Hypersymmetric extensions of Maxwell-Chern-Simons gravity in 2 + 1 dimensions

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We present a consistent way of coupling three-dimensional Maxwell-Chern-Simons gravity theory with massless spin-$\frac{5}{2}$ gauge fields. We first introduce the simplest hyper-Maxwell-Chern-Simons gravity generically containing two massless spin-2 fields coupled with a massless Majorana fermion of spin $\frac{5}{2}$ whose novel underlying superalgebra is explicitly constructed. We then present three alternative hypersymmetric extensions of the Maxwell algebra which are shown to emerge from the Inönü-Wigner contraction procedure of precise combinations of the $\mathfrak{osp}(1|4)$ and the $\mathfrak{sp}(4)$ algebras. This allows us to construct distinct types of hyper-Maxwell-Chern-Simons theories that extend to include generically interacting nonpropagating spin-4 fields accompanied by one or two spin-$\frac{5}{2}$ gauge fields.

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I. INTRODUCTION

Hypergravity was an early alternative to supergravity theory [1,2] proposed by Aragone and Deser [3] that deems a spin-$5/2$ field as the superpartner of the graviton. Although this proposal initially attracted some attention [3–6], it was promptly discarded due to the incompatibility of the minimal coupling between gravity (spin 2) and the spin-$5/2$ field with higher-spin (HS) gauge invariance. This obstruction relies on the fact that the HS gauge variation of the Einstein-Hilbert action is proportional to the Ricci tensor so that it cannot be canceled by means of the minimally coupled spin-$5/2$ field, which is instead proportional to the full Riemann tensor. Nonetheless, owing to the particular relationship between the Riemann tensor and the Ricci tensor in three spacetime dimensions, Aragone and Deser managed to formulate the first consistent interacting and nonpropagating HS theory [7].

In $2+1$ dimensions, anti–de Sitter hypergravity theory was studied in [8–10]. The theory is constructed in terms of a Chern-Simons (CS) action for two copies of the $\mathfrak{osp}(4|1)$ superalgebra[1] and contains a spin-2 field, a spin-4 field, and a spin-$5/2$ field. The asymptotic structure analysis of this model was performed in [10], where it was shown that its asymptotic symmetry algebra is given by two copies of the hypersymmetric extension of the $W(2,4)$ algebra, known as the WB_2 superalgebra [20] and the $W(2,5/2,4)$ superalgebra [21]. Furthermore, this study also revealed the existence of hypersymmetry bounds involving a nonlinear function of the mass, angular momentum, and bosonic higher-spin charges, as well as an interesting class of HS solutions including solitons and extremal HS black holes with unbroken hypersymmetries [10,22]. In the vanishing cosmological constant limit it was verified that the spin-4 field decouples, thus reproducing the hypergravity theory studied in [23,24].

These results were explored more recently by means of an explicit extension of the Poincaré algebra with

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[1] Along these lines, (super)conformal gravity formulated as a CS gauge theory for $[\mathfrak{osp}(1|4)]$ $\mathfrak{sp}(4)$ (super)algebra was done in [11–15], while the corresponding asymptotic structure was studied in [16–19].

half-integer spin generators in any dimension that allowed one to reformulate the hypergravity theory of Aragone and Deser as a genuine gauge theory using standard fiber bundle structure in terms of CS actions [25] in three and five spacetime dimensions [23,24,26].[2] Indeed, the asymptotic structure analysis in $2+1$ dimensions led to a nonlinear hypersymmetric extension of the $\mathfrak{bms}_3$ algebra, along with nonlinear bounds for the energy. Moreover, despite the fact that this theory involves HS fields, the absence of bosonic HS fields allows one to describe the theory in the standard Riemann-Cartan geometry. A natural question that then arises is whether it is possible to construct consistent extensions of the hypergravity theories based on extensions of the hyper-Poincaré algebra. It should be mentioned that, in order to improve the likelihood of this possibility, it is mandatory to have full control of the building blocks—namely, a consistent algebra with a nondegenerate invariant tensor. In what follows we address this question by considering a nontrivial extension of the Poincaré algebra that has long been known as the Maxwell algebra [28,29], which is associated with the symmetry group of the Dirac (Klein-Gordon) equation minimally coupled to a constant electromagnetic field in Minkowski space in $3+1$ dimensions [30]. In any dimension, this algebra is characterized by the commutator

$$[P_a, P_b] = Z_{ab}, \tag{1.1}$$

modifying in this way the commutator of the momentum generators, which vanishes for the Poincaré algebra. The study of different aspects of the Maxwell algebra in four or more dimensions, including its derivation as an S expansion on the anti–de Sitter (AdS) algebra, can be found in [31,32]. The Maxwell group symmetries and its generalizations have been useful for extending standard general relativity through CS and Born-Infeld gravity theories in odd and even spacetime dimensions, respectively [33–36]. Deformations of this algebra and their dynamics through nonlinear realizations have been investigated [37–39], as have other interesting applications; see, e.g., [40–45].

In three spacetime dimensions, Maxwell-CS gravity appears to be a very appealing alternative theory of gravity in vacuum introduced in [46–48][3] and subsequently studied in [54–56]. The asymptotic structure of the Maxwell-CS theory was investigated in [55] by imposing a set of suitable boundary conditions resulting in an asymptotic symmetry algebra given by a deformation of the $\mathfrak{bms}_3$ algebra, known to emerge from the asymptotic symmetry analysis of general relativity at null infinity [57–61], which goes in line with the result obtained in [62] by expanding the Virasoro algebra (see also [63,64]). Interestingly, the presence of the gravitational Maxwell gauge field modifies not only the asymptotic symmetry but also the vacuum of the theory [55]. Physical implications of the gravitational Maxwell gauge field have also been explored in the context of spin-3 gravity [65], nonrelativistic gravity [66–69], and supergravity [70,71]. It is worth noting that in three dimensions the extension of Poincaré algebra found by Hietarinta [27] becomes isomorphic to three-dimensional Maxwell algebra that amounts to a simple interchanging of roles between the translation generators P_a and Z_a in the Maxwell algebra [72–74]. Indeed, CS gravity theories based on both the Hietarinta and Maxwell algebras were explored in [72,75], showing in particular that, upon spontaneous breaking of a local symmetry, they lead precisely to the topologically massive gravity theory [76] and the minimal massive gravity [77].

One of the main advantages of working in three spacetime dimensions is that the Maxwell algebra can alternatively be recovered as an Inönü-Wigner (IW) contraction of three copies of the $\mathfrak{so}(2,1)$ algebra which also provides a nontrivial invariant form that is imperative for the construction of extensions of Maxwell-CS gauge theories [78]. In particular, the spin-3 Maxwell algebra as well as its corresponding invariant bilinear form can be derived by contracting three copies of the $\mathfrak{sl}(3,\mathbb{R})$ symmetry [65].[4] One may then ask whether a hypersymmetric extension of the Maxwell algebra with fermionic spin-$\frac{5}{2}$ generators, which transform in a spin-$\frac{3}{2}$ irreducible representation of the Lorentz group, can be obtained by contracting diverse combinations of the $\mathfrak{osp}(1|4)$ and the $\mathfrak{sp}(4)$ algebras. In this work, we show that not one but three distinct hypersymmetric extensions of the Maxwell algebra, including their invariant bilinear forms, can be effectively derived through the IW procedure. The obtained hyper-Maxwell algebras indeed require the presence of spin-4 generators and allow us to construct CS hypersymmetric gravity theories. Furthermore, we also show that a remarkable hyper-Maxwell-CS gravity theory without spin-4 gauge fields can be constructed, and whose underlying superalgebra results in a sub-superalgebra of one of the hyper-Maxwell algebras that include spin-4 generators, which transform in an spin-3 irreducible representation of the Lorentz group.

The paper is organized as follows: In Sec. II, we briefly review the Maxwell-CS gravity theory defined on three spacetime dimensions. Sections III–V contain our main results. In Sec. III, we present the simplest CS hypergravity theory invariant under a hypersymmetric extension of the

[2] Fermionic HS generalizations of the Poincaré superalgebra were previously studied in [27] for any dimension. See [23] for further studies on these extensions of the hyper-Poincaré algebra in gauge theories beyond general relativity in odd dimensions [25].

[3] A CS gravity theory based on Maxwell algebra in $2+1$ was initially considered in [46,47] as a prominent model leading to the two-dimensional linear gravities referred to in [49–53] as a dimensional reduction.

[4] In [79–82] following suitable IW contractions allowed the authors to formulate the three-dimensional higher-spin gravity with vanishing cosmological constant as a CS gauge theory similar to their well-known counterparts in AdS_3 [79,83–86].

Maxwell algebra. In Sec. IV, we introduce three alternative hyper-Maxwell algebras including spin-4 generators by considering the IW contraction procedure. Section V is devoted to the construction of CS hypersymmetric gravity theories based on the aforementioned hyper-Maxwell symmetries. Section VI concludes our work with some discussions about future developments.

II. THREE-DIMENSIONAL MAXWELL-CHERN-SIMONS GRAVITY THEORY

In this section, we briefly review the three-dimensional Maxwell-CS gravity theory [46–48] (see also [54,55,70]). This alternative theory of gravity is based on the so-called Maxwell algebra, which can be seen as an extension and deformation of the Poincaré algebra $\mathfrak{iso}(2,1)$, which turns out to be a non-semisimple group. In addition to the usual local rotations J_a and local translation generators P_a, the Maxwell symmetry is characterized by the presence of three additional Abelian generators Z_a. In particular, the Maxwell generators satisfy the following nonvanishing commutation relations:

$$\begin{aligned} [J_a, J_b] &= \epsilon^c{}_{ab} J_c, \qquad [J_a, P_b] = \epsilon^c{}_{ab} P_c, \\ [J_a, Z_b] &= \epsilon^c{}_{ab} Z_c, \qquad [P_a, P_b] = \epsilon^c{}_{ab} Z_c, \end{aligned} \tag{2.1}$$

where $a, b, c = 0, 1, 2$ are Lorentz indices which are lowered and raised with the Minkowski metric $\eta_{ab} = (-1, 1, 1)$ and ϵ_{abc} is the three-dimensional Levi-Civita tensor which satisfies $\epsilon_{012} = -\epsilon^{012} = 1$.

The most general quadratic Casimir invariant for the Maxwell algebra is [46–48]

$$C = \alpha_0 J^a J_a + \alpha_1 P^a J_a + \alpha_2 (P^a P_a + J^a Z_a), \tag{2.2}$$

where α_0, α_1, and α_2 are arbitrary constants. The Maxwell algebra then admits the following nonvanishing components of the invariant tensor:

$$\begin{aligned} \langle J_a J_b \rangle &= \alpha_0 \eta_{ab}, \qquad \langle J_a P_b \rangle = \alpha_1 \eta_{ab}, \\ \langle J_a Z_b \rangle &= \alpha_2 \eta_{ab}, \qquad \langle P_a P_b \rangle = \alpha_2 \eta_{ab}. \end{aligned} \tag{2.3}$$

The gauge connection one-form A can be conveniently chosen as follows[5]:

$$A = \omega^a J_a + e^a P_a + k^a Z_a, \tag{2.4}$$

where ω^a is the (dualized) spin connection, e^a denotes the dreibein and k^a is the so-called gravitational Maxwell gauge field. The corresponding curvature two-form is given by

$$F = R^a J_a + T^a P_a + F^a Z_a, \tag{2.5}$$

where

$$\begin{aligned} R^a &= d\omega^a + \frac{1}{2}\epsilon^a{}_{bc}\omega^b\omega^c, \\ T^a &= de^a + \epsilon^a{}_{bc}\omega^b e^c, \\ F^a &= dk^a + \epsilon^a{}_{bc}\omega^b k^c + \frac{1}{2}\epsilon^a{}_{bc} e^b e^c. \end{aligned} \tag{2.6}$$

Considering the gauge connection (2.4) and the invariant tensor (2.3), the corresponding action for the Maxwell algebra can then be described in terms of the three-dimensional CS action,

$$I_{CS} = \frac{k}{4\pi}\int \left\langle AdA + \frac{2}{3}A^3 \right\rangle, \tag{2.7}$$

with $k = \frac{1}{4G}$ being the CS level of the theory related to the gravitational constant G. Indeed, the action reads [46–48] (see also [54,55,70])

$$I_{\text{Maxwell}} = \frac{k}{4\pi}\int 2\alpha_1 R^a e_a + \alpha_2(e^a T_a + 2R^a k_a) + \alpha_0 L(\omega), \tag{2.8}$$

where

$$L(\omega) = \left(d\omega^a + \frac{1}{3}\epsilon^a{}_{bc}\omega^b\omega^c\right)\omega_a \tag{2.9}$$

is the Lorentz-Chern-Simons form.

The Maxwell-CS action contains three independent sectors proportional to α_0, α_1, and α_2. The parity-odd term given by the Lorentz-CS three-form [87,88] appears along the α_0 constant while the Einstein-Hilbert term is related to the α_1 constant. On the other hand, the additional gauge field k^a contributes only to the α_2 sector.

In particular, the equations of motion are given by

$$\begin{aligned} \delta\omega_a: &\quad \alpha_0 R^a + \alpha_1 T^a + \alpha_2 F^a = 0, \\ \delta k_a: &\quad \alpha_2 R^a = 0, \\ \delta e_a: &\quad \alpha_1 R^a + \alpha_1 T^a = 0. \end{aligned} \tag{2.10}$$

It follows that when $\alpha_2 \neq 0$, the previous equations can be equivalently written as the vanishing of the curvature two-forms (2.6). It is worth mentioning that the suitable choice of the gauge field in Eq. (2.4) allows one to generically describe the theory in Riemannian geometry (torsionless). The standard $(2+1)$-dimensional gravity in vacuum is then recovered for the case $\alpha_0 = \alpha_2 = 0$, which is the well-known CS theory for $\mathfrak{iso}(2,1)$ [87,88].

In this work, we will "hypersymmetrize" the three-dimensional Maxwell gravity. As we will see, some

[5]More general choices of the gauge field can be considered leading to a more general Maxwell-CS theory that includes a nonvanishing torsion term.

hypersymmetric extensions will also require the presence of spin-4 gauge fields. The construction of the simplest hypersymmetric extension of the Maxwell-CS gravity is discussed in the next section.

III. HYPER-MAXWELL-CHERN-SIMONS GRAVITY THEORY

Here we present the simplest hypersymmetric extension of the Maxwell-CS gravity theory. To this end, we construct a hyper-Maxwell algebra by introducing fermionic generators which transform into an spin-$\frac{3}{2}$ irreducible representation of the Lorentz group. Therefore, the hyper-Maxwell algebra is spanned by the set $\{J_a, P_a, Z_a, Q_{\alpha a}\}$, whose generators satisfy the following nonvanishing (anti)commutation relations:

$$
\begin{aligned}
[J_a, J_b] &= \epsilon^m{}_{ab} J_m, \qquad [J_a, P_b] = \epsilon^m{}_{ab} P_m, \\
[J_a, Z_b] &= \epsilon^m{}_{ab} Z_m, \qquad [P_a, P_b] = \epsilon^m{}_{ab} Z_m, \\
[J_a, Q_{\alpha b}] &= \frac{1}{2}(\Gamma_a)^\beta{}_\alpha Q_{\beta b} + \epsilon_{abc} Q_\beta^{\ c}, \\
\{Q_{\alpha a}, Q_{\beta b}\} &= -\frac{4}{3}\eta_{ab} Z_c (C\Gamma^c)_{\alpha\beta} + \frac{5}{3}\epsilon_{abc} C_{\alpha\beta} Z^c \\
&\quad + \frac{2}{3} Z_{(a|}(C\Gamma_{|b)})_{\alpha\beta},
\end{aligned} \tag{3.1}
$$

where $Q_{\alpha a}$ are Γ-traceless vector-spinor generators that fulfill $(\Gamma_a)^\beta{}_\alpha Q_{\alpha\beta} = \Gamma^a Q_a = 0$. Here C is the charge conjugation matrix

$$
C_{\alpha\beta} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \tag{3.2}
$$

which satisfies $C^T = -C$ and $C\Gamma^a = (C\Gamma^a)^T$, with Γ^a being the Dirac matrices in three spacetime dimensions. We shall denote this algebra as $\widehat{\mathfrak{hm}}$, which, like the hyper-Poincaré algebra [23], requires neither an enlargement of the Lorentz group nor the introduction of bosonic HS generators to satisfy the Jacobi identities. One can notice that the algebra $\widehat{\mathfrak{hm}}$ has a structure similar to the so-called nonstandard Maxwell superalgebra [89,90], in which the translational generators P_a are not expressed as bilinear expressions of fermionic generators. Nonetheless, since the sub-superalgebra spanned by the generators J_a, Z_a, and $Q_{\alpha a}$ is indeed the hyper-Poincaré algebra, as mentioned before, a simple interchanging of roles between the translation generators P_a and Z_a relates the superalgebra in Eq. (3.1) to its Hietarinta form in [27]. Therefore, one naturally expects gauge theories based on the hyper-Maxwell or Hietarinta version of the algebra to certainly be endowed with quite different physical implications; see, e.g., [72–74]. In the present work, we will not consider this last reinterpretation of the theory since our purpose is to study the coupling of three-dimensional Maxwell gravity with massless spin-$\frac{5}{2}$ fields.

Let us now consider a hypersymmetric Maxwell-CS action that is invariant under the hyper-Maxwell algebra $\widehat{\mathfrak{hm}}$ (3.1). The CS action can be constructed from the gauge field

$$
A = e^a P_a + \omega^a J_a + k^a Z_a + \bar{\psi}^a Q_a, \tag{3.3}
$$

whose components are the dreibein, the (dualized) spin connection, the gravitational Maxwell field and a Majorana spin-$\frac{5}{2}$ field. In particular, the Majorana conjugate reads $\bar{\psi}_{\alpha a} = \psi_a^\beta C_{\beta\alpha}$. The corresponding curvature two-form $F = dA + \frac{1}{2}[A, A]$ is given by

$$
F_{\widehat{\mathfrak{hm}}} = T^a P_a + R^a J_a + \tilde{F}^a Z_a + D\bar{\psi}^a Q_a, \tag{3.4}
$$

with

$$
\begin{aligned}
\tilde{F}^a &= F^a - \frac{3}{2} i\bar{\psi}_b \Gamma^a \psi^b, \\
D\psi^a &= d\psi^a + \frac{3}{2}\omega^b \Gamma_b \psi^a - \omega_b \Gamma^a \psi^b,
\end{aligned} \tag{3.5}
$$

where T^a, R^a, and F^a are as defined in Eq. (2.6). The fermionic fields are assumed to be Γ traceless, i.e, $\Gamma^a \psi_a = 0$. On the other hand, one can easily check to see that, in addition to $C_0 = J^a J_a$ and $C_1 = P^a J_a$, the hyper-Maxwell algebra $\widehat{\mathfrak{hm}}$ admits another quadratic Casimir invariant given by

$$
C_2 = P^a P_a + J^a Z_a + Q_\alpha^a C^{\alpha\beta} Q_{\beta a}. \tag{3.6}
$$

Hence, the $\widehat{\mathfrak{hm}}$ algebra admits the following nonvanishing components of an invariant tensor:

$$
\begin{aligned}
\langle J_a J_b \rangle &= \alpha_0 \eta_{ab}, \qquad \langle J_a P_b \rangle = \alpha_1 \eta_{ab}, \\
\langle J_a Z_b \rangle &= \alpha_2 \eta_{ab}, \qquad \langle P_a P_b \rangle = \alpha_2 \eta_{ab}, \\
\langle Q_{\alpha a} Q_{\beta b} \rangle &= 2\alpha_2 \left(\frac{2}{3} C_{\alpha\beta}\eta_{ab} - \frac{1}{3}\epsilon_{abc}(C\Gamma^c)_{\alpha\beta} \right),
\end{aligned} \tag{3.7}
$$

where α_0, α_1, and α_2 are arbitrary constants. The invariance of such a bilinear form under the action of the hyper-Maxwell algebra requires that $\langle J_a Z_b \rangle$, $\langle P_a P_b \rangle$, and $\langle Q_{\alpha a} Q_{\beta b} \rangle$ have the same global coefficient. When one uses the gauge connection one-form (3.3) and the invariant bilinear form (3.7) in the general form of a three-dimensional CS action (2.7), it then reduces, up to boundary terms, to

$$I_{\widehat{\mathfrak{hm}}} = \frac{k}{4\pi}\int 2\alpha_1 R^a e_a + \alpha_2(2R^a k_a + T^a e_a + 2i\bar{\psi}_a D\psi^a) + \alpha_0 L(\omega). \tag{3.8}$$

The previous CS action is invariant under the $\widehat{\mathfrak{hm}}$ algebra, extending the Maxwell gravity theory with a massless spin-$\frac{5}{2}$ gauge field. Note that the terms along the arbitrary constants α_0 and α_1 are not affected by the extension and coincide with the terms appearing in the bosonic action (2.8). The spin-$\frac{5}{2}$ field only appears in the term proportional to the α_2 constant. The action can then be seen as an "exotic" hypersymmetric gravity theory which does not require the presence of spin-4 gauge fields, which turns out to be as the natural Maxwellian extension of hypergravity given by Aragone and Deser in [7] but formulated as a genuine gauge theory. As we shall see later, there are other hypersymmetric extensions of the Maxwell gravity, but they require the presence of spin-4 gauge fields.

For completeness, we provide the equations of motion, which are given as

$$\begin{aligned} \delta\omega_a:&\quad \alpha_0 R^a + \alpha_1 T^a + \alpha_2 \tilde{F}^a = 0,\\ \delta k_a:&\quad \alpha_2 R^a = 0,\\ \delta e_a:&\quad \alpha_1 R^a + \alpha_1 T^a = 0,\\ \delta\bar{\psi}_a:&\quad \alpha_2 D\psi^a = 0. \end{aligned} \tag{3.9}$$

When $\alpha_2 \neq 0$, the previous equations can be equivalently written as the vanishing of the curvature two-forms (3.5). By construction, the CS action (3.8) is invariant under the local hypersymmetry transformation laws given by

$$\delta e^a = 0, \qquad \delta\omega^a = 0, \qquad \delta k^a = 3i\bar{\epsilon}_b\Gamma^a\psi^b,$$
$$\delta\psi^a = d\epsilon^a + \frac{3}{2}\omega^b\Gamma_b\epsilon^a - \omega_b\Gamma^a\epsilon^b, \tag{3.10}$$

where ϵ^a is the fermionic gauge parameter.

The CS action (3.8) is the simplest hypersymmetric extension of the Maxwell gravity without spin-4 gauge fields. As will be discussed, the previous hypersymmetric extension of the Maxwell algebra is not unique. However, to our knowledge, it seems that the hyper-Maxwell $\widehat{\mathfrak{hm}}$ is the only consistent way to accommodate spin-$\frac{5}{2}$ generators to the Maxwell algebra without including spin-4 generators.

A different analysis can be done if we choose for the vierbein e^a to accompany the Z_a generator (or, equivalently, by performing $Z_a \leftrightarrow P_a$). The analysis will lead to a hypersymmetrization of the "Hietarinta gravity" [75] which, although interesting, is beyond the scope of this work.

The extension of the hyper-Maxwell gravity theory in Eq. (3.8) can then be extended to include fermionic fields of spin $(n+3/2)$ which become suitably described by a completely symmetric and (triple) Γ-traceless one-form $\bar{\psi}_{a_1\cdots a_n}$ so that the action reads

$$I_{\widehat{\mathfrak{hm}}} = \frac{k}{4\pi}\int 2\alpha_1 R^a e_a + \alpha_2(2R^a k_a + T^a e_a + 2i\bar{\psi}_{a_1\cdots a_n} D\psi^{a_1\cdots a_n}) + \alpha_0 L(\omega), \tag{3.11}$$

with

$$D\psi^{a_1\cdots a_n} = d\psi^{a_1\cdots a_n} + \left(n+\frac{1}{2}\right)\omega_b\Gamma^b\psi^{a_1\cdots a_n} - \omega_b\Gamma^{(a_1}\psi^{a_2\cdots a_n)b},$$

and, being left invariant by the local hypersymmetry transformations, is given by

$$\delta e^a = 0, \tag{3.12}$$

$$\delta\omega^a = 0, \tag{3.13}$$

$$\delta k^a = 2\left(n+\frac{1}{2}\right)i\bar{\epsilon}_{a_1\cdots a_n}\Gamma^a\psi^{a_1\cdots a_n}, \tag{3.14}$$

$$\delta\psi^{a_1\cdots a_n} = D\epsilon^{a_1\cdots a_n}. \tag{3.15}$$

Thus, the latter action naturally extends the hypergravity theory in [7] to include the Maxwell-CS gravity dynamics that amounts to describing two interacting nonpropagating gravitons and a spin-$(n+3/2)$ gauge field.

IV. ON THE EXTENSION OF THE MAXWELL ALGEBRA WITH SPIN-$\frac{5}{2}$ AND SPIN-4 GENERATORS

In this section, we present three alternative hypersymmetric extensions of the Maxwell algebra, which are obtained through the IW contraction procedure [91]. As we shall see, such hyper-Maxwell algebras require the presence of spin-4 generators and appear upon the contraction of diverse combinations of $\mathfrak{osp}(1|4)$ and $\mathfrak{sp}(4)$ algebras. Interestingly, we show that the hyper-Maxwell algebra (3.1) without spin-4 generators (transforming in a spin-3 irreducible representation of the Lorentz group) appears as a subalgebra of one of the alternative hyper-Maxwell algebras.

To start with, and in order to fix our notation, we will briefly review the $\mathfrak{osp}(1|4)$ superalgebra.

A. The $\mathfrak{osp}(1|4)$ superalgebra

The $\mathfrak{osp}(1|4)$ superalgebra with $\mathfrak{sl}(2|R)$ principal embedded in $\mathfrak{sp}(4)$ is spanned by the set of generators $\{T_a, T_{abc}, \mathcal{G}_{\alpha a}\}$, which satisfy the following nonvanishing (anti)commutation relations:

$$
\begin{aligned}
[T_a, T_b] &= \epsilon^m{}_{ab} T_m, \\
[T_a, T_{bcd}] &= 3\epsilon^m{}_{a(b} T_{cd)m}, \\
[T_a, \mathcal{G}_{\alpha b}] &= \frac{1}{2}(\Gamma_a)^\beta{}_\alpha \mathcal{G}_{\beta b} + \epsilon_{abc}\mathcal{G}_\beta^{\ c}, \\
[T_{abc}, T_{mnk}] &= -6(\eta_{(ab}\epsilon^l{}_{c)(m}\eta_{nk)} + 5\epsilon^l{}_{(m|(a}\delta^d_b\eta_{c)|n}\eta_{k)d})T_l \\
&\quad + 2(5\epsilon^l{}_{(m|(a}\delta^d_b T_{c)l|n}\eta_{k)d} - \epsilon^l{}_{(m|(a}\eta_{bc)}T_{|nk)l} \\
&\quad - \epsilon^l{}_{(m(a|}T_{bc)l}\eta_{|nk)}), \\
[T_{abc}, \mathcal{G}_{\alpha d}] &= (\delta^k_d \eta_{(ab|} - 5\eta_{d(a}\delta^k_{b|})(\Gamma_{|c)})^\beta{}_\alpha \mathcal{G}_{\beta k} \\
&\quad + \eta_{(ab|}(\Gamma_d)^\beta{}_\alpha \mathcal{G}_{\beta|c)}, \\
\{\mathcal{G}_{\alpha a}, \mathcal{G}_{\beta b}\} &= \left(T_{abc} - \frac{4}{3}\eta_{ab}T_c\right)(C\Gamma^c)_{\alpha\beta} + \frac{5}{3}\epsilon_{abc}C_{\alpha\beta}T^c \\
&\quad + \frac{2}{3}T_{(a|}(C\Gamma_{|b)})_{\alpha\beta},
\end{aligned}
\tag{4.1}
$$

where $a, b, \ldots = 0,\ 1,\ 2$ are Lorentz indices lowered and raised with the off-diagonal Minkowski metric η_{ab} and $\epsilon^m{}_{bc}$ is the three-dimensional Levi-Civita tensor. Here T_a span the $\mathfrak{sl}(2,R)$ subalgebra and stand for the spin-2 generators, while T_{abc} and the fermionic $\mathcal{G}_{\alpha a}$ generators yield, respectively, to spin-4 and spin-$\frac{5}{2}$ fields in the Chern-Simons theory. Let us note that T_{abc} are traceless and totally symmetric generators satisfying $\eta^{ab}T_{abc} = 0$, while $\mathcal{G}_{\alpha a}$ are Γ-traceless vector-spinor generators satisfying $(\Gamma^a)^\beta{}_\alpha \mathcal{G}_{\beta a} = \Gamma^a \mathcal{G}_a = 0$. In particular, the subalgebra spanned by the set $\{T_a, T_{abc}\}$ defines a $\mathfrak{sp}(4)$ algebra.

The $\mathfrak{osp}(1|4)$ superalgebra has the following nonvanishing components of the invariant tensor

$$
\begin{aligned}
\langle T_a T_b \rangle &= \frac{1}{2}\eta_{ab}, \\
\langle T_{abc} T_{mnk} \rangle &= 5\eta_{m(a}\eta_{b|n}\eta_{|c)k} - 3\eta_{(ab}\eta_{|c)(m}\eta_{nk)}, \\
\langle \mathcal{G}_{\alpha a} \mathcal{G}_{\beta b} \rangle &= \frac{2}{3}C_{\alpha\beta}\eta_{ab} - \frac{1}{3}\epsilon_{abc}(C\Gamma^c)_{\alpha\beta}.
\end{aligned}
\tag{4.2}
$$

Let us note that the hyper-Poincaré algebra with and without spin-4 generators can be obtained by considering different IW contractions of the $\mathfrak{osp}(1|4)\otimes\mathfrak{sp}(4)$ superalgebra [24]. Here we shall see that three alternative hyper-Maxwell algebras appear when we consider the IW contractions of the $\mathfrak{osp}(1|4)\otimes\mathfrak{osp}(1|4)\otimes\mathfrak{sp}(4)$ and $\mathfrak{osp}(1|4)\otimes\mathfrak{sp}(4)\otimes\mathfrak{sp}(4)$ superalgebras.

B. Hyper-Maxwell algebra with spin-4 generators

Let us first consider the $\mathfrak{osp}(1|4)\otimes\mathfrak{osp}(1|4)\otimes\mathfrak{sp}(4)$ superalgebra. One can then show that the IW contraction of such a structure allows us to obtain a hyper-Maxwell algebra with spin-4 generators. To this end, let us consider the following redefinition of the generators:

$$
\begin{aligned}
J_a &= T_a + T_a^+ + T_a^-, & P_a &= \frac{1}{\ell}(T_a^+ - T_a^-), & Z_a &= \frac{1}{\ell^2}(T_a^+ + T_a^-), \\
J_{abc} &= T_{abc} + T_{abc}^+ + T_{abc}^-, & P_{abc} &= \frac{1}{\ell}(T_{abc}^+ - T_{abc}^-), & Z_{abc} &= \frac{1}{\ell^2}(T_{abc}^+ + T_{abc}^-), \\
Q_{\alpha a} &= \frac{1}{\sqrt{\ell}}(\mathcal{G}_{\alpha a}^+ - i\mathcal{G}_{\alpha a}^-), & \Sigma_{\alpha a} &= \frac{1}{\sqrt{\ell^3}}(\mathcal{G}_{\alpha a}^+ + i\mathcal{G}_{\alpha a}^-),
\end{aligned}
\tag{4.3}
$$

where $\{T_a^+, T_{abc}^+, \mathcal{G}_{\alpha a}^+\}$ and $\{T_a^-, T_{abc}^-, \mathcal{G}_{\alpha a}^-\}$ each span an $\mathfrak{osp}(1|4)$ superalgebra, while $\{T_a, T_{abc}\}$ are $\mathfrak{sp}(4)$ generators. On the other hand, ℓ is a length parameter related to the cosmological constant through $\Lambda \propto \pm\frac{1}{\ell^2}$. It is simple to verify that the set of generators $\{J_a, P_a, Z_a, J_{abc}, P_{abc}, Z_{abc}, Q_{\alpha a}, \Sigma_{\alpha a}\}$ satisfies a hypersymmetric version of the so-called AdS-Lorentz algebra [92–96]. It is then straightforward to show that the algebra obtained after the vanishing cosmological constant limit $\ell \to \infty$ leads to the following hypersymmetric algebra:

$$
\begin{aligned}
[J_a, J_b] &= \epsilon^m{}_{ab}J_m, & [J_a, P_b] &= \epsilon^m{}_{ab}P_m, \\
[J_a, Z_b] &= \epsilon^m{}_{ab}Z_m, & [P_a, P_b] &= \epsilon^m{}_{ab}Z_m, \\
[J_a, J_{bcd}] &= 3\epsilon^m{}_{a(b}J_{cd)m}, & [J_a, P_{bcd}] &= 3\epsilon^m{}_{a(b}P_{cd)m}, \\
[P_a, J_{bcd}] &= 3\epsilon^m{}_{a(b}P_{cd)m}, & [J_a, Z_{bcd}] &= 3\epsilon^m{}_{a(b}Z_{cd)m}, \\
[Z_a, J_{bcd}] &= 3\epsilon^m{}_{a(b}Z_{cd)m}, & [P_a, P_{bcd}] &= 3\epsilon^m{}_{a(b}Z_{cd)m}, \\
[J_a, Q_{\alpha b}] &= \frac{1}{2}(\Gamma_a)^\beta{}_\alpha Q_{\beta b} + \epsilon_{abc}Q_\beta^{\ c}, \\
[J_a, \Sigma_{\alpha b}] &= \frac{1}{2}(\Gamma_a)^\beta{}_\alpha \Sigma_{\beta b} + \epsilon_{abc}\Sigma_\beta^{\ c}, \\
[P_a, Q_{\alpha b}] &= \frac{1}{2}(\Gamma_a)^\beta{}_\alpha \Sigma_{\beta b} + \epsilon_{abc}\Sigma_\beta^{\ c},
\end{aligned}
\tag{4.4}
$$

$$
\begin{aligned}
[J_{abc}, J_{mnk}] &= -6(\eta_{(ab}\epsilon^{l}{}_{c)(m}\eta_{nk)} + 5\epsilon^{l}{}_{(m|(a}\delta^{d}{}_{b}\eta_{c)|n}\eta_{k)d})J_{l} \\
&\quad + 2(5\epsilon^{l}{}_{(m|(a}\delta^{d}{}_{b}J_{c)l|n}\eta_{k)d} - \epsilon^{l}{}_{(m|(a}\eta_{bc)}J_{|nk)l} - \epsilon^{l}{}_{(m(a|}J_{bc)l}\eta_{|nk)}), \\
[J_{abc}, P_{mnk}] &= -6(\eta_{(ab}\epsilon^{l}{}_{c)(m}\eta_{nk)} + 5\epsilon^{l}{}_{(m|(a}\delta^{d}{}_{b}\eta_{c)|n}\eta_{k)d})P_{l} \\
&\quad + 2(5\epsilon^{l}{}_{(m|(a}\delta^{d}{}_{b}P_{c)l|n}\eta_{k)d} - \epsilon^{l}{}_{(m|(a}\eta_{bc)}P_{|nk)l} - \epsilon^{l}{}_{(m(a|}P_{bc)l}\eta_{|nk)}), \\
[J_{abc}, Z_{mnk}] &= -6(\eta_{(ab}\epsilon^{l}{}_{c)(m}\eta_{nk)} + 5\epsilon^{l}{}_{(m|(a}\delta^{d}{}_{b}\eta_{c)|n}\eta_{k)d})Z_{l} \\
&\quad + 2(5\epsilon^{l}{}_{(m|(a}\delta^{d}{}_{b}Z_{c)l|n}\eta_{k)d} - \epsilon^{l}{}_{(m|(a}\eta_{bc)}Z_{|nk)l} - \epsilon^{l}{}_{(m(a|}Z_{bc)l}\eta_{|nk)}), \\
[P_{abc}, P_{mnk}] &= -6(\eta_{(ab}\epsilon^{l}{}_{c)(m}\eta_{nk)} + 5\epsilon^{l}{}_{(m|(a}\delta^{d}{}_{b}\eta_{c)|n}\eta_{k)d})Z_{l} \\
&\quad + 2(5\epsilon^{l}{}_{(m|(a}\delta^{d}{}_{b}Z_{c)l|n}\eta_{k)d} - \epsilon^{l}{}_{(m|(a}\eta_{bc)}Z_{|nk)l} - \epsilon^{l}{}_{(m(a|}Z_{bc)l}\eta_{|nk)}),
\end{aligned}
\tag{4.5}
$$

$$
\begin{aligned}
[J_{abc}, Q_{\alpha d}] &= (\delta^{k}{}_{d}\eta_{(ab|} - 5\eta_{d(a}\delta^{k}{}_{b|})(\Gamma_{|c)})^{\beta}{}_{\alpha}Q_{\beta k} + \eta_{(ab|}(\Gamma_{d})^{\beta}{}_{\alpha}Q_{\beta|c)}, \\
[J_{abc}, \Sigma_{\alpha d}] &= (\delta^{k}{}_{d}\eta_{(ab|} - 5\eta_{d(a}\delta^{k}{}_{b|})(\Gamma_{|c)})^{\beta}{}_{\alpha}\Sigma_{\beta k} + \eta_{(ab|}(\Gamma_{d})^{\beta}{}_{\alpha}\Sigma_{\beta|c)}, \\
[P_{abc}, Q_{\alpha d}] &= (\delta^{k}{}_{d}\eta_{(ab|} - 5\eta_{d(a}\delta^{k}{}_{b|})(\Gamma_{|c)})^{\beta}{}_{\alpha}\Sigma_{\beta k} + \eta_{(ab|}(\Gamma_{d})^{\beta}{}_{\alpha}\Sigma_{\beta|c)}, \\
\{Q_{\alpha a}, Q_{\beta b}\} &= \left(P_{abc} - \frac{4}{3}\eta_{ab}P_{c}\right)(C\Gamma^{c})_{\alpha\beta} + \frac{5}{3}\epsilon_{abc}C_{\alpha\beta}P^{c} + \frac{2}{3}P_{(a|}(C\Gamma_{|b)})_{\alpha\beta}, \\
\{Q_{\alpha a}, \Sigma_{\beta b}\} &= \left(Z_{abc} - \frac{4}{3}\eta_{ab}Z_{c}\right)(C\Gamma^{c})_{\alpha\beta} + \frac{5}{3}\epsilon_{abc}C_{\alpha\beta}Z^{c} + \frac{2}{3}Z_{(a|}(C\Gamma_{|b)})_{\alpha\beta}.
\end{aligned}
\tag{4.6}
$$

The newly obtained algebra, which we shall denote as $\mathfrak{hm}_{(4)}$, corresponds to a hyper-Maxwell algebra in the presence of the spin-4 generators $\{J_{abc}, P_{abc}, Z_{abc}\}$. This hypersymmetric extension of the Maxwell algebra is characterized by two fermionic generators $Q_{\alpha a}$ and $\Sigma_{\alpha a}$. The presence of a second spinorial charge is not arbitrary but is required to satisfy the Jacobi identities. Let us note that the presence of a second fermionic charge has already been discussed in the context of $D = 11$ supergravity [97] and superstring theory [98]. Subsequently, it was also studied in the context of the supersymmetric extension of the Maxwell algebra in [99–106]. As we will see in the next section, this novel algebra will allow us to construct a different Maxwell hypergravity theory in three spacetime dimensions.

C. Nonstandard hyper-Maxwell algebra with spin-4 generators

A different hyper-Maxwell algebra with one vector-spinor generator can be recovered from the $\mathfrak{osp}(1|4) \otimes \mathfrak{sp}(4) \otimes \mathfrak{sp}(4)$ superalgebra. Let us first consider the following redefinition of the generators:

$$
\begin{aligned}
J_{a} &= T_{a} + T_{a}^{+} + T_{a}^{-}, & P_{a} &= \frac{1}{\ell}(T_{a}^{+} - T_{a}^{-}), & Z_{a} &= \frac{1}{\ell^{2}}(T_{a}^{+} + T_{a}^{-}), \\
J_{abc} &= T_{abc} + T_{abc}^{+} + T_{abc}^{-}, & P_{abc} &= \frac{1}{\ell}(T_{abc}^{+} - T_{abc}^{-}), & Z_{abc} &= \frac{1}{\ell^{2}}(T_{abc}^{+} + T_{abc}^{-}), \\
Q_{\alpha a} &= \frac{\sqrt{2}}{\ell}\mathcal{G}_{\alpha a},
\end{aligned}
\tag{4.7}
$$

where the subset $\{T_{a}^{+}, T_{abc}^{+}, \mathcal{G}_{\alpha a}\}$ satisfies an $\mathfrak{osp}(1|4)$ superalgebra, while $\{T_{a}^{-}, T_{abc}^{-}\}$ and $\{T_{a}, T_{abc}\}$ each span an $\mathfrak{sp}(4)$ algebra. After considering the above redefinition, the set of generators $\{J_{a}, P_{a}, Z_{a}, J_{abc}, P_{abc}, Z_{abc}, Q_{\alpha a}\}$ satisfies an alternative hypersymmetric version of the AdS-Lorentz algebra [92–96] whose flat limit leads to the following algebra:

$$
\begin{aligned}
&[J_a, J_b] = \epsilon^m{}_{ab} J_m, \qquad && [J_a, P_b] = \epsilon^m{}_{ab} P_m, \\
&[J_a, Z_b] = \epsilon^m{}_{ab} Z_m, \qquad && [P_a, P_b] = \epsilon^m{}_{ab} Z_m, \\
&[J_a, J_{bcd}] = 3\epsilon^m{}_{a(b} J_{cd)m}, \qquad && [J_a, P_{bcd}] = 3\epsilon^m{}_{a(b} P_{cd)m}, \\
&[P_a, J_{bcd}] = 3\epsilon^m{}_{a(b} P_{cd)m}, \qquad && [J_a, Z_{bcd}] = 3\epsilon^m{}_{a(b} Z_{cd)m}, \\
&[Z_a, J_{bcd}] = 3\epsilon^m{}_{a(b} Z_{cd)m}, \qquad && [P_a, P_{bcd}] = 3\epsilon^m{}_{a(b} Z_{cd)m}, \\
&[J_a, \mathcal{Q}_{\alpha b}] = \frac{1}{2} (\Gamma_a)^\beta{}_\alpha \mathcal{Q}_{\beta b} + \epsilon_{abc} \mathcal{Q}_\beta{}^c, &&
\end{aligned} \tag{4.8}
$$

$$
\begin{aligned}
[J_{abc}, J_{mnk}] &= -6(\eta_{(ab}\epsilon^l{}_{c)(m}\eta_{nk)} + 5\epsilon^l{}_{(m|(a}\delta^d{}_b\eta_{c)|n}\eta_{k)d}) J_l \\
&\quad + 2(5\epsilon^l{}_{(m|(a}\delta^d{}_b J_{c)l|n}\eta_{k)d} - \epsilon^l{}_{(m|(a}\eta_{bc)} J_{|nk)l} - \epsilon^l{}_{(m|(a} J_{bc)l}\eta_{|nk)}), \\
[J_{abc}, P_{mnk}] &= -6(\eta_{(ab}\epsilon^l{}_{c)(m}\eta_{nk)} + 5\epsilon^l{}_{(m|(a}\delta^d{}_b\eta_{c)|n}\eta_{k)d}) P_l \\
&\quad + 2(5\epsilon^l{}_{(m|(a}\delta^d{}_b P_{c)l|n}\eta_{k)d} - \epsilon^l{}_{(m|(a}\eta_{bc)} P_{|nk)l} - \epsilon^l{}_{(m|(a} P_{bc)l}\eta_{|nk)}), \\
[J_{abc}, Z_{mnk}] &= -6(\eta_{(ab}\epsilon^l{}_{c)(m}\eta_{nk)} + 5\epsilon^l{}_{(m|(a}\delta^d{}_b\eta_{c)|n}\eta_{k)d}) Z_l \\
&\quad + 2(5\epsilon^l{}_{(m|(a}\delta^d{}_b Z_{c)l|n}\eta_{k)d} - \epsilon^l{}_{(m|(a}\eta_{bc)} Z_{|nk)l} - \epsilon^l{}_{(m|(a} Z_{bc)l}\eta_{|nk)}), \\
[P_{abc}, P_{mnk}] &= -6(\eta_{(ab}\epsilon^l{}_{c)(m}\eta_{nk)} + 5\epsilon^l{}_{(m|(a}\delta^d{}_b\eta_{c)|n}\eta_{k)d}) Z_l \\
&\quad + 2(5\epsilon^l{}_{(m|(a}\delta^d{}_b Z_{c)l|n}\eta_{k)d} - \epsilon^l{}_{(m|(a}\eta_{bc)} Z_{|nk)l} - \epsilon^l{}_{(m|(a} Z_{bc)l}\eta_{|nk)}),
\end{aligned} \tag{4.9}
$$

$$
\begin{aligned}
[J_{abc}, \mathcal{Q}_{\alpha d}] &= (\delta^k{}_d \eta_{(ab|} - 5\eta_{d(a}\delta^k{}_{b|})(\Gamma_{|c)})^\beta{}_\alpha \mathcal{Q}_{\beta k} + \eta_{(ab|}(\Gamma_d)^\beta{}_\alpha \mathcal{Q}_{\beta|c)}, \\
\{\mathcal{Q}_{\alpha a}, \mathcal{Q}_{\beta b}\} &= \left(Z_{abc} - \frac{4}{3}\eta_{ab} Z_c\right)(C\Gamma^c)_{\alpha\beta} + \frac{5}{3}\epsilon_{abc} C_{\alpha\beta} Z^c + \frac{2}{3} Z_{(a|}(C\Gamma_{|b)})_{\alpha\beta}.
\end{aligned} \tag{4.10}
$$

This algebra, which we will denote as $\widetilde{\mathfrak{hm}}_{(4)}$, corresponds to a nonstandard hyper-Maxwell algebra where P_{abc} and J_{abc} transform into a spin-3 irreducible representation of the Lorentz group. We refer to this algebra as nonstandard since the translational generators P_a are not expressed as bilinear expressions of fermionic generators $\mathcal{Q}_{\alpha a}$. As we will see in the next section, this feature will imply that the hypersymmetric CS action based on this algebra will describe an exotic hypersymmetric action. This is analogous to what happens in the case of the nonstandard Maxwell superalgebra introduced in [89] and to the simplest hyper-Maxwell algebra obtained in the previous section.

An alternative nonstandard hyper-Maxwell algebra can also be obtained from the $\mathfrak{osp}(1|4) \otimes \mathfrak{sp}(4) \otimes \mathfrak{sp}(4)$ superalgebra by considering a different redefinition of the $\mathfrak{osp}(1|4)$ and $\mathfrak{sp}(4)$ generators:

$$
\begin{aligned}
&J_a = T_a + T_a^+ + T_a^-, \qquad && P_a = \frac{1}{\ell}(T_a^+ - T_a^-), \qquad && Z_a = \frac{1}{\ell^2}(T_a^+ + T_a^-), \\
&J_{abc} = \frac{1}{\ell}(T_{abc} + T_{abc}^+ + T_{abc}^-), \qquad && P_{abc} = \frac{1}{\ell}(T_{abc}^+ - T_{abc}^-), \qquad && Z_{abc} = \frac{1}{\ell}(T_{abc}^+ + T_{abc}^-), \\
&\mathcal{Q}_{\alpha a} = \frac{\sqrt{2}}{\ell}\mathcal{G}_{\alpha a}. && &&
\end{aligned} \tag{4.11}
$$

Such a redefinition differs from the previous one [Eq. (4.7)] at the level of the length parameter ℓ. The set of generators $\{J_a, P_a, Z_a, J_{abc}, P_{abc}, Z_{abc}, \mathcal{Q}_{\alpha a}\}$ satisfies a different hypersymmetric version of the AdS-Lorentz algebra. In this case, the flat limit $\ell \to \infty$ reproduces a different nonstandard hyper-Maxwell algebra whose nonvanishing (anti)-commutators read

$$
\begin{aligned}
&[J_a, J_b] = \epsilon^m{}_{ab} J_m, \qquad [J_a, P_b] = \epsilon^m{}_{ab} P_m,\\
&[J_a, Z_b] = \epsilon^m{}_{ab} Z_m, \qquad [P_a, P_b] = \epsilon^m{}_{ab} Z_m,\\
&[J_a, J_{bcd}] = 3\epsilon^m{}_{a(b} J_{cd)m}, \qquad [J_a, P_{bcd}] = 3\epsilon^m{}_{a(b} P_{cd)m},\\
&[J_a, Z_{bcd}] = 3\epsilon^m{}_{a(b} Z_{cd)m},\\
&[J_a, Q_{\alpha b}] = \frac{1}{2}(\Gamma_a)^\beta{}_\alpha Q_{\beta b} + \epsilon_{abc} Q_\beta^{\ c},\\
&[J_{abc}, Z_{mnk}] = -6(\eta_{(ab}\epsilon^l{}_{c)(m}\eta_{nk)} + 5\epsilon^l{}_{(m|(a}\delta^d{}_b\eta_{c)|n}\eta_{k)d}) Z_l,\\
&[P_{abc}, P_{mnk}] = -6(\eta_{(ab}\epsilon^l{}_{c)(m}\eta_{nk)} + 5\epsilon^l{}_{(m|(a}\delta^d{}_b\eta_{c)|n}\eta_{k)d}) Z_l,\\
&[Z_{abc}, Z_{mnk}] = -6(\eta_{(ab}\epsilon^l{}_{c)(m}\eta_{nk)} + 5\epsilon^l{}_{(m|(a}\delta^d{}_b\eta_{c)|n}\eta_{k)d}) Z_l,\\
&\{Q_{\alpha a}, Q_{\beta b}\} = -\frac{4}{3}\eta_{ab} Z_c (C\Gamma^c)_{\alpha\beta} + \frac{5}{3}\epsilon_{abc} C_{\alpha\beta} Z^c\\
&\qquad + \frac{2}{3} Z_{(a|}(C\Gamma_{|b)})_{\alpha\beta}.
\end{aligned} \tag{4.12}
$$

Interestingly, one can see that the subset $\{J_a, P_a, Z_a, Q_{\alpha a}\}$ defines a hyper-Maxwell subalgebra without spin-4 generators which coincides with the simplest hyper-Maxwell algebra $\widehat{\mathfrak{hm}}$ obtained previously [Eq. (3.1)].

V. HYPERSYMMETRIC EXTENSION OF THE MAXWELL GRAVITY THEORY WITH SPIN-4 GAUGE FIELDS

In this section, we construct corresponding three-dimensional CS theories that are invariant under the hyper-Maxwell algebra $\mathfrak{hm}_{(4)}$ with spin-4 generators. As we shall see, we shall require not only the presence of spin-4 gauge fields but also the inclusion of a second Majorana spin-$\frac{5}{2}$ gauge field. For completeness, we also present the construction of a nonstandard hyper-Maxwell-CS gravity based on the $\widetilde{\mathfrak{hm}}_{(4)}$ hyperalgebra.

A. $\mathfrak{hm}_{(4)}$ Chern-Simons hypergravity

Let us first consider the CS hypergravity theory that is invariant under the hyper-Maxwell algebra $\mathfrak{hm}_{(4)}$ [Eqs. (4.4)–(4.6)], which is spanned by the set $\{P_a, J_a, Z_a, P_{abc}, J_{abc}, Z_{abc}, Q_\alpha^a, \Sigma_\alpha^a\}$. The CS action can be constructed from the gauge field

$$
\begin{aligned}
A = {} & e^a P_a + \omega^a J_a + k^a Z_a + e^{abc} P_{abc} + \omega^{abc} J_{abc}\\
& + k^{abc} Z_{abc} + \bar{\psi}^a Q_a + \bar{\xi}^a \Sigma_a,
\end{aligned} \tag{5.1}
$$

whose components are the dreibein, the (dualized) spin connection, the gravitational Maxwell field, three spin-4 gauge fields, and two Majorana spin-$\frac{5}{2}$ gauge fields. The corresponding curvature two-form reads

$$
F_{\mathfrak{hm}_{(4)}} = \hat{\mathcal{T}}^a P_a + \mathcal{R}^a J_a + \mathcal{F}^a_{(\xi)} Z_a + \hat{\mathcal{T}}^{abc} P_{abc} + \mathcal{R}^{abc} J_{abc} + \mathcal{F}^{abc}_{(\xi)} Z_{abc} + \hat{D}\bar{\psi}^a Q_a + D\bar{\xi}^a \Sigma_a, \tag{5.2}
$$

where

$$
\begin{aligned}
\hat{\mathcal{T}}^a &= T^a + 30\epsilon^a{}_{bc}\omega^{bmn} e^c{}_{mn} - \frac{3}{2} i\bar{\psi}_b \Gamma^a \psi^b,\\
\mathcal{R}^a &= R^a + 15\epsilon^a{}_{bc}\omega^{bmn}\omega^c{}_{mn},\\
\mathcal{F}^a_{(\xi)} &= F^a + 30\epsilon^a{}_{bc}\omega^{bmn} k^c{}_{mn} + 15\epsilon^a{}_{bc} e^{bmn} e^c{}_{mn} - 3i\bar{\psi}_b\Gamma^a\xi^b,\\
\hat{\mathcal{T}}^{abc} &= de^{abc} - 10\epsilon^{(a}{}_{mn}\omega^{mk|b} e^{c)n}{}_k + 3\epsilon^{(a}{}_{mn} e^m \omega^{n|bc)} + 3\epsilon^{(a}{}_{mn}\omega^m e^{n|bc)} + \frac{i}{2}\bar{\psi}^{(a}\Gamma^{|b}\psi^{c)},\\
\mathcal{R}^{abc} &= d\omega^{abc} - 5\epsilon^{(a}{}_{mn}\omega^{mk|b}\omega^{c)n}{}_k + 3\epsilon^{(a}{}_{mn}\omega^m\omega^{n|bc)},\\
\mathcal{F}^{abc}_{(\xi)} &= dk^{abc} - 10\epsilon^{(a}{}_{mn}\omega^{mk|b} k^{c)n}{}_k - 5\epsilon^{(a}{}_{mn} e^{mk|b} e^{c)n}{}_k + 3\epsilon^{(a}{}_{mn}\omega^m k^{n|bc)} + 3\epsilon^{(a}{}_{mn} k^m \omega^{n|bc)}\\
&\quad + 3\epsilon^{(a}{}_{mn} e^m e^{n|bc)} + i\bar{\psi}^{(a}\Gamma^{|b}\xi^{c)},\\
\hat{D}\psi^a &= d\psi^a + \frac{3}{2}\omega^b\Gamma_b\psi^a - \omega_b\Gamma^a\psi^b - 5\omega^{bca}\Gamma_b\psi_c,\\
D\xi^a &= d\xi^a + \frac{3}{2}\omega^b\Gamma_b\xi^a - \omega_b\Gamma^a\xi^b - 5\omega^{bca}\Gamma_b\xi_c + \frac{3}{2} e^b\Gamma_b\psi^a - e_b\Gamma^a\psi^b - 5e^{bca}\Gamma_b\psi_c.
\end{aligned} \tag{5.3}
$$

The fermionic fields and generators are assumed to be Γ traceless, i.e, $\Gamma^a\psi_a = \Gamma^a\xi_a = 0$ and $Q^a\Gamma_a = \Sigma^a\Gamma_a = 0$. The $\mathfrak{hm}_{(4)}$ hyperalgebra admits the following nonvanishing components of an invariant bilinear form:

$$
\begin{aligned}
\langle J_a J_b\rangle &= \alpha_0\eta_{ab}, \qquad \langle J_a P_b\rangle = \alpha_1\eta_{ab},\\
\langle P_a P_b\rangle &= \alpha_2\eta_{ab}, \qquad \langle J_a Z_b\rangle = \alpha_2\eta_{ab},\\
\langle J_{abc}J_{mnk}\rangle &= 2\alpha_0(5\eta_{m(a}\eta_{b|n}\eta_{|c)k} - 3\eta_{(ab|}\eta_{|c)(m|}\eta_{|nk)}),\\
\langle J_{abc}P_{mnk}\rangle &= 2\alpha_1(5\eta_{m(a}\eta_{b|n}\eta_{|c)k} - 3\eta_{(ab|}\eta_{|c)(m|}\eta_{|nk)}),\\
\langle J_{abc}Z_{mnk}\rangle &= 2\alpha_2(5\eta_{m(a}\eta_{b|n}\eta_{|c)k} - 3\eta_{(ab|}\eta_{|c)(m|}\eta_{|nk)}),\\
\langle P_{abc}P_{mnk}\rangle &= 2\alpha_2(5\eta_{m(a}\eta_{b|n}\eta_{|c)k} - 3\eta_{(ab|}\eta_{|c)(m|}\eta_{|nk)}),\\
\langle Q_{\alpha a}Q_{\beta b}\rangle &= \frac{2\alpha_1}{3}(2C_{\alpha\beta}\eta_{ab} - \epsilon_{abc}(C\Gamma^c)_{\alpha\beta}),\\
\langle Q_{\alpha a}\Sigma_{\beta b}\rangle &= \frac{2\alpha_2}{3}(2C_{\alpha\beta}\eta_{ab} - \epsilon_{abc}(C\Gamma^c)_{\alpha\beta}). \qquad (5.4)
\end{aligned}
$$

Here α_0, α_1, and α_2 are arbitrary constants which are related to the $\mathfrak{osp}(1|4)$ and $\mathfrak{sp}(4)$ constants given by (ν,ρ) and μ, respectively. Indeed, the invariant tensor (5.4) can be obtained from the invariant tensor of the $\mathfrak{osp}(1|4)\otimes\mathfrak{osp}(1|4)\otimes\mathfrak{sp}(4)$ superalgebra by considering the redefinition (4.3) along with

$$
\alpha_0 = \frac{\mu+\nu+\rho}{2}, \qquad \alpha_1 = \frac{\nu-\rho}{2\ell}, \qquad \alpha_2 = \frac{\nu+\rho}{2\ell^2}, \qquad (5.5)
$$

and the flat limit $\ell\to\infty$.

If one replaces the gauge connection one-form (5.1) and the invariant tensor (5.4) in the general expression for a CS action (2.7), it then reduces, up to a surface term, to

$$
\begin{aligned}
I_{\mathfrak{hm}_{(4)}} = \frac{k}{4\pi}\int &\alpha_1(2\mathcal{R}^a e_a + 20\mathcal{R}^{abc}e_{abc} + 2i\bar{\psi}_a\hat{D}\psi^a)\\
&+ \alpha_2(2\mathcal{R}^a k_a + e^a\mathcal{T}_a + 20\mathcal{R}^{abc}k_{abc} + 10e^{abc}\mathcal{T}_{abc}\\
&+ 2i\bar{\psi}_a D\xi^a + 2i\bar{\xi}_a\hat{D}\psi^a) + \alpha_0 L(\Omega), \qquad (5.6)
\end{aligned}
$$

where $\mathcal{T}^a$ and $\mathcal{T}^{abc}$ are given by

$$
\begin{aligned}
\mathcal{T}^a &= T^a + 30\epsilon^a{}_{bc}\omega^{bmn}e^c{}_{mn},\\
\mathcal{T}^{abc} &= de^{abc} - 10\epsilon^{(a}{}_{mn}\omega^{mk|b}e^{c)n}{}_k + 3\epsilon^{(a}{}_{mn}e^m\omega^{n|bc)}\\
&\quad + 3\epsilon^{(a}{}_{mn}\omega^m e^{n|bc)}, \qquad (5.7)
\end{aligned}
$$

and

$$
\begin{aligned}
L(\Omega) = L(\omega) + 10\bigg(&d\omega^{abc} - \frac{10}{3}\epsilon^a{}_{mn}\omega^{mkb}\omega^{cn}{}_k\\
&+ 3\epsilon^a{}_{mn}\omega^m\omega^{nbc}\bigg)\omega_{abc}, \qquad (5.8)
\end{aligned}
$$

with $L(\omega)$ being the Lorentz-CS form (2.9).

The CS action (5.6) is invariant under the $\mathfrak{hm}_{(4)}$ hyperalgebra and is split into three independent sectors proportional to the three arbitrary constants. The piece along α_0 contains the parity-odd Lorentz-CS term [87,88] together with its spin-4 version. Along the α_1 constant appears the Einstein-Hilbert term, contributions of the spin-4 fields, and a fermionic term. The term along α_2 extends the Maxwell gravity with fermionic terms and also has spin-4 field contributions. It is straightforward to see that the previous CS action reduces to the Maxwell-CS gravity (2.8) when the spin-4 and spin-$\frac{5}{2}$ fields are switched off. Let us note that, unlike with hyper-Poincaré gravity [23], the presence of a second spinor gauge field ξ^a is required to ensure the proper extension of the Maxwell gravity with spin-$\frac{5}{2}$ and spin-4 gauge fields.

The field equations for $\alpha_2\neq 0$ are given by the vanishing of the curvature two-form in Eq. (5.2), i.e., $F_{\mathfrak{hm}_{(4)}}=0$, whose components transform covariantly with respect to the hypersymmetry transformation laws

$$
\begin{aligned}
\delta e^a &= 3i\bar{\epsilon}_b\Gamma^a\psi^b, \qquad \delta\omega^a = 0, \qquad \delta k^a = 3i\bar{\epsilon}_b\Gamma^a\xi^b + 3i\bar{\varrho}_b\Gamma^a\psi^b,\\
\delta e^{abc} &= -i\bar{\epsilon}^a\Gamma^b\psi^c, \qquad \delta\omega^{abc} = 0, \qquad \delta k^{abc} = -i\bar{\varrho}^a\Gamma^b\psi^c - i\bar{\epsilon}^a\Gamma^b\xi^c,\\
\delta\psi^a &= d\epsilon^a + \frac{3}{2}\omega^b\Gamma_b\epsilon^a - \omega_b\Gamma^a\epsilon^b - 5\omega^{bca}\Gamma_b\epsilon_c,\\
\delta\xi^a &= d\varrho^a + \frac{3}{2}\omega^b\Gamma_b\varrho^a - \omega_b\Gamma^a\varrho^b - 5\omega^{bca}\Gamma_b\varrho_c + \frac{3}{2}e^b\Gamma_b\epsilon^a - e_b\Gamma^a\epsilon^b - 5e^{bca}\Gamma_b\epsilon_c, \qquad (5.9)
\end{aligned}
$$

where ϵ^a and ϱ^a are the fermionic gauge parameters related to the respective Q_a and Σ_a generators.

The novel hypergravity theory can be seen as a Maxwellian generalization of the hyper-Poincaré gravity in the presence of spin-4 gauge fields. In particular, the hyper-Poincaré-CS action coupled to spin-4 gauge fields and its exotic counterpart appear as particular subcases along the α_1 and α_0 constants, respectively. Let us note that, in the absence of spin-4 gauge fields, the term along α_1 constant corresponds to the usual hypergravity introduced in [7,23]. Although the $\mathfrak{hm}_{(4)}$ hyperalgebra can be seen as an enlargement and deformation of the hyper-Poincaré symmetry, the present hypergravity theory does not contain a cosmological constant term.

B. Nonstandard hypersymmetric Maxwell-Chern-Simons action

Now we shall construct the CS action that is invariant under the nonstandard hyper-Maxwell algebra in the presence of spin-4 generators given by Eqs. (4.8)–(4.10), which is spanned by $\{P_a, J_a, Z_a, J_{abc}, P_{abc}, Z_{abc}, Q_\alpha^a\}$. The gauge field one-form is given by

$$A = e^a P_a + \omega^a J_a + k^a Z_a + e^{abc} P_{abc} + \omega^{abc} J_{abc} + k^{abc} Z_{abc} + \bar{\psi}^a Q_a, \tag{5.10}$$

whose components are the dreibein, the (dualized) spin connection, the gravitational Maxwell field, three spin-4 gauge fields, and one Majorana spin-5/2 field. The curvature two-form is given in this case by

$$F_{\widetilde{\mathfrak{hm}}_{(4)}} = \mathcal{T}^a P_a + \mathcal{R}^a J_a + \mathcal{F}^a_{(\psi)} Z_a + \mathcal{T}^{abc} P_{abc} + \mathcal{R}^{abc} J_{abc} + \mathcal{F}^{abc}_{(\psi)} Z_{abc} + \hat{D}\bar{\psi}^a Q_a, \tag{5.11}$$

where $\mathcal{T}^a$, $\mathcal{T}^{abc}$ are defined in Eq. (5.7), while $\mathcal{R}^a$, $\mathcal{R}^{abc}$, $\hat{D}\psi^a$ can be found in Eq. (5.3), and

$$\begin{aligned} \mathcal{F}^a_{(\psi)} &= F^a + 30\epsilon^a{}_{bc}\omega^{bmn}k^c{}_{mn} + 15\epsilon^a{}_{bc}e^{bmn}e^c{}_{mn} - \frac{3}{2}i\bar{\psi}_b\Gamma^a\psi^b, \\ \mathcal{F}^{abc}_{(\psi)} &= dk^{abc} - 10\epsilon^{(a}{}_{mn}\omega^{mk|b}k^{c)n}{}_k - 5\epsilon^{(a}{}_{mn}e^{mk|b}e^{c)n}{}_k + 3\epsilon^{(a}{}_{mn}\omega^m k^{n|bc)} + 3\epsilon^{(a}{}_{mn}k^m\omega^{n|bc)} \\ &\quad + 3\epsilon^{(a}{}_{mn}e^m e^{n|bc)} + \frac{i}{2}\bar{\psi}^{(a}\Gamma^{|b}\psi^{c)}. \end{aligned} \tag{5.12}$$

The algebra $\widetilde{\mathfrak{hm}}_{(4)}$ admits the following nonvanishing components of an invariant bilinear form:

$$\begin{aligned} \langle J_a J_b\rangle &= \alpha_0\eta_{ab}, \qquad \langle J_a P_b\rangle = \alpha_1\eta_{ab}, \\ \langle P_a P_b\rangle &= \alpha_2\eta_{ab}, \qquad \langle J_a Z_b\rangle = \alpha_2\eta_{ab}, \\ \langle J_{abc}J_{mnk}\rangle &= 2\alpha_0(5\eta_{m(a}\eta_{b|n}\eta_{|c)k} - 3\eta_{(ab|}\eta_{|c)(m|}\eta_{|nk)}), \\ \langle J_{abc}P_{mnk}\rangle &= 2\alpha_1(5\eta_{m(a}\eta_{b|n}\eta_{|c)k} - 3\eta_{(ab|}\eta_{|c)(m|}\eta_{|nk)}), \\ \langle J_{abc}Z_{mnk}\rangle &= 2\alpha_2(5\eta_{m(a}\eta_{b|n}\eta_{|c)k} - 3\eta_{(ab|}\eta_{|c)(m|}\eta_{|nk)}), \\ \langle P_{abc}P_{mnk}\rangle &= 2\alpha_2(5\eta_{m(a}\eta_{b|n}\eta_{|c)k} - 3\eta_{(ab|}\eta_{|c)(m|}\eta_{|nk)}), \\ \langle Q_{\alpha a}Q_{\beta b}\rangle &= 2\alpha_2\left(\frac{2}{3}C_{\alpha\beta}\eta_{ab} - \frac{1}{3}\epsilon_{abc}(C\Gamma^c)_{\alpha\beta}\right). \end{aligned} \tag{5.13}$$

Here α_0, α_1, and α_2 are arbitrary constants which are related to the $\mathfrak{osp}(1|4)$ and $\mathfrak{sp}(4)$ constants given by ν and (μ, ρ), respectively. Indeed, the invariant tensor (5.13) can be obtained from the invariant tensor of the $\mathfrak{osp}(1|4) \otimes \mathfrak{sp}(4) \otimes \mathfrak{sp}(4)$ superalgebra by considering the redefinition (4.7) along with Eq. (5.5) and the flat limit $\ell \to \infty$.

The CS action is then given, up to boundary terms, by

$$\begin{aligned} I_{\widetilde{\mathfrak{hm}}_{(4)}} = \frac{k}{4\pi}\int & \alpha_1(2\mathcal{R}^a e_a + 20\mathcal{R}^{abc}e_{abc}) \\ & + \alpha_2(2\mathcal{R}^a k_a + e^a\mathcal{T}_a + 20\mathcal{R}^{abc}k_{abc} \\ & + 10e^{abc}\mathcal{T}_{abc} + 2i\bar{\psi}_a\hat{D}\psi^a) + \alpha_0 L(\Omega). \end{aligned} \tag{5.14}$$

The CS action based on the $\widetilde{\mathfrak{hm}}_{(4)}$ hyperalgebra describes an exotic hypersymmetric gravity theory. One can see that the spin-$\frac{5}{2}$ gauge fields do not contribute to the Einstein-Hilbert sector. This is due mainly to the structure of the $\widetilde{\mathfrak{hm}}_{(4)}$ hyperalgebra (4.8)–(4.10), in which $\{Q, Q\} \sim P$. Let us note that the same behavior appears in the simplest hyper-Maxwell gravity (3.8) and in the nonstandard supersymmetric extension of the Maxwell gravity [90]. It would be worth studying the Hietarinta version [72–74] of the nonstandard hyper-Maxwell-CS action (5.14)—in which the role of the dreibein and the gravitational Maxwell gauge field are interchanged—and its physical implications. The field equations for $\alpha_2 \neq 0$ are given by the vanishing of the curvature two-form in Eq. (5.11), i.e., $F_{\widetilde{\mathfrak{hm}}_{(4)}} = 0$. In the present case, the curvatures transform covariantly under the hypersymmetry transformation laws as follows:

$$\begin{aligned} \delta e^a &= 0, \qquad \delta\omega^a = 0, \qquad \delta k^a = 3i\bar{\epsilon}_b\Gamma^a\psi^b, \\ \delta e^{abc} &= 0, \qquad \delta\omega^{abc} = 0, \qquad \delta k^{abc} = -i\bar{\epsilon}^a\Gamma^b\psi^c, \\ \delta\psi^a &= d\epsilon^a + \frac{3}{2}\omega^b\Gamma_b\epsilon^a - \omega_b\Gamma^a\epsilon^b - 5\omega^{bca}\Gamma_b\epsilon_c, \end{aligned} \tag{5.15}$$

where ϵ^a is the fermionic gauge parameter related to Q_a.

VI. CONCLUDING REMARKS

In this work, we have presented a consistent way of coupling three-dimensional Maxwell-CS gravity theory with a spin-$\frac{5}{2}$ gauge field. To this end, we have constructed the simplest hypersymmetric extension of the Maxwell algebra, which was carried out with a consistent insertion of new fermionic generators into the three-dimensional Maxwell group which transforms into a spin-$\frac{3}{2}$ irreducible representation of the Lorentz group. The respective CS theory that is invariant under the aforementioned hyper-Maxwell algebra was then introduced and can be seen as an exotic hypersymmetric gravity theory that extends the hypergravity of Aragone and Deser [7] that has allowed us to write its corresponding extension to include massless gauge fields of spin $(n + 3/2)$. Interestingly, it has also been shown that the hypersymmetric extension of the Maxwell algebra is not unique but can be further extended to include spin-4 generators that are accompanied by additional fermionic spin-$\frac{3}{2}$ ones. Indeed, it has been shown explicitly that three different hyper-Maxwell algebras can be derived by considering the IW contraction of diverse combinations of the $\mathfrak{osp}(1|4)$ and the $\mathfrak{sp}(4)$ algebras. The first of these superalgebras obtained here [Eqs. (4.4)–(4.6)] has allowed us to construct a consistent CS hypergravity theory that amounts to endowing Maxwell gravity with interacting massless spin-4 fields and two independent fermionic gauge fields of spin $\frac{5}{2}$. A second alternative hypersymmetric algebra [Eqs. (4.8)–(4.10)], named here as a nonstandard algebra, has allowed us to construct the nonstandard hypersymmetric Maxwell-CS action, which is characterized as having just one spin-$\frac{5}{2}$ fermionic gauge field. Notably, in the third case (4.12) the spin-4 gauge fields can be consistently truncated, which is consistent with the fact that the set of spin-4 generators indeed form a subalgebra, so the simplest hyper-Maxwell gravity theory that includes a single massless spin-$\frac{5}{2}$ gauge field is effectively recovered.

Note that for our simplest hyper-Maxwell-Chern-Simons gravity, as in the case of the Poincaré hypergravity [23], the inclusion of additional HS generators of half-integer spin into the Maxwell algebra does not affect the causal structure underlying Maxwell-CS gravity theory, which distinguishes it from the more familiar infinite-dimensional higher-spin algebras, which require one to extend the Lorentz group in order to include generators of higher spins $s > 2$.

The results obtained here could serve as a starting point for various further studies. In particular, one could explore the possibility of including a cosmological constant in our theory. Indeed, with regard to the bosonic sector of the theory, it is encouraging to know that the Maxwell gravity theory can alternatively be recovered as a vanishing cosmological constant limit of the so-called AdS-Lorentz gravity [70,78,94] and the Poincaré-Lorentz gravity theory [56]. A cosmological constant could then be incorporated in the hyper-Maxwell gravity theory by constructing a CS hypergravity theory based on a hyper-AdS-Lorentz or a hyper-Poincaré-Lorentz symmetry. Although they probably should appear as deformations of the hyper-Maxwell algebras presented here, one could expect different underlying geometries. Indeed, unlike the AdS-Lorentz gravity theory, the inclusion of a cosmological constant through the Poincaré-Lorentz symmetry implies the presence of a nonvanishing torsion [56]. It would be worth exploring the physical implications of considering a nonvanishing torsion in a hypergravity theory.

It is worth stressing that the study of suitable asymptotic symmetries for the extensions of hyper-Maxwell theories found here become crucial due to their inherent topological character, which is pointed out by the absence of local bulk degrees of freedom [86]. Thus, one can expect that suitable asymptotic conditions could be performed in each case along the lines of [85,86], giving rise to adequate asymptotic symmetries being canonically realized by deformations of the hyper-$\mathfrak{bms}_3$ asymptotic algebra. One might then wonder whether interesting hypersymmetry bounds could be derived from these asymptotic algebras, thereby implying prominent properties for solutions such as solitonlike ones, as well as those with a sensible thermodynamics; see, e.g., [22,23]. Indeed, since the hyper-Maxwell algebra appears as the IW contraction of diverse combinations of the $\mathfrak{osp}(1|4)$ and the $\mathfrak{sp}(4)$ algebras, it seems natural to expect that the respective asymptotic algebras could alternatively be recovered as a precise combination of the $W_{(2,\frac{5}{2},4)}$ and $W_{(2,4)}$ algebras (this is a work in progress).

Another interesting aspect that deserves exploration is the derivation of the hyper-Maxwell algebras introduced here by considering the S-expansion method. In three spacetime dimensions, the Maxwell algebra and its supersymmetric extension can be obtained by expanding the $\mathfrak{so}(2,1)$ and $\mathfrak{osp}(2,1)$ algebras, respectively [70]. One could extend the S-expansion procedure at the hypersymmetric level by studying the S expansion of the $\mathfrak{osp}(1|4)$ superalgebra considering different semigroups. It would be interesting to recover not only known algebras (such as the hyper-Poincaré algebra and the ones presented here) but also novel algebras that are the hypersymmetric extensions of known (or unknown) bosonic algebras. A particular advantage of considering the S-expansion approach is that it provides us not only with the expanded (anti)commutation relations but also with the nonvanishing components of the invariant tensor of the expanded algebra, which are essentials in the construction of a CS action.

It is well known that three-dimensional CS actions possess higher-dimensional generalizations. Following [26], it would be worthwhile to extend our construction to five or more odd spacetime dimensions. As a concluding remark, let us mention that the $\mathcal{N}$ extension of our results can be done by considering an appropriate IW contraction of diverse combinations of the $\mathfrak{osp}(M|4)$ and $\mathfrak{sp}(4)$ algebras. In particular, one would expect that an $\mathcal{N} = (M,N)$ extended version of the simplest hyper-Maxwell algebra presented here could be derived from the $\mathfrak{osp}(M|4) \times \mathfrak{osp}(N|4) \times \mathfrak{sp}(4)$ superalgebra. Let us note that the $\mathfrak{osp}(M|4) \times \mathfrak{osp}(N|4)$ superalgebra corresponds to the $\mathcal{N}$-extended hyper-AdS algebra studied in [22].

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