Conformal transformation with multiple scalar fields and geometric property of field space with Einstein-like solutions

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(Received 18 May 2021; accepted 15 August 2021; published 15 September 2021)

Multiple scalar fields appear in vast modern particle physics and gravity models. When they couple to gravity nonminimally, conformal transformation is utilized to bring the theory into the Einstein frame. However, the kinetic terms of scalar fields are usually not canonical, which makes analytic treatment difficult. Here, we investigate under what conditions the theories can be transformed to the quasicanonical form, in which case the effective metric tensor in field space is conformally flat. We solve the relevant nonlinear partial differential equations for an arbitrary number of scalar fields and present several solutions that may be useful for future phenomenological model building, including the σ model with a particular nonminimal coupling. We also find conformal flatness can always be achieved in some modified gravity theories, for example, the Starobinsky model.

DOI: 10.1103/PhysRevD.104.064042

I. INTRODUCTION

Scalar fields are ubiquitous in modern particle physics and gravity models, including inflation theories and dark energy scenarios [1–8]. In many cases [9–27], the scalar fields are nonminimally coupled to gravity through a term $f(\phi^i)R$, where f is some function of scalar fields ϕ^i and R is the Ricci scalar curvature. For example, the function $f(\phi)$ is proportional to ϕ^2 in the Jordan-Brans-Dicke theory [28,29] and Higgs inflation model [30]. In modified gravity theories where only functions of R are introduced [31], it is equivalent to treat as introducing a scalar field, for instance, $f(\phi) \sim \phi^2$ in the Starobinsky model [32].

The Lagrangian with the $f(\phi^i)R$ term is usually referred to as the one in the Jordan frame, in which the kinetic terms of scalar fields are canonical. To compare with experimental observations, it is standard to perform a conformal transformation [33–35] on the metric tensor to Einstein frame such that $f(\phi^i)R$ is transformed into $\tilde{R}$. However, conformal transformation would induce a noncanonical kinetic term of scalars in the Einstein frame, which makes analytic treatment rather complicated, and various approximate methods have to be utilized. If there is only one scalar field in the theory, it is always possible to redefine the field variable and make the kinetic term canonical. However, it is not clear whether such redefinition always exists in theories with multiple scalar fields.

The systematic investigation on conformal transformation with multiple scalar fields was conducted in Ref. [36], in which the analysis was done with the effective metric tensor $\mathcal{G}_{ij}$ in the field space defined by the kinetic term $\mathcal{G}_{ij}d\phi^i d\phi^j$. It is found that generally the field metric is not flat; therefore, the kinetic terms are not canonical. Only one solution was found for $f(\phi^i)$ with two scalar fields such that the associated $\mathcal{G}_{ij}$ is conformally flat. This finding is closely related to analytic analysis in phenomenological model building involving multiple scalar fields. For example, a standard model Higgs doublet composes four real scalar fields, and the σ model has N fields with SO(N) symmetry. It is then unclear whether physical models with a Higgs doublet, σ model, and other multiple scalar fields might induce noncanonical kinetic terms that result in unstable systems. Is it possible to find $f(\phi^i)$ with a conformally flat field metric in the Einstein frame such that the equations of motion and energy-momentum tensor of scalar fields are simpler?

In this paper, we intend to answer the above question and present several new solutions for $f(\phi^i)$ with the corresponding $\mathcal{G}_{ij}$ conformally flat. We solve the relevant nonlinear partial differential equations for the requirements on $f(\phi^i)$ for any number of scalar fields and tabulate the solutions in Table I, which might be useful for future model building. Our results suggest that for the σ model with N scalar fields the field space can be conformally flat if the

TABLE I. The analytic solutions $f(\phi^i)$ in the first row that give conformally flat metrics, with the corresponding $\hat{\mathcal{G}}_{ij}$, $\hat{\Gamma}^i_{jk}$, $\hat{\mathcal{R}}_{ijkl}$, and $\hat{\mathcal{C}}_{ijkl}$. The metrics in the third column are solutions to the Einstein-like equation in vacuum of field space. In some cases, the constants a, b_i, and c should satisfy some conditions that allow $f(\phi^i) > 0$ (see the text for details).

$f(\phi^i) =$	Constant	$\begin{cases} a(c+b_i\phi^i)^{\frac{1}{1-p}}, & p\neq 1 \\ a\exp(b_i\phi^i), & p=1\end{cases}$	$a+b_i\phi^i-\frac{1}{12}\delta_{ij}\phi^i\phi^j$
$\hat{\mathcal{G}}_{ij} =$	δ_{ij}	$\begin{cases} \delta_{ij}+\frac{3ab_ib_j}{(1-p)^2}(c+b_k\phi^k)^{\frac{2p-1}{1-p}}, & p\neq 1 \\ \delta_{ij}+3ab_ib_j\exp(b_k\phi^k), & p=1\end{cases}$	$\delta_{ij}+\frac{3(b_i-\frac{\phi^i}{6})(b_j-\frac{\phi^j}{6})}{a+b_k\phi^k-\frac{1}{12}\delta_{kl}\phi^k\phi^l}$
$\hat{\Gamma}^i_{jk} =$	0	$\propto (p-\frac{1}{2})b_ib_jb_k$	$\propto (-\frac{1}{6}f_i\delta_{jk}-\frac{1}{2f}f_if_jf_k)$
$\hat{\mathcal{R}}_{ijkl} =$	0	0	$\propto (\delta_{i[k}\delta_{l]j}-\frac{6}{f}f_{[i}\delta_{j][k}f_{l]})$
$\hat{\mathcal{C}}_{ijkl} =$	0	0	0

coupling has a particular form, in which local scaling symmetry is evident. We also find in some modified gravity theories that involve a function of R and scalar fields that the associated field spaces are always conformally flat.

This paper is organized as follows. In Sec. II, we establish our theoretical formalism along with the notations. Then, in Sec. III, we analyze the structure of field space and solve the differential equations for a conformally flat metric tensor. Later, in Sec. IV, we discuss a particular case in which a local scaling symmetry is present for the σ model that couples to gravity nonminimally. After that, in Sec. V, we show in modified gravity theories, such as the Starobinsky model, that the field space is always conformally flat. Finally, we give our conclusion.

Throughout the paper, we use the four-dimensional space-time metric $g_{\mu\nu}$ with a sign convention $(-1,1,1,1)$ and the natural unit $M_p \equiv 1/\sqrt{8\pi G} = 1$. Greek letters $(\mu,\nu,\rho,\ldots)$ denotes the space-time indices, while Latin letters $(i,j,I,J,\ldots)$ refer to field variables in the field space. Riemann tensor is defined by $R^\rho_{\sigma\mu\nu} = \partial_\mu\Gamma^\rho_{\sigma\nu} - \partial_\nu\Gamma^\rho_{\sigma\mu} + \Gamma^\rho_{\mu\tau}\Gamma^\tau_{\sigma\nu} - \Gamma^\rho_{\nu\tau}\Gamma^\tau_{\sigma\mu}$, where the connection is given by $\Gamma^\rho_{\mu\nu} = \frac{1}{2}g^{\rho\tau}[\partial_\mu g_{\nu\tau} + \partial_\nu g_{\mu\tau} - \partial_\tau g_{\mu\nu}]$, and Ricci scalar R is defined through $R_{\sigma\nu} \equiv R^\rho{}_{\sigma\rho\nu}$ and $R \equiv g^{\sigma\nu}R_{\sigma\nu}$. We may easily check $R^\rho{}_{\sigma\mu\nu} = R_{\nu\mu\sigma}{}^\rho$, where the latter is also widely used in the literature.

II. FORMALISM AND NOTATIONS

We shall first consider the following general Lagrangian $\mathcal{L}$ in four-dimensional space-time for N nonminimally coupled scalar fields, ϕ^i, $i = 1,\ldots,N$:

$$\frac{\mathcal{L}}{\sqrt{-g}} = f(\phi^i)R - \frac{1}{2}g^{\mu\nu}\delta_{ij}\nabla_\mu\phi^i\nabla_\nu\phi^j - V(\phi^i), \tag{1}$$

where g are the determinant of $g_{\mu\nu}$ and the covariant derivative is denoted by ∇. V is the scalar potential that can be neglected in our main theoretical discussions but would be relevant for phenomenological studies. This Lagrangian is referred to as the one in the Jordan frame where nonminimal coupling $f(\phi^i)R$ is present. In the cases of phenomenological interest, $f(\phi^i)$ should satisfy $f(\phi^i) > 0$ in the relevant parameter regions.

We make the standard conformal transformation on the metric tensor:

$$\tilde{g}_{\mu\nu} = \Omega^2(x)g_{\mu\nu}, \qquad \Omega^2(x) = 2f(\phi^i). \tag{2}$$

After using the transformation relations in the Appendix A and denoting $\omega \equiv \ln\Omega$, we can get the Lagrangian in the Einstein frame:

$$\begin{aligned}\frac{\mathcal{L}}{\sqrt{-\tilde{g}}} &\supseteq \frac{1}{2}\tilde{R} - 3\tilde{g}^{\mu\nu}\tilde{\nabla}_\mu\omega\tilde{\nabla}_\nu\omega - \frac{1}{2\Omega^2}\tilde{g}^{\mu\nu}\delta_{ij}\tilde{\nabla}_\mu\phi^i\tilde{\nabla}_\nu\phi^j \\ &= \frac{1}{2}\tilde{R} - \frac{1}{2}\tilde{g}^{\mu\nu}\mathcal{G}_{ij}\tilde{\nabla}_\mu\phi^i\tilde{\nabla}_\nu\phi^j,\end{aligned} \tag{3}$$

where an effective metric tensor in field space $\mathcal{G}_{ij}$ appears and is given by

$$\mathcal{G}_{ij} = \frac{1}{2f}\left(\delta_{ij} + \frac{3}{f}f_if_j\right). \tag{4}$$

Here and after, we use the short notation for derivatives of f, $f_i = \frac{\partial f}{\partial\phi^i}$, $f_{ij} = \frac{\partial^2 f}{\partial\phi^i\partial\phi^j}$, etc.

Because of the curved metric $\mathcal{G}_{ij}$ in field space, the scalar fields generally have noncanonical kinetic terms [unless we begin with noncanonical ones in Eq. (1), $\delta_{ij} \to 2f\delta_{ij} - \frac{3}{f}f_if_j$]. If there is only one scalar field ϕ in the theory, $\mathcal{G}_{11}$ is positive definite for $f(\phi^i) > 0$. Then, we can always redefine a new field variable Φ by the differential equation $d\Phi(x)/d\phi(x) = \pm\sqrt{\mathcal{G}_{11}}$ and, therefore, make Φ's kinetic term canonical. For the cases with multiple scalars, it would be much more complicated, as we shall present below.

III. STRUCTURE OF THE FIELD SPACE

With the metric tensor $\mathcal{G}_{ij}(\phi^k)$ in field space, we would like to know whether there exists a set of field coordinates $\varphi^I = \varphi^I(\phi^k)$ such that the associated metric tensor is flat, $\mathcal{G}_{JK}(\varphi^I) = \delta_{JK}$. One necessary condition for the existence is that the Ricci scalar curvature $\mathcal{R}(\mathcal{G}_{ij}) = 0$, because $\mathcal{R}$ is an invariant quantity under coordinate transformation. We can easily check that $\mathcal{R} \equiv 0$ for $N = 1$ due to the antisymmetric properties of $\mathcal{R}_{ijkl}$, which confirms our analysis above that a canonical kinetic term can always be achieved after the redefinition of the field for $N = 1$. However, in general, $\mathcal{R} \neq 0$ for $N > 1$; therefore, field coordinates φ^i may not exist such that their kinetic terms are canonical.

It is modest to ask whether the geometry of field space is conformally flat, $\mathcal{G}_{JK}(\varphi^I) \propto \delta_{JK}$. In such cases, the kinetic terms may be referred as quasicanonical. In such forms, the energy-momentum tensor and equation of motion would be simpler (neglecting the potential term):

$$\tilde{T}_{\mu\nu} = -\frac{1}{2}\mathcal{G}_{IJ}\tilde{g}_{\mu\nu}\tilde{g}^{\alpha\beta}\tilde{\nabla}_\alpha\varphi^I\tilde{\nabla}_\beta\varphi^J + \mathcal{G}_{IJ}\tilde{\nabla}_\mu\varphi^I\tilde{\nabla}_\nu\varphi^J,$$
$$\tilde{\Box}\varphi^I + \Gamma^I_{JK}\tilde{g}^{\mu\nu}\tilde{\nabla}_\mu\varphi^J\tilde{\nabla}_\nu\varphi^K = 0.$$

As we have learned from Riemannian geometry, manifolds with $N \leq 3$ are always conformally flat. But for $N > 3$ it is no longer true, and conformal flatness is determined by the Weyl tensor, which is defined as

$$\mathcal{C}_{ijkl} \equiv \mathcal{R}_{ijkl} - \frac{2}{N-2}(\mathcal{G}_{i[k}\mathcal{R}_{l]j} - \mathcal{G}_{j[k}\mathcal{R}_{l]i}) + \frac{2\mathcal{R}}{(N-1)(N-2)}\mathcal{G}_{i[k}\mathcal{G}_{l]j}. \quad (5)$$

If $\mathcal{C}_{ijkl} = 0$, the field space is conformally flat. To compute $\mathcal{C}_{ijkl}$ in the cases of our interest, we conformally transform $\mathcal{G}_{ij}$ into

$$\hat{\mathcal{G}}_{ij} = 2f \times \mathcal{G}_{ij} = \delta_{ij} + \frac{3}{f}f_i f_j \quad (6)$$

and calculate the corresponding $\hat{\mathcal{R}}_{ijkl}, \hat{\mathcal{R}}_{ij}, \hat{\mathcal{R}}$, and $\hat{\mathcal{C}}_{ijkl}$. The relation $\hat{\mathcal{C}}^i{}_{jkl} = \mathcal{C}^i{}_{jkl}$ enables us to reach the condition that the field space with metric $\mathcal{G}_{ij}$ is conformally flat if $\hat{\mathcal{C}}_{ijkl} = 0$.

First, we compute the determinant of $\hat{\mathcal{G}}_{ij}$, $\hat{\mathcal{G}}$, and the inverse metric $\hat{\mathcal{G}}^{ij}$:

$$\hat{\mathcal{G}} = 1 + \frac{3}{f}\sum_{i=1}^{N} f_i^2, \qquad \hat{\mathcal{G}}^{ij} = \delta_{ij} - \frac{3}{f\hat{\mathcal{G}}}f_i f_j, \quad (7)$$

which are surprisingly simple. The calculation details can be found in the Appendix B. Note that the indices of inverse metric are in the subscript, which allows us to do tensor analysis just as the usual matrix manipulation. We can easily check that $\hat{\mathcal{G}}^{ij}\hat{\mathcal{G}}_{jk} = \delta_{ik} \equiv \delta^i_k$. Based on the symmetric property, we can evaluate that the metric field $\hat{\mathcal{G}}_{ij}$ is positive definite for $f(\phi^i) > 0$. Therefore, there is no ghost in such physical systems. This conclusion is independent of which parameterization of $f(\phi^i)$ is used, since the determinant $\hat{\mathcal{G}}^{ij}$ does not change sign under the field transformations.

We can also obtain the following geometric quantities after tedious calculations:

$$\hat{\Gamma}^i_{jk} = \frac{1}{2}\mathcal{G}^{il}(\partial_j\mathcal{G}_{kl} + \partial_k\mathcal{G}_{jl} - \partial_l\mathcal{G}_{jk}) = \frac{3}{f\hat{\mathcal{G}}}f_i\left(f_{jk} - \frac{1}{2f}f_j f_k\right), \quad (8)$$

$$\hat{\mathcal{R}}_{ijkl} = \frac{3}{f\hat{\mathcal{G}}}\Big[(f_{ik}f_{jl} - f_{il}f_{jk}) + \frac{1}{2f}(f_i f_l f_{jk} + f_j f_k f_{il} - f_i f_k f_{jl} - f_j f_l f_{ik})\Big] = \frac{6}{f\hat{\mathcal{G}}}\left(f_{i[k}f_{l]j} + \frac{1}{f}f_{[i}f_{j][k}f_{l]}\right), \quad (9)$$

$$\hat{\mathcal{R}}_{ij} = \frac{3}{f\hat{\mathcal{G}}}\Big[(f_{ij}f_{kk} - f_{ik}f_{jk}) + \frac{1}{2f}(f_i f_k f_{jk} + f_j f_k f_{ik} - f_i f_j f_{kk} - f_{ij}f_k^2) + \frac{3}{f\hat{\mathcal{G}}}f_k f_l(f_{ik}f_{jl} - f_{ij}f_{lk}) - \frac{3}{2f^2\hat{\mathcal{G}}}f_k f_l(f_j f_l f_{ik} + f_i f_k f_{lj} - f_i f_j f_{lk} - f_k f_l f_{ij})\Big], \quad (10)$$

$$\hat{\mathcal{R}} = \frac{3}{f^2\hat{\mathcal{G}}^2}[f\hat{\mathcal{G}}(f_{ii}f_{jj} - f_{ij}^2) + (f_i f_j f_{ij} - f_k^2 f_{jj}) + 6(f_{ij}f_{jk} - f_{ik}f_{jj})f_i f_k], \quad (11)$$

where $f_k^2 = \sum_k f_k f_k, f_{ij}^2 = \sum_{ij} f_{ij}f_{ij}$, and all the repeated indices are summed. $\hat{\mathcal{C}}_{ijkl}$ can be obtained straightforwardly with Eq. (5).

Now we are in a position to discuss the conditions for conformal flatness of the field space. One solution with $\hat{\mathcal{R}}_{ijkl} = 0$ was found in the literature [36], with $f(\phi_1, \phi_2) = \xi_1(\phi^1)^2 + \xi_2(\phi^2)^2$ for $N = 2$, where ξ_i are arbitrary positive constants. This can easily be checked by calculating the $\hat{\mathcal{R}}_{1212} = 0$, which is the only independent component for $N = 2$. One is tempted to extend the case to $N > 2$, since the σ model would fall in this category. Unfortunately, extension of such a form for $N > 2$ gives $\hat{\mathcal{R}}_{1212} \neq 0$, in general, except all ξ_i are equal to some particular value, as we shall show below.

Here we present new solutions with $\hat{\mathcal{R}}_{ijkl} = 0$, or $\hat{\mathcal{R}}_{ijkl} \neq 0$ but $\hat{\mathcal{C}}_{ijkl} = 0$. Completely solving f from the nonlinear partial differential equation $\hat{\mathcal{R}}_{ijkl} = 0$ is notoriously difficult and unpractical. Besides, there is no unique solution for such nonlinear equations. We have known a similar case from solving the Einstein field equation, which is also nonlinear and has multiple solutions. In this paper, we shall present several solutions based on the symmetric properties of $\hat{\mathcal{R}}_{ijkl}$. We enumerate several cases below.

(1) $\hat{\Gamma}^i_{jk} = 0$ *and* $\hat{\mathcal{R}}_{ijkl} = 0$.—This is the simplest case and can be easily verified from the definition of $\hat{\mathcal{R}}_{ijkl}$. We may further divide this category into two cases, after observing the feature in Eq. (8).

(a) $f_i = 0$.—This is the trivial solution with $f =$ positive constant, in which case the scalar fields are minimally coupled with gravity.

(b) $f_{jk} = \frac{1}{2f} f_j f_k$.—We can solve the equation by taking a further derivative, using the above relation recursively and getting an additional condition $f_{ijk} = 0$, which indicates f is a quadratic function of ϕ_i:

$$f(\phi^i) = a(c + b_i\phi^i)^2, \tag{12}$$

where $a > 0$, b_i, and c are arbitrary nonzero constants.

(2) $\hat{\Gamma}^i_{jk} \neq 0$ *but* $\hat{\mathcal{R}}_{ijkl} = 0$.—Inspired by the second case above, we notice that taking a form $f_{ij} = \frac{p}{f} f_i f_j$ would give a vanishing Riemann tensor. Solving the differential equation gives the solutions

$$f(\phi^i) = a(c + b_i\phi^i)^{\frac{1}{1-p}},$$
$$f_i = \frac{ab_i}{1-p}(c + b_j\phi^j)^{\frac{p}{1-p}}, \quad \text{for } p \neq 1, \tag{13}$$

and

$$f(\phi^i) = a \exp(b_i\phi^i),$$
$$f_i = ab_i \exp(b_j\phi^j), \quad \text{for } p = 1. \tag{14}$$

The solution for $p = 0$ is included above, in which case f is a linear function of ϕ^i. The metrics in this category solve Einstein-like equations in vacuum at any dimension N:

$$\hat{\mathcal{G}}_{ij} = \begin{cases} \delta_{ij} + \frac{3ab_ib_j}{(1-p)^2}(c + b_k\phi^k)^{\frac{2p-1}{1-p}}, & p \neq 1, \\ \delta_{ij} + 3ab_ib_j \exp(b_k\phi^k), & p = 1. \end{cases} \tag{15}$$

For phenomenological studies, the existence of $f(\phi^i) > 0$ should be imposed to constrain the parameters a, b_i, and c. In the case of $p = 1$, $a > 0$ and b_i is arbitrary constant. In the case of general p except for some fractions (for instances, $p = 1/2, 3/4, 5/6, \ldots$), there are no general conditions for a, b_i, and c. The reason is that linear function $c + b_j\phi^j$ can go from $-\infty$ to ∞. As long as for our physical interests there exists $f(\phi^i) > 0$ at some domains of ϕ^i, which are determined by the explicit shape and minimum of potential $V(\phi^i)$, the theories can recover Einstein's gravity.

(3) $\hat{\mathcal{R}}_{ijkl} \neq 0$ *but* $\hat{\mathcal{C}}_{ijkl} = 0$.—Even if the Riemann tensor does not vanish but has the following structure:

$$\hat{\mathcal{R}}_{ijkl} \propto \hat{\mathcal{G}}_{i[k}\hat{\mathcal{G}}_{l]j}, \tag{16}$$

we would obtain $\hat{\mathcal{C}}_{ijkl} = 0$ as well. Contracting with $\mathcal{G}^{ik}\mathcal{G}^{jl}$ gives the proportional factor $\hat{\mathcal{R}}/[N(N-1)]$. Observing that

$$\hat{\mathcal{G}}_{i[k}\hat{\mathcal{G}}_{l]j} = \delta_{i[k}\delta_{l]j} - \frac{6}{f} f_{[i}\delta_{j][k}f_{l]} \tag{17}$$

and comparing with $\hat{\mathcal{R}}_{ijkl}$, we would have the following relation:

$$f_{ij} = -\frac{1}{6}\delta_{ij}. \tag{18}$$

The general solution of the above equation would be

$$f(\phi_i) = a + b_i\phi^i - \frac{1}{12}\delta_{ij}\phi^i\phi^j, \quad f_i = b_i - \frac{1}{6}\phi^i. \tag{19}$$

Similarly, the existence of $f(\phi^i) > 0$ over some parameter ranges of ϕ^i would constrain a and b_i. Put it another way, $f(\phi^i)$ cannot be negative definite. We can write $f(\phi^i) = -\frac{1}{12}\sum_i(\phi^i - 6b_i)^2 + a + 3\sum_i b_i^2$. Therefore, as long as $a + 3\sum_i b_i^2 > 0$, $f(\phi^i) > 0$ can be satisfied at some parameter ranges of ϕ^i. In the case of nonpositive f, we cannot make the conformal transformation in Eq. (2) and a physical theory as Einstein gravity would be missing. Again, we note that $f > 0$ in the whole parameter spaces might be too restrictive. As long as there exist parameter spaces with $f > 0$ around the field domain we are interested in, for instance, the inflation regime and the potential minimum, conformal transformation is still valid in the finite domain that Einstein gravity can be recovered.

We point out that, by redefining a, b_i, and c, the forms of $f(\phi^i)$ in all the above solutions do not change under the field shift $\phi^i \to \phi^i + d^i$, where d^i are arbitrary constant. The solutions are summarized in Table I, where $f(\phi^i)$ and its corresponding $\hat{\mathcal{G}}_{ij}$, $\hat{\Gamma}^i_{jk}$, $\hat{\mathcal{R}}_{ijkl}$, and $\hat{\mathcal{C}}_{ijkl}$ are listed. These $f(\phi^i)$ might be useful for future model building due to their simple forms.

So far, we have focused on the Riemannian metric. If one of the scalar fields has opposite sign for the kinetic term, we would get the Lorentzian metric $\hat{\mathcal{G}}_{ij} = \eta_{ij} + 3f_if_j/f$, where $\eta_{11}=-1, \eta_{ii}=1$ for $i\neq 1$, and $\eta_{ij}=0$ if $i\neq j$. Note that the opposite sign in the Jordan frame does not necessarily lead to a ghost in the presence of nonminimal coupling. The reason is that conformal transformation induces an additional kinetic term in the Einstein frame with the total coefficient proportional to $\eta_{ij}+\frac{3}{f}f_i f_j$. As long as the field metric tensor $\eta_{ij}+\frac{3}{f}f_i f_j$ is positive definite, we have normal scalars. In fact, viable theories with opposite sign were discussed in α-attractor inflation models; see Refs. [37–39].

Similarly, for the Lorentzian field metric, we calculate

$$\mathcal{G}=\det\hat{\mathcal{G}}_{ij}=-\left(1+\frac{3}{f}\eta_{ij}f_i f_j\right)=-\left(1+\frac{3}{f}f_i f^i\right), \tag{20}$$

$$\mathcal{G}^{ij}=\eta_{ij}+\frac{3}{f\mathcal{G}}(-1)^{\delta_{1i}+\delta_{1j}}f_i f_j=\eta^{ij}+\frac{3}{f\mathcal{G}}f^i f^j, \tag{21}$$

$$\hat{\Gamma}^i_{jk}=-\frac{1}{\mathcal{G}}\eta_{il}f_l\left(f_{jk}-\frac{1}{2f}f_j f_k\right)=-\frac{1}{\mathcal{G}}f^i\left(f_{jk}-\frac{1}{2f}f_j f_k\right). \tag{22}$$

Here, the subscripts of f denote the usual derivatives, upgraded by η^{ij}. Note that the similar tensor structures of $\hat{\mathcal{G}}_{ij}$ and $\hat{\Gamma}^i_{jk}$ to previous cases lead to the same tensor structure of $\hat{\mathcal{R}}_{ijkl}$. Therefore, the solutions to $\hat{\mathcal{R}}_{ijkl}=0$ barely change, except the replacement $\delta_{ij}\to\eta_{ij}$.

The absence of a ghost requires $\hat{\mathcal{G}}_{ij}$ to be positive definite, which imposes some conditions on $f(\phi^i)$ in addition to $f(\phi^i)>0$. Namely, we shall have the following constraints:

$$-\left(1-\frac{3f_1^2}{f}\right)>0,$$
$$-\left(1-\frac{3f_1^2}{f}+\frac{3}{f}\sum_{i=2}^{k}f_i^2\right)>0, \qquad k=2,\ldots,N.$$

These constraints restrict a, b_i, and c further, which can be obtained straightforwardly. Since there are no transparent solutions, we do not list them here.

We note that the function f is not invariant under nonlinear field redefinition, and $\hat{\mathcal{G}}_{ij}$, $\hat{\Gamma}^i_{jk}$, and $\hat{\mathcal{R}}_{ijkl}$ are also not invariant. Still, our results are useful for theoretical consistency check and phenomenological studies, in a sense that we may make field redefinition to transform the theory into the form we present, namely, in the Jordan frame where the kinetic terms are canonical. Then, we can calculate whether f satisfies the differential equations and the corresponding $\hat{\mathcal{G}}_{ij}$ is positive definite. In the Einstein frame, reparametrization of the field ϕ^i does not alter the results due to the change rules of $\hat{\mathcal{G}}_{ij}$ and $\hat{\mathcal{R}}_{ijkl}$.

IV. LOCAL SCALING SYMMETRY

In previous sections, we have presented several $f(\phi^i)$ with conformally flat field space by showing the associated $\mathcal{C}_{ijkl}=0$. However, we have not demonstrated explicitly the new field coordinate $\varphi^I(\phi^i)$ of the field space whose metric $\mathcal{G}_{IJ}(\varphi^K)$ is proportional to δ_{IJ}. In most cases, such coordinate transformations $\phi^i\to\varphi^I$ have no compact analytic solutions. Here, we have found an interesting case:

$$f(\phi^i)=a-\frac{1}{12}\delta_{ij}\phi^i\phi^j, \tag{23}$$

where the system has a global SO(N) symmetry for ϕ^i and is also called as σ model in field theory. We can introduce an auxiliary field χ and rewrite the relevant Lagrangian as

$$\frac{\mathcal{L}}{\sqrt{-g}}\supseteq a\left[g^{\mu\nu}\nabla_\mu\chi\nabla_\nu\chi+\frac{1}{6}R\chi^2\right] -\frac{1}{2}\left[g^{\mu\nu}\delta_{ij}\nabla_\mu\phi^i\nabla_\nu\phi^j+\frac{1}{6}R\delta_{ij}\phi^i\phi^j\right]. \tag{24}$$

Because there is a local scaling symmetry in the above Lagrangian,

$$\begin{aligned} g_{\mu\nu}(x)&\to\bar{g}_{\mu\nu}(x)=\lambda^2(x)g_{\mu\nu}(x),\\ \phi^i(x)&\to\bar{\phi}^i(x)=\lambda^{-1}(x)\phi(x),\\ \chi(x)&\to\bar{\chi}(x)=\lambda^{-1}(x)\chi(x), \end{aligned} \tag{25}$$

where $\lambda(x)$ is an arbitrary positive function, the original theory can be recovered by setting $\lambda(x)=\chi(x)/\sqrt{6}$. This symmetry is also called Weyl or conformal symmetry in the literature and has wide applications in model building; see Refs. [40,41] for recent examples.

When $a=1/2$, this theory has an extended global symmetry SO$(1,N)$, $\chi^2-\delta_{ij}\phi^i\phi^j=6$. The geometry of the field space or kinetic term is related to the distance element in the field space, $ds^2=d\chi^2-\delta_{ij}d\phi^i d\phi^j$. Introducing field coordinates (T,φ^i), we can parameterize

$$\begin{aligned} \chi&=\sqrt{6}\left(\frac{1}{2}\delta_{ij}\phi^i\phi^j e^T+\cosh T\right),\\ \varphi^N&=\sqrt{6}\left(\frac{1}{2}\delta_{ij}\phi^i\phi^j e^T-\sinh T\right),\\ \varphi^i&=\sqrt{6}\phi^i e^T, \qquad i,j=1,2,\ldots,N-1. \end{aligned} \tag{26}$$

We can easily show

$$d\chi = \sqrt{6}\left[\left(\frac{1}{2}\delta_{ij}\phi^i\phi^j e^T + \sinh T\right)dT + \delta_{ij}\phi^i e^T\, d\phi^j\right],$$
$$d\varphi^N = \sqrt{6}\left[\left(\frac{1}{2}\delta_{ij}\phi^i\phi^j e^T - \cosh T\right)dT + \delta_{ij}\phi^i e^T\, d\phi^j\right],$$
$$d\varphi^i = \sqrt{6}(e^T\, d\phi^i + \phi^i e^T dT). \tag{27}$$

In the new field coordinates (T, φ^i), the element is given by $ds^2 = -dT^2 - e^{2T}\delta_{ij}d\varphi^i d\varphi^j$. Refining $dT = -e^T d\tau$ or $\tau = e^{-T}$, we obtain $ds^2 = -\frac{1}{\tau^2}(d\tau^2 + \delta_{ij}d\varphi^i d\varphi^j)$. Therefore, the geometry of field space is shown explicitly to be conformally flat in the coordinate of (τ, φ^i).

Note that, although the geometric structure is determined by the hypersurface in field space, $\chi^2 - \delta_{ij}\phi^i\phi^j = 6$, in phenomenological studies one usually adopts a particular parametrization in which only one field variable is responsible for the physical effects of interest. For example, if we are interested in only the radial part as the inflaton, we can use the following parametrization of the field coordinates (φ, θ^i):

$$\chi = \sqrt{6}\cosh\varphi,$$
$$\phi^1 = \sqrt{6}\sinh\varphi\cos\theta^1,$$
$$\phi^2 = \sqrt{6}\sinh\varphi\sin\theta^1\cos\theta^2,$$
$$\vdots$$
$$\phi^{N-1} = \sqrt{6}\sinh\varphi\sin\theta^1\ldots\sin\theta^{N-2}\cos\theta^{N-1},$$
$$\phi^N = \sqrt{6}\sinh\varphi\sin\theta^1\ldots\sin\theta^{N-2}\sin\theta^{N-1}. \tag{28}$$

In this case, radial field φ would have a canonical kinetic term from the very beginning, while the angular field variables θ^i would not. Analysis of the dynamics of θ^i might involve approximations such as expanding by the θ^i/M_p as small parameters, etc. We have actually encountered the special case for $N = 1$ in the α-attractor inflation model [37,38] and its extensions with Weyl gauge field [39,42,43]. The confirmation from the $N = 1$ case also partially suggests the correctness of our calculation for general N.

V. $F(R)$ GRAVITY

In this section, we extend our discussions by considering the following Lagrangian of modified gravity in four-dimensional space-time for N nonminimally coupled scalar fields, ϕ^i, $i = 1, 2, \ldots, N$:

$$\frac{\mathcal{L}}{\sqrt{-g}} \supseteq F(R, \phi^i) - \frac{1}{2}g^{\mu\nu}\delta_{ij}\nabla_\mu\phi^i\nabla_\nu\phi^j, \tag{29}$$

where $F(R, \phi^i)$ is a function of R and ϕ^i without derivatives. Such terms are motivated from quantum corrections and cosmological models. For example, in the Starobinsky-like model, we have $F(R, \phi^i) = f(\phi^i)R + \alpha R^2/2$, which can give viable inflation scenarios.

We can similarly use the auxiliary field χ and rewrite the Lagrangian

$$\frac{\mathcal{L}}{\sqrt{-g}} = F(\chi^2, \phi^i) + F_R(\chi^2, \phi^i)\,(R - \chi^2) - \frac{1}{2}g^{\mu\nu}\delta_{ij}\nabla_\mu\phi^i\nabla_\nu\phi^j, \tag{30}$$

where F_R denotes the derivative of $F(R, \phi)$ over R and $F(\chi^2, \phi) \equiv F(R \to \chi^2, \phi)$. One may check that the equation of motion for χ still gives $\chi^2 = R$. Then denoting the 0-component $\phi^0 \equiv \chi$ and new f function

$$f(\phi^I) = F_R(\chi^2, \phi^i), \qquad I = 0, 1, 2, \ldots, N, \tag{31}$$

we have reduced the system into the case we discussed in previous sections. However, there is a crucial difference that ϕ^0 has no kinetic term. It turns out that this difference leads to significantly different results.

Following the similar procedures and omitting the potential term, we can obtain

$$\frac{\mathcal{L}}{\sqrt{-\tilde{g}}} \supseteq \frac{1}{2}\tilde{R} - 3\tilde{g}^{\mu\nu}\tilde{\nabla}_\mu\omega\tilde{\nabla}_\nu\omega - \frac{1}{2\Omega^2}\tilde{g}^{\mu\nu}\delta_{ij}\tilde{\nabla}_\mu\phi^i\tilde{\nabla}_\nu\phi^j$$
$$= \frac{1}{2}\tilde{R} - \frac{1}{2}\tilde{g}^{\mu\nu}\mathcal{G}_{IJ}\tilde{\nabla}_\mu\phi^I\tilde{\nabla}_\nu\phi^J, \tag{32}$$

where $\omega = \ln\Omega$, $\Omega^2 = 2f(\phi^I)$, and $\mathcal{G}_{IJ} = \frac{1}{2f}\bar{\mathcal{G}}_{IJ}$,

$$\bar{\mathcal{G}}_{IJ} = \delta_{IJ} + \frac{3}{f}f_I f_J - \delta_{0I}\delta_{0J}. \tag{33}$$

Here we have used $\bar{\mathcal{G}}_{IJ}$ to distinguish from $\hat{\mathcal{G}}_{ij}$ in the R-gravity case. We calculate

$$\bar{\mathcal{G}} = \frac{3}{f}f_0^2,$$
$$\bar{\mathcal{G}}^{IJ} = \delta_{IJ} - \frac{3}{f\bar{\mathcal{G}}}f_0(f_I\delta_{0J} + f_J\delta_{0I}) + \frac{\hat{\mathcal{G}}}{\bar{\mathcal{G}}}\delta_{0I}\delta_{0J}, \tag{34}$$

$$\bar{\Gamma}^I_{JK} = \frac{1}{f_0}\left(f_{JK} - \frac{1}{2f}f_J f_K\right)\delta_{0I}, \tag{35}$$

$$\bar{\mathcal{R}}_{IJKL} = \bar{\mathcal{R}}_{IJ} = \bar{\mathcal{R}} = 0, \qquad \bar{\mathcal{C}}_{IJKL} = 0. \tag{36}$$

All the curvature components vanish identically, which means the $\bar{\mathcal{G}}_{IJ}$ in Eq. (33) with any nonzero $f(\phi^I)$ solves the Einstein-like equations in vacuum. This conclusion does not depend on the form of $F(R, \phi^i)$ as long as $F(R, \phi^i)$ depends on R nonlinearly, a surprising result at first glance. Actually, there is a transparent way to understand this result by observing the first line of Eq. (32). Because $F(R,\phi^i)$ gravity is associated with an additional scalar degree of freedom, it is justified to introduce a new field variable φ with $d\varphi/d\omega = \sqrt{6}\Omega(\phi^I)$. The kinetic terms can be organized as

$$\frac{1}{2\Omega^2}\tilde{g}^{\mu\nu}\delta_{IJ}\tilde{\nabla}_\mu\phi^I\tilde{\nabla}_\nu\phi^J, \qquad I,J=1,2,\ldots N+1, \tag{37}$$

where $\phi^{N+1}=\varphi$. It is obvious that the metric tensor in field space δ_{IJ}/Ω^2 is conformally flat.

VI. CONCLUSION

We have investigated the conformal transformation with multiple scalar fields that nonminimally couple with gravity. These theories are ubiquitous in modern particle physics and cosmological models. Conformal transformation is employed to transform the Lagrangian from the Jordan frame to the Einstein frame, which also makes the kinetic terms of scalar fields noncanonical. We have found that if the number of scalar fields is larger than one, in general, it is not possible to redefine the field variables to make all the kinetic terms canonical.

We have discussed under what conditions the kinetic terms are positive definite (therefore, without a ghost) and whether they could be brought into quasicanonical, namely, different from canonical by a common factor. The latter is equivalent to the problem of finding a conformally flat metric tensor in the field space of scalars. We have solved the nonlinear partial differential equations in arbitrary dimensions and presented several solutions in Table I that give conformally flat metric tensors. The σ model with a particular nonminimal coupling is one of the solutions. These solutions may be useful for future phenomenological model building for inflation and dark energy. We have also shown that in some modified gravity theories, including the Starobinsky model, the metric tensor in field space is always conformally flat.

ACKNOWLEDGMENTS

Y. T. is supported by National Natural Science Foundation of China (NSFC) under Grant No. 11851302 and by the Fundamental Research Funds for the Central Universities. Y.-L. W. is supported in part by the National Key Research and Development Program of China under Grant No. 2020YFC2201501, NSFC under Grants No. 11851302, No. 11851303, No. 11690022, and No. 11747601, and the Strategic Priority Research Program of the Chinese Academy of Sciences under Grant No. XDB23030100. Y. T. is also supported by the Key Research Program of the Chinese Academy of Sciences, Grant No. XDPB15.

APPENDIX A: CONFORMAL TRANSFORMATION

The conformal transformation of metric tensor $\tilde{g}_{\mu\nu}=\Omega^2(x)g_{\mu\nu}$ leads to various relations between various geometric quantities:

$$\tilde{\Gamma}^\rho_{\mu\nu} = \Gamma^\rho_{\mu\nu} + (\delta^\rho_\mu\nabla_\nu\omega + \delta^\rho_\nu\nabla_\mu\omega - g_{\mu\nu}\nabla^\rho\omega), \tag{A1}$$

$$\begin{aligned}\tilde{R}^\rho{}_{\sigma\mu\nu} = {}& R^\rho{}_{\sigma\mu\nu} + 2\delta^\rho_{[\nu}\nabla_{\mu]}\nabla_\sigma\omega - 2g^{\rho\alpha}g_{\sigma[\nu}\nabla_{\mu]}\nabla_\alpha\omega \\ &+ 2\nabla_{[\nu}\omega\delta^\rho_{\mu]}\nabla_\sigma\omega - 2\nabla_{[\nu}\omega g_{\mu]\sigma}g^{\rho\beta}\nabla_\beta\omega \\ &- 2g_{\sigma[\nu}\delta^\rho_{\mu]}g^{\alpha\beta}\nabla_\alpha\omega\nabla_\beta\omega,\end{aligned} \tag{A2}$$

$$\begin{aligned}\tilde{R}_{\mu\nu} = {}& R_{\mu\nu} - 2\nabla_\mu\nabla_\nu\omega - g_{\mu\nu}g^{\alpha\beta}\nabla_\alpha\nabla_\beta\omega + 2\nabla_\mu\omega\nabla_\nu\omega \\ &- 2g_{\mu\nu}g^{\alpha\beta}\nabla_\alpha\omega\nabla_\beta\omega,\end{aligned} \tag{A3}$$

$$\tilde{R} = \tilde{g}^{\mu\nu}\tilde{R}_{\mu\nu} = \Omega^{-2}[R - 6\Box\omega - 6g^{\mu\nu}\nabla_\mu\omega\nabla_\nu\omega], \tag{A4}$$

where $\omega\equiv\ln\Omega$, $\Box\equiv g^{\mu\nu}\nabla_\mu\nabla_\nu$, and $[\cdots]$ in the subscripts indicate antisymmetrization of the included indices. Note that conformal transformation does not have effects on the coordinates x^μ and the usual partial derivative $\partial_\mu\equiv\frac{\partial}{\partial x^\mu}$. Then using the relation between $\tilde{R}$ and R, we can rewrite the Lagrangian $\mathcal{L}$ as

$$\begin{aligned}\frac{\mathcal{L}}{\sqrt{-\tilde{g}}} = {}& \frac{1}{2}\tilde{R} + \frac{3}{\Omega^2}\Box\omega + 3\tilde{g}^{\mu\nu}\tilde{\nabla}_\mu\omega\tilde{\nabla}_\nu\omega \\ &- \frac{1}{2\Omega^2}\tilde{g}^{\mu\nu}\delta_{ij}\tilde{\nabla}_\mu\phi^i\tilde{\nabla}_\nu\phi^j - \frac{V(\phi^i)}{\Omega^4}.\end{aligned} \tag{A5}$$

The second term in the right-hand side of the equation can be written as

$$\Omega^{-2}\Box\omega = \tilde{\Box}\omega - 2\tilde{g}^{\mu\nu}\tilde{\nabla}_\mu\omega\tilde{\nabla}_\nu\omega. \tag{A6}$$

In the action which is the space-time integral of $\mathcal{L}$, $\tilde{\Box}\omega$'s contribution is a surface term due to the following identity:

$$\tilde{\Box}\omega = \frac{1}{\sqrt{-\tilde{g}}}\partial_\mu(\sqrt{-\tilde{g}}\tilde{g}^{\mu\nu}\partial_\nu\omega) \tag{A7}$$

and, therefore, can be neglected in the cases we are considering in this paper.

APPENDIX B: DETERMINANT AND INVERSE METRIC

Here, we present the details of calculating the determinant $\mathcal{G}$ and the inverse metric tensor $\hat{\mathcal{G}}^{ij}$. The computation of $\mathcal{G}$ is done as follows. Writing $\hat{\mathcal{G}}_{ij}=\delta_{ij}+Af_if_j$, $A\equiv 3/f$, we have

$$\hat{\mathcal{G}} \equiv \det \hat{\mathcal{G}}_{ij} = \frac{1}{\Pi_i f_i} \det \begin{pmatrix} \frac{1}{f_1} + Af_1 & Af_2 & \cdots \\ Af_1 & \frac{1}{f_2} + Af_2 & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix} = \frac{1}{\Pi_i f_i} \det \begin{pmatrix} \frac{1}{f_1} + Af_1 & Af_2 & \cdots \\ -1/f_1 & \frac{1}{f_2} & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix}$$

$$= \frac{1}{\Pi_i f_i} \det \begin{pmatrix} \frac{1}{f_1} + Af_1 + \frac{A}{f_1}(f_2^2 + \cdots + f_N^2) & 0 & \cdots \\ -1/f_1 & \frac{1}{f_2} & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix} = 1 + A \sum_{i=1}^{N} f_i^2. \tag{B1}$$

For the inverse metric, we first compute the diagonal elements $\hat{\mathcal{G}}^{ii}$. It is straightforward to obtain by using the analog to the determinant of $\hat{\mathcal{G}}_{ij}$:

$$\hat{\mathcal{G}}^{ii} = \frac{1}{\hat{\mathcal{G}}} \left(1 + A \sum_{j \neq i} f_j^2 \right). \tag{B2}$$

For the off-diagonal elements $\hat{\mathcal{G}}^{ij} (i \neq j)$, without showing all the elements in the matrix, we have

$$\hat{\mathcal{G}}^{ji} = \frac{(-1)^{i+j}}{\hat{\mathcal{G}}} \det \begin{pmatrix} \cdots & 1 + Af_{i-1}^2 & Af_{i-1}f_i & Af_{i-1}f_{i+1} & \cdots & & & \\ \cdots & Af_{i+1}f_{i-1} & Af_{i+1}f_i & 1 + Af_{i+1}^2 & \cdots & \iddots & & \\ & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \\ & \vdots & \vdots & \vdots & \cdots & 1 + Af_{j-1}^2 & Af_{j-1}f_{j+1} & \cdots \\ \cdots & Af_j f_{i-1} & Af_j f_i & Af_j f_{i+1} & \cdots & Af_j f_{j-1} & Af_j f_{j+1} & \cdots \\ & \vdots & \vdots & \vdots & \cdots & Af_{j+1}f_{j-1} & 1 + Af_{j+1}^2 & \cdots \end{pmatrix}$$

$$= \frac{\prod_{k \neq i} f_k}{(-1)^{i+j} \hat{\mathcal{G}}} \det \begin{pmatrix} \cdots & \frac{1}{f_{i-1}} + Af_{i-1} & Af_i & Af_{i+1} & \cdots & & & \\ \cdots & Af_{i-1} & Af_i & \frac{1}{f_{i+1}} + Af_{i+1} & \cdots & \iddots & & \\ & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \\ & \vdots & \vdots & \vdots & \cdots & \frac{1}{f_{j-1}} + Af_{j-1} & Af_{j+1} & \cdots \\ \cdots & Af_{i-1} & Af_i & Af_{i+1} & \cdots & Af_{j-1} & Af_{j+1} & \cdots \\ & \vdots & \vdots & \vdots & \cdots & Af_{j-1} & \frac{1}{f_{j+1}} + Af_{j+1} & \cdots \end{pmatrix}$$

$$= \frac{\prod_{k \neq i} f_k}{(-1)^{i+j} \hat{\mathcal{G}}} \det \begin{pmatrix} 0 & \frac{1}{f_{i-1}} & 0 & 0 & \cdots & & & \\ 0 & 0 & 0 & \frac{1}{f_{i+1}} & \cdots & \iddots & & \\ & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \\ & \vdots & \vdots & \vdots & \cdots & \frac{1}{f_{j-1}} & 0 & \cdots \\ \cdots & Af_{i-1} & Af_i & Af_{i+1} & \cdots & Af_{j-1} & Af_{j+1} & \cdots \\ & \vdots & \vdots & \vdots & \cdots & 0 & \frac{1}{f_{j+1}} & \cdots \end{pmatrix} = -\frac{Af_i f_j}{\hat{\mathcal{G}}}. \tag{B3}$$

The above two results can be written in a unified form:

$$\hat{\mathcal{G}}^{ij} = \delta_{ij} - \frac{A}{\hat{\mathcal{G}}} f_i f_j. \tag{B4}$$

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