **26**, 252 (1977).
- [42] T. Damour and A. Vilenkin, *Phys. Rev. D* **53**, 2981 (1996).
- [43] M. Giovannini, *Phys. Rev. D* **58**, 083504 (1998).
- [44] D. J. H. Chung, E. W. Kolb, and A. Riotto, *Phys. Rev. D* **59**, 023501 (1998).
- [45] D. J. H. Chung, P. Crotty, E. W. Kolb, and A. Riotto, *Phys. Rev. D* **64**, 043503 (2001).
- [46] N. D. Birrell and P. C. W. Davies, *J. Phys. A* **13**, 2109 (1980).
- [47] S. Hashiba and J. Yokoyama, *J. Cosmol. Astropart. Phys.* **01** (2019) 028.
- [48] S. Hashiba and Y. Yamada, *J. Cosmol. Astropart. Phys.* **05** (2021) 022.
- [49] B. Spokoiny, *Phys. Lett. B* **315**, 40 (1993).
- [50] Y. Akrami, R. Kallosh, A. Linde, and V. Vardanyan, *J. Cosmol. Astropart. Phys.* **06** (2018) 041.
- [51] J. Haro, J. Amorós, and S. Pan, *Eur. Phys. J. C* **79**, 505 (2019).
- [52] J. de Haro and L. A. Saló, *Phys. Rev. D* **95**, 123501 (2017).
- [53] S. Hashiba and J. Yokoyama, *Phys. Rev. D* **99**, 043008 (2019).
- [54] L. Kofman, A. Linde, and A. Starobinsky, *Phys. Rev. D* **56**, 3258 (1997).
- [55] D. Benisty and E. I. Guendelman, *Int. J. Mod. Phys. D* **29**, 2042002 (2020).
- [56] D. Benisty and E. I. Guendelman, *Eur. Phys. J. C* **80**, 577 (2020).
- [57] L. A. Saló, D. Benisty, E. I. Guendelman, and J. d. Haro, *J. Cosmol. Astropart. Phys.* **07** (2021) 007.
- [58] L. A. Saló, D. Benisty, E. I. Guendelman, and J. D. Haro, *Phys. Rev. D* **103**, 123535 (2021).
- [59] M. Joyce, *Phys. Rev. D* **55**, 1875 (1997).
- [60] A. A. Grib, S. G. Mamayev, and V. M. Mostepanenko, *Friedmann Laboratory Publishing for Theoretical Physics* (St. Petersburg, 1994).
- [61] G. F. Giudice, E. W. Kolb, and A. Riotto, *Phys. Rev. D* **64**, 023508 (2001).
- [62] J. Ellis, D. V. Nanopoulos, and S. Sarkar, *Nucl. Phys.* **B259**, 175 (1985).
- [63] J. Ellis, A. Linde, and D. Nanopoulos, *Phys. Lett.* **118B**, 59 (1982).
- [64] J. de Haro, S. Pan, and L. A. Saló, *J. Cosmol. Astropart. Phys.* **06** (2019) 056.
- [65] Y. B. Zeldovich and A. A. Starobinsky, *Sov. Phys. JETP* **34**, 1159 (1972).
- [66] N. D. Birrell and C. P. W. Davies, *Quantum Fields in Curved Space* (Cambridge University Press, Cambridge, England, 1982).
- [67] T. S. Bunch, *J. Phys. A* **13**, 1297 (1980).
- [68] A. Linde, *Contemp. Concepts Phys.* **5**, 1 (2005).
- [69] M. Giovannini, *Phys. Rev. D* **60**, 123511 (1999).
- [70] K. Dimopoulos, L. D. Wood, and C. Owen, *Phys. Rev. D* **97**, 063525 (2018).
- [71] J. Rubio and C. Wetterich, *Phys. Rev. D* **96**, 063509 (2017).
- [72] M. Maggiore, *Phys. Rep.* **331**, 283 (2000).
- [73] M. Giovannini, *Phys. Rev. D* **60**, 123511 (1999).
- [74] M. Giovannini, *Phys. Lett. B* **759**, 528 (2016).
- [75] M. Giovannini, *Classical Quantum Gravity* **26**, 045004 (2009).
- [76] M. Giovannini, *Phys. Lett. B* **668**, 44 (2008).