Indirect detection of gravitons through quantum entanglement

Sugumi Kanno,[1] Jiro Soda,[2] and Junsei Tokuda[2]
[1]*Department of Physics, Kyushu University, 744 Motooka, Nishi-ku, Fukuoka 819-0395, Japan*
[2]*Department of Physics, Kobe University, Kobe 657-8501, Japan*

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We propose an experiment that the entanglement between two macroscopic mirrors suspended at the end of an equal-arm interferometer is destroyed by the noise of gravitons through bremsstrahlung. By calculating the correlation function of the noise, we obtain the decoherence time from the decoherence functional. We estimate that the decoherence time induced by the noise of gravitons in squeezed states stemming from inflation is approximately 20 s for 40 km long arms and 40 kg mirrors. Our analysis shows that observation of the decoherence time of quantum entanglement has the potential to detect gravitons indirectly. This indirect detection of gravitons would give strong evidence of quantum gravity.

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I. INTRODUCTION

It is widely believed that gravity should be quantized because other forces are quantized. Nevertheless, there is no satisfactory quantum theory of gravity. Some physicists think that gravity may not be quantized after all [1]. Thus, to find experimental evidence of quantum gravity is quite important. A clear consequence of quantum theory of gravity would be the existence of gravitons. Hence, it is desired to come up with a novel way to observe gravitons experimentally.

It has been known that gravitons are imperceptible. In fact, Dyson has conjectured that no conceivable experiment in our Universe can detect a single graviton [2,3]. If it is true, we need to seek alternative ways for confirming the existence of gravitons. One possible way is to focus on an inflationary scenario. It is believed that primordial gravitational waves can be generated during inflation from quantum fluctuations of geometry. If we succeed in observing the primordial gravitational waves, it would imply a discovery of gravitons [4–6]. However, even if the primordial gravitational waves arrive at the interferometers, the direct detection of the gravitons is difficult with current experimental techniques. Another possible way is to focus on the statistical property of primordial gravitational waves. The state of the primordial gravitational waves becomes squeezed during inflation [7,8]. Unfortunately, it is pointed out that observing the squeezed states is also practically impossible [9].

Let us recall the history of discovery of atoms. Einstein used Brownian motion to deduce the existence of atoms. In the same way, instead of direct detection of gravitons, indirect search for gravitons might be possible. Recently, the noise induced by gravitons has been discussed in Refs. [10–13]. An indirect detection of gravitons by making use of the process of decoherence through the noise of gravitons is also proposed in Refs. [14–16]. However, no feasible experiment has been proposed yet. The goal of this paper is to propose a feasible experimental setup for indirect detection of gravitons in the squeezed states stemming from inflation. In the following, we work in the natural unit: $c = \hbar = 1$.

II. EXPERIMENTAL SETUPS

The proposed setup, shown in Fig. 1 consists of an equal-arm Michelson interferometer which has a macroscopic suspended mirror at the end of each arm. An incident photon (photon beam with very low intensity) is injected from left, and the beam splitter converts the photon into a superposition state of being in both the upper and lower arms simultaneously until it is detected by the oscillation of either mirror [17].[1] In this study, we assume that the upper and lower mirrors (2 and 1) are in position eigenstates. Then, the oscillations of two mirrors are specified by their positions as

$$\vec{\xi_1}(t) = (\xi_1, 0, 0), \qquad \xi_1 = A\cos\omega t,$$
$$\vec{\xi_2}(t) = (0, \xi_2, 0), \qquad \xi_2 = A\cos\omega t, \tag{1}$$

where the amplitude of the oscillation A and the angular frequency ω of both mirrors are set to the same value for simplicity. Let H_1 and H_2 denote each Hilbert space of mirrors. Then, the Hilbert space for the combined system of the two mirrors is $H_1 \otimes H_2$. Let H_1 be spanned by a basis

[1]Here, we considered an incident photon for simplicity. In fact, photon beams with high intensity create the entangled mirrors in laboratory [18,19].

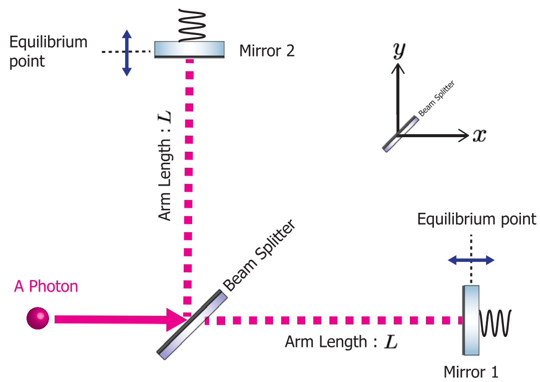

FIG. 1. The proposed setup: an equal-arm Michelson interferometer for a single photon where there is a macroscopic suspended mirror at the end of each arm.

$\{|0\rangle, |\overrightarrow{\xi_1}\rangle\}$ and H_2 be spanned by $\{|0\rangle, |\overrightarrow{\xi_2}\rangle\}$, where $|0\rangle$ is the vacuum state in equilibrium and $|\overrightarrow{\xi_a}\rangle$ $(a = 1,\ 2)$ represents a coherent state as an excited state induced by a single photon. After an incident photon is injected, the state of both mirrors with mass m is described by the maximally entangled state where mirror 1 oscillates and mirror 2 is in equilibrium or mirror 1 is in equilibrium and mirror 2 oscillates,

$$\frac{1}{\sqrt{2}}|\overrightarrow{\xi_1}\rangle \otimes |0\rangle + \frac{1}{\sqrt{2}}|0\rangle \otimes |\overrightarrow{\xi_2}\rangle. \tag{2}$$

Now, let us consider the influence of environmental quantum gravitational fields (gravitons) on the system of the mirrors. We assume that the initial total system is given by $|0\rangle \otimes |0\rangle \otimes |h\rangle$, where $|h\rangle$ represents the initial state of gravitons. As we will explain later, we consider squeezed states of gravitons stemming from inflation. After the incident photon is injected, the system of the mirrors gets entangled, and the total system becomes

$$|\psi(t_{\rm i})\rangle = \left\{\frac{1}{\sqrt{2}}|\overrightarrow{\xi_1}\rangle \otimes |0\rangle + \frac{1}{\sqrt{2}}|0\rangle \otimes |\overrightarrow{\xi_2}\rangle\right\} \otimes |h\rangle, \tag{3}$$

where $t_{\rm i}$ is the initial time. We impose normalization conditions as

$$\langle\overrightarrow{\xi_1}|\overrightarrow{\xi_1}\rangle = \langle\overrightarrow{\xi_2}|\overrightarrow{\xi_2}\rangle = \langle 0|0\rangle = \langle h|h\rangle = 1. \tag{4}$$

If we focus on the system of the mirrors, the reduced density operator is obtained by tracing out the degree of freedom of environmental gravitons such as

$$\begin{aligned}\rho_{\rm m}(t_{\rm i}) &= {\rm Tr}_{\rm grav}|\psi(t_{\rm i})\rangle\langle\psi(t_{\rm i})| \\ &= \frac{1}{2}[\rho_{11}(t_{\rm i}) + \rho_{22}(t_{\rm i}) + \rho_{12}(t_{\rm i}) + \rho_{21}(t_{\rm i})], \end{aligned}\tag{5}$$

where we defined

$$\begin{aligned}\rho_{11}(t) &\equiv |\overrightarrow{\xi_1}(t)\rangle|0\rangle\langle\overrightarrow{\xi_1}(t)|\langle 0|, \\ \rho_{12}(t) &\equiv |\overrightarrow{\xi_1}(t)\rangle|0\rangle\langle 0|\langle\overrightarrow{\xi_2}(t)|, \\ \rho_{21}(t) &\equiv |0\rangle|\overrightarrow{\xi_2}(t)\rangle\langle\overrightarrow{\xi_1}(t)|\langle 0|, \\ \rho_{22}(t) &\equiv |0\rangle|\overrightarrow{\xi_2}(t)\rangle\langle 0|\langle\overrightarrow{\xi_2}(t)|.\end{aligned}$$

The presence of interference term $\rho_{12} + \rho_{21}$ shows the initial entangled state between the mirrors.

Next, let us consider the time evolution. Since gravitons couple to each mirror as we will see in Eq. (16), the quantum state of gravitons changes due to bremsstralung in accordance with the oscillation of the mirror on either side. The quantum state of the total system at the time $t > t_{\rm i}$ is then formally written as

$$\begin{aligned}|\psi(t)\rangle = &\frac{1}{\sqrt{2}}|\overrightarrow{\xi_1}(t)\rangle \otimes |0\rangle \otimes |h;\overrightarrow{\xi_1}\rangle \\ &+ \frac{1}{\sqrt{2}}|0\rangle \otimes |\overrightarrow{\xi_2}(t)\rangle \otimes |h;\overrightarrow{\xi_2}\rangle,\end{aligned}\tag{6}$$

where $|h;\overrightarrow{\xi_a}\rangle$ represents the state of gravitons which is labeled by the oscillations of each mirror $a = 1,\ 2$. The norm of $|h;\overrightarrow{\xi_a}\rangle$ is normalized to unity. We ignored the force of radiation reaction of the mirror because the gravitational backreaction is negligible and then the state of each mirror remains unchanged. The point here is that the system of mirrors gets entangled with environmental gravitons due to the matter-gravity interaction. The effect of the bremsstralung of gravitons is expressed by the reduced density operator of the form

$$\begin{aligned}\rho_{\rm m}(t) = &\frac{1}{2}[\rho_{11}(t) + \rho_{22}(t) \\ &+ \exp(i\Phi)\rho_{12}(t) + \exp(-i\Phi^*)\rho_{21}(t)],\end{aligned}\tag{7}$$

where the influence functional $\Phi(t)$ is defined in terms of $|h;\overrightarrow{\xi_a}\rangle$ as

$$\exp(i\Phi) \equiv \langle h;\overrightarrow{\xi_2}|h;\overrightarrow{\xi_1}\rangle. \tag{8}$$

The functional Φ expresses the influence of the gravitons on the reduced system. The imaginary part of the influential functional suppress the interference term,[2] and that is referred to as the decoherence functional $\Gamma = {\rm Im}\Phi$ [20,21]. Once Γ is calculated, we can read off the decoherence timescale, which is given by $\Gamma(t_{\rm f}) = 1$.

The decoherence leads to the loss of the entanglement between the mirror 1 and the mirror 2. To quantify the time

[2]As we will see in Sec. V, this decoherence is induced by the noise of gravitons. The presence of the noise leads to the Langevin equation of geodesic deviation of the mirrors [10,16].

evolution of the entanglement, the entanglement negativity $\mathcal{N}$ as a measure of entanglement is useful. The negativity $\mathcal{N}$ is defined by summing over all the negative eigenvalues of $\rho_{\mathrm{m}}^{\mathrm{T}}(t)$, where $\rho_{\mathrm{m}}^{\mathrm{T}}(t)$ denotes a partial transpose of $\rho_{\mathrm{m}}(t)$. To compute $\mathcal{N}$, we express the density matrix in the form of 4×4 matrix as

$$\rho_{\mathrm{m}}(t)=\frac{1}{2}\begin{pmatrix}0 & 0 & 0 & 0\\ 0 & 1 & e^{i\Phi} & 0\\ 0 & e^{-i\Phi^*} & 1 & 0\\ 0 & 0 & 0 & 0\end{pmatrix}.$$

Taking a partial transpose with respect to the subsystem of mirror 1, we have

$$\rho_{\mathrm{m}}^{\mathrm{T}}(t)=\frac{1}{2}\begin{pmatrix}0 & 0 & 0 & e^{i\Phi}\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ e^{-i\Phi^*} & 0 & 0 & 0\end{pmatrix}. \tag{9}$$

Then, by computing eigenvalues of $\rho_{\mathrm{m}}^{\mathrm{T}}(t)$, we obtain

$$\mathcal{N}=\frac{1}{2}\exp(-\Gamma). \tag{10}$$

By observing the decoherence time or the change of entanglement over time, we could detect gravitons indirectly. This is the key idea of this paper. In the next section, we see how the mirrors interact with gravitons.

III. ACTION FOR THE TOTAL SYSTEM

Let us consider the action for gravitational waves in the Minkowski space first. The metric describing gravitational waves in the transverse traceless gauge is expressed as

$$ds^2=-dt^2+(\delta_{ij}+h_{ij})dx^i dx^j, \tag{11}$$

where t is the time, x_i are spatial coordinates, and δ_{ij} and h_{ij} are the Kronecker delta and the metric perturbations which satisfy the transverse traceless conditions $h_{ij,j}=h_{ii}=0$. The indices (i,j) run from 1 to 3. Substituting the metric Eq. (11) into the Einstein-Hilbert action, we obtain the quadratic action for the metric perturbations,

$$S_g=\frac{M_{\mathrm{p}}^2}{8}\int \mathrm{d}^4x\,[\dot{h}^{ij}\dot{h}_{ij}-h^{ij,k}h_{ij,k}], \tag{12}$$

where the reduced Planck mass is defined by $M_{\mathrm{p}}^{-2}=8\pi G$ and a dot denotes the derivative with respect to the time. We can expand the metric field $h_{ij}(x^i,t)$ in terms of the Fourier modes

$$h_{ij}(x^i,t)=\frac{2}{M_{\mathrm{p}}\sqrt{V}}\sum_{\mathbf{k},B}h_{\mathbf{k}}^B(t)e^{i\mathbf{k}\cdot\mathbf{x}}e_{ij}^B(\mathbf{k}), \tag{13}$$

where we introduced the polarization tensor $e_{ij}^B(\mathbf{k})$ normalized as $e_{ij}^{*B}(\mathbf{k})e_{ij}^C(\mathbf{k})=\delta^{BC}$. Here, the index B denotes the linear polarization modes $B=+,\times$. Note that we consider finite volume $V=L_xL_yL_z$ and discretize the $\mathbf{k}$-mode with a width $\mathbf{k}=(2\pi n_x/L_x,2\pi n_y/L_y,2\pi n_z/L_z)$ where $\mathbf{n}=(n_x,n_y,n_z)$ are integers. Note that we used $k=|\mathbf{k}|$. We then see that a gravitational wave consists of an infinite number of harmonic oscillators.

Next, we consider the action for two mirrors. In this study, we regard the mirror as a point particle effectively because the dynamical degree of the center of mass is essential for discussing the noise of gravitons later. Then, we consider how the particle feels gravitational waves. A single particle does not feel the gravitational waves because of Einstein's equivalence principle at least classically. We need to consider geodesic deviation between the two particles in order to see the effect of gravity on them. We introduce the Fermi normal coordinates for calculating the geodesic deviation between the timelike geodesics of two particles γ_τ and $\gamma_{\tau'}$. The Fermi normal coordinates are local inertial coordinates that are adapted to a geodesic. We expand the coordinate along the timelike and spacelike geodesics that are orthogonal to each other at the position of the beam splitter as shown in Fig. 1, in which the spacelike geodesics are simplified as the (x,y) plane. The dynamics of the γ_τ and $\gamma_{\tau'}$ is described by the position $\xi_a(t)$, $a=1,2$, respectively, in the vicinity of the origin of the beam splitter. The action for a particle along γ_τ is given by

$$\begin{aligned}S_p&=-m\int_{\gamma_\tau}d\tau\\ &=-m\int_{\gamma_\tau}dt\sqrt{-g_{\mu\nu}(t,\xi^i(t))\dot{\xi}^\mu\dot{\xi}^\nu},\end{aligned} \tag{14}$$

where τ is proper time and the position of the particle is represented by $\xi^\mu=(t,\xi^i(t))$. The metric $g_{\mu\nu}$ up to the second order of arbitrary position x^i in the Fermi normal coordinates is computed as

$$\begin{aligned}ds^2\simeq{}&(-1-R_{0i0j}x^ix^j)dt^2-\frac{4}{3}R_{0jik}x^jx^kdtdx^i\\ &+\left(\delta_{ij}-\frac{1}{3}R_{ikj\ell}x^kx^\ell\right)dx^idx^j.\end{aligned} \tag{15}$$

Here, the Riemann tensor is evaluated at the origin $x^i=0$ in the Fermi normal coordinate system. Because the Riemann tensor R_{0i0j} is gauge invariant at the leading order in the metric fluctuation h_{ij}, we can evaluate it in the transverse traceless gauge and obtain $R_{0i0j}(0,t)=-\ddot{h}_{ij}(0,t)/2$. Substituting the metric (15) into the action (14), we can read off the interaction between the particles and gravitational waves as

$$S_{\rm int} \simeq \int dt \left[\frac{m}{4}\ddot{h}_{11}(0,t)\xi_1^{12} + \frac{m}{4}\ddot{h}_{22}(0,t)\xi_2^{22}\right], \quad (16)$$

where x, y components are expressed by 1,2, respectively. We can then derive the Langevin equation of geodesic deviation of the mirrors in the presence of gravitons from this action [10,16]. Note that the geodesic deviation of particles in the graviton background is studied in Refs. [22,23]. Also note that a particle motion in the graviton background is discussed in Ref. [24,25], in which a different form of interaction is used.

IV. DECOHERENCE FUNCTIONAL AND THE DECOHERENCE TIME

A. Quantization and the noise induced by gravitons

Now, we canonically quantize the total system. We work in the interaction picture below. We promote the free metric field $h^A(\mathbf{k},t)$ to the operator $\hat{h}^A(\mathbf{k},t)$ in terms of the creation and annihilation operators such as

$$\hat{h}^A(\mathbf{k},t) = \hat{a}_A(\mathbf{k})u_k(t) + \hat{a}_A^\dagger(-\mathbf{k})u_k^*(t), \quad (17)$$

where the creation and annihilation operators satisfy the standard commutation relations $[\hat{a}_A(\mathbf{k}),\hat{a}^\dagger_{A'}(\mathbf{k}')] = \delta_{\mathbf{k},\mathbf{k}'}\delta_{AA'}$ and $u_k(t)$ denotes a mode function properly normalized as $\dot{u}_k(t)u_k^*(t) - u_k(t)\dot{u}_k^*(t) = -i$. The Minkowski vacuum $|0\rangle$ is defined by $\hat{a}_A(\mathbf{k})|0\rangle = 0$, with choosing the mode function as $u_k(t) = \frac{1}{\sqrt{2k}}e^{-ikt} \equiv u_k^{\rm M}(t)$. From the Langevin equation of geodesic deviation of the mirrors in the presence of gravitons, the noise of gravitons is identified as [16]

$$\hat{N}_{ij}(t) \equiv \frac{1}{M_{\rm p}\sqrt{V}}\sum_A \sum_{\mathbf{k}\le\Omega_{\rm m}} k^2 e^A_{ij}(\mathbf{k})\hat{h}^A(\mathbf{k},t), \quad (18)$$

where $\sum_{\mathbf{k}\le\Omega_{\rm m}}$ represents the mode sum with the UV cutoff. This noise of gravitons always exists if the gravitational waves are quantized.

B. Decoherence functional

Since we found that gravitons are quantum fluctuations of gravitational waves in the form of the noise Eq. (18), we calculate the decoherence functional and then find the necessary experimental setups for obtaining a measurable time of decoherence due to the noise of gravitons. The decoherence functional reads[3] [16,21]

[3]Strictly speaking, when gravitons have nonzero expectation value $\langle\hat{N}_{ij}\rangle \neq 0$, we need to replace $\hat{N}_{ij}$ by $\delta N_{ij} \equiv \hat{N}_{ij} - \langle\hat{N}_{ij}\rangle$ as discussed in Ref. [16]. We have $\langle\hat{N}_{ij}\rangle \neq 0$ when considering squeezed-coherent state, for instance.

$$\Gamma(t_{\rm f}) \approx \frac{m^2}{8}\int_0^{t_{\rm f}} {\rm d}t\Delta(\xi^i\xi^j)(t)\int_0^{t_{\rm f}} {\rm d}t'\Delta(\xi^k\xi^\ell)(t') \times \langle\{\hat{N}_{ij}(t),\hat{N}_{k\ell}(t')\}\rangle. \quad (19)$$

Here, $\Delta(\xi^i\xi^j)(t) \equiv \xi_1^i(t)\xi_1^j(t) - \xi_2^i(t)\xi_2^j(t)$ denotes a difference of $\xi^i(t)\xi^j(t)$ in the superposition. The bracket $\langle\hat{O}\rangle$ denotes an expectation value of an arbitrary operator $\hat{O}$ for a given quantum state. This expression shows that one can compute the decoherence rate due to gravitons once the anticommutator correlation function of $N_{ij}(t)$ is given. Because the state of the primordial gravitational waves becomes squeezed during inflation [7,8], we consider squeezed states $|\zeta\rangle$ below. Although the precise functional form of the anticommutator correlation function of $N_{ij}(t)$ depends on inflationary models, we evaluate it in conventional inflationary scenarios for illustration.

The anticommutator correlation function of $N_{ij}(t)$ in the squeezed state can be computed in the infinite volume limit $L_x, L_y, L_z \to \infty$ as

$$\langle\zeta|\{\hat{N}_{ij}(t),\hat{N}_{k\ell}(t')\}|\zeta\rangle = \left(\delta_{ik}\delta_{j\ell} + \delta_{i\ell}\delta_{jk} - \frac{2}{3}\delta_{ij}\delta_{k\ell}\right)\frac{F(\Omega_m(t-t'))}{10\pi^2 M_{\rm p}^2}, \quad (20)$$

where we defined the anticommutator symbol $\{\cdot,\cdot\}$ as $\{\hat{X},\hat{Y}\} \equiv (\hat{X}\hat{Y} + \hat{Y}\hat{X})/2$ and

$$F(\Omega_m(t-t')) \equiv \int_0^{\Omega_m} {\rm d}k k^6 {\rm Re}[u_k^{\rm sq}(t)u_k^{\rm sq*}(t')]. \quad (21)$$

Here, the mode function in the squeezed state is given in terms of the squeezing parameter r_k and the phase φ_k as

$$u_k^{\rm sq}(t) \equiv u_k^{\rm M}(t)\cosh r_k - e^{-i\varphi_k}u_k^{\rm M*}(t)\sinh r_k. \quad (22)$$

In the conventional inflationary scenario, as to short-wavelength primordial gravitational waves shorter than 10^{-16} Hz, the expected number density in terms of the physical frequency at present is given by [26]

$$N_k|_{k=2\pi f} = \sinh^2 r_k = \frac{1}{4}\left(\frac{f_{\rm c}}{f}\right)^4, \quad (23)$$

where $f_{\rm c}$ is the cutoff frequency of primordial gravitational waves given by

$$f_{\rm c} = 10^9\sqrt{\frac{H}{10^{-4}M_{\rm p}}}\,{\rm Hz}. \quad (24)$$

Then, the graviton density parameter today reads

$$\Omega_{\rm g} \simeq 10^{-14}\left(\frac{H}{10^{-4}M_{\rm p}}\right)^2. \quad (25)$$

Note that the bounds become $f_c \lesssim 4.3 \times 10^{10}$ Hz for alternatives to inflation [26]. Equation (23) leads to $r_k \gg 1$ for $f < f_c$, which explains the large squeezing of primordial gravitons. We then have $\sinh 2r_k \simeq \cosh 2r_k \simeq (f_c/f)^4$. In general, the phase φ_k depends on k, but in this study, we assume $\varphi_k = \pi$ for simplicity. Depending on the actual dependence of φ_k, the effect of the noise correlation may become small. Then, in terms of parameters introduced above, we can evaluate the integral (21) as

$$F(y) = (2\pi f_c)^4 \Omega_m^2 \frac{y \sin y + \cos y - 1}{y^2}. \tag{26}$$

This result clearly shows the non-Markovian nature of the decoherence process, $F(y) \neq 0$ for $y > 0$, meaning that we have nonlocal time correlations.

By substituting Eq. (26) into Eq. (19), we have the decoherence functional (19) in our setup,

$$\begin{aligned}\Gamma = &\frac{m^2}{120\pi^2 M_p^2}\int_0^{t_f} dt \Delta\xi_1^2(t)\int_0^{t_f} dt' \Delta\xi_1^2(t') F(t-t') \\ &+ \frac{m^2}{120\pi^2 M_p^2}\int_0^{t_f} dt \Delta\xi_2^2(t)\int_0^{t_f} dt' \Delta\xi_2^2(t') F(t-t') \\ &- \frac{m^2}{120\pi^2 M_p^2}\int_0^{t_f} dt \Delta\xi_1^2(t)\int_0^{t_f} dt' \Delta\xi_2^2(t') F(t-t'),\end{aligned} \tag{27}$$

where $\Delta\xi_1^2$ and $\Delta\xi_2^2$ are given by

$$\Delta\xi_1^2(t) = (L + A\cos\omega t)^2 - L^2, \tag{28}$$

$$\Delta\xi_2^2(t) = L^2 - (L + A\cos\omega t)^2. \tag{29}$$

Here, we stress that the decoherence process we considered is non-Markovian; that is, nonlocal time correlations of graviton noise are taken into account in (27). This is in contrast to the computation done in the literature [27,28].

Let us evaluate the integrals in Eq. (27). Because the amplitude of oscillation induced by a photon is much smaller than the arm length $A \ll L$, $|\Delta\xi_a^2| \sim 2LA\cos\omega t$, we obtain

$$\Gamma = \frac{m^2}{10\pi^2 M_p^2}(AL)^2 I, \tag{30}$$

where

$$I = \frac{1}{\Omega_m^2}\int_0^{x_f} dx \cos\beta x \int_0^{x_f} dx' \cos\beta x' F(x - x'). \tag{31}$$

Here, we defined the dimensionless time and frequency $x_f = \Omega_m t_f$, $\beta = \omega/\Omega_m$, respectively. In our setup, we have $\beta \ll 1$. Under this condition, we perform the integration and find the following terms become dominant for large x_f,

$$\begin{aligned}\frac{I(x_f)}{(2\pi f_c)^4} \simeq &-\frac{1}{2}\beta x_f \,\mathrm{Si}((1+\beta)x_f) \\ &+ \frac{1}{2}\beta x_f \,\mathrm{Si}((1-\beta)x_f) + \beta x_f \mathrm{Si}(\beta x_f),\end{aligned} \tag{32}$$

where the sine integral is defined by $\mathrm{Si}(x) = \int_0^x \frac{\sin t}{t} dt$ whose value at $x = \infty$ is $\pi/2$. Then, by picking up the leading-order terms of (32) at large x_f, the decoherence functional is found to be

$$\Gamma = \frac{4\pi^3}{5}\left(\frac{m}{M_p}\right)^2 (Lf_c)^4 \left(\frac{A}{L}\right)^2 N, \tag{33}$$

where we defined $N = \omega t_f$. The resultant decoherence functional Γ is independent of the UV cutoff Ω_m. This is expected from the observation that the current setup is insensitive to gravitons whose frequency is higher than $\omega/2\pi$, the frequency of mirrors. This Ω_m independence suggests that the analysis based on the Fermi-normal coordinates is good enough at least in the current setup. Also, we find the linear growth of Γ in N. Although the decoherence is correlated in time (non-Markovian), the decoherence appears Markovian on a timescale larger than the correlation time. This result is expected from the fact that the number of gravitons excited by the mirror oscillations is proportional to the number of oscillations at a late time.

C. Decoherence time

Now, let us estimate the decoherence time. When considering conventional inflation and taking the parameters, $\omega = 1$ kHz, $L = 40$ km, $m = 40$ kg, $f_c = 10^9$ Hz, and $N = 2 \times 10^4$, we obtain $\Gamma \simeq 1$, where the amplitude of oscillation is supposed to be about ten times zero-point fluctuations $A = 10/\sqrt{2m\omega}$ [29]. Then, the decoherence time t_f is approximately 20 s. In this case, the time evolution of negativity is plotted in Fig. 2. If we increase the arm length L by 10^2, the decoherence time becomes 2 ms. For alternatives to inflation ($f_c = 4.3 \times 10^{10}$ Hz

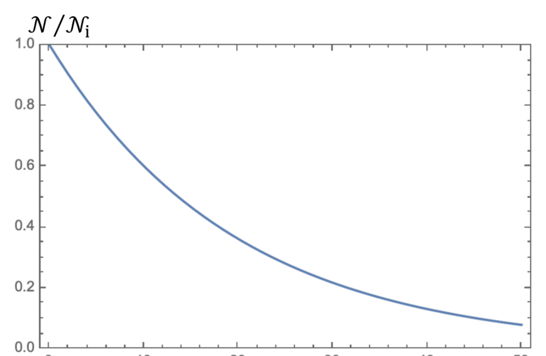

FIG. 2. Time evolution of negativity normalized by the initial value $\mathcal{N}_i$ for $\omega = 1$ kHz, $L = 40$ km, $m = 40$ kg, and $f_c = 10^9$ Hz. The negativity decays with the decoherence time 20 s.

[26]), the decoherence time becomes 6 μs with the same parameters.

V. DISCUSSIONS

In this section, we compare the decoherence time due to gravitons with that due to other possible sources of decoherence. Let us consider decoherence due to interactions with thermal photons and air molecules. The typical wavelength of the thermal photons in room temperature ($T = 300$ K) is much shorter than the size of mirrors. In this case, the decoherence induced by scattering thermal photons is much smaller than the one induced by scattering air molecules. Hence, scattering air molecules becomes the dominant source of decoherence in our setup. In this situation, the decoherence time τ is written as [30]

$$\tau = \frac{1}{\Lambda(\Delta x)^2}, \tag{34}$$

where Δx is a separation of a coherent superposition of the mirror, which is supposed to be about ten times zero-point fluctuations $10/\sqrt{2m\omega}$ in our setup. A parameter Λ is defined by

$$\Lambda = \frac{8}{3\hbar^2} n\sqrt{2\pi M} R^2 (k_B T)^{\frac{3}{2}}. \tag{35}$$

Here, n, M, T, and k_B are the number density of air molecules, the mass of an individual air molecule, the temperature of the environment, and the Boltzmann constant, respectively. The number density is $n \sim 10^{12}$ per 1 m^3 for ultrahigh vacuum 10^{-10} Pa. Let us consider the mirror with the radius $R = 0.17$ m and $m = 40$ kg weight just for reference, which mimics the setups of advanced LIGO (aLIGO) [31]. Then, the decoherence time due to the scattering air molecules is estimated as

$$\tau \sim 1200 \left(\frac{R}{0.17\ \mathrm{m}}\right)^{-2} \left(\frac{T}{10\ \mathrm{K}}\right)^{-\frac{3}{2}} \mathrm{s}. \tag{36}$$

By contrast, the decoherence time due to gravitons is 20 s in the conventional scenario of inflation and 2 ms in scenarios alternative to inflation with the same values of parameters. The difference between the decoherence time induced by gravitons and the one by air molecules becomes more significant if we perform these experiments in space where space pressure will be on the order of 10^{-15} Pa. In this case, the scattering-induced decoherence time is longer than the one in Eq. (36) by factor 10^5. Hence, our setup is not disturbed by the common scattering-induced decoherence, which is usually regarded as dominant sources of decoherence.

We found that the noises of primordial gravitons can dominate over other decoherence sources such as air molecules. This suggests that one can detect primordial gravitons via a loss of entanglement between the mirror 1 and the mirror 2, in principle. As a measure of entanglement, we computed the entanglement negativity $\mathcal{N}$. We need to study how to measure the entanglement. In fact, over the past few years, quantum optomechanics has emerged as a new research field, coupling mechanical oscillators to optical fields. This field makes it possible to create matter entanglement by using interferometers. As the witness of the entanglement, a bell test is performed in Ref [32], and the second-order coherence is measured in Ref. [33], and the authors experimentally observe entanglement between the two remote mechanical oscillators with a confidence level above 99.8%. Also, logarithmic negativity of order 10^{-2} has been observed between mechanical motion and microwave cavity in Ref. [34]. This resolution is equivalent to the detection of negativity of order 10^{-1}. More recently, some more experimental setups are proposed in Ref. [35]. Hence, although their setups are not the same as ours, we expect that it would be possible to witness the entanglement via negativity using such methodology. We will study how to measure the negativity in our setup in the future.

VI. CONCLUSION

We proposed the experimental setup for detecting gravitons indirectly by observing the decoherence time of the entanglement between two macroscopic mirrors suspended at the end of an equal-arm interferometer. It is found that longer arms or heavier mirrors make the decoherence time shorter. In particular, the decoherence time induced by the noise of primordial gravitons stemming from inflation reads approximately 20 s for 40 km long arms and 40 kg mirrors when assuming the conventional inflationary scenario. This timescale can be much shorter than the timescale of the decoherence induced by scatterings between mirrors and air molecules/thermal photons. Our result opens the possibility of detecting gravitons indirectly by using the proposed setup.

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