Helicity polarization in relativistic heavy ion collisions

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We discuss helicity polarization, which can be locally induced from both vorticity and helicity charge in noncentral heavy ion collisions. Helicity charge redistribution can be generated in viscous fluid, and contributes to azimuthal asymmetry of the polarization along global angular momentum or beam momentum. We also discuss detecting the initial net helicity charge from topological charge fluctuation or initial color longitudinal field by the helicity correlation of two hyperons and the helicity alignment of vector mesons in central heavy ion collisions.

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I. INTRODUCTION

The spin polarization effect has drawn much attention in relativistic heavy ion collisions recently. Spin degrees of freedom provide us a unique probe to detect the feature of quark gluon plasma in the quantum level. Much development has been made along this direction on either the experimental aspect [1–8] or the theoretical aspect [9–43]. Some relevant reviews on spin effects in relativistic heavy ion collisions are available in Refs. [44–50]. Most of these works concentrate on the global or local polarization along the global angular momentum (transverse polarization) or beam momentum (longitudinal polarization) for hyperons or vector mesons. In this paper, we will discuss another possible spin polarization along the momentum of final hadrons. In order to distinguish such polarization with the longitudinal polarization along the beam momentum, we denote it as helicity polarization. Other earlier works associated with the helicity in heavy ion collisions can be found in Refs. [51–55].

II. HELICITY POLARIZATION

In relativistic heavy ion collisions, the single-particle mean spin vector $S^\mu(p)$ is given by [14]

$$S^\mu(p) = -\frac{1}{8m}\epsilon^{\mu\nu\rho\sigma}p_\nu \frac{\int d\Sigma_\alpha p^\alpha \varpi_{\rho\sigma} f(1-f)}{\int d\Sigma_\alpha p^\alpha f}, \quad (1)$$

where f is Fermion-Dirac distribution function

$$f = \frac{1}{e^{\beta_\mu p^\mu}+1}, \quad \text{with} \quad \beta_\mu = \frac{u_\mu}{T} \quad \text{and} \quad u^2 = 1, \quad (2)$$

and $\varpi_{\rho\sigma}$ is thermal vorticity

$$\varpi_{\mu\nu} = -\frac{1}{2}(\partial_\mu\beta_\nu - \partial_\nu\beta_\mu). \quad (3)$$

In this paper, we also need the temperature vorticity tensor $\Omega^{\mu\nu}$ defined by

$$\Omega^{\mu\nu} = \partial^\mu(Tu^\nu) - \partial^\nu(Tu^\mu), \quad (4)$$

and its dual tensor

$$\tilde{\Omega}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\Omega_{\rho\sigma}. \quad (5)$$

In previous works [9,23,26], the polarization along the global angular momentum $-y$, impact parameter x, or the beam momentum z are all investigated. Now let us consider the polarization along the direction of the particle's momentum—helicity polarization. We dot the unit 3-vector momentum $\hat{\mathbf{p}}$ with Eq. (1) and obtain

$$S^h \equiv \hat{\mathbf{p}}\cdot\mathbf{S}(p) = \frac{1}{8m}\frac{\int d\Sigma_\alpha p^\alpha \hat{\mathbf{p}}\cdot(\nabla\times\beta\mathbf{u})f(1-f)}{\int d\Sigma_\alpha p^\alpha f}, \quad (6)$$

where the superscript h denotes helicity, $\beta = 1/T$, and $\mathbf{u}$ is the spatial component of 4-vector u^μ. It is very interesting that only the spatial components of vorticity tensor in the lab frame are involved, and the time component does not contribute at all. Hence, we can obtain the information of

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the pure spatial vorticity by measuring the helicity polarization if we neglect the helicity charge at the beginning.

Following the assumption used in Ref. [23] that the temperature vorticity vanishes at all times for ideal uncharged fluid, and that the temperature on the decoupling hypersurface $d\Sigma_\alpha$ only depends on the Bjorken time, Eq. (1) can be reduced into

$$S^\mu(p) = \frac{1}{4mT}\frac{dT}{d\tau}\frac{p_\nu \partial_\sigma^p \int d\Sigma_\alpha p^\alpha f}{\int d\Sigma_\alpha p^\alpha f} \times (\epsilon^{\mu\nu 0\sigma}\cosh\eta - \epsilon^{\mu\nu 3\sigma}\sinh\eta). \tag{7}$$

It is obvious that the first term contributes to the longitudinal polarization which has been fully discussed by Becattini and Karpenko in Ref. [23], and it can be approximated at rapidity $Y = 0$ as

$$S^z \approx \frac{1}{mT}\frac{dT}{d\tau} v_2(p_T)\sin 2\phi, \tag{8}$$

when only the elliptic flow term v_2 is retained. The second term contributes to the helicity polarization and can be approximated as

$$S^h \approx \frac{Y}{mT}\frac{dT}{d\tau} v_2(p_T)\sin 2\phi = YS^z, \tag{9}$$

at small rapidity. Under such approximation and neglecting the dependence on Y for S_z, we note that the helicity polarization must be rapidity odd and vanish at $Y = 0$, which is different from the longitudinal polarization. Hence, we should detect the helicity polarization locally with $Y > 0$ or $Y < 0$ separately.

III. HELICITY CHARGE REDISTRIBUTION

The helicity charge could contribute to helicity polarization as well. For simplicity, we will restrict ourselves to the chiral limit so that we can use the chiral kinetic theory to deal with it. At the chiral limit, the helicity coincides with the chirality, except for a trivial opposite sign for antiparticles. In the chiral kinetic theory, the polarization vector of fermion particles are proportional to the axial component of the Wigner function. In a chiral system with free fermions in local equilibrium, the axial component of the Wigner function is given by [56]

$$\mathcal{A}^\mu(x,p) = \frac{p^\mu}{E_p}\sum_\lambda \lambda f_\lambda - \frac{\epsilon^{\mu\nu\rho\sigma}p_\nu \varpi_{\rho\sigma}}{4E_p}\sum_\lambda f_\lambda(1-f_\lambda), \tag{10}$$

where $E_p = |\mathbf{p}|$ and $\lambda = \pm 1$ denotes the particle's helicity or chirality with right hand ($\lambda = +1$) or left hand ($\lambda = -1$). Here f_λ represents the Fermi-Dirac distribution with helicity λ

$$f_\lambda = \frac{1}{e^{\beta_\mu p^\mu - \lambda\bar{\mu}_5} + 1}, \tag{11}$$

where $\bar{\mu}_5 = \mu_5/T$ with μ_5 being axial chemical potential. When there is no axial charge with $\mu_5 = 0$, the first term will vanish and the second term will give rise to the polarization in Eq. (1). However, the chiral separation effect or the local polarization effect from the vorticity [57] can induce a redistribution of axial charge, and initial zero axial charge can evolve into a dipole distribution along the vorticity direction. Then the axial charge can exist locally. If we assume the axial charge density or axial chemical potential $\bar{\mu}_5$ is small, we can neglect the axial chemical potential in the second term in Eq. (10) because this term is the first order contribution, but we should keep the linear term of $\bar{\mu}_5$ in the first term because it is actually from the zeroth order contribution.

Now let us consider how the axial charge can be separated and redistributed by using relativistic hydrodynamics. Since the axial charge is zero initially, we can deal with the evolution of the axial charge in the background of the relativistic uncharged hydrodynamics. When there are no conserving charges, the relativistic hydrodynamic equation is just energy-momentum conservation,

$$\partial_\nu T^{\mu\nu} = 0, \tag{12}$$

where $T^{\mu\nu}$ is the energy-momentum tensor and the constitutive equation is given by

$$T^{\mu\nu} = (\varepsilon + P)u^\mu u^\nu - Pg^{\mu\nu} + \pi^{\mu\nu}, \tag{13}$$

where ε is the energy density, P is the pressure of the fluid, and the symmetric tensor $\pi^{\mu\nu}$ has all possible dissipative terms such as shear tensor or bulk tensor and so on. In Landau frame, the dissipative tensor $\pi^{\mu\nu}$ is orthogonal to the fluid velocity, i.e., $\pi^{\mu\nu}u_\nu = \pi^{\mu\nu}u_\mu = 0$. From the hydrodynamic equation (12), we can obtain the following equation directly,

$$u^\nu\partial_\nu(Tu^\mu) - \partial^\mu T = -\frac{1}{s}\Delta^{\mu\alpha}\partial^\nu\pi_{\alpha\nu}, \tag{14}$$

where $s = (\varepsilon + P)/T$ is entropy density in a fluid comoving frame. With the definition of temperature vorticity in Eq. (4), this equation implies

$$\Omega^{\nu\mu}u_\nu = -\frac{1}{s}\Delta^{\mu\alpha}\partial^\nu\pi_{\alpha\nu}. \tag{15}$$

When there is no electromagnetic field imposed on the chiral system, the axial current is conserved as

$$\partial_\mu j_5^\mu = 0. \tag{16}$$

From the well-known constitutive equation of axial current [57,58]

$$j_5^\mu = n_5 u^\mu + \frac{1}{2}\xi_5 \epsilon^{\mu\nu\rho\sigma} u_\nu \partial_\rho u_\sigma, \tag{17}$$

where for the free fermion system, the axial charge density n_5 and anomalous transport coefficient ξ_5 are given by

$$n_5 = \frac{1}{3}\mu_5 T^2, \qquad \xi_5 = \frac{1}{6}T^2, \tag{18}$$

we have neglected the high order term of μ_5. Substituting Eq. (17) into Eq. (16) gives rise to

$$\partial_\mu (n_5 u^\mu) = -\frac{1}{24}\Omega_{\mu\nu}\tilde{\Omega}^{\mu\nu}. \tag{19}$$

It is very interesting to note that the equation (19) is very similar to the chiral anomaly of the axial current induced from the electromagnetic field in QED, with the electromagnetic field tensor replaced by the temperature vorticity tensor up to a constant factor. By using Eq. (15), we obtain

$$\begin{aligned} \partial_\mu (n_5 u^\mu) &= -\frac{1}{6s}\omega_{T\mu}\partial_\nu \pi^{\mu\nu} \\ &\approx -\frac{1}{6s}\omega_{T\mu}\Delta_{\nu\lambda}(\partial^\lambda \pi^{\mu\nu} - \pi^{\mu\nu}\partial^\lambda \ln T), \end{aligned} \tag{20}$$

where we have introduced the temperature vorticity vector $\omega_{T\mu} = \tilde{\Omega}_{\mu\nu} u^\nu$. In order to arrive at the final equation in Eq. (20), we have used Eq. (14) again and neglected the dissipative terms. This result shows that the redistribution of axial charge in uncharged fluid only happens for viscous fluid from initial zero axial charge. The redistribution can be generated from the coupling between the spatial vorticity and spatial gradient of dissipative tensor, or between the spatial vorticity, spatial gradient of temperature, and dissipative tensor.

When the axial charge is redistributed, the first term in Eq. (10) will lead to extra contribution for the local helicity polarization as

$$\Delta S^h(p_T, Y, \phi) \approx \frac{\int d\Sigma_\alpha p^\alpha \bar{\mu}_5 f(1-f)}{\int d\Sigma_\alpha p^\alpha f}. \tag{21}$$

Hence, the final polarization depends on the final distribution of the axial chemical potential, and this can be determined by the relativistic hydrodynamical simulation. From the axial charge redistribution, we can also obtain the extra contribution to the spin polarization along x, y, and z axis, respectively,

$$\begin{aligned} \Delta S^x(p_T, Y, \phi) &= \frac{\cos\phi}{\cosh Y}\Delta S^h(p_T, Y, \phi), \\ \Delta S^y(p_T, Y, \phi) &= \frac{\sin\phi}{\cosh Y}\Delta S^h(p_T, Y, \phi), \\ \Delta S^z(p_T, Y, \phi) &= \tanh Y \Delta S^h(p_T, Y, \phi), \end{aligned} \tag{22}$$

where ϕ is the azimuthal angle of the particle's momentum. It should be noted that in Refs. [28,33], the authors simulated the spin polarization with axial charge redistribution from the chiral kinetic equation, and found that the quark local spin polarizations exhibit an azimuthal asymmetry similar to the experimental data for the Λ hyperon. Then it will be very valuable to investigate the spin polarization from the helicity charge redistribution in the scenario of relativistic hydrodynamics.

IV. HELICITY CORRELATION

In the previous discussion, we only restricted ourselves to the axial charge induced locally by redistribution from initial zero value. In the heavy ion collisions at very high energy, the net helicity could exist at the beginning due to classical color longitudinal fields just after the collision [59,60] or QCD sphaleron transitions in the quark gluon plasma [61–65]. Such net initial axial charge could lead to the well-known chiral magnetic effect [66–68], and result in electric charge separation along the angular momentum in noncentral heavy ion collisions. Because the net initial axial charge with positive and negative sign should be produced with equal probability in many events, the helicity polarization of one particle after averaging over these different events will vanish. However, we can detect this net axial charge by measuring the helicity correlation of two hyperons event-by-evently [17,55].

As we all know, the polarization of hyperons can be determined from the angular distribution of hyperon decay products in the hyperon rest frame

$$\frac{dN}{d\cos\theta^*} = \frac{1}{2}(1 + \alpha_H P_H^h \cos\theta^*), \tag{23}$$

where θ^* is the angle between the polarization direction (here it is just the hyperon's momentum in the lab frame) and the momentum of the hyperon's decay products in the hyperon's rest frame, and P_H^h denotes the helicity polarization of hyperon H. Now, we choose two hyperons H_1 and H_2 which must be in the same event, and calculate the average value event by event:

$$\begin{aligned} C_{H_1 H_2} &\equiv \left\langle \frac{dN_1}{d\cos\theta_1^*}\frac{dN_1}{d\cos\theta_2^*}\cos\theta_1^*\cos\theta_2^* \right\rangle \\ &= \frac{1}{4}\alpha_{H_1}\alpha_{H_2}P_{H_1}^h P_{H_2}^h. \end{aligned} \tag{24}$$

If the two hyperons are the same, then the correlation will be given by

$$C_{HH} = \frac{1}{4}\alpha_H^2 (P_H^h)^2, \tag{25}$$

which is positive. If the two hyperons are particle and antiparticle, assuming the polarization for them is the same, then the correlation will be given by

$$C_{H\bar{H}} = -\frac{1}{4}\alpha_H^2 (P_H^h)^2, \tag{26}$$

which is negative due to $\alpha_H = -\alpha_{\bar{H}}$. In measuring such initial helicity polarization, we do not need to determine the reaction plane and we can even detect it in central heavy ion collisions. After taking an average over many events without determining the reaction plane, we expect all the spin polarization from the vorticity or the helicity charge redistribution will vanish and only initial net helicity charge survives. Helicity polarization correlation measures the global helicity polarization.

V. HELICITY ALIGNMENT

Similarly, we can measure the spin alignment along the direction of the vector meson's momentum, which can be denoted by the helicity alignment. The spin alignment for vector mesons can be described by a Hermitian 3×3 spin-density matrix ρ. The matrix element ρ_{00} can be determined from the angular distribution of the decay products [69],

$$\frac{dN}{d\cos\theta^*} = \frac{3}{4}[1 - \rho_{00} + (3\rho_{00} - 1)\cos^2\theta^*], \tag{27}$$

where we take the vector meson's momentum direction $\hat{\mathbf{p}} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$ as the quantization axis. When ρ_{00} deviates from $1/3$, spin alignment will arise. We assume that the spin density matrix for quarks or antiquarks is diagonal as [10]

$$\rho^h_{q/\bar{q}} = \frac{1}{2}\begin{pmatrix} 1 + P^h_{q/\bar{q}} & 0 \\ 0 & 1 - P^h_{q/\bar{q}} \end{pmatrix}, \tag{28}$$

where $P^h_{q/\bar{q}}$ denotes the polarization for quarks or antiquarks along the particle's momentum. In the recombination scenario, we assume that the quark and antiquark combine into a vector meson only when their momentum is along a similar direction. For simplicity, we will set the quark and antiquark in the same direction. Then we can obtain the density matrix for vector meson from Eq. (28) [10]

$$\rho = \begin{pmatrix} \rho_{11} & 0 & 0 \\ 0 & \rho_{00} & 0 \\ 0 & 0 & \rho_{-1-1} \end{pmatrix}, \tag{29}$$

where

$$\rho_{11} = \frac{(1 + P_q^h)(1 + P_{\bar{q}}^h)}{3 + P_q^h P_{\bar{q}}^h}, \tag{30}$$

$$\rho_{00} = \frac{1 - P_q^h P_{\bar{q}}^h}{3 + P_q^h P_{\bar{q}}^h}, \tag{31}$$

$$\rho_{-1-1} = \frac{(1 - P_q^h)(1 - P_{\bar{q}}^h)}{3 + P_q^h P_{\bar{q}}^h}. \tag{32}$$

Then the deviation from $1/3$ for ρ_{00} is given by

$$\Delta \equiv \rho_{00} - \frac{1}{3} = \frac{1 - P_q^h P_{\bar{q}}^h}{3 + P_q^h P_{\bar{q}}^h} - \frac{1}{3} \approx -\frac{4}{9} P_q^h P_{\bar{q}}^h, \tag{33}$$

when the polarization of the quark or antiquark is very small. Given the helicity density matrix, we can calculate the spin density matrix along any spin-quantization axis and obtain the spin alignment along these different directions. We use ρ^x, ρ^y, and ρ^z to denote the density matrices when we quantize the spin in x, y, and z axis, respectively. The matrix elements ρ^x_{00}, ρ^y_{00}, and ρ^z_{00} are given by

$$\rho^x_{00} = \frac{1 - \rho_{00}}{2} + \frac{3\rho_{00} - 1}{2}\sin^2\theta\cos^2\phi, \tag{34}$$

$$\rho^y_{00} = \frac{1 - \rho_{00}}{2} + \frac{3\rho_{00} - 1}{2}\sin^2\theta\sin^2\phi, \tag{35}$$

$$\rho^z_{00} = \rho_{00} + \frac{1 - 3\rho_{00}}{2}\sin^2\theta. \tag{36}$$

We note that

$$\rho^x_{00} + \rho^y_{00} + \rho^z_{00} = 1. \tag{37}$$

Actually, this identity holds for any normalized spin-1 density matrix, i.e., the sum of the 00-components of normalized spin-density matrices in three orthogonal spin-quantization axes must be unity. With the diagonal density matrix (29) and the results (34)–(36), we can easily obtain the 00-component of the transverse density matrix along any direction orthogonal to the particle's momentum

$$\rho^T_{00} = \frac{1 - \rho_{00}}{2}. \tag{38}$$

In heavy ion collisions, we can choose this transverse direction as the normal vector of the reaction plane. If we assume the momentum distribution of the vector meson in the form

$$E\frac{dN}{d^3p} = N_0(1 + 2v_2\cos 2\phi), \tag{39}$$

where only the elliptic flow v_2 is retained. Then the averaged ρ^x_{00}, ρ^y_{00}, and ρ^z_{00} at $Y = 0$ are given by, respectively,

$$\bar{\rho}^x_{00} = \frac{1}{3} + \frac{1}{4}\Delta + \frac{3}{4}v_2\Delta, \tag{40}$$

$$\bar{\rho}^y_{00} = \frac{1}{3} + \frac{1}{4}\Delta - \frac{3}{4}v_2\Delta, \tag{41}$$

$$\bar{\rho}_{00}^{z} = \frac{1}{3} - \frac{1}{2}\Delta. \quad (42)$$

These results explicitly show how the helicity alignment from helicity charge can result in the spin alignment along the transverse or longitudinal directions.

Similar to the polarization correlation, the deviation is also proportional to the square of the quark's polarization. However, the difference between them is that the helicity polarization correlation must be from the average or net global helicity charge, yet the spin alignment could be from the local polarization or event-by-event fluctuation [35,36]. The relevant quantity for hyperon polarization correlation is the square of mean polarization $\langle P_{q/\bar{q}} \rangle^2$, while the relevant quantity for spin alignment is the average of the square of polarization $\langle P_{q/\bar{q}}^2 \rangle$. Hence, in general the spin alignment should be larger than the polarization for the hyperon. Because of this local polarization effect or fluctuation, it is hard to detect the initial net helicity charge by helicity alignment, even in central heavy ion collisions, if there exists strong topological charge fluctuation. The strong alignment measured by STAR and ALICE, which is not consistent with hyperon polarization, might be due to the fact that the global polarization is very small while the local polarization fluctuation is very large.

VI. SUMMARY AND OUTLOOK

We have discussed the possible spin polarization along the direction of the detected hadrons—helicity polarization. We find that the vorticity produced in noncentral heavy-ion collisions can also lead to the helicity polarization, which could emerge locally with azimuthal asymmetry, and at approximately rapidity-odd dependence. The viscous hydrodynamics could induce the temperature vorticity from initial zero value and lead to the helicity charge redistribution. Such helicity charge redistribution can result in helicity polarization, and longitudinal or transverse polarization as well. When the topological charge or initial color longitudinal field fluctuates event by event, the global net helicity charge could be measured by the correlation of helicity polarization among two hyperons, and local helicity charge fluctuation could be measured by the spin alignment along the vector meson momentum in central heavy ion collisions at very high energies.

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