Treating divergence in quark matter by using energy projectors

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We calculate the gluon self-energy using quark energy projectors in a general quark-gluon plasma. By separating the quark field into positive- and a negative-energy modes, the quark loop constructed with the same mode is always convergent, and the divergence appears only in the mixed loop with different modes and is medium independent. After removing the divergence in vacuum, we obtain the one-loop gluon self-energy at finite temperature, chemical potential, and quark mass without approximation. With the method of quark-loop resummation, we calculate nonperturbatively the gluon Debye mass and thermodynamic potential. In the limit of small gluon momentum in comparison with temperature, chemical potential, and quark mass, our calculation comes back to the known HTL/HDL results in literature.

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I. INTRODUCTION

The properties of QCD matter at finite temperature and chemical potential, especially the deconfinement and chiral symmetry phase transitions [1–26] and their realization in high-energy nuclear collisions [27–39] and compact stars [40–54], have been widely studied for decades. As an often used method in theoretical calculations, one introduces energy projectors to divide the quark field into positive- and negative-energy modes, and therefore any Feynman diagram is separated into two groups: the pure fraction constructed from the same modes, and the mixed fraction contructed from the two modes [55–61]. The energy-projector method is widely used in the study of color superconductivity at extremely high baryon density [60–64]. In the nonrelativistic limit with heavy quark mass (NRQCD), the positive- and negative-energy modes become, respectively, the relevant and irrelevant modes, and by integrating out the irrelevant mode, the relevant mode becomes the dominant one and controls the behavior of heavy quark systems [65–82].

The one-loop diagram plays a crucial role in nonperturbative calculations of QCD, like the approaches of hard thermal loop resummation (HTL) [83–88] and hard dense loop resummation (HDL) [89]. A key problem in these calculations is how to treat the divergence at the one-loop level. In some limits, like extremely high temperature or high baryon density, the divergence is properly removed in HTL and HDL [90–92]. In this paper we focus on the divergence problem in the calculation of the in-medium one-loop gluon self-energy at finite temperature, chemical potential, and quark mass, using the quark energy-projector method. We will see that the divergence appears only in the mixed loop and is medium independent. Therefore, it does not change the thermodynamic properties relative to the vacuum and can be directly removed.

The paper is organized as follows. We rewrite the quark sector of the QCD Lagrangian density in terms of energy projectors in Nambu-Gorkov space in Sec. II, and then calculate the gluon self-energy at the one-loop level by separating the quark loop into a pure loop without divergence and a mixed loop with vacuum divergence in Sec. III. After taking the often used loop resummation, in Sec. IV we calculate the thermodynamic properties of the quark matter, like the gluon Debye mass and gluon thermodynamic potential, and compare our calculations under some extreme conditions with the known results in the literature. We finally summarize in Sec. V.

II. ENERGY-PROJECTOR METHOD

We first consider particle-antiparticle symmetry of the strong interaction. By taking the charge-conjugation operator C which changes quark fields $\psi(x)$ and $\bar{\psi}(x)$ to $\psi_C(x) = C\bar{\psi}^T(x)$ and $\bar{\psi}_C(x) = \psi^T(x)C$, one introduces the Nambu-Gorkov space [93,94]

$$\Psi = \begin{pmatrix} \psi \\ \psi_C \end{pmatrix}, \qquad \bar{\Psi} = (\bar{\psi}, \bar{\psi}_C). \tag{1}$$

To make the fields in momentum space dimensionless, the normalization factors in the Fourier transformation from coordinate space to momentum space for the quark field Ψ and gluon field A_μ^a are chosen as

$$\Psi(x) = \frac{1}{\sqrt{V}}\sum_k e^{-ik\cdot x}\Psi(k),$$
$$A^a_\mu(x) = \frac{1}{\sqrt{TV}}\sum_q e^{-iq\cdot x}A^a_\mu(q), \tag{2}$$

where V and T are the volume and temperature of the thermal system, $k_0 = -i(2n_k+1)\pi T$ and $q_0 = -i2n_q\pi T$ with $n_k, n_q = 0, \pm 1, \pm 2, \ldots$ appearing in $k\cdot x = k_0 t - \mathbf{k}\cdot\mathbf{x}$ and $q\cdot x = q_0 t - \mathbf{q}\cdot\mathbf{x}$ are the quark and gluon frequencies in the imaginary time formalism of finite-temperature field theory, and the summations $\sum_k$ and $\sum_q$ denote the frequency summation and momentum integration.

The quark sector of the QCD Lagrangian density in coordinate space,

$$\mathcal{L} = \bar\psi(i\gamma^\mu\partial_\mu + g\gamma^\mu A^a_\mu T_a + \mu_f\gamma_0 - m_f)\psi, \tag{3}$$

with quark mass m_f, quark chemical potential μ_f, Gell-Mann matrices T_a ($a = 0, 1, \ldots, 8$), and quark-gluon coupling constant g, can be expressed as

$$\mathcal{L} = \frac{1}{2}\sum_p \bar\Psi(k)G^{-1}(k,p)\Psi(p) \tag{4}$$

in momentum space and Nambu-Gorkov space, where the full quark propagator

$$G^{-1} = G_0^{-1} + g\mathcal{A} \tag{5}$$

contains the free propagator

$$G_0^{-1}(k,p) = \frac{1}{T}\begin{pmatrix} [G_0^+]^{-1}(k) & 0 \\ 0 & [G_0^-]^{-1}(k)\end{pmatrix}\delta(k-p),$$
$$[G_0^\pm]^{-1}(k) = \gamma^\mu k_\mu \pm \mu_f\gamma_0 - m_f \tag{6}$$

and the modified gauge field

$$\mathcal{A}(k,p) = \frac{1}{\sqrt{VT^3}}\Gamma^\mu_a A^a_\mu(k-p),$$
$$\Gamma^\mu_a = \gamma^\mu\begin{pmatrix} T_a & 0 \\ 0 & -T_a^T\end{pmatrix}. \tag{7}$$

We further separate the quark fields into two parts with positive and negative energy. The energy projectors onto states of positive and negative energy for free massive quarks are defined as [59,95–98]

$$\Lambda_\pm(\tilde k) = \frac{1}{2\epsilon_k}[\epsilon_k \pm \gamma_0(\boldsymbol{\gamma}\cdot\mathbf{k} + m_f)], \tag{8}$$

with the quark energy $\epsilon_k = \sqrt{m_f^2 + \mathbf{k}^2}$. Note that $\tilde k = (\epsilon_k, \mathbf{k})$ is an on-shell four-momentum which is different from the general four-momentum $k = (k_0, \mathbf{k})$. Taking into account the orthogonal and complete properties

$$\Lambda_+\Lambda_- = \Lambda_-\Lambda_+ = 0, \qquad \Lambda_+ + \Lambda_- = 1 \tag{9}$$

and the relations

$$\Lambda_\pm^\dagger = \Lambda_\pm, \qquad \Lambda_\pm^2 = \Lambda_\pm, \tag{10}$$

it is easy to check that the states

$$\Psi_\pm(k) = \Lambda_\pm(\tilde k)\Psi(k) \tag{11}$$

satisfy the Dirac equation

$$H\Psi_\pm(k) = \pm\epsilon_k\Psi_\pm(k), \tag{12}$$

with the free Hamiltonian $H = \gamma_0(\boldsymbol{\gamma}\cdot\mathbf{k} + m_f)$. This is the reason why we call $\Psi_\pm$ the positive- and negative-energy states. Using the energy projectors, the Lagrangian density (4) can be rewritten as

$$\mathcal{L} = \frac{1}{2}\sum_{m,n=\pm}\sum_p \bar\Psi_m(k)G^{-1}_{mn}(k,p)\Psi_n(p), \tag{13}$$

with the matrix elements of the full quark propagator in energy space

$$G^{-1}_{mn}(k,p) = [G_0^{-1}]_{mn}(k,p) + g\mathcal{A}_{mn}(k,p),$$
$$[G_0^{-1}]_{mn}(k,p) = \gamma_0\Lambda_m(\tilde k)\gamma_0 G_0^{-1}(k,p)\Lambda_n(\tilde p),$$
$$\mathcal{A}_{mn}(k,p) = \gamma_0\Lambda_m(\tilde k)\gamma_0\mathcal{A}(k,p)\Lambda_n(\tilde p). \tag{14}$$

The matrix elements of the free propagator can be explicitly expressed as

$$[G_0^{-1}]_{++}(k,p) = \frac{1}{T}\begin{pmatrix} k_0 + \mu_f - \epsilon_k & 0 \\ 0 & k_0 - \mu_f - \epsilon_k\end{pmatrix} \times \gamma_0\Lambda_+(\tilde k)\delta(k-p),$$
$$[G_0^{-1}]_{--}(k,p) = \frac{1}{T}\begin{pmatrix} k_0 + \mu_f + \epsilon_k & 0 \\ 0 & k_0 - \mu_f + \epsilon_k\end{pmatrix} \times \gamma_0\Lambda_-(\tilde k)\delta(k-p),$$
$$[G_0^{-1}]_{+-}(k,p) = [G_0^{-1}]_{-+}(k,p) = 0. \tag{15}$$

While the free quark propagator is diagonal in energy space, the full propagator has off-diagonal elements due to the coupling between quark/quark hole and antiquark hole/antiquark.

Since heavy quarks are so heavy, they can be treated nonrelativistically. To go from a relativistic theory with positive- and negative-energy modes to a nonrelativistic theory with only positive-energy modes, one obtains the nonrelativistic Lagrangian density by integrating out the negative-energy mode (the irrelevant mode). This is usually used in NRQCD [99]. For light flavors, both the positive- and negative-energy modes are relevant and should be explicitly included in the Lagrangian density.

We now extract, from the Lagrangian density (13), the Feynman rules for the positive- and negative-energy quark fields in Nambu-Gorkov space, which will be used in the calculation of the gluon self-energy later. Considering the diagonal property (15) of the free propagator G_0^{-1} in energy space, the Lagrangian density becomes

$$\mathcal{L} = \frac{1}{2}\sum_p \left[\sum_{m=\pm} \bar{\Psi}_m(k)[G_0^{-1}]_{mm}(k,p)\Psi_m(p) + g\sum_{m,n=\pm} \bar{\Psi}_m(k)\mathcal{A}_{mn}(k,p)\Psi_n(p)\right]. \tag{16}$$

By taking into account the relations (7), (14), and (15) for the quark and gauge fields and the projection properties

$$\Lambda_\pm \Psi_\pm = \Psi_\pm, \qquad \Lambda_\mp \Psi_\pm = 0, \tag{17}$$

the two free propagators for the positive- and negative-energy quarks can be expressed as

$$\mathbb{G}_0^\pm(k) = T\begin{pmatrix} \frac{1}{k_0+\mu_f\mp\epsilon_k} & 0 \\ 0 & \frac{1}{k_0-\mu_f\mp\epsilon_k} \end{pmatrix}, \tag{18}$$

which satisfy the completeness condition

$$\left[\sum_n \mathbb{G}_0^n \Lambda_n\right]\gamma_0 = G_0, \tag{19}$$

and the four kinds of coupling vertices among gluon and positive- and negative-energy quarks can be generally represented as

$$\mathbb{V}_{mn}^{\mu,a}(\tilde{k},\tilde{p}) = \frac{g}{\sqrt{VT^3}}\Lambda_m(\tilde{k})\gamma^0\Gamma_a^\mu\Lambda_n(\tilde{p}). \tag{20}$$

III. ONE-LOOP GLUON SELF-ENERGY

In this section we calculate the gluon self-energy, using the quark propagators (18) and coupling vertices (20). Since a quark loop can be constructed from two positive-energy quarks, two negative-energy quarks, and one positive- and one negative-energy quark, the quark contribution $\Pi_Q^{\mu\nu,ab}(q)$ to the gluon self-energy $\Pi^{\mu\nu,ab}(q)$ at the one-loop level contains three parts (see Fig. 1):

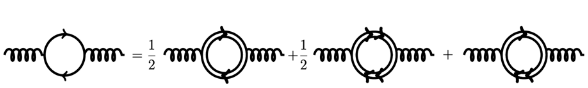

FIG. 1. Gluon self-energy at the one quark loop level. The double lines indicate quark modes with positive (one arrow) and negative (double arrows) energies.

$$\Pi_Q^{\mu\nu,ab} = \Pi_{++}^{\mu\nu,ab} + \Pi_{--}^{\mu\nu,ab} + 2\Pi_{+-}^{\mu\nu,ab},$$
$$\Pi_{mn}^{\mu\nu,ab}(q) = -\frac{1}{2}\frac{(-1)^2}{2!}(2-1)!\sum_k \mathrm{Tr}[\mathbb{G}_0^m(k_+) \times \mathbb{V}_{mn}^{\mu,a}(\tilde{k}_+,\tilde{k}_-)\mathbb{G}_0^n(k_-)\mathbb{V}_{nm}^{\nu,b}(\tilde{k}_-,\tilde{k}_+)], \tag{21}$$

with the two quark momenta $k_+^\mu = (k_0, \mathbf{k}+\mathbf{q}/2)$, $k_-^\mu = (k_0-q_0, \mathbf{k}-\mathbf{q}/2)$, on-shell momenta $\tilde{k}_\pm^\mu = (\epsilon_\pm, \mathbf{k}\pm\mathbf{q}/2)$, and energies $\epsilon_\pm = \sqrt{m_f^2+(\mathbf{k}\pm\mathbf{q}/2)^2}$. The first coefficient $1/2$ in $\Pi_{mn}^{\mu\nu,ab}(q)$ is from the normalization in Nambu-Gorkov space, and the second coefficient $(-1)^n(n-1)!/n!$ $(n=2)$ is from the topological number of the Feynman diagram. Here we have used the symmetry $\Pi_{+-}^{\mu\nu,ab} = \Pi_{-+}^{\mu\nu,ab}$ in energy space.

Taking into account the projection properties

$$\mathbb{G}_0^n\Lambda_n = \Lambda_n\mathbb{G}_0^n \tag{22}$$

and $\Lambda_n^2 = \Lambda_n$, there is

$$\Pi_{mn}^{\mu\nu,ab}(q) = -\frac{g^2}{4VT^3}\sum_k \mathrm{Tr}[\mathbb{G}_0^m(k_+)\Lambda_m(\tilde{k}_+)\gamma^0\Gamma_a^\mu \times \mathbb{G}_0^n(k_-)\Lambda_n(\tilde{k}_-)\gamma^0\Gamma_b^\nu]. \tag{23}$$

A. $\Pi_{--}^{\mu\nu,ab}(q)$

We now take $\Pi_{--}^{\mu\nu,ab}$ as an example to show the calculation and simplification of the gluon self-energy. After considering the trace in color space $\mathrm{Tr}(T_aT_b) = \mathrm{Tr}(T_a^TT_b^T) = \delta_{ab}/2$, the self-energy in color and spin spaces at the one-loop level can be factorized into two parts:

$$\Pi_{--}^{\mu\nu,ab} = -\frac{1}{T^2}\frac{\delta_{ab}}{2}\Pi_{--}^{\mu\nu},$$
$$\Pi_{--}^{\mu\nu} = \frac{g^2T}{4V}\sum_{k,s=\pm}\frac{\mathrm{Tr}[\Lambda_-(\tilde{k}_+)\gamma^0\gamma^\mu\Lambda_-(\tilde{k}_-)\gamma^0\gamma^\nu]}{(k_+^0+s\mu_f+\epsilon_+)(k_-^0+s\mu_f+\epsilon_-)}, \tag{24}$$

where the summation $\sum_{s=\pm}$ is over the quark and antiquark in Nambu-Gorkov space. By considering the trace in spin space,

$$\mathrm{Tr}[\Lambda_-(\tilde{k}_+)\gamma^0\gamma^\mu\Lambda_-(\tilde{k}_-)\gamma^0\gamma^\nu] = \frac{1}{\epsilon_+\epsilon_-}[g^{\mu\nu}(m_f^2-\bar{k}_+^\sigma\bar{k}_\sigma^-)+\bar{k}_+^\mu\bar{k}_-^\nu+\bar{k}_+^\nu\bar{k}_-^\mu], \tag{25}$$

where the two new on-shell quark momenta $\bar{k}_\pm$ are defined as $\bar{k}_\pm = (-\epsilon_\pm, \mathbf{k}\pm\mathbf{q}/2)$, and summarizing the quark frequencies

$$\sum_{k_0,s=\pm}\frac{1}{(k_+^0+s\mu_f+\epsilon_+)(k_-^0+s\mu_f+\epsilon_-)}$$
$$=\frac{\sum_{s=\pm}[f_F(\epsilon_++s\mu_f)-f_F(\epsilon_-+s\mu_f)]}{T(q_0+\epsilon_+-\epsilon_-)}$$
$$\equiv\frac{\epsilon_-\epsilon_+}{2T}F(q_0,\epsilon_+,\epsilon_-), \tag{26}$$

with the Fermi-Dirac distribution $f_F(x)=1/(e^{x/T}+1)$, where we have used the relation $\tanh(x+in_q\pi)=\tanh(x)$, the gluon self-energy in spin space $\Pi^{\mu\nu}_{--}(q)$ can be further separated into two parts:

$$\Pi^{\mu\nu}_{--}=g^{\mu\nu}\bar{\Pi}_{--}+\bar{\Pi}^{\mu\nu}_{--}, \tag{27}$$

with the scalar function $\bar{\Pi}_{--}(q)$ and tensor function $\bar{\Pi}^{\mu\nu}_{--}(q)$,

$$\bar{\Pi}_{--}=\frac{g^2}{8}\int\frac{d^3\mathbf{k}}{(2\pi)^3}(m_f^2-\bar{k}_+^\sigma\bar{k}_\sigma^-)F(q_0,\epsilon_+,\epsilon_-),$$
$$\bar{\Pi}^{\mu\nu}_{--}=\frac{g^2}{8}\int\frac{d^3\mathbf{k}}{(2\pi)^3}(\bar{k}_+^\mu\bar{k}_-^\nu+\bar{k}_+^\nu\bar{k}_-^\mu)F(q_0,\epsilon_+,\epsilon_-), \tag{28}$$

where we have taken the continuous integration $\int d^3\mathbf{k}/(2\pi)^3$ over the quark three-momentum $\mathbf{k}$, instead of the discrete summation $\sum_\mathbf{k}/V$. Note that the function $F(q_0,\epsilon_+,\epsilon_-)$ defined in Eq. (26) contains only the Fermi-Dirac distribution, and therefore any ultraviolet divergence of the integral will be suppressed by the exponential function $e^{-\epsilon_\pm/T}$ in the distribution and the integral is always convergent.

A key question for the calculation of the gluon self-energy is the divergence analysis. If there exists any infrared or ultraviolet divergence, a renormalization procedure is required. To see this clearly, we further simplify the three-momentum integration in Eq. (28). We take the component k_z of the quark momentum $\mathbf{k}$ along the gluon momentum $\mathbf{q}$,

$$\mathbf{k}=|\mathbf{k}|\cos\theta\hat{\mathbf{q}}+|\mathbf{k}|\sin\theta\cos\phi\hat{\mathbf{k}}_x+|\mathbf{k}|\sin\theta\sin\phi\hat{\mathbf{k}}_y, \tag{29}$$

where $\mathbf{k}_x$ and $\mathbf{k}_y$ are perpendicular to the gluon momentum $\mathbf{q}$. Taking into account the symmetry in the transverse plane, $\Pi^{\mu\nu}_{--}(q)$ is independent of the choice of the directions $\hat{\mathbf{k}}_x$ and $\hat{\mathbf{k}}_y$. The integration over the azimuth angle ϕ is easy. Considering that the quark energies $\epsilon_\pm=\sqrt{m_f^2+|\mathbf{k}|^2+|\mathbf{q}|^2/4\pm|\mathbf{k}||\mathbf{q}|\cos\theta}$ and in turn the function $F(q_0,\epsilon_+,\epsilon_-)$ are independent of the angle ϕ, the scalar and tensor functions $\bar{\Pi}_{--}(q)$ and $\bar{\Pi}^{\mu\nu}_{--}(q)$ can be easily written as

$$\bar{\Pi}_{--}=\frac{g^2}{32\pi^2}\int d|\mathbf{k}|d\cos\theta|\mathbf{k}|^2(m_f^2-\bar{k}_+^\mu\bar{k}_\mu^-)F(q_0,\epsilon_+,\epsilon_-),$$
$$\bar{\Pi}^{\mu\nu}_{--}=\frac{g^2}{32\pi^2}\int d|\mathbf{k}|d\cos\theta|\mathbf{k}|^2H^{\mu\nu}(q,k)F(q_0,\epsilon_+,\epsilon_-),$$
$$H^{00}=2\epsilon_+\epsilon_-,$$
$$H^{0i}=H^{i0}=-\sum_{n=\pm}\epsilon_n\left(|\mathbf{k}|\cos\theta-n\frac{|\mathbf{q}|}{2}\right)\hat{q}^i,$$
$$H^{ij}=\left(2\mathbf{k}^2\cos^2\theta-\frac{1}{2}|\mathbf{q}|^2\right)\hat{q}^i\hat{q}^j+\mathbf{k}^2\sin^2\theta(\delta^{ij}-\hat{q}^i\hat{q}^j), \tag{30}$$

with the function $F(q_0,\epsilon_+,\epsilon_-)$ defined in Eq. (26).

Then we perform a variable substitution from $(|\mathbf{k}|,\cos\theta)$ to (ϵ_-,ϵ_+). From the relations

$$\mathbf{k}^2=(\epsilon_+^2+\epsilon_-^2)/2-m_f^2-\mathbf{q}^2/4,$$
$$\cos\theta=\frac{\epsilon_+^2-\epsilon_-^2}{2|\mathbf{q}||\mathbf{k}|} \tag{31}$$

and the corresponding Jacobian determinant

$$\left|\frac{\partial(|\mathbf{k}|,\cos\theta)}{\partial(\epsilon_-,\epsilon_+)}\right|=\frac{\epsilon_-\epsilon_+}{|\mathbf{q}||\mathbf{k}|}, \tag{32}$$

we finally obtain the scalar and tensor parts of the one-loop gluon self-energy in terms of the integration over ϵ_- and ϵ_+,

$$\bar{\Pi}_{--}=\frac{g^2}{32\pi^2}\int_R d\epsilon_-d\epsilon_+\frac{\epsilon_-\epsilon_+}{|\mathbf{q}|}\frac{(\epsilon_--\epsilon_+)^2-\mathbf{q}^2}{2}F_S(q_0,\epsilon_+,\epsilon_-),$$
$$\bar{\Pi}^{\mu\nu}_{--}=\frac{g^2}{32\pi^2}\int_R d\epsilon_-d\epsilon_+\frac{\epsilon_-\epsilon_+}{|\mathbf{q}|}H^{\mu\nu}(q,\epsilon_+,\epsilon_-)F_S(q_0,\epsilon_+,\epsilon_-),$$
$$H^{00}=2\epsilon_+\epsilon_-,$$
$$H^{0i}=H^{i0}=\frac{(\epsilon_++\epsilon_-)^2-\mathbf{q}^2}{2|\mathbf{q}|}q_0\hat{q}^i,$$
$$H^{ij}=\frac{1}{2}\left(\frac{(\epsilon_-^2-\epsilon_+^2)^2}{\mathbf{q}^2}-\mathbf{q}^2\right)\hat{q}^i\hat{q}^j+\left[\frac{1}{2}(\epsilon_-^2+\epsilon_+^2-2m_f^2)-\frac{1}{4}\left(\frac{(\epsilon_-^2-\epsilon_+^2)^2}{\mathbf{q}^2}+\mathbf{q}^2\right)\right]\times(\delta^{ij}-\hat{q}^i\hat{q}^j), \tag{33}$$

where the integration region R is controlled by the requirement that the three momenta $\mathbf{q},\mathbf{k}+\mathbf{q}/2$ and $\mathbf{k}-\mathbf{q}/2$ should form a triangle, which leads to the constraints on the quark energies,

$$\sqrt{\epsilon_-^2-m_f^2}+\sqrt{\epsilon_+^2-m_f^2}\geq|\mathbf{q}|,$$
$$\left|\sqrt{\epsilon_-^2-m_f^2}-\sqrt{\epsilon_+^2-m_f^2}\right|\leq|\mathbf{q}|, \tag{34}$$

and the function F_S defined as

$$F_S(q_0, \epsilon_+, \epsilon_-) = \frac{1}{2}[F(q_0, \epsilon_+, \epsilon_-) + F(q_0, \epsilon_-, \epsilon_+)] \tag{35}$$

satisfies the symmetry when exchanging the quark energies ϵ_- and ϵ_+.

The exchange symmetry of the gluon self-energy helps us a lot for its divergence analysis. The lower limit of the integration in Eq. (33) is $\epsilon_- = \epsilon_+ = m_f$ and the upper limit is $\epsilon_- = \epsilon_+ = \infty$. Considering that $(\epsilon_- - \epsilon_+)^2 - \mathbf{q}^2$ in $\bar{\Pi}_{--}$ and $H^{\mu\nu}(q, \epsilon_+, \epsilon_-)$ in $\bar{\Pi}^{\mu\nu}_{--}$ are finite at the lower limit and only finite polynomials at the upper limit, the problem of divergence is controlled by the factor

$$\epsilon_-\epsilon_+ F_S \sim \frac{f_F(\epsilon_+ \pm \mu_f) - f_F(\epsilon_- \pm \mu_f)}{q_0 + \epsilon_+ - \epsilon_-}, \tag{36}$$

which is finite at the lower limit for any q_0 and goes to zero exponentially at the upper limit. Therefore, there are no infrared (for massless quarks) or ultraviolet divergences for the self-energy $\Pi^{\mu\nu}_{--}$.

B. $\Pi_Q^{\mu\nu,ab}(q)$

We now consider the contribution of the first loop in Fig. 1 constructed from two positive-energy quarks to the gluon self-energy. Similar to Eq. (24), we first separate the trivial color part from the spin- and momentum-dependent part, and then perform the variable substitutions $k \to -k$ and $\tilde{k} \to -\tilde{k}$. Considering the relations $\epsilon_{-k+q/2} = \epsilon_{k-q/2} = \epsilon_-, \epsilon_{-k-q/2} = \epsilon_{k+q/2} = \epsilon_+$ for the quark energies and $\Lambda_+(-\tilde{k}, m_f) = \Lambda_-(\tilde{k}, -m_f)$ for the projectors, and taking into account the property that the trace (25) in spin space is symmetric under the exchange of μ and ν and the replacement of m_f by $-m_f$, we have

$$\begin{aligned}&\mathrm{Tr}[\Lambda_+(-\tilde{k} + q/2)\gamma^0\gamma^\nu\Lambda_+(-\tilde{k} - q/2)\gamma^0\gamma^\mu] \\ &\quad = \mathrm{Tr}[\Lambda_-(\tilde{k}_-)\gamma^0\gamma^\mu\Lambda_-(\tilde{k}_+)\gamma^0\gamma^\nu],\end{aligned} \tag{37}$$

which leads to the result that the two loops constructed from positive- or negative-energy quarks have exactly the same contribution to the gluon self-energy,

$$\Pi^{\mu\nu,ab}_{++}(q) = \Pi^{\mu\nu,ab}_{--}(q). \tag{38}$$

The third loop in Fig. 1 contains vacuum divergence, arising from the mixing between the positive- and negative-energy quarks. Excluding the same color factor, its contribution to the gluon self-energy is represented as

$$\Pi^{\mu\nu}_{+-} = \frac{g^2 T}{4V}\sum_{k,s=\pm}\frac{\mathrm{Tr}[\Lambda_+(\tilde{k}_+)\gamma^0\gamma^\mu\Lambda_-(\tilde{k}_-)\gamma^0\gamma^\nu]}{(k_+^0 + s\mu_f - \epsilon_+)(k_-^0 + s\mu_f + \epsilon_-)}. \tag{39}$$

We take the trace in spin space

$$\begin{aligned}&\mathrm{Tr}[\Lambda_+(\tilde{k}_+)\gamma^0\gamma^\mu\Lambda_-(\tilde{k}_-)\gamma^0\gamma^\nu] \\ &\quad = \frac{-1}{\epsilon_+\epsilon_-}[g^{\mu\nu}(m_f^2 - \tilde{k}_+^\sigma\bar{k}_\sigma^-) + \tilde{k}_+^\mu\bar{k}_-^\nu + \tilde{k}_+^\nu\bar{k}_-^\mu]\end{aligned} \tag{40}$$

and the summation over quark frequencies

$$\begin{aligned}&\sum_{k_0,s=\pm}\frac{1}{(k_+^0 + s\mu_f - \epsilon_+)(k_-^0 + s\mu_f + \epsilon_-)} \\ &\quad = \frac{\sum_{s=\pm}[f_F(\epsilon_+ + s\mu_f) + f_F(\epsilon_- + s\mu_f)] - 2}{T(q_0 + \epsilon_+ + \epsilon_-)}, \\ &\quad \equiv \frac{\epsilon_-\epsilon_+}{2T}\left[J(q_0, \epsilon_+, \epsilon_-) - \frac{4}{\epsilon_-\epsilon_+(q_0 + \epsilon_+ + \epsilon_-)}\right],\end{aligned} \tag{41}$$

where $\Pi^{\mu\nu}_{+-}(q)$ can again be separated into two parts:

$$\Pi^{\mu\nu}_{+-} = g^{\mu\nu}\bar{\Pi}_{+-} + \bar{\Pi}^{\mu\nu}_{+-}. \tag{42}$$

Integrating out the azimuth angle ϕ, performing a variable substitution from $(|\mathbf{k}|, \cos\theta)$ to (ϵ_-, ϵ_+), and considering the exchange symmetry between ϵ_- and ϵ_+, the scalar and tensor functions $\bar{\Pi}_{+-}$ and $\bar{\Pi}^{\mu\nu}_{+-}$ are written as

$$\begin{aligned}\bar{\Pi}_{+-} &= \frac{g^2}{32\pi^2}\int_R d\epsilon_- d\epsilon_+ \frac{\epsilon_-\epsilon_+}{|\mathbf{q}|}\frac{\mathbf{q}^2 - (\epsilon_- + \epsilon_+)^2}{2}J_S(q_0, \epsilon_+, \epsilon_-), \\ \bar{\Pi}^{\mu\nu}_{+-} &= \frac{g^2}{32\pi^2}\int_R d\epsilon_- d\epsilon_+ \frac{\epsilon_-\epsilon_+}{|\mathbf{q}|}I^{\mu\nu}(q, \epsilon_+, \epsilon_-)J_S(q_0, \epsilon_+, \epsilon_-), \\ I^{00} &= H^{00}, \\ I^{0i} &= I^{i0} = \frac{\mathbf{q}^2 - (\epsilon_+ - \epsilon_-)^2}{2|\mathbf{q}|}q_0\hat{q}^i, \\ I^{ij} &= -H^{ij},\end{aligned} \tag{43}$$

where the function J_S is defined as

$$J_S(q_0, \epsilon_+, \epsilon_-) = \frac{1}{2}[J(q_0, \epsilon_+, \epsilon_-) + J(-q_0, \epsilon_+, \epsilon_-)], \tag{44}$$

with the function $J(q_0, \epsilon_+, \epsilon_-)$ defined in Eq. (41). Like the function F, J contains only the Fermi-Dirac distribution, and therefore the integral is convergent at any temperature. However, here we have neglected the second term in the square brackets of Eq. (41) which is temperature independent but leads to the divergence of $\bar{\Pi}_{+-}$ and $\bar{\Pi}^{\mu\nu}_{+-}$ when doing the energy integral. Since this term is medium independent, the divergence can be removed through a renormalization procedure in vacuum (see, for instance, Ref. [100]). The temperature-dependent part is always convergent, due to the Boltzmann factor $e^{-\epsilon/T}$ in the particle distribution function. Considering that all thermodynamic functions are relative to the vacuum, the divergence treatment in vacuum will not affect the thermodynamic properties of the system.

It can be checked that the total quark contribution to the gluon self-energy at the one-loop level,

$$\Pi_Q^{\mu\nu,ab}(q) = -\frac{1}{T^2}\frac{\delta_{ab}}{2}\Pi_Q^{\mu\nu}(q),$$
$$\Pi_Q^{\mu\nu}(q) = 2\Pi_{--}^{\mu\nu}(q) + 2\Pi_{+-}^{\mu\nu}(q) \equiv \bar{\Pi}_Q^{\mu\nu}(q), \quad (45)$$

satisfies the Ward-identity [101] at any temperature T, chemical potential μ_f, and quark mass m_f,

$$q_\mu \bar{\Pi}_Q^{\mu\nu}(q) = 0. \quad (46)$$

The total quark loop $\bar{\Pi}_Q^{\mu\nu}$ can be separated into the longitudinal and transverse parts by using the tensor projectors $P_L^{\mu\nu}$ and $P_T^{\mu\nu}$ [90],

$$\bar{\Pi}_Q^{\mu\nu}(q) = \bar{\Pi}_Q^T(q)P_T^{\mu\nu} + \bar{\Pi}_Q^L(q)P_L^{\mu\nu},$$
$$P_T^{00} = P_T^{0i} = P_T^{i0} = 0, \qquad P_T^{ij} = -\delta^{ij} + \frac{q^i q^j}{\mathbf{q}^2},$$
$$P_L^{\mu\nu} = g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} - P_T^{\mu\nu}. \quad (47)$$

Using the relations $\bar{\Pi}_Q^{00} = (1 - q_0^2/q^2)\bar{\Pi}_Q^L$ and $\bar{\Pi}_{Q\mu}^{\mu} = 2\bar{\Pi}_Q^T + \bar{\Pi}_Q^L$, the transverse and longitudinal self-energies $\bar{\Pi}_Q^T(q)$ and $\bar{\Pi}_Q^L(q)$ are expressed as

$$\bar{\Pi}_Q^T = \frac{g^2}{4\pi^2}\frac{1}{|\mathbf{q}|}\int_R d\epsilon_- d\epsilon_+ \sum_{s=\pm} s \frac{\epsilon_- + s\epsilon_+}{q_0^2 - (\epsilon_- + s\epsilon_+)^2} \times \left\{ m_f^2 + \frac{1}{4\mathbf{q}^2}\prod_{s'=\pm}[(\epsilon_- + ss'\epsilon_+)^2 - s'\mathbf{q}^2] \right\} \times f_F^s(\epsilon_+, \epsilon_-),$$
$$\bar{\Pi}_Q^L = -\frac{g^2}{8\pi^2}\frac{q^2}{|\mathbf{q}|^3}\int_R d\epsilon_- d\epsilon_+ \sum_{s=\pm} s(\epsilon_- + s\epsilon_+) \times \frac{(\epsilon_- - s\epsilon_+)^2 - \mathbf{q}^2}{q_0^2 - (\epsilon_- + s\epsilon_+)^2} f_F^s(\epsilon_+, \epsilon_-),$$
$$f_F^s = \frac{1}{2}\sum_{s'=\pm}[f_F(\epsilon_- + s'\mu_f) + s f_F(\epsilon_+ + s'\mu_f)]. \quad (48)$$

C. Gluon loop and ghost loop

We now calculate the contribution from the gluon loop and ghost loop to the gluon self-energy; see the diagrams in Fig. 2. Similar to the treatment in Sec. III B for the quark loop, we separate the temperature-independent part from the medium part, and then take renormalization process [100,102] to remove the vacuum divergence. After the renormalization, the convergent medium part satisfies the Ward identity and can be represented by the projectors $P_L^{\mu\nu}$ and $P_T^{\mu\nu}$,

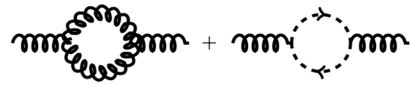

FIG. 2. Contribution from the gluon loop and ghost loop to the gluon self-energy.

$$\bar{\Pi}_G^{\mu\nu}(q) = \bar{\Pi}_G^T(q)P_T^{\mu\nu} + \bar{\Pi}_G^L(q)P_L^{\mu\nu}, \quad (49)$$

with the transverse and longitudinal self-energies

$$\bar{\Pi}_G^T = \frac{3g^2}{8\pi^2}\frac{1}{|\mathbf{q}|^3}\int_{R'} d\epsilon_- d\epsilon_+ \sum_{s=\pm} s(\epsilon_- + s\epsilon_+) \times \frac{q^2((\epsilon_- - s\epsilon_+)^2 + \mathbf{q}^2) + 2\mathbf{q}^2((\epsilon_- + s\epsilon_+)^2 - \mathbf{q}^2)}{q_0^2 - (\epsilon_- + s\epsilon_+)^2} \times f_B^s(\epsilon_+, \epsilon_-),$$
$$\bar{\Pi}_G^L = -\frac{3g^2}{4\pi^2}\frac{q^2}{|\mathbf{q}|^3}\int_{R'} d\epsilon_- d\epsilon_+ \sum_{s=\pm} s(\epsilon_- + s\epsilon_+) \times \frac{(\epsilon_- - s\epsilon_+)^2 - 2\mathbf{q}^2}{q_0^2 - (\epsilon_- + s\epsilon_+)^2} f_B^s(\epsilon_+, \epsilon_-),$$
$$f_B^s = \frac{1}{2}[f_B(\epsilon_-) + s f_B(\epsilon_+)]. \quad (50)$$

As is often used in the literature [102], we have chosen the $\xi = 1$ covariant gauge in the calculation (for reviews on covariant gauge, see Refs. [103–105]). While the calculation process is very similar to that in Sec. III B, there are important differences in the results. 1) The gluon and ghost are massless particles with energy $\epsilon_\pm = |\mathbf{k} \pm \mathbf{q}/2|$, and the integration region R' is the massless limit of the region R. 2) The gluon and ghost fields are not separated into positive- and negative-energy modes. Note that the subscript $\pm$ in $\epsilon_\pm$ throughout the paper (not only here) does not mean positive and negative modes. $\mathbf{k} \pm \mathbf{q}/2$ are the two quark/gluon/ghost momenta of the loop. 3) Quarks satisfy the Fermi-Dirac distribution f_F but gluons and ghosts satisfy the Bose-Einstein distribution f_B.

With the total gluon self-energy,

$$\bar{\Pi}^{\mu\nu}(q) = \bar{\Pi}_Q^{\mu\nu}(q) + \bar{\Pi}_G^{\mu\nu}(q), \quad (51)$$

the gluon propagator under the covariant gauge condition at the one-loop level has the well-known form [90,106]

$$\Delta^{\mu\nu} = \frac{P_T^{\mu\nu}}{q^2 + \bar{\Pi}^T(q)} + \frac{P_L^{\mu\nu}}{q^2 + \bar{\Pi}^L(q)} + \frac{\xi}{q^2}E^{\mu\nu}, \quad (52)$$

with $E^{\mu\nu} = q^\mu q^\nu/q^2$, $\bar{\Pi}^T = \bar{\Pi}_Q^T + \bar{\Pi}_G^T$, and $\bar{\Pi}^L = \bar{\Pi}_Q^L + \bar{\Pi}_G^L$.

IV. APPLICATION

A. Debye mass

In the HTL and HDL approaches, the ring diagram resummation technique in QED and QCD [107,108] is employed to consider nonperturbative effects. With the resummed gluon propagator, the Debye screening mass m_D is defined as [2,91,109]

$$m_D^2 = -\bar{\Pi}_{00}(q_0 = 0, |\mathbf{q}| \to 0). \tag{53}$$

In extremely hot and dense QCD for massless quarks ($T, \mu_f \gg m_f$), it is represented as [90,110]

$$m_D^2 = g^2\left(\frac{N_c}{3} + \frac{N_f}{6}\right)T^2 + g^2\sum_f \frac{\mu_f^2}{2\pi^2}, \tag{54}$$

with the number of colors N_c and number of flavors N_f.

We now take our obtained total gluon self-energy $\Pi^{\mu\nu}(q)$ to calculate the Debye screening mass in the general case at finite temperature and density. We consider first the contribution from the loop constructed from two negative-energy quarks to the Debye mass,

$$m_{--}^2 = -g^{00}\bar{\Pi}_{--}(q_0 = 0, |\mathbf{q}| \to 0) - \bar{\Pi}_{--}^{00}(q_0 = 0, |\mathbf{q}| \to 0). \tag{55}$$

In the massless case with $m_f = 0$ the integration region R for the quark energies ϵ_+ and ϵ_- is reduced to R'. For a function $A(q, \epsilon_+, \epsilon_-)$ with exchange symmetry between ϵ_+ and ϵ_-, the integration can be written as

$$\begin{aligned}\int_{R'} d\epsilon_- d\epsilon_+ A(q, \epsilon_+, \epsilon_-) &= \int_0^\infty d\epsilon_- \int_0^\infty d\epsilon_+ A(q, \epsilon_+, \epsilon_-) \\ &\quad \times \Theta(\epsilon_- + \epsilon_+ - |\mathbf{q}|)\Theta(|\mathbf{q}| - |\epsilon_- - \epsilon_+|) \\ &= 2\int_0^\infty d\epsilon_- \int_0^{\epsilon_-} d\epsilon_+ A(q, \epsilon_+, \epsilon_-) \\ &\quad \times \Theta(\epsilon_- + \epsilon_+ - |\mathbf{q}|)\Theta(|\mathbf{q}| - (\epsilon_- - \epsilon_+)).\end{aligned} \tag{56}$$

Taking a Taylor expansion for the two step functions around $|\mathbf{q}| = 0$,

$$\begin{aligned}&\Theta(\epsilon_- + \epsilon_+ - |\mathbf{q}|)\Theta(|\mathbf{q}| - (\epsilon_- - \epsilon_+)) \\ &= \Theta(\epsilon_+ - \epsilon_-)\Theta(\epsilon_+ + \epsilon_-) \\ &\quad + \frac{1}{|\mathbf{q}|}[\Theta(\epsilon_- + \epsilon_+)\delta(\epsilon_+ - \epsilon_-) - \Theta(\epsilon_+ - \epsilon_-)\delta(\epsilon_+ + \epsilon_-)],\end{aligned} \tag{57}$$

and considering the restriction $\epsilon_- \geq \epsilon_+$ and $\epsilon_- + \epsilon_+ \geq 0$, only the second term with $\delta(\epsilon_+ - \epsilon_-)$ contributes. From the limits

$$\begin{aligned}&\lim_{\epsilon_+ \to \epsilon_-}\left[\frac{1}{2}(\epsilon_- - \epsilon_+)^2 + 2\epsilon_-\epsilon_+\right] = 2\epsilon_-^2, \\ &\lim_{\epsilon_+ \to \epsilon_-} F_S(q_0 = 0, \epsilon_+, \epsilon_-) \\ &\quad = -\frac{2}{T\epsilon_-^2}\sum_{s=\pm} f_F(\epsilon_- + s\mu_f)(1 - f_F(\epsilon_- + s\mu_f)),\end{aligned} \tag{58}$$

we finally derive the expression for the Debye mass,

$$\begin{aligned}m_{--}^2 &= -\frac{g^2}{32\pi^2}\int_0^\infty d\epsilon_- \int_0^{\epsilon_-} d\epsilon_+ \epsilon_-\epsilon_+ \delta(\epsilon_+ - \epsilon_-) \\ &\quad \times [(\epsilon_- - \epsilon_+)^2 + 4\epsilon_-\epsilon_+]F_S(q_0 = 0, \epsilon_+, \epsilon_-) \\ &= \frac{g^2}{2}\left(\frac{T^2}{6} + \frac{\mu_f^2}{2\pi^2}\right).\end{aligned} \tag{59}$$

Taking into account the positive-energy quark loop $\Pi_{++}^{\mu\nu}$ which contributes the same to the Debye mass, the mixed quark loop $\Pi_{+-}^{\mu\nu}$ which makes no contribution due to the fact that the trace (40) in spin space disappears in the limit $|\mathbf{q}| \to 0$ at $\mu = \nu = 0$, the gluon and ghost loops $\Pi_G^{\mu\nu}$ which contribute

$$m_G^2 = -\Pi_G^{00}(q_0 = 0, |\mathbf{q}| \to 0) = \frac{N_c}{3}g^2T^2, \tag{60}$$

and the summation over all quark flavors, we obtain

$$m_D^2 = m_Q^2 + m_G^2 = \sum_f (2m_{--}^2) + m_G^2, \tag{61}$$

which is exactly the same as that shown in Eq. (54).

In the general case with nonzero quark mass, the contribution from the negative-energy quark loop to the gluon self-energy at $q_0 = 0$ becomes

$$\begin{aligned}\bar{\Pi}_{--} &= \mathcal{O}(|\mathbf{q}|/m_f), \\ \bar{\Pi}_{--}^{0i} &= \mathcal{O}((|\mathbf{q}|/m_f)^2)\hat{q}^i, \\ \bar{\Pi}_{--}^{00} &= -\frac{g^2m_f^3}{8\pi^2T}\int_1^\infty dx x\sqrt{x^2 - 1} \\ &\quad \times \sum_{s=\pm} f_F(x_s)(1 - 2f_F(x_s)) + \mathcal{O}(|\mathbf{q}|/m_f), \\ \bar{\Pi}_{--}^{ij} &= -\delta^{ij}\frac{g^2m_f^3}{24\pi^2T}\int_1^\infty dx\frac{(x^2 - 1)^{3/2}}{x} \\ &\quad \times \sum_{s=\pm} f_F(x_s)(1 - 2f_F(x_s)) + \mathcal{O}(|\mathbf{q}|/m_f),\end{aligned} \tag{62}$$

with $x_s = m_f x + s\mu_f$. Considering the similar contribution from the other quark loops and gluon and ghost loops, we derive the total Debye mass in the limit $|\mathbf{q}| = 0$,

$$m_D^2 = m_G^2 + \sum_f \frac{g^2 m_f^3}{4\pi^2 T}\int_1^\infty dx x\sqrt{x^2-1} \times \sum_{s=\pm} f_F(x_s)(1-2f_F(x_s)), \tag{63}$$

which reduces to the familiar result (61) for massless quarks.

B. High-density limit

Quark matter at low temperature and high baryon density can be realized in compact stars and nuclear collisions at intermediate energy. In the limit of zero temperature, the Bose-Einstein distribution f_B disappears and the Fermi-Dirac distribution f_F becomes a step function of the baryon chemical potential μ_f. Therefore, the gluon self-energy comes only from the quark loops. For massless quarks, with the symmetric quark distributions

$$f_F^\pm(\epsilon_+, \epsilon_-) = \frac{1}{2}[\Theta(\mu_f - \epsilon_-) \pm \Theta(\mu_f - \epsilon_+)], \tag{64}$$

and the continuation from Matsubara frequency summation to the Wick rotation integral $T\sum_{q_0} \to \int_{-\infty}^{+\infty} d\omega_q/(2\pi)$ ($q_0 = -i\omega_q$), the integration over the quark momentum $\mathbf{k}$ can be done easily, and the total transverse and longitudinal parts of the gluon self-energy can be explicitly expressed as

$$\begin{aligned}\bar{\Pi}^T &= -\frac{g^2\mu_f^2}{12\pi^2}\left(\frac{2q_0^2}{\mathbf{q}^2}+1\right) + \frac{g^2}{384\pi^2|\mathbf{q}|^3}\sum_{n,s=\pm}\mathcal{F}_T(nq_0, |\mathbf{q}|, s\mu_f),\\ \bar{\Pi}^L &= \frac{g^2\mu_f^2}{3\pi^2}\left(\frac{q_0^2}{\mathbf{q}^2}-1\right) - \frac{g^2}{192\pi^2|\mathbf{q}|^3}\sum_{n,s=\pm}\mathcal{F}_L(nq_0, |\mathbf{q}|, s\mu_f),\end{aligned} \tag{65}$$

with the functions $\mathcal{F}_T(nq_0, |\mathbf{q}|, s\mu_f)$ and $\mathcal{F}_L(nq_0, |\mathbf{q}|, s\mu_f)$ defined as

$$\begin{aligned}\mathcal{F}_T &= (q_0^2 + nq_0|\mathbf{q}| + 4\mathbf{q}^2 + 4s\mu_f q_0 + 2s\mu_f|\mathbf{q}| + 4\mu_f^2) \times (q_0^2 - \mathbf{q}^2)(nq_0 - |\mathbf{q}| + 2s\mu_f) \times \ln\frac{(nq_0 - |\mathbf{q}| + 2s\mu_f)^2}{(nq_0 - |\mathbf{q}|)^2},\\ \mathcal{F}_L &= (nq_0 + 2|\mathbf{q}| + 2s\mu_f)(q_0^2 - \mathbf{q}^2) \times (nq_0 - |\mathbf{q}| + 2s\mu_f)^2 \ln\frac{(nq_0 - |\mathbf{q}| + 2s\mu_f)^2}{(nq_0 - |\mathbf{q}|)^2}.\end{aligned} \tag{66}$$

Under the condition that $q_0, |\mathbf{q}| \ll \mu_f$ at extremely high baryon density, which is usually considered in the HDL approach, the above transverse and longitudinal self-energies are reduced to the well-known result [5,90]

$$\begin{aligned}\Pi^T &= -\frac{m_E^2}{2}\frac{q_0}{|\mathbf{q}|}\left[\left(1-\frac{q_0^2}{\mathbf{q}^2}\right)H\left(\frac{q_0}{|\mathbf{q}|}\right) + \frac{q_0}{|\mathbf{q}|}\right],\\ \Pi^L &= -m_E^2\left(1-\frac{q_0^2}{\mathbf{q}^2}\right)\left[1-\frac{q_0}{|\mathbf{q}|}H\left(\frac{q_0}{|\mathbf{q}|}\right)\right],\end{aligned} \tag{67}$$

where the effective gluon mass m_E and function $H(x)$ are

$$\begin{aligned}m_E^2 &= \frac{g^2\mu_f^2}{2\pi^2},\\ H(x) &= \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right).\end{aligned} \tag{68}$$

One can also check the gluon screening mass and dynamical mass in quark matter at high density by calculating the following limits of the gluon self-energies:

$$\begin{aligned}-\bar{\Pi}^T(q_0 = 0, |\mathbf{q}| \to 0) &= 0,\\ -\bar{\Pi}^L(q_0 = 0, |\mathbf{q}| \to 0) &= \frac{g^2\mu_f^2}{2\pi^2},\\ -\bar{\Pi}^T(q_0 \to 0, |\mathbf{q}| = 0) &= \frac{g^2\mu_f^2}{6\pi^2},\\ -\bar{\Pi}^L(q_0 \to 0, |\mathbf{q}| = 0) &= \frac{g^2\mu_f^2}{6\pi^2}.\end{aligned} \tag{69}$$

C. Thermodynamic potential

We now close the resummed gluon propagator to see the gluon contribution to the thermodynamic potential which controls the global behavior of the hot and dense quark-gluon plasma. The gluon contribution with ring diagrams can be written as [111]

$$\Omega = \Omega_0 + \Omega_1 + \Omega_{\mathrm{ring}}, \tag{70}$$

where Ω_0 is the free gluon contribution. To use the standard definition of Ω_{ring}, we have separated Ω_1 with only one self-energy on the ring from Ω_{ring}. Taking into account the separation of a general gluon propagator into transverse and longitudinal parts [see Eq. (52)], Ω_{ring} can be divided into two parts [85]:

$$\Omega_{\mathrm{ring}} = -(N_c^2 - 1)\left(R_T + \frac{1}{2}R_L\right). \tag{71}$$

Calculations using HTL/HDL approximations can be found in Refs. [86,112,113]. Here we calculate R_L and R_T with the derived gluon self-energy without any restriction on the medium temperature T and quark chemical potential μ_f. To simplify the calculation we consider the massless

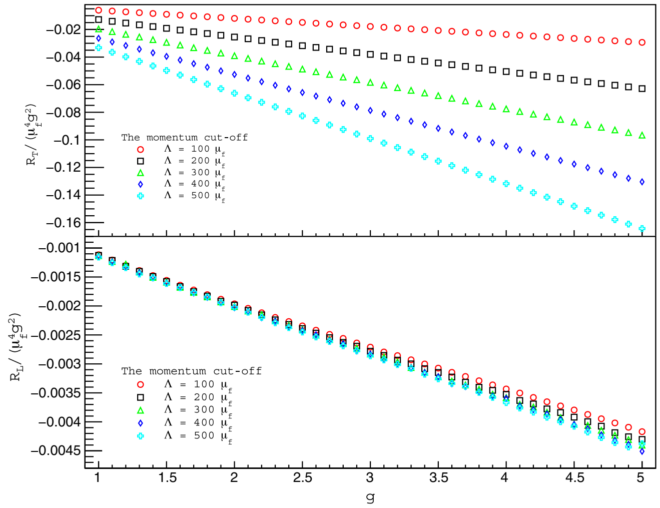

FIG. 3. Scaled longitudinal and transverse thermodynamic potentials $R_L/(\mu_f^4 g^2)$ and $R_T/(\mu_f^4 g^2)$ as functions of the coupling constant g for cold quark matter with temperature $T = 0$ and different gluon momentum cutoffs Λ.

limit. In this case, the thermodynamic potential is proportional to μ_f^4. After integrating out the azimuth angles ϕ and θ of the gluon momentum $\mathbf{q}$, the temperature, chemical potential, and coupling constant dependence of the transverse and longitudinal thermodynamic potentials $R_T(T,\mu_f,g)$ and $R_L(T,\mu_f,g)$ can be explicitly expressed as

$$R_T = \int \frac{d\omega_q}{2\pi} \int d|\mathbf{q}| \frac{4\pi\mathbf{q}^2}{(2\pi)^3} \times \left[\ln\left(1 - \frac{\Pi^T(-i\omega_q, |\mathbf{q}|)}{\omega_q^2 + \mathbf{q}^2}\right) + \frac{\Pi^T(-i\omega_q, |\mathbf{q}|)}{\omega_q^2 + \mathbf{q}^2}\right],$$
$$R_L = \int \frac{d\omega_q}{2\pi} \int d|\mathbf{q}| \frac{4\pi\mathbf{q}^2}{(2\pi)^3} \times \left[\ln\left(1 - \frac{\Pi^L(-i\omega_q, |\mathbf{q}|)}{\omega_q^2 + \mathbf{q}^2}\right) + \frac{\Pi^L(-i\omega_q, |\mathbf{q}|)}{\omega_q^2 + \mathbf{q}^2}\right]. \quad (72)$$

Considering $\Pi \sim g^2$, the Taylor expansion of the logarithm function leads to $R_T, R_L \sim g^4$ in the weak coupling region with $g \ll 1$. In the intermediate coupling region $1 \lesssim g \lesssim 5$, a detailed calculation shows an approximate dependence $R_T, R_L \sim g^2 + \lambda g^3$. Figure 3 shows the scaled thermodynamic potentials at different gluon momentum cutoffs $|\mathbf{q}| < \Lambda$. Note that R_T and R_L are convergent and it is not necessary to take a cutoff; Fig. 3 just shows that the coupling dependence is independent of the gluon momentum.

V. SUMMARY

In this paper we focused on the gluon self-energy at finite temperature, chemical potential, and quark mass in the frame of QCD. With the method of quark energy projectors, the divergence of the self-energy $\Pi^{\mu\nu}(q)$ appears only in the mixed loop constructed from the positive- and negative-energy modes and is medium independent. Therefore, the divergence can be removed through a renormalization procedure in vacuum. After a resummation of the gluon self-energy which is often used in literature, we calculated nonperturbatively the gluon Debye mass and thermodynamic potential. Since the Debye mass is irrelevant to the mixed quark loop, it is renormalization independent. In the limits of massless quarks and extremely high temperature and high density, our calculations come back to the well-known results from HTL and HDL. The quark energy-projector method used in this paper can be straightforwardly extended to treat the divergence of other QCD diagrams with more quark loops.

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