Generating a seed magnetic field *à la* the chiral Biermann battery

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Cosmological and astrophysical observations indicate the presence of a magnetic field over all scales. In order to explain these magnetic fields, it is assumed that there exists a seed magnetic field which gets amplified by dynamos. These seed fields may have been produced during inflation, at phase transitions, or during some turbulent phase of the early Universe. One well-known mechanism to get the seed field is the Biermann battery, which was originally discussed in the context of generation in an astrophysical object. Requirements for this mechanism to work are (i) a nonzero gradient of the electron number density and pressure, and (ii) they are nonparallel to each other. In the present article we propose a similar mechanism to generate the seed field but in an inhomogeneous chiral plasma. Our mechanism works, in the presence of chiral anomaly, by the virtue of inhomogeneity in the chiral chemical potential and temperature. We discuss various scenarios where inhomogeneities in the chemical potential and temperature can arise. We find that, depending on the epoch of generation, the strength of the seed magnetic fields varies from a few nanogauss (nG) to a few hundred nG.

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I. INTRODUCTION

Magnetic fields are ubiquitous in our observable Universe and are observed at all length scales, starting from our Solar System to the Milky Way to galaxy clusters and superclusters and even in voids of the large-scale structure. The pervading magnetic fields are expected to produce important effects on various processes including baryogenesis [1], primordial nucleosynthesis [2], and on the physics of the cosmic microwave background [3,4] (for a review, see Ref. [5]). Even after having so many important effects, the origin of the seed magnetic field remains an open-ended problem in modern cosmology. It is well known that the observed magnetic fields in astrophysical structures of different sizes are produced by amplification of seed magnetic fields [6,7]. The weakest "seed" magnetic fields amplified by the first dynamos in the early Universe could have worked at a cosmological phase transition, including the electroweak (EW) and quark confinement (QCD) phase transitions, during inflation or some turbulent phase in the primordial plasma due to some asymmetry [8–11]. Recently, the asymmetric models of the chiral plasma, where there is a finite difference in the number densities of the left-handed and right-handed massless electrons, have attracted a lot of quite interesting attention [12–15] (for more references see Ref. [16] and references therein). In the present work, we have considered a mechanism to generate the seed magnetic field in the early Universe due to inhomogeneities, in the chiral chemical potential and temperature, present in the chiral plasma.

The dynamics of relativistic chiral matter has been a subject of interest from both theoretical and experimental points of view [17,18]. The chiral matter is realized in various systems including the electroweak plasma in the early Universe [19], quark-gluon plasma in heavy ion collisions [20], Weyl semimetals [21], electron plasma in neutron stars [22,23], and the interior neutrino medium of the core-collapse supernova explosion [24] (see Ref. [25] and references therein). Chiral plasma exhibits interesting transport properties which are not seen in normal plasma. For instance, the triangle anomaly [26], which arises in the context of quantum field theory, leads to the chiral magnetic effect [27] and the chiral vortical effect [28], to mention a few. Processes related to the chiral plasma dynamics are affected by the Abelian anomaly [29,30] of the minimal standard model (MSM) and it is given by the anomaly equation [31] $\partial_\mu j^\mu_R = -\frac{g'^2 y_R^2}{64\pi^2}\mathcal{F}_{\mu\nu}\tilde{\mathcal{F}}^{\mu\nu}$. $\mathcal{F}$ and $\tilde{\mathcal{F}}$ are the $U_Y(1)$ hypercharge field strength and their duals respectively, j^μ_R is the current for the right-handed particles, g' is the associated coupling constants, and y_R is the hypercharge of the right electron. The helicity of the gauge fields are shown to be related to the Chern-Simons (CS) number $N_{\rm CS} = -\frac{g'^2}{32\pi^2}\int d^3x\epsilon_{ijk}\mathcal{F}_{ij}\mathcal{A}_k$ of the hypercharge field potential. The asymmetry in the number density of

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right-handed electrons changes with the CS number as $\Delta n_R = \frac{1}{2} y_R^2 \Delta N_{\rm CS}$ [31,32]. Origin of the asymmetry in the number density of right electrons $\delta_R = \Delta n_R / s$, where $s = \frac{2}{45}\pi^2 T^3 N_{\rm eff}$ is the entropy density and $N_{\rm eff}$ is the effective degree of freedom of the MSM, is usually attributed to the out of equilibrium processes at the grand unified theory scale [33] (for a review, see [34]). Similar expressions can also be written for the number asymmetry of the left-handed particles and their antiparticles. The total number asymmetry of the chiral particles is a summation of the left- and right-handed particle number asymmetries and is known as chiral asymmetry. Chiral asymmetry is commonly parametrized by the chiral chemical potential $\mu_5 = \mu_R - \mu_L$, where R is for the right-handed particles and L is for the left-handed particles. The anomalous coupling of the chirality and hypermagnetic helicity leads to an exponential growth of thermal fluctuations of these fields up to a value where it is in equipartition with the chirality. This phase of exponential growth is known as "chiral plasma instability." The evolution of the effective magnetic helicity of the hypermagnetic fields is given by the kinetic equation [12] $\frac{\partial}{\partial t}(\mu_5 + \frac{\alpha}{\pi}\mathcal{H}_B) = -\Gamma_f \mu_5$. In this equation, Γ_f represents the chiral flipping rate and $\mathcal{H}_B$ is the magnetic helicity of the (hypercharge) magnetic fields. It is to be noted here that the mentioned magnetic fields are not standard-model electromagnetic fields, but instead hypercharge magnetic fields. In subsequent parts of the paper, unless we refer to this as a standard-model magnetic field, the magnetic fields noted are hypercharge magnetic fields. In the absence of the reactions that flip the chirality of the interacting particles, the chiral number densities are conserved. Flipping starts at temperature T_f, when the chiral flipping rates Γ_f becomes equivalent to the expansion rate of the Universe. For instance, for the processes at the EW phase transition $T_f \sim \text{TeV}$ and for the QCD scale $T_f \sim \text{few GeV}$. At temperatures $T > T_f$, the asymmetry in the number densities of the massless electrons remains in the thermal equilibrium via its coupling with the hypercharge gauge bosons. Therefore, it is expected that in this regime, helical magnetic fields are generated and they grow at the cost of chiral asymmetry in the plasma. These helical magnetic fields and the chiral symmetry of the leptons support each other in the process of "inverse cascading," transferring magnetic energy from a small length scale to a large length scale. In Ref. [12], it is shown that chiral asymmetry could survive till $T \sim 10$ MeV.

So far, the generation of magnetic fields has been discussed mostly in a homogeneous chiral plasma [11,35]. In the present work, we exploit chiral magnetohydrodynamics (MHD) equations used to describe the dynamics of the inhomogeneous chiral plasma to generate the seed magnetic field. We show that inhomogeneities in the chiral chemical potential and temperature of the fluid lead to a sufficiently large seed field through a Biermann batterylike mechanism [36]. These seeds can be further amplified, by dynamos as well as instabilities in the chiral plasma, to currently the observed strength of magnetic fields in voids. We call this a *chiral Biermann battery mechanism*. In the present work, we have considered the flat Friedmann-Lemaître-Robertson-Walker metric $\eta^{\mu\nu}$ with signature $(-,+,+,+)$ and used our units in such a way that $\hbar = c = k_B = 1$. This manuscript is structured as follows: Sec. II provides an overview of chiral dynamics and also discusses the generation of magnetic fields by the Biermann battery like mechanism. Section III discusses the scenario in which inhomogeneities in chiral chemical potential and temperature can arise. This also discusses the condition for the Biermann battery to be operative. A summary and future prospects of the work are given in Sec. IV.

II. CHIRAL BIERMANN BATTERY MECHANISM

Generation of seed magnetic fields by a cosmic battery is commonly based on the fact that in a charge neutral universe, the positive and negative charge particles have different behaviors due their mass difference. For a given pressure gradient in the gas, electrons would be accelerated much more than the ions due to their small mass compared to ions. This leads to a current and hence an electric field $\vec{E} = -\vec{\nabla} p_e/(en_e)$. If the curl of the thermally generated electric field has a nonzero value, then from Faraday's law of induction, the magnetic field can grow. The resulting battery is termed a Biermann battery mechanism [36]. This mechanism is mostly explored in the context of stellar objects and early Universe processes at the time of recombination. However, before the recombination epoch, a similar mechanism can be operative in chiral plasma provided there is an inhomogeneity in chiral chemical potential and temperature. Before proceeding further, we provide a brief overview of relativistic MHD equations required to describe the dynamic of chiral fluids. Later, we use them to derive an equation which looks similar to the Biermann battery.

A. Overview of chiral fluid

To provide an overall description of chiral plasma, we assume that an external magnetic field is present in the beginning. Later on, we will come to the case where the initial magnetic field is absent and the seed field is generated. In the presence of an external magnetic field the hydrodynamic equations that govern the time evolution of the anomalous chiral fluid are given by the following set of equations [37]:

$$\nabla_\mu T^{\mu\nu} = F^{\nu\lambda} j_\lambda, \tag{1}$$

$$\nabla_\mu j^\mu = 0, \tag{2}$$

$$\nabla_\mu j_5^\mu = C E_\mu B^\mu, \tag{3}$$

where $T^{\mu\nu}$ is the energy-momentum tensor of an ideal fluid and $F^{\nu\lambda}$ is the electromagnetic field strength tensor. The electric and magnetic field four-vectors are represented by E^μ and B^μ respectively. The vector current is given by $j^\mu = j_R^\mu + j_L^\mu$ and the chiral current is represented by $j_5^\mu = j_R^\mu - j_L^\mu$. The chiral anomaly coefficient is denoted by C.

In the state of local equilibrium, the energy-momentum tensor $T^{\mu\nu}$, the vector current j^μ, and the chiral current j_5^μ can be expressed in terms of the four velocity of the fluid u^μ, energy density ρ, vector charge density $n_{\rm v}$, and axial charge density n_5. In the absence of electromagnetic fields, the local equilibrium reached at a length scale of spatial variation of chemical potential μ, i.e., $\ell_{\rm LTE} \ll \ell_\mu \sim \mu(\vec{x})/|\vec{\nabla}\mu(\vec{x})|$ (here ℓ_μ is the scale over which chemical potential varies significantly). Thus, the local distribution function of the fermions is given by the local expression $f_i^{\rm eq}(t,\vec{x},\vec{p}) = [{\rm Exp}(\frac{\epsilon_p - \mu_i(t,\vec{x})}{T(t,\vec{x})}) + 1]^{-1}$, where $\epsilon_p = c|\vec{p}|$ (here i = right-/left- handed particles). Spatial variation of electromagnetic fields and matters occurs at much larger scale than $\ell_{\rm LTE}$. Since the chiral anomaly relation is local, the electric and chiral chemical potentials should be space-time dependent. The relation between axial charge density ρ_5 and the zeroth component of axial current j_5^μ is given by $\rho_5(t,\vec{x}) = \langle j_5^0(t,\vec{x})\rangle_{T,\mu_5}$, which is valid for $\mu_5 \ll T$ [38]. For a small deviation from local equilibrium, vector current and chiral current respectively take the form [39,40]

$$j^\mu = n_{\rm v} u^\mu - \frac{\sigma}{2} T \Delta^{\mu\nu}\partial_\nu\left(\frac{\mu_{\rm v}}{T}\right) + \sigma E^\mu + \xi_{\rm v}\omega^\mu + \xi_{\rm v}^{(B)} B^\mu, \tag{4}$$

$$j_5^\mu = n_5 u^\mu - \frac{\sigma}{2} T \Delta^{\mu\nu}\partial_\nu\left(\frac{\mu_5}{T}\right) + \xi_5\omega^\mu + \xi_5^{(B)} B^\mu, \tag{5}$$

where $\Delta^{\mu\nu} = (\eta^{\mu\nu} + u^\mu u^\nu)$ is the projection operator, $n_{{\rm v},5} = n_R \pm n_L$, $\mu_{{\rm v},5} = \mu_R \pm \mu_L$, $\xi_{{\rm v},5} = \xi_R \pm \xi_L$, $\xi_{{\rm v},5}^{(B)} = \xi_R^{(B)} \pm \xi_L^{(B)}$, and σ is the conductivity. The vorticity four-vector is represented by ω^μ. The mathematical expression of the transport coefficients $\xi_{{\rm v},5}$ and $\xi_{{\rm v},5}^{(B)}$ in Eqs. (4) and (5) have been calculated by many authors and it has been shown that these terms are not only allowed but also required for anomalies [10,30,39]. The second term in Eqs. (4) and (5) arise only when there is inhomogeneity in chemical potential or temperature or both. For the present study, inhomogeneity in both chemical potential and temperature are important. The total current ($j_{\rm tot}^\mu = j^\mu + j_5^\mu$) from the right-handed chiral particles is given by

$$j_{\rm tot}^\mu = 2 n_{\rm R} u^\mu + \sigma E^\mu + 2\xi_{\rm R}\omega^\mu + 2\xi_{\rm R}^{(B)} B^\mu - \sigma T \Delta^{\mu\nu}\partial_\nu\left(\frac{\mu_{\rm R}}{T}\right). \tag{6}$$

The coefficients $\xi_{\rm R}$ and $\xi_{\rm R}^{(B)}$ are given as [39,41]

$$\xi_{\rm R} = C\mu_{\rm R}^2\left[1 - \frac{2n_{\rm R}\mu_{\rm R}}{3(\rho+p)}\right] + \frac{DT^2}{2}\left[1 - \frac{2n_{\rm R}\mu_{\rm R}}{(\rho+p)}\right], \tag{7}$$

$$\xi_{\rm R}^{(B)} = C\mu_{\rm R}\left[1 - \frac{n_{\rm R}\mu_{\rm R}}{2(\rho+p)}\right] - \frac{D}{2}\left[\frac{n_{\rm R}T^2}{(\rho+p)}\right]. \tag{8}$$

The second terms in these equations are uniquely fixed by the requirement on the entropy current s^μ to satisfy $\partial_\mu s^\mu \geq 0$ [39]. The coefficient D in the above equations cannot be derived solely from hydrodynamics [41], which is a manifestation of additional microscopic properties of the chiral degrees of freedom. Values of the coefficient D is derived by considering gauge-gravitational duality by many authors [30]. In the simplest case of noninteracting chiral fermions, values of the coefficients C and D are given by $C = 1/4\pi^2$ and $D = 1/12$.

B. Seed magnetic field generation

Using the expression for $\vec{j}_{\rm tot}$, given in Eq. (6) and the Maxwell's equation $\vec{\nabla}\times\vec{B} = \vec{j}_{\rm tot}$, we get

$$\begin{aligned}\vec{\nabla}\times\vec{B} &= 2n_{\rm R}\vec{v} - \sigma T[\vec{\nabla}(\mu_{\rm R}/T) + \vec{v}\partial_t(\mu_{\rm R}/T) \\ &\quad + \vec{v}(\vec{v}\cdot\vec{\nabla})(\mu_{\rm R}/T)] \\ &\quad + \sigma(\vec{E} + \vec{v}\times\vec{B}) + 2\xi_{\rm R}\vec{\omega} + 2\xi_{\rm R}^{(B)}\vec{B}.\end{aligned} \tag{9}$$

To obtain the evolution equation for the magnetic fields, we first eliminate $\vec{E}$ from the above equation as

$$\begin{aligned}\vec{E} &= -\vec{v}\times\vec{B} + \frac{\vec{\nabla}\times\vec{B}}{\sigma} - \frac{2n_{\rm R}}{\sigma}\vec{v} - \frac{2\xi_{\rm R}}{\sigma}\vec{\omega} - \frac{2\xi_{\rm R}^{(B)}}{\sigma}\vec{B} \\ &\quad + T[\vec{\nabla}(\mu_{\rm R}/T) + \vec{v}\partial_t(\mu_{\rm R}/T) + \vec{v}(\vec{v}\cdot\vec{\nabla})(\mu_{\rm R}/T)].\end{aligned} \tag{10}$$

Now taking the curl of the above equation and using $\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$, the evolution equation for $\vec{B}$ field is given as

$$\frac{\partial\vec{B}}{\partial t}=\underbrace{\frac{1}{\sigma}\nabla^2\vec{B}}_{\text{I}}+\underbrace{\vec{\nabla}\times(\vec{v}\times\vec{B})}_{\text{II}}+\underbrace{\frac{2}{\sigma}[n_{\text{R}}\vec{\omega}+\vec{\nabla}n_{\text{R}}\times\vec{v}]}_{\text{III}}+\underbrace{\frac{2}{\sigma}[\vec{\nabla}\xi_{\text{R}}\times\vec{\omega}+\xi_{\text{R}}\vec{\nabla}\times\vec{\omega}]}_{\text{IV}}+\underbrace{\frac{2}{\sigma}[\vec{\nabla}\xi_{\text{R}}^{(B)}\times\vec{B}+\xi_{\text{R}}^{(B)}(\vec{\nabla}\times\vec{B})]}_{\text{V}}+\underbrace{\frac{\vec{\nabla}\sigma}{\sigma^2}\times(\vec{\nabla}\times\vec{B})}_{\text{VI}}$$
$$-\underbrace{\frac{2n_{\text{R}}}{\sigma^2}(\vec{\nabla}\sigma\times\vec{v})}_{\text{VII}}-\underbrace{\frac{2\xi_{\text{R}}}{\sigma^2}(\vec{\nabla}\sigma\times\vec{\omega})}_{\text{VIII}}-\underbrace{\frac{2\xi_{\text{R}}^{(\text{B})}}{\sigma^2}(\vec{\nabla}\sigma\times\vec{B})}_{\text{IX}}-\underbrace{\frac{1}{T}[\vec{\nabla}T\times\vec{\nabla}\mu_{\text{R}}]}_{\text{X}}-\underbrace{\vec{\nabla}\times[T\{\vec{v}\partial_t(\mu_{\text{R}}/T)+\vec{v}(\vec{v}\cdot\vec{\nabla})(\mu_{\text{R}}/T)\}]}_{\text{XI}}. \quad (11)$$

It is important to note here that this equation is valid for the case when chiral plasma is inhomogeneous. In the case of homogeneous chiral plasma, all terms with $\vec{\nabla}\sigma$, $\vec{\nabla}n_R$, $\vec{\nabla}\xi_R$, $\vec{\nabla}\xi_R^{(B)}$, $\vec{\nabla}T$, and $\vec{\nabla}\mu_R$ will vanish. Before proceeding further, we estimate the order of magnitude of each term in the right-hand side of the above equation. In order to do so we take $\sigma\sim T/4\pi\alpha$ [42], $n_{\text{R}}\sim\mu T^2/6$, $\xi_{\text{R}}\sim(\mu^2/\pi)\sqrt{\alpha/4\pi}$, and $\xi_{\text{R}}^{(B)}\sim 2\alpha\mu/\pi$. If L is the length scale of interest, the term by term order in Eq. (11) is $\text{I}\sim 4\pi\alpha(\frac{B}{T^2})(\frac{T}{L^2})=\text{VI}$, $\text{II}\sim\frac{Bv}{L}=(\frac{B}{T^2})(\frac{T^2}{L})v$, $\text{III}\sim\frac{2n_R v}{\sigma L}\sim\frac{4\pi}{3}\alpha(\frac{\mu}{T})(\frac{T^2}{L})v=\text{VII}$, $\text{IV}\sim\frac{2\xi_{\text{R}}v}{\sigma L^2}\sim\frac{4}{\pi}\alpha^{3/2}(\frac{\mu}{T})^2(\frac{T}{L^2})v=\text{VIII}$, $\text{V}\sim\frac{2\xi_{\text{R}}^{(B)}B}{\sigma L}\sim 16\alpha^2(\frac{\mu}{T})(\frac{B}{T^2})(\frac{T^2}{L})=\text{IX}$, $\text{X}\sim(\frac{\mu}{T})(\frac{T}{L^2})=\text{VIII}$, $\text{XI}\sim(\frac{\mu}{T})(\frac{T}{L^2})v^2$. Since $\alpha\sim 10^{-2}$, $\frac{\mu}{T}\sim(10^{-4}–10^{-6})$, $v\ll 1$ we can ignore all other terms compared to I, II, and X. Further at temperature scale of our interest, the variation in conductivity is also small and the VI, VII, VIII, and IX terms can be dropped. Hence, Eq. (11) will reduce to the following form with the above approximation

$$\frac{\partial\vec{B}}{\partial t}=\frac{1}{\sigma}\nabla^2\vec{B}+\vec{\nabla}\times(\vec{v}\times\vec{B})-\frac{1}{T}[\vec{\nabla}T\times\vec{\nabla}\mu_{\text{R}}]. \quad (12)$$

The above equation represents the magnetic induction equation for the magnetic fields in the case of inhomogeneous chiral plasma. The first two terms on the right-hand side represent diffusion and convection respectively. The first term signifies the transport of the magnetic field via diffusion. However, the second term describes how the magnetic field in a conducting fluid changes with time under the influence of a velocity field v. In the absence of initial electromagnetic fields i.e., $\vec{E}=0=\vec{B}$, Eq. (12) reduces to

$$\frac{\partial\vec{B}}{\partial t}=-\frac{1}{T}[\vec{\nabla}T\times\vec{\nabla}\mu_{\text{R}}]. \quad (13)$$

It is important to note here that seed magnetic fields are produced via this mechanism only in the case of inhomogeneous chiral plasma. Along with this, the following conditions should also be satisfied: $\vec{\nabla}T\times\vec{\nabla}\mu_{\text{R}}\neq 0$ or the nonparallel components of $\vec{\nabla}T$ and $\vec{\nabla}\mu_{\text{R}}$. This equation looks exactly like the Biermann mechanism. Therefore, we call this mechanism a chiral Biermann battery mechanism.

III. CONDITIONS FOR THE CHIRAL BATTERY

For the chiral Biermann mechanism to work, the following conditions must be satisfied (i) $\vec{\nabla}\mu_5\neq 0$, $\vec{\nabla}T\neq 0$ and (ii) $\vec{\nabla}T\nparallel\vec{\nabla}\mu_{\text{R}}$. Here we have discussed three important scenarios where all three conditions are satisfied.

a. One of the most promising scenarios to achieve all three conditions is the first order phase transition or during any turbulent phase of the early Universe [43]. Our Universe has gone through several phase transitions (PTs) including electroweak, at around $T_{\text{EW}}\sim 100$ GeV, and at the QCD phase transition occurring around the critical temperature $T_{\text{QCD}}=150$ MeV. Although the opinion is divided, various arguments raise the possibility that these transitions might be first order. In this work, we will assume that the QCD phase transition is first order. If the QCD transition is first order then the Universe has to cool somewhat below the critical temperature T_{QCD} before any regions of hadron matter appear. The Universe supercools a finite amount before the appearance of small nucleation sites. These are bubbles of hadronic phase which consist mostly of pions. It is important to highlight the fact that there are two different timescales involved, namely (i) the QCD timescale, which is of the order of $\tau_{\text{QCD}}\sim 1/T_{\text{QCD}}$, and the Hubble timescale, which is $\tau_{\text{H}}\sim 10^{19}/T_{\text{c}}\gg\tau_{\text{QCD}}$. Thus, nucleation is a local phenomena. After nucleation, the bubble grows explosively like a deflagration bubble. For small supercoolings (of the order 2% i.e., $T_s\sim 0.98\,T_{\text{QCD}}$) the deflagration front travels slowly i.e., $v_{\text{front}}\ll c_s=1/\sqrt{3}$ [19]. However, the front is preceded by a supersonic shock which moves with a velocity $v_{\text{sh}}>c_{\text{s}}$. The propagation of shock leads to heating and compression in the quark matter. With increasing time more and more bubbles are nucleated, they grow, and the shock fronts preceding the bubbles begin to collide. At this stage, the Universe enters into a turbulent phase. If the supercooling is small, then the turbulence dies out, the Universe outside the hadron bubbles is reheated to T_{QCD}, and the explosive bubble growth is halted. At this time hadronic bubbles are roughly $1/10$ of the average distance between nucleation sites. In the case of small supercooling, collisions between two shock fronts and between a shock and a deflagration front may lead to inhomogeneity in the temperature as well as the chemical potential. These inhomogeneities exist over a scale of coexisting phases. Moreover, it was assumed that the hardonic phase includes spheres of the same size. This is only an approximation. In fact, there is a complex distribution of sizes due to the fact that the nucleation sites do not appear at exactly the same time. Also, their shapes are not exactly spherical and may include ripples when the surface tension becomes unimportant. Thus, when the shock fronts collide, a turbulent phase begins and vorticity is generated [44]. During this phase, all three conditions required for the generation of the seed field are met. An estimate of the generated seed field can be given as follows: The duration of the QCD phase transition $t_{\rm PT} \sim 0.22\tau_{\rm H} \simeq 43\ \mu$s, the temperature will be inhomogeneous over the scale of the coexisting phase, and so is the chemical potential. $\Delta T/T_{\rm QCD} \sim (\ell/\tau_{\rm H})^2$, where ℓ is the scale over which the temperature and chemical potential will be inhomegeneous [19]. This scale is typically the size of different coexisting phases. Using these number, we can estimate the strength of the seed field generated at the source as follows: $B_{\rm QCD} \sim t_{\rm pt} \times (\frac{\Delta T}{T_{\rm QCD}})(\frac{\Delta\mu_5}{T_{\rm QCD}})(\frac{T_{\rm QCD}}{\ell^2}) \sim 0.26$ nG.

b. Another interesting scenario which can generate inhomogeneity in the temperature and chemical potential is that of the inhomogeneous QCD phase transition proposed in Ref. [45]. This is possible when the temperature is inhomogeneous. The inhomogeneous temperature depends on two parameters, (i) density perturbation $\Delta_T^{(\rm rms)}$ and (ii) the temperature interval of nucleation $\Delta_{\rm nuc}$. It has been proposed that, when $\Delta_T^{\rm rms} > \Delta_{\rm nuc}$, the nucleation of the bubbles at a given time will be inhomogeneous [45]. Since the inflation produced density perturbation which leads to temperature fluctuation of the order of $\Delta_T^{(\rm rms)} \sim 10^{-5}$ and results of the lattice simulation with quenched QCD (no dynamical quarks) give the value of the (dimensionless) temperature interval of nucleation $\Delta_{\rm nuc} \sim 10^{-6}$, the nucleation is thus inhomogeneous. Initially, cold spheres of the diameter $\ell_{\rm smooth} \sim 10^{-4} d_{\rm H}$ (where $d_{\rm H} = c/H \sim 10$ km is the Hubble distance at the QCD transition [46]) with equal and uniform temperature are distributed randomly, which is $\Delta_T^{(\rm rms)} \times T_{\rm QCD}$ less than the rest of the uniform Universe. When the temperature of the cold spot decreases to the value of the actual nucleation temperature $T_{\rm n}$, homogeneous nucleation takes place within it. However, the Hubble expansion would result in the cooling of the Universe and would take $\Delta t_{\rm cool} = (\Delta_T^{(\rm rms)}/3c_s^2)\tau_H$ time to cool down to $T_{\rm n}$. Inside each cold spot, there will be large number of tiny hadron bubbles. These bubbles merge within $\Delta t_{\rm cool}$ if $\Delta_{\rm nuc} < (v_{\rm def}/v_{\rm heat})\Delta_T^{\rm rms}$, where $v_{\rm def}$ is the speed of the deflagration front and $v_{\rm heat}$ is the effective speed by which released latent heat propagates to stop nucleation. The length scale of temperature propagation is determined by the latent heat released in the cold spot, which propagates in all directions and is given as $\ell_{\rm heat} = 2v_{\rm heat}\Delta t_{\rm cool}$ (which is of the order of a few meters [47]). Thus, in this scenario we can have a temperature gradient of the order of $\Delta T_{\rm QCD}/\ell_{\rm heat} \sim 10^{-1}$–$10^{-4}$ MeV/km (when $\ell_{\rm heat}/\ell_{\rm smooth} = 1, 2, 5, 10$). In this scenario, obtained values of the magnetic fields strength are again in the range of ∼nG.

c. There is one more viable scenario, around electroweak scale, where all three conditions can be satisfied, and it is when a few hypermagnetic modes grow exponentially in a chiral plasma. It has been shown that in chiral plasma, due to finite chiral asymmetry, the quantum effects lead to the production of hypercharge magnetic fields. Few modes of these fields show exponential growth due to the parity odd interaction of the fermions with the Abelian fields. The exponential growth of those modes with wave number k during a chiral plasma instability occurs at a timescale of $t \sim \sigma/k(4\mu - k)$, where the mode with wave number $k_{\rm max} = 2\mu$ has the maximum growth rate [48]. During this phase, chiral plasma goes through a turbulent evolution. The timescale of the maximum growth rate corresponding to the chemical potential $\mu_{\rm max} = 100T/4\mu^2$. The requirement that the growth timescale should be smaller than the Hubble time τ_H gives the constraint on $\mu/T = \delta$ as $\delta = 2 \times 10^{-6}(T/T_f)^{1/2}$. If the temperature at which left-right asymmetry is generated at a temperature $T < T_f$, the time available for the generation of the magnetic fields is Γ_f^{-1}, rather than the Hubble time. This means we can consider asymmetry $\delta \geq 2 \times 10^{-6}$ [31,49]. In this case, for the typical values of $T \sim 100$ GeV, $\ell_\mu \sim \ell_{\rm heat} \sim {\rm GeV}^{-1}$, $t_{\rm EW} \sim {\rm GeV}^{-1}$, and $\delta_{\rm EW} \sim 10^{-6}$, strength can be calculated by using $B(t_{\rm EW}) \sim \delta_{\rm EW} \times (T/\ell_{\rm heat})(t/\ell_\mu) \sim 10^{-3}$ nG.

IV. SUMMARY AND FUTURE PROSPECTS OF THE WORK

In this work, we have used the Biermann battery like mechanism to explore the possibility of the generation of a seed magnetic field in the early Universe. Our mechanism works when the chiral chemical potential and the temperature have spatial dependence. Magnetic field generation via this mechanism may work at phase transitions or during the turbulent phase of exponential growth of the chiral modes (this is known as chiral plasma instability) in the presence of a finite amount of chiral asymmetry.

The strength of the generated magnetic fields via this mechanism are of the order of ~nG. The helical magnetic fields produced by this mechanism subsequently evolve through various turbulent phases, which preserves the helicity and ultimately produces the primordial standard-model magnetic fields surviving till the present epoch. Once the primordial helical magnetic fields are generated by the chiral Biermann battery mechanism at a length scale larger than the "frozen-in" scale $L_f \sim \sqrt{t/4\pi\sigma}$ (here t is the cosmic time), the first and second terms in Eq. (12) dominate. At high temperature the conductivity of the chiral plasma, $\sigma \sim T/\alpha \ln(1/\alpha)$ (see Ref. [42]), is very large and hence dissipation by the first term in Eq. (12) can be ignored. The dynamics of the fields are solely governed by the convection term. At length scales $\ell > L_f$, magnetic fields are coherent and are said to be frozen in. This means that the helicity of the magnetic fields is conserved both locally and globally. When the fluid velocity is small, it is also possible for the first term in Eq. (12) to dominate over the second term. In this case the magnetic fields dies out exponentially and helicity is no longer conserved. If the fluid velocity is not small, the evolution of the fields is governed by both terms. In this situation, it has been argued that the local helicity of the magnetic fields changes due to the reconnection of the field lines, and therefore they are not conserved. However, the global helicity, a summation of random local changes, remains conserved [50]. The Reynolds number $R_M = 4\pi\sigma l v \gg 1$ determines the conservation of global helicity at a characteristic length scale $l \sim 1/e^2 T$ (which characterizes gauge field configurations such as sphalerons). Whether this criteria is satisfied depends on the dynamics of the fluid during baryogenesis and requires significant departure from the thermal equilibrium. Under these circumstances, magnetic fields evolve, conserving the global magnetic helicity even though the field is not frozen in.

So far in this work, we have focused our attention on the production of the seed field in an inhomogeneous chiral plasma. To glean a complete picture of the generated magnetic field and processes involved in it, we need to study the evolution of generated magnetic field to today's epoch and the power spectrum of the magnetic field. These will be topics of future studies.

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