Neutrino as the dark force

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We point out a novel role for the Standard Model neutrino in dark matter phenomenology where the exchange of neutrinos generates a long-range potential between dark matter particles. The resulting dark matter self interaction could be sufficiently strong to impact small-scale structure formation, without the need of any light dark force carrier. This is a generic feature of theories where dark matter couples to the visible sector through the neutrino portal. It is highly testable with improved decay rate measurements at future Z, Higgs, and τ factories, as well as precision cosmology.

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Dark matter (DM) is a key ingredient throughout the evolution of our universe, yet its identity remains unknown. The nature of DM is under careful scrutiny at various experimental frontiers from laboratories to the cosmos, and some hints already exist. Nongravitational self-interaction of DM could compete with gravity and impact the formation of structures. Such a new force can help alleviating tensions between numerical simulations and the observed small-scale structure of the universe, known as the "core-cusp" and "too big to fail" problems [1–3]. It could also yield important consequences such as seeding supermassive black hole formation [4–8]. The dynamics of self-interacting particle DM have been explored in a broad range of theories [9–25], which typically host more degrees of freedom than the DM itself. Light dark force carriers are often introduced to mediate the DM self interaction whose potential imprint on the visible universe is tightly constrained [26–30]. A separate small-scale challenge, known as the "missing satellite" problem [31–34], favors warm DM candidates that can erase heretofore-unobserved small structures [35–41]. Although these puzzles might be relaxed with known physics such as baryonic feedback [42–47], they serve as good motivations for building and testing novel DM models.

Because neutrinos are the lightest known particles other than the photon and their properties remain to be fully understood, it is natural to speculate on the possible role of neutrinos to address the above puzzles. In this article, we demonstrate that DM self-interactions can be mediated exclusively by Standard Model (SM) neutrinos, without the introduction of light dark force carriers. This is a generic possibility within the class of neutrino portal theories. There are several attractive outcomes. First, DM self interaction proceeds via the exchange of two neutrinos. At separations shorter than the inverse neutrino mass, the potential governing DM self interaction is long range, of the form $1/r^5$. For asymmetric DM, the interaction is repulsive, and the low-energy scattering can be solved within quantum mechanics, independent of short distance physics. Second, the DM-neutrino interaction establishes a thermal history of the dark states and allows robust constraints to be set on their mass scale. It could also keep the two species in kinetic equilibrium for an extended period, enabling the warm DM scenario. Last, unlike many dark sector models that are secluded from the visible sector, the DM candidate considered here must interact with known particles through neutrinos. It is highly testable by precision SM decay rate measurements. Laboratory and cosmological measurements provide complementary future probes of such a novel target.

Our starting point is the effective interacting Lagrangian

$$\mathcal{L}_{\rm int} = \frac{(\bar{L}_\alpha H)(\chi\phi)}{\Lambda_\alpha} + \text{H.c.}, \tag{1}$$

where $L_\alpha (\alpha = e, \mu, \tau)$ is a SM lepton doublet in the flavor basis and H is the Higgs doublet. The dark fermion χ and scalar ϕ are SM gauge singlets but charged under a global $U(1)$ or Z_2 symmetry. The lighter is stable and serves as the DM candidate, which we assume to be χ hereafter. The operator is dimension five, having a cutoff scale Λ_α. Interestingly, this neutrino portal operator has been introduced and explored for a number of other motivations [41,48–56]. Below the electroweak symmetry scale, a Yukawa interaction is generated between the neutrino and dark particles,

$$\mathcal{L}_{\rm int} = y_\alpha \bar{\nu}_\alpha \chi\phi + \text{H.c.}, \tag{2}$$

where $y_\alpha = v/\sqrt{2}\Lambda_\alpha$ and $v = 246$ GeV is the vacuum expectation value of the Higgs field.

I. DARK MATTER SELF-INTERACTION

We first explore DM self interaction of relevance to structure formation. Because the DM particles are already nonrelativistic when they self interact in galaxies and clusters, the heavier ϕ field could be integrated out from Eq. (2), leading to

$$\mathcal{L}_{\rm int} \simeq \frac{|y_\alpha|^2}{2(m_\phi^2 - m_\chi^2)} (\bar\chi\gamma^\mu \mathbb{P}_R \chi)(\bar\nu_\alpha \gamma_\mu \mathbb{P}_L \nu_\alpha), \quad (3)$$

where $\mathbb{P}_{L,R} = (1 \mp \gamma_5)/2$ and $m_{\chi,\phi}$ are the masses of the dark states. This effective Lagrangian is valid when the mass difference between ϕ and χ is much larger than the energy/momentum transfer carried by neutrinos. The mass square difference factor downstairs captures an enhancement effect when the dark state masses are near. The nonrelativistic matrix element for $\chi\chi \to \chi\chi$ scattering via two-neutrino exchange (Fig. 1) is

$$i\mathcal{M}(t) = \frac{-i|y_\alpha|^4}{24\pi^2(m_\phi^2 - m_\chi^2)^2}\left(\frac{1}{\epsilon} + \log\frac{\mu^2}{-t} + \frac{5}{3}\right)t, \quad (4)$$

where $t = q^2 < 0$ with q^μ the four-momentum transfer and μ is the renormalization scale. The ultraviolet (UV) divergence ($\epsilon \to 0$) is regularized in the full theory (including ϕ).

The long-range part of the potential after resumming the multiple two-neutrino exchange can be derived using the dispersion theory technique [57–59],

$$V(r) = \int \frac{d^3q}{(2\pi)^3} \frac{-1}{2\pi i} e^{-i\mathbf{q}\cdot\mathbf{r}} \int_0^\infty dt' \frac{{\rm disc}\mathcal{M}(t')}{t' - q^2} = \frac{|y_\alpha|^4}{128\pi^3(m_\phi^2 - m_\chi^2)^2 r^5}, \quad (5)$$

where ${\rm disc}\mathcal{M}(t')$ is the discontinuity of $\mathcal{M}$ across its branch cut in the complex plane of t'. A similar potential from neutrino exchange within the SM has also been explored [60–65]. At very short distances where ϕ cannot be integrated out, and at very large distances where neutrino masses are non-negligible, the $1/r^5$ form of the above potential will break down [66]. Thanks to the repulsive nature of the potential and the nonrelativistic nature of DM scattering considered here, the probability for two DM particles to find each other at distances $r \lesssim m_\phi^{-1}$ is highly suppressed. More concretely, the probability density $r^2 R(r)^2$ (where R is the radial wavefunction) scales as r^2 in this region.[1] This motivates us to proceed by assuming only χ is present in the universe, which coincides with the asymmetric DM idea [67].[2] In addition, the potential energy at $r \gtrsim m_\nu^{-1}$ (inverse of neutrino mass) is much lower than the typical DM kinetic energy in galaxies and clusters. A direct comparison shows $V(r = m_\nu^{-1})/K \sim 10^{-37}(10\ {\rm MeV}/m_\chi)^5$ for an order one coupling y, where $K = m_\chi v^2/2 \sim 10^{-6} m_\chi$ is the typical dark matter kinetic energy in galaxies. Based on these observations, we conclude it is a good approximation to simply consider the potential in Eq. (5) for the discussions here.

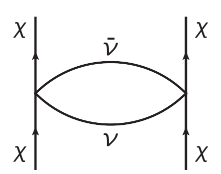

FIG. 1. Two-neutrino exchange diagram that can generate a long-range potential between DM particles.

The low energy scattering problem for a repulsive $1/r^5$ potential is well defined in quantum mechanics. It is free from UV dependence in spite of being singular [69,70]. In Refs. [71,72], the analytic expression of the scattering phase shift has been derived for all partial waves. In particular, the S-wave phase shift takes the form

$$\tan\delta_0 = \frac{3^{-2/3}\Gamma(-1/3)}{\Gamma(1/3)} f^{5/3} + \mathcal{O}(f^5), \qquad f = k^{3/5}\left[\frac{\mu_{\chi\chi}|y_\alpha|^4}{64\pi^3(m_\phi^2 - m_\chi^2)^2}\right]^{1/5}, \quad (6)$$

where Γ is the Euler gamma function, $\mu_{\chi\chi} = m_\chi/2$ is the reduced mass of the $\chi\chi$ system, and k is the relative momentum of scattering. For nonrelativistic DM and perturbative values of y_α, we find the expansion parameter $f \ll 1$. Thus, the $f^{5/3}$ term dominates. Higher partial wave ($\ell \geq 1$) phase shifts begin from order f^5 or higher and are not important. The scattering cross section is S-wave dominated and well approximated by

$$\sigma_{\chi\chi\to\chi\chi} \simeq \frac{4\pi}{k^2}\sin^2\delta_0 \simeq 0.027\left[\frac{m_\chi|y_\alpha|^4}{(m_\phi^2 - m_\chi^2)^2}\right]^{2/3}. \quad (7)$$

The resulting cross section is insensitive to the relative velocity as long as $f \ll 1$. This implies the same prediction of the DM self-interaction cross section applies to various astrophysical objects, from dwarf galaxies to clusters. It is important to note that the Born approximation does not work. Indeed, the resulting cross section goes as $|y_\alpha|^{8/3}$ rather than $|y_\alpha|^8$, indicating the importance of resumming multiple neutrino bubble exchange contributions [71]. This is the key for generating a sizable DM interaction in spite of

[1] To numerically solve the Schrödinger equation, the potential energy term needs to be regularized near the origin (at $r \ll m_\phi^{-1}$). We find the resulting probability and cross section are independent of the regularization scheme.

[2] Had we considered symmetric DM, the $\chi\bar\chi$ interaction potential would be attractive and require detailed knowledge of short distance physics [68].

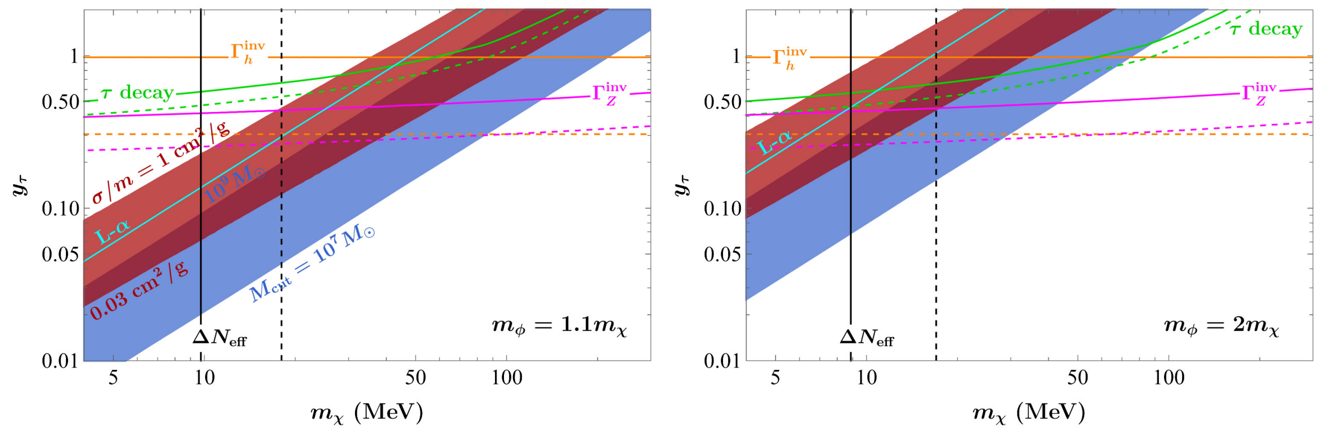

FIG. 2. The region of parameter space where the long-range force due to neutrino exchange can generate a sufficiently large DM self-interaction cross section for addressing puzzles in small-scale structure formation. The dark red shaded band corresponds to $0.03\ \mathrm{cm}^2/\mathrm{gram} \le \sigma_{\chi\chi\to\chi\chi}/m_\chi \le 1\ \mathrm{cm}^2/\mathrm{gram}$. The dark blue band corresponds to χ serving as a warm DM. The two regions intersect in the darkest shaded region. The cyan curve sets an upper bound on neutrino-χ interactions from Lyman-α. The lower bound on the DM mass set by $\Delta N_{\rm eff}$ is shown by the vertical black line. Existing upper bounds on the Yukawa coupling $|y_\tau|$ are set from the invisible decays of the Z boson (magenta curve) and Higgs boson (orange curve) and the leptonic decay of the τ (green curve). Current (projected) bounds are solid (dashed). All bounds are at 95% confidence level.

the loop-level origin of the potential, Eq. (5). Numerically, we have verified the above results by solving the Schrödinger equation using the shooting method (see, e.g., [9,73]).

With the cross section in Eq. (7), we derive the parameter space for self-interacting DM. In Fig. 2, the red shaded band corresponds to $0.03\ \mathrm{cm}^2/\mathrm{gram} \le \sigma_{\chi\chi\to\chi\chi}/m_\chi \le 1\ \mathrm{cm}^2/\mathrm{gram}$, potentially relevant for solving the various small-scale puzzles [74]. The upper bound is set by the Bullet Cluster observation [75–77]. Here we focus on the coupling of the ν_τ neutrino with the dark sector, which receives the least constraints compared to other flavor choices (see discussions below). In the left and right panels, we choose the mass ratios $m_\phi/m_\chi = 1.1$ and 2, respectively. When m_ϕ and m_χ are closer, the effective coupling in Eq. (3) is more enhanced, allowing for smaller values of y_α.

II. OVERLAP WITH WARM DARK MATTER

The interaction of Eq. (2) has another significant cosmological implication. A sufficiently large y_α can keep DM and neutrinos in kinetic equilibrium with each other for an extended period of time, leading to a suppressed DM density power spectrum at small length scales via collisional damping. The cutoff mass scale of the smallest gravitationally bound DM halo is [41]

$$M_{\rm cut} \simeq 10^8\ M_\odot \left[\frac{|y_\alpha|}{0.3}\right]^3 \left[\frac{20\ \mathrm{MeV}}{m_\chi}\right]^{\frac{3}{4}} \left[\frac{26\ \mathrm{MeV}}{\sqrt{m_\phi^2 - m_\chi^2}}\right]^3, \quad (8)$$

where $M_\odot$ is the solar mass. With $10^7\ M_\odot \lesssim M_{\rm cut} \lesssim 10^9\ M_\odot$, χ is a warm DM candidate and can shed light on the "missing satellite" problem. The favored parameter space is depicted by the blue shaded band in Fig. 2. Remarkably, there is an overlap with the self-interacting DM region, allowing all puzzles in small-scale structure formation to be tied to this simple framework.

The neutrino-DM interaction can also be constrained by larger scale probes, including the cosmic microwave background (CMB) [78–80] and large scale structure [81,82]. The constraint from Lyman-α [81] is the strongest among these, setting an upper limit on the elastic scattering cross section of $\sigma_{\rm el}/m_\chi < 10^{-36}\ \mathrm{cm}^2/\mathrm{MeV}$ for neutrino energies of around 100 eV. This bound is shown by the cyan curve in Fig. 2. Other constraints from higher energy neutrinos, e.g., those detected from SN1987A [83] or at IceCube [84–87], do not set a bound on the plotted parameter space.

III. EARLY UNIVERSE CONSTRAINTS

To derive the above self-interaction results, we have made the assumption that DM is asymmetric. This has the advantage of making the DM self interaction repulsive, thus the low energy observables are free from UV dependence. A number of dark sector asymmetry generation options exist which would extend the above minimal setup [88,89]. Here we show the strength of the DM-neutrino interaction is compatible with such an assumption. When the temperature of the universe is higher than the χ and ϕ masses, the Yukawa interaction of Eq. (2) thermalizes them with neutrinos. The key cross section for annihilating away the $\bar{\chi}$ particles through a t-channel ϕ exchange is

$$(\sigma v_{\rm Ml})_{\chi\bar{\chi}\to\nu\bar{\nu}} = \frac{|y_\alpha|^4 m_\chi^2}{32\pi(m_\chi^2 + m_\phi^2)^2}. \quad (9)$$

The annihilation of $\phi\phi^* \to \nu\bar{\nu}$ is P-wave suppressed. For $m_\phi \sim m_\chi$, the condition for efficiently depleting the symmetric population (i.e., $(\sigma v_{\rm Ml})_{\chi\bar{\chi}\to\nu\bar{\nu}} \gg 3 \times 10^{-26}\ {\rm cm}^3/{\rm s}$) corresponds to

$$|y_\alpha| \gg 0.004 \left(\frac{m_\chi}{10\ {\rm MeV}}\right)^{1/2}. \tag{10}$$

Clearly, this requirement is easily satisfied for the coupling values of interest in Fig. 2. There is also a requirement from particle asymmetry transfer considerations. Because χ is light, its asymmetry is much larger than the baryon asymmetry. The neutrino portal operator in Eq. (1) can convert the χ asymmetry into a neutrino-antineutrino asymmetry. To avoid overproducing the cosmic baryon asymmetry, the χ-asymmetry-generating mechanism must occur after the electroweak phase transition with the sphaleron process turned off.

There is an important constraint on the lightness of DM from $\Delta N_{\rm eff}$, the excess radiation degrees of freedom in the universe, during the big bang nucleosynthesis and recombination epochs [90–92]. To support a $U(1)$ stabilizing symmetry for the dark sector, we must assume χ is a Dirac fermion and ϕ a complex scalar. For the two mass ratios considered in Fig. 2, lower limits on the χ mass are 9.7 and 8.9 MeV, respectively, applying a conservative 2σ limit $\Delta N_{\rm eff} < 0.5$ [93,94]. The upcoming CMB Stage-IV experiment [95] can probe the DM mass up to 18 and 17 MeV, respectively.

IV. LABORATORY CONSTRAINTS

The interaction strength of the neutrino portal to the dark sector can be probed by a number of precision measurements of known particles. The effective operator of Eq. (1) contributes to the invisible decay width of the Higgs boson,

$$\Gamma_{h\to\bar{\nu}_\alpha\chi\phi} = \frac{|y_\alpha|^2 G_F m_h^3}{256\sqrt{2}\pi^3} \int_{(\sqrt{z_\chi}+\sqrt{z_\phi})^2}^1 dx f_h(x, z_\chi, z_\phi),$$
$$f_h(x, z_\chi, z_\phi) = x^{-2}(1-x)^2(x - z_\phi + z_\chi) \times \sqrt{x^2 - 2x(z_\chi + z_\phi) + (z_\chi - z_\phi)^2}, \tag{11}$$

where $z_\chi = m_\chi^2/m_h^2$ and $z_\phi = m_\phi^2/m_h^2$. The f_h integral evaluates to 1/3 in the limit $z_\chi = z_\phi = 0$. An upper bound on y_α is derived by requiring the branching ratio of this decay (adding the charge-conjugation channel) to be less than 24% at 95% confidence level (CL) [96,97], as shown by the horizontal orange curve in Fig. 2. An optimistic projected sensitivity for the Higgs invisible decay branching ratio at the HL-LHC of 3% [98] would strengthen this limit to $|y_\alpha| < 0.3$.

The Yukawa interaction of Eq. (2) contributes to the invisible decay width of the Z boson,

$$\Gamma_{Z\to\bar{\nu}_\alpha\chi\phi} = \frac{|y_\alpha|^2 G_F m_Z^3}{768\sqrt{2}\pi^3} \int_{(\sqrt{z_\chi}+\sqrt{z_\phi})^2}^1 dx f_Z(x, z_\chi, z_\phi),$$
$$f_Z(x, z_\chi, z_\phi) = x^{-1}(2+x) f_h(x, z_\chi, z_\phi), \tag{12}$$

where here $z_\chi = m_\chi^2/m_Z^2$ and $z_\phi = m_\phi^2/m_Z^2$. The existing constraint on the Z boson invisible width requires new physics contribute $\Gamma < 2.8$ MeV at 95% CL [99], setting an upper bound on y_α as shown by the magenta curve in Fig. 2. Future lepton colliders could improve this sensitivity to $\Gamma < 1$ MeV [100]. Both the Higgs and Z decay constraints apply universally to all neutrino flavors $\alpha = e, \mu, \tau$.

For $\alpha = \tau$, the Yukawa interaction of Eq. (2) leads to a new decay mode $\tau^- \to \mu^-\bar{\nu}_\mu\chi\phi$ which mimics the normal leptonic decay. A similar process was considered in [101]. We simulate this four-body decay with FeynRules [102] and MadGraph [103] and obtain an upper bound on y_τ as a function of the DM mass, as shown by the green curve in Fig. 2. The projected bound assumes an improvement on the sensitivity of this channel from 0.04% [99] to 0.014% at Belle II [104]. For other flavor choices $\alpha = \mu$, e, much stronger constraints arise from leptonic decays of charged kaons and pions. In those cases, the parameter space of interest to cosmology has already been excluded.

V. NEUTRINO SELF-INTERACTION

The neutrino portal coupling could also lead to nonstandard neutrino self interaction, which arises from a box diagram with χ and ϕ in the loop. The low-energy effective operator takes the form

$$\mathcal{L}_{{\rm SI}\nu} = G_{\rm eff}(\bar{\nu}\gamma^\mu P_L \nu)(\bar{\nu}\gamma_\mu P_L \nu),$$
$$G_{\rm eff} = \frac{|y_\alpha|^4 (m_\phi^4 - m_\chi^4 - 2m_\phi^2 m_\chi^2 \log\frac{m_\phi^2}{m_\chi^2})}{64\pi^2 (m_\phi^2 - m_\chi^2)^3}. \tag{13}$$

In the limit $m_\phi \simeq m_\chi$, $G_{\rm eff} \simeq |y_\alpha|^4/(192\pi^2 m_\chi^2)$. A sizable neutrino self interaction has been suggested as an ingredient for solving the Hubble tension [105,106]. However, the relevant parameter regions are already ruled out by laboratory and $\Delta N_{\rm eff}$ constraints in this model.

VI. UV COMPLETION

Eq. (1) can be generated in a UV-complete model by integrating out a gauge singlet vectorlike fermion that couples to both the visible and dark sectors. The interacting Lagrangian takes the form $\mathcal{L}_{\rm UV} = \lambda_V \bar{L}_\alpha H N_R + \lambda_D \bar{N}_L \chi\phi + M\bar{N}_L N_R + {\rm H.c.}$ The first Yukawa term allows for a heavy-light neutrino mixing below the electroweak scale, $N_L = \sqrt{1 - |U_{\alpha 4}|^2}\hat{N}_L + U_{\alpha 4}\hat{\nu}_\alpha$, where the hat fields are physical states and $U_{\alpha 4} = \lambda_V v/\sqrt{M^2 + \lambda_V^2 v^2}$. Together with the second Yukawa term, we obtain the relationship, $y_\alpha = \lambda_D U_{\alpha 4}$, where y_α is the Yukawa coupling introduced in Eq. (2). In this UV completion, there are additional constraints on the mixing parameter $U_{\alpha 4}$. For $\alpha = \tau$ flavor and N mass around/above the electroweak scale, the strongest constraints are from τ lepton decays, $|U_{\tau 4}| \lesssim 0.2$ [52,107]. If $|U_{\tau 4}|$ is close to this upper bound, the favored range of y_τ in Fig. 2 can be obtained with an order one fundamental Yukawa coupling λ_D.

VII. OTHER POSSIBILITIES

If the mass difference between ϕ and χ is tuned to be comparable to the typical kinetic energy of DM, the effective Lagrangian Eq. (3) will break down. In galaxies, ϕ particles could be produced on shell via $\chi\chi$ collisions. In this case, ϕ could still decay quickly back to χ and a neutrino within the cosmological timescale, leading to dissipative DM of the inelastic kind [108,109]. In the limiting case where ϕ and χ are degenerate, both will serve as DM. The Born level momentum-transfer cross section [11] of $\chi - \phi$ scattering via neutrino exchange is $\sigma_{\chi\phi\to\chi\phi} \simeq |y_\alpha|^4/(32\pi m_\chi^2 v^2)$ in the limit $m_\chi v \gg m_\nu$, where v is the relative velocity. The Bullet Cluster constraint on this cross section implies $|y_\alpha| < 0.026(m_\chi/10\ \mathrm{MeV})^{3/4}(v/10^{-3})^{1/2}$. This constraint could be affected by nonperturbative effects due to multiple neutrino exchanges.

VIII. SUMMARY

The findings of this work demonstrate that the simple neutrino portal offers a rich DM phenomenology. Self-interacting DM can occur without introducing light dark force carriers, but rather via the exchange of SM neutrinos. We identify a parameter space where DM has sufficiently strong self interactions to influence small-scale structure formation. Meanwhile, the interaction between DM and neutrinos could accommodate a warm DM candidate. This interplay allows for a unified solution to all the puzzles in small-scale structure formation. The corresponding neutrino portal interaction is a well-motivated target for precision measurements of decay rates at future collider experiments [110–115] as well as precision measurements by the upcoming CMB-S4 project. Similar physics as discussed above can be generalized to DM self interaction via the exchange of other motivated light fermions, such as the sterile neutrino [116–121].

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