Universal 10^{20} Hz stochastic gravitational waves from photon spheres of black holes

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We show that photon spheres of supermassive black holes generate high-frequency stochastic gravitational waves through photon-graviton conversion. Remarkably, the frequency is universally determined as $m_e\sqrt{m_e/m_p} \simeq 10^{20}$ Hz in terms of the proton mass m_p and the electron mass m_e. It turns out that the density parameter of the stochastic gravitational waves $\Omega_{\rm gw}$ could be 10^{-12}. Since the existence of the gravitational waves from photon spheres is robust, it is worth seeking methods of detecting high-frequency gravitational waves around 10^{20} Hz.

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I. INTRODUCTION

The era of gravitational wave astronomy commenced [1] when the first gravitational wave event from a binary black hole merger was detected [2]. Gravitational waves provide us with new information about the Universe even before the recombination epoch, which cannot be observed with electromagnetic waves. However, the frequency range of gravitational waves that we have observed is rather narrow, 10 Hz ∼ 10 kHz, while that of electromagnetic waves has covered the frequency range from kHz (radio) to 10^{26} Hz (gamma ray). Apparently, it is important to extend the frequency frontier of gravitational wave observations.

There are many plans for observing gravitational waves from nHz to 100 MHz [3]. Beyond 100 MHz, however, the detection method of gravitational waves is different from that of gravitational wave interferometry [4]. For example, magnons in cavities are used to obtain the sensitivity at 8 and 14 GHz [5]. Also detectors based on the photon-graviton conversion phenomenon [6] have been proposed to observe gravitational waves from planetary-mass primordial black hole binaries (typically 200 MHz) [7]. The photon-graviton conversion is also used as a way to constrain the stochastic background gravitational waves with frequencies above 1 THz [8].

In order to boost the study of high-frequency gravitational wave detectors, the existence of guaranteed sources which emit high-frequency gravitational waves is essential. So far, various high-frequency gravitational wave sources have been proposed [4,9–20]. In particular, primordial black holes (PBHs) [21] are a possible source of high-frequency gravitational waves [22–25]. PBHs evaporated before the big bang nucleosynthesis produced stochastic background around $10^{15} \sim 10^{19}$ Hz with its typical density parameter $\Omega_{\rm gw} \sim 10^{-7.5}$. At present, however, it would be fair to say that the existence of these sources are not guaranteed. There is a more plausible source of high-frequency waves, namely those emitted by thermal fluctuations in the hot plasma, first proposed in [17] and further developed recently in [18,19]. These lead to peak emissions around 80 GHz, with an amplitude that scales with the reheating temperature, reaching $\Omega_{\rm gw} \sim 10^{-10}$ for grand unified theory-scale temperatures.

In this paper, we propose a novel source of high-frequency gravitational waves, namely, magnetospheres of supermassive black holes which is currently suggested by the Event Horizon Telescope observation [26]. The stochastic gravitational waves can be generated through the photon-graviton conversion from photon spheres of supermassive black holes. Indeed, in the photon sphere of a black hole, the steady photon accretion from accretion disk effectively increases the conversion probability, and sufficient amount of gravitons are produced. Since the existence of supermassive black holes has been already proved, they are guaranteed sources. We show that the frequency of these gravitational waves does not depend on the mass of black hole and strength of magnetic field under the assumption of equipartition of energies. Remarkably, the frequency is universally determined as $m_e\sqrt{m_e/m_p} \simeq 10^{20}$ Hz in terms of the proton mass m_p and the electron mass m_e. We estimate contribution from supermassive black holes (SMBHs) to stochastic gravitational wave background and find that the density parameter of the stochastic gravitational waves $\Omega_{\rm gw}$ could be of the order of 10^{-12}. It encourages us to seek detectors for high-frequency gravitational waves.

II. PHOTON SPHERES OF BLACK HOLES

In this section, we relate the photon intensity emitted from the accretion disk to the flux flowing into the vicinity of the photon sphere. These photons can travel for large lengths without experiencing scatterings or being absorbed by the plasma of the accretion disk, and such long traveling distances are needed for efficient conversion in gravitational waves as will be explained later.

For simplicity, we assume the system to be a Schwarzschild black hole with a thin accretion disk. The Schwarzschild metric is given by

$$ds^2 = -f(r)c^2dt^2 + \frac{dr^2}{f(r)} + r^2(d\theta^2 + \sin^2\theta d\phi^2), \quad (1)$$

with $f(r) = 1 - r_g/r$ and gravitational radius $r_g := 2GM/c^2$. According to Lambert's law, the number of photons emitted from an area element dS_e of an accretion disk per unit time and passing through the solid angle $d\Omega_e$ is given by

$$d^3\left(\frac{dN}{d\tau_e}\right) = I_e^{(N)}(\omega)|\cos\theta_e|d\Omega_e dS_e d\omega, \quad (2)$$

where $d\tau_e$ is the proper time at the area element dS_e located at the radius r_e, and $I_e^{(N)}(\omega)$ is a photon number intensity with an frequency ω. For the angular coordinates used here, see Fig. 1. We use subscript e to refer to coordinates on the accretion disk.

It is useful to introduce an impact parameter $b := \frac{r_e}{\sqrt{f(r_e)}}\sin\Theta \sim \frac{r_e}{\sqrt{f(r_e)}}\sin(\frac{\pi}{2} - \theta_e)$ instead of θ_e [27]. The area element can be written as $dS_e = 2\pi r_e dr_e/\sqrt{f(r_e)}$. Then, we can rewrite Eq. (2) as

$$d^3\left(\frac{dN}{dt}\right) = (2\pi)^2 I_e^{(N)}(\omega)\left(\frac{f(r_e)}{r_e}b^2\right) \times \frac{db}{\sqrt{r_e{}^2/f(r_e) - b^2}} dr_e d\omega, \quad (3)$$

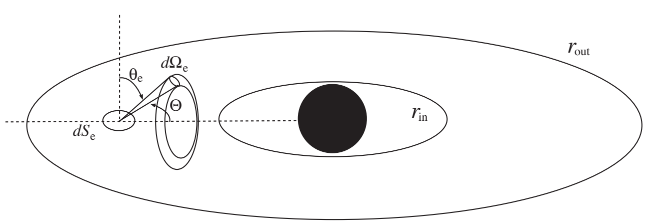

FIG. 1. Photon radiation from a thin accretion disk. r_{in} and r_{out} denote the inner and outer radius of the accretion disk, respectively. Since massive matter has the innermost stable circular orbit (ISCO) at $3r_g$ in the Schwarzschild background, we set the inner radius of the accretion disk to be $r_{in} = 3r_g$.

where t is the Schwarzschild time. To evaluate this, we replace b with the critical impact parameter $b_{crit} = 3\sqrt{3}r_g/2$ which corresponds to the impact parameter of circularly orbiting photons (see, e.g., p. 144 of Ref. [28]). Integrating radial coordinate r_e from $3r_g$ to infinity, we obtain the photon number emitted to $b = b_{crit} + db$ per unit time and unit frequency

$$d\left(\frac{d^2N}{dtd\omega}\right) \sim 27 r_g I_e{}^{(N)}(\omega)db. \quad (4)$$

This equation is useful in Sec. IV to estimate the number of gravitons produced around the photon sphere.

III. PHOTON-GRAVITON CONVERSION

In this section, we shall briefly review the conversion phenomenon between photon and graviton proposed by Gertsenshtein [6], with which we shall explore the possibility of gravitational wave emission from black hole magnetospheres. For more detailed derivation, see [29].

We consider the Einstein-Hilbert action with the Euler-Heisenberg effective Lagrangian for electromagnetic fields minimally coupled to the gravity,

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{16\pi G}R - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right] + \frac{\alpha^2}{90m_e^4}\int d^4x\sqrt{-g}\left[(F_{\mu\nu}F^{\mu\nu})^2 + \frac{7}{4}(\tilde{F}_{\mu\nu}F^{\mu\nu})^2\right], \quad (5)$$

where $\alpha = 7.297... \times 10^{-3}$ is the fine structure constant and m_e is the electron mass. The field strength is defined by $F_{\mu\nu} := \partial_\mu A_\nu - \partial_\nu A_\mu$. In order to describe propagations of gravitons and photons, we consider perturbations around background fields,

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}, \qquad A_\mu = \bar{A}_\mu + \mathcal{A}_\mu, \quad (6)$$

where $\eta_{\mu\nu}$ is the Minkowski metric and $\kappa := \sqrt{16\pi G}$. Hereafter, we impose the TT gauge for $h_{\mu\nu}$ (i.e., $h_{0i} = \partial_i h^i{}_j = h^i{}_i = 0$) and the radiation gauge for A_μ (i.e., $\mathcal{A}_0 = \partial_i \mathcal{A}^i = 0$), where $i, j = 1, 2, 3$. The background magnetic field is assumed to be static and uniform and to be aligned in the y direction of the Cartesian coordinates.

The conversion phenomenon occurs when gravitational waves propagate in the z direction, i.e., a direction perpendicular to the magnetic field. The plane-wave configurations along the z direction are expanded as

$$\mathcal{A}_i(z,t) = i\mathcal{A}_+(z)u_i e^{i(kz-\omega t)} + i\mathcal{A}_\times(z)v_i e^{i(kz-\omega t)}, \quad (7)$$

$$h_{ij}(z,t) = h_+(z)e^{+}_{ij}e^{i(kz-\omega t)} + h_\times(z)e^{\times}_{ij}e^{i(kz-\omega t)}, \quad (8)$$

where $u_i, v_i, e^+_{ij}, e^\times_{ij}$ are the polarization vectors and tensors. Substituting this plane-wave configuration for the linearized equation of motion obtained from Eq. (5), one gets a Schrödinger-type equation

$$i\frac{d}{dz}\psi(z) = \mathcal{M}\psi(z), \tag{9}$$

where

$$\psi(z) := \begin{pmatrix} h_\lambda(z) \\ \mathcal{A}_\lambda(z) \end{pmatrix}, \qquad \mathcal{M} = \begin{pmatrix} 0 & \Delta_{g\gamma} \\ \Delta_{g\gamma} & \Delta_\gamma \end{pmatrix} \tag{10}$$

and λ stands for the polarization, $\lambda = +, \times$. Here, $\mathcal{M}$ is called a mixing matrix which describes the effective photon mass, Δ_γ, and the coupling between gravitons and photons, $\Delta_{g\gamma} := 2\sqrt{\pi}B/M_{\rm pl}$. Δ_γ consists of two parts as

$$\Delta_\gamma = \Delta_{\mathrm{QED},\lambda} + \Delta_p. \tag{11}$$

Here, $\Delta_{\mathrm{QED},\lambda}$ describes the effective photon mass that originates from the effect of the quantum electrodynamics (QED). The QED contribution comes from the Euler-Heisenberg effective action in Eq. (5), and depends on the polarization λ as

$$\Delta_{\mathrm{QED},\lambda} = -k(\lambda)\varrho\omega B^2, \qquad \varrho = \frac{4\alpha^2}{45m_e^4}, \tag{12}$$

where $k(+) = 2, k(\times) = 7/2$. Since $k(\lambda)$ is the factor of order one, it will be often abbreviated in the following discussion since it does not change the order of magnitude of our results. The term Δ_p originates from the fact that in the presence of plasma, electromagnetic waves have an effective mass that corresponds to the plasma frequency ω_p. It is given as

$$\Delta_p = \frac{\omega_p^2}{2\omega} \quad \text{and} \quad \omega_p^2 = 4\pi\alpha\frac{n_e}{m_e} \tag{13}$$

with the electron number density n_e. Since the mixing matrix $\mathcal{M}$ is constant, under the initial condition $h_\lambda(0) = 0, \mathcal{A}_\lambda(0) = 1$, Eq. (9) is easily solved as

$$\begin{aligned} h_\lambda(z) &= \frac{1}{2}(e^{-i\kappa_+ z} - e^{-i\kappa_- z})\sin(2\theta), \\ \mathcal{A}_\lambda(z) &= e^{-i\kappa_+ z}\sin^2\theta + e^{-i\kappa_- z}\cos^2\theta, \end{aligned} \tag{14}$$

where θ is the mixing angle given by

$$\cos 2\theta = -\frac{\Delta_\gamma}{\Delta_{\rm osc}}, \qquad \sin 2\theta = \frac{2\Delta_{g\gamma}}{\Delta_{\rm osc}},$$
$$\text{and} \quad \Delta_{\rm osc} := \sqrt{\Delta_\gamma^2 + (2\Delta_{g\gamma})^2}. \tag{15}$$

Here, we defined the eigenvalues of $\mathcal{M}$ by $\kappa_\pm := (\Delta_\gamma \pm \Delta_{\rm osc})/2$. Therefore, the conversion probability from gravitons to photons after propagating the distance z becomes

$$P(z) = \left(\frac{2\Delta_{g\gamma}}{\Delta_{\rm osc}}\right)^2 \sin^2\left(\frac{\Delta_{\rm osc}}{2}z\right). \tag{16}$$

The complete conversion is possible only when the coefficient $(2\Delta_{g\gamma}/\Delta_{\rm osc})^2$ becomes unity. In other words, it is only possible at the resonance frequency where the effective photon mass vanishes. By solving $\Delta_\gamma = 0$, the resonance frequency is given by

$$\omega_r^2 = \frac{\omega_p^2}{2}\frac{1}{k(\lambda)\varrho B^2} = \frac{45\pi}{2k(\lambda)\alpha}\frac{n_e m_e^3}{B^2}. \tag{17}$$

This is completely determined by $k(\lambda)$, the magnetic field B and the plasma density n_e. At the resonance frequency, conversion phenomenon occurs when a phase of the conversion probability of Eq. (16) become $\pi/2$. This determines the conversion length as

$$\begin{aligned} L_{\gamma\leftrightarrow g} &= 1.7\left(\frac{10^{12}\ \mathrm{G}}{B}\right)10^{-6}\ \mathrm{pc} \\ &= 5.4\left(\frac{10^{12}\ \mathrm{G}}{B}\right)10^{10}\ \mathrm{m}. \end{aligned} \tag{18}$$

The fact that this conversion length is very long is the reason why the photon-graviton conversion rarely occurs. However, as we will see in the next section, the conversion occurs effectively in the vicinity of a photon sphere of a black hole.

Let us estimate the frequency range that contributes to the conversion phenomenon. First, the conversion phenomenon occurs sufficiently (i.e., $P \sim 1$) even at frequency $\omega = \omega_r + \Delta\omega_r$, which is close to the resonance frequency. Then, let us estimate the order of $\Delta\omega_r$. Since the factor of $(2\Delta_{g\gamma}/\Delta_{\rm osc})^2$ in Eq. (16) can be written as $1/[1 + (\Delta_\gamma/2\Delta_{g\gamma})^2]$, the frequency ω should satisfy

$$\Delta_\gamma \lesssim 2\Delta_{g\gamma} \tag{19}$$

for the conversion probability to become $O(1)$. In this inequality, Δ_γ depends on $\omega = \omega_r + \Delta\omega_r$. Expanding around ω_r, we obtain the following inequality for $\Delta\omega_r$:

$$\Delta\omega_r \lesssim \frac{2\sqrt{\pi}}{k(\lambda)}\frac{1}{M_{\rm pl}\varrho B} = \frac{45\sqrt{\pi}}{2k(\lambda)\alpha^2}\frac{m_e^4}{M_{\rm pl}B}. \tag{20}$$

If we use $k(\lambda) \sim 3$ this becomes

$$\Delta\omega_r \lesssim 0.36\left(\frac{10^{12}\ \mathrm{G}}{B}\right)\ \mathrm{MHz}. \tag{21}$$

This upper bound determines the frequency band where the conversion occurs sufficiently, and depends only on the magnetic field B.

IV. UNIVERSAL GRAVITATIONAL WAVES FROM PHOTON SPHERES

In the following, we will apply the photon-graviton conversion phenomena discussed in the previous section to the system of a single black hole with mass M and the accretion disk around it, and show that gravitational waves with the universal frequency are emitted from the vicinity of the photon sphere. The idea is quite simple; photons from the accretion disk steadily accrete around the photon sphere, and those with the resonance frequency ω_r are converted into gravitons with the same frequency ω_r by the magnetic field of the magnetosphere.

In an inner disk, we assume the thermal population of electrons. Then, the equipartition of energy between the plasma and the magnetic fields is expected be satisfied [30–32],

$$\frac{B^2}{8\pi} \sim m_p c^2 n_e, \tag{22}$$

where m_p is the proton mass. Surprisingly, under the assumption of the equipartition of energy, the resonance frequency of Eq. (17) is independent of the magnetic field B and the plasma density n_e, and it is estimated as

$$\omega_r \sim \left(\frac{45}{16k(\lambda)\alpha}\frac{m_e^3}{m_p c^2}\right)^{1/2} \sim 2.1\times 10^{20}\ \mathrm{Hz}. \tag{23}$$

Therefore, the typical frequency of gravitational waves emitted from the magnetosphere is 10^{20} Hz, regardless of the details of the black holes such as the mass M and the magnetic field B. This is a very robust and universal result.

Let us estimate the luminosity of gravitational waves by counting the number of gravitons produced by the conversion. First, we focus on a given impact parameter b around b_{crit}. A photon with the impact parameter b will stay around the photon sphere for a period $T(b) \coloneqq -\frac{3\sqrt{3}}{2c} r_g \log|2(b-b_{\mathrm{crit}})/r_g|$ [27]. Therefore, for a photon with the impact parameter b, the conversion probability from a photon to a graviton is given by $P(cT(b))$. Taking into account the photon flux with the impact parameter b given by Eq. (4), the number of produced gravitons is given by

$$d\left(\frac{d^2N_{\gamma\to g}}{dtd\omega}\right) \sim 27 r_g I_e{}^{(N)}(\omega_r)db \times P(cT(b)). \tag{24}$$

Here we have extrapolated the result of conversion probability in the flat background. This approximation is plausible, because the curvature of the spacetime is negligible under the geometrical optics approximation. In the photon-graviton conversion, only the magnetic field orthogonal to the photon sphere contributes to the conversion, and in realistic black hole magnetospheres, the direction of the magnetic field may vary. However, such variation of the magnetic field does not change the order of magnitude of our result. Integrating the above quantity with respect to $b \in [b_{\mathrm{crit}} - r_g/2, b_{\mathrm{crit}} + r_g/2]$,[1] we obtain the number of gravitons produced in the vicinity of the photon sphere per unit time,

$$\frac{dN_{\gamma\to g}}{dt} = \frac{27}{2}\frac{27(r_g\Delta_{g\gamma})^2}{1+27(r_g\Delta_{g\gamma})^2} r_g^2 I_e{}^{(N)}(\omega_r)\Delta\omega_r. \tag{25}$$

Here, $r_g\Delta_{g\gamma}$ is a useful dimensionless parameter given by

$$r_g\Delta_{g\gamma} = 2.59\times 10^{-20}\left(\frac{M}{M_\odot}\right)\left(\frac{B}{1\ \mathrm{G}}\right). \tag{26}$$

As long as the equipartition principle holds, this parameter is very small compared to unity. Therefore, we can approximate Eq. (25) as

$$\frac{dN_{\gamma\to g}}{dt} = \frac{729}{2}\xi(r_g\Delta_{g\gamma})^2 L(\omega_r)\frac{\Delta\omega_r}{\hbar\omega_r}. \tag{27}$$

Here, we introduced a spectral photon luminosity $L(\omega_r)$ which represents the photon energy flux from the accretion disk per unit time and unit frequency. The dimensionless factor ξ is defined by $\xi \coloneqq r_g^2/A(r_g)$, with the area of the accretion disk $A(r_g)$ such that it emits x rays near the resonance frequency $\sim 10^{20}$ Hz. Multiplying Eq. (27) by the energy of a graviton, we obtain the luminosity of gravitational waves as

$$\frac{dE_{\gamma\to g}}{dt} = \frac{729}{2}\xi(r_g\Delta_{g\gamma})^2 L(\omega_r)\Delta\omega_r. \tag{28}$$

To evaluate this luminosity, we use a band given by Eq. (21) where the conversion sufficiently occurs; $\Delta\omega_r \coloneqq 0.36(\frac{10^{12}\ \mathrm{G}}{B})$ MHz. According to the deep x-ray survey data from Chandra observatory [33], active galactic nuclei (AGN) x-ray luminosity is typically $L_X(2\text{–}10\ \mathrm{keV}) \sim 10^{43}\text{–}10^{46}\ \mathrm{erg\,sec^{-1}}$. This roughly corresponds to the Eddington luminosity of a supermassive black hole. For this reason, we will assume that the luminosity of the x-ray band is the Eddington luminosity. Thus, the spectral luminosity of a typical AGN reads

[1] Here, we used the analytic formula $\int_{-1}^{1}\sin^2(a\log|y|)\times dy = \frac{(2a)^2}{1+(2a)^2}$.

$$L(\omega_r \sim 10^{20}\ \mathrm{Hz}) \sim \frac{L_{\mathrm{Edd}}(M)}{10^{20}\ \mathrm{Hz}} \sim 10^{24}\left(\frac{M}{10^6\ M_\odot}\right)\ \mathrm{erg\,sec^{-1}\,Hz^{-1}}, \quad (29)$$

where M is the mass of the central black hole. Finally, we obtain the typical luminosity of gravitational waves emitted from the photon sphere of a single black hole

$$\frac{dE}{dt} \sim 8.80\xi \times 10^{22}\left(\frac{B}{10^6\ \mathrm{G}}\right)\left(\frac{M}{10^6\ M_\odot}\right)^3\ \mathrm{erg\,sec^{-1}}. \quad (30)$$

V. HIGH-FREQUENCY STOCHASTIC GRAVITATIONAL WAVES

As we saw in the previous section, the frequency of gravitational waves from a photon sphere of a black hole is universally given by 10^{20} Hz, which does not depend on the details of black holes. Apparently, the luminosity of gravitational waves emitted by each black hole is rather faint. Nevertheless, we can observe them as background gravitational waves. In the following, we will calculate the density parameter of the stochastic gravitational waves.

For simplicity, we ignore the effects of the expansion of the universe and assume the equipartition of the energy between the energy of magnetic fields and the Eddington luminosity (see pp. 132–133 of Ref. [34]). Then, we obtain

$$\frac{B^2}{8\pi}4\pi\left(\frac{3}{2}r_g\right)^2 c = L_{\mathrm{Edd}}(M). \quad (31)$$

Thus, the magnetic field can be estimated as

$$B \sim 0.241 \times 10^7\sqrt{\frac{10^6\ M_\odot}{M}}\mathrm{G}. \quad (32)$$

Under this assumption, the luminosity of gravitational waves from a single black hole can be written as $\frac{dE}{dt}(B(M), M)$. We set our position at the origin and consider the radiation from a point $\boldsymbol{x}$. Assume that there are $n(\boldsymbol{x}, M)d^3\boldsymbol{x}dM$ black holes of mass M at the point $\boldsymbol{x}$. Since the luminosity $dE/dt(B(M), M)$ is conserved on the sphere surrounding the point $\boldsymbol{x}$, the energy density of gravitational waves we observe reads

$$\rho_{\mathrm{gw}}(\boldsymbol{x}, M) = \frac{1}{4\pi c}d^3\boldsymbol{x}dM\frac{1}{|\boldsymbol{x}|^2}n(\boldsymbol{x}, M)\frac{dE}{dt}(B(M), M). \quad (33)$$

Here, the arguments of the left-hand side imply that $\rho_{\mathrm{gw}}(\boldsymbol{x}, M)$ is a contribution from a point $\boldsymbol{x}$ and a given mass M. Integrating this over the entire observable region and the mass of the black holes, and dividing it by the critical energy density of the Universe, we obtain the density parameter of the stochastic gravitational waves as

$$h_0{}^2\Omega_{\mathrm{gw}} = \frac{2Gh_0{}^2}{3c^3H_0^2}\int dM\frac{dE}{dt}(B(M), M) \times \int_{|\boldsymbol{x}|\le cH_0{}^{-1}} d^3\boldsymbol{x}\frac{n(\boldsymbol{x}, M)}{|\boldsymbol{x}|^2}, \quad (34)$$

where h_0 is the dimensionless Hubble constant.

From the recent observations of Refs. [35–37], we see that the mass range of supermassive black holes is $10^6\ M_\odot \sim 10^{11}\ M_\odot$. It is natural to assume the distribution of black holes to be spatially homogeneous and to take the power law with respect to the mass,

$$n(\boldsymbol{x}, M) = C(\beta)\frac{N_{\mathrm{galaxy}}}{\frac{4\pi}{3}(cH_0^{-1})^3}M^{-\beta}, \quad (35)$$

where $\beta \ge 0$ and $N_{\mathrm{galaxy}} = 2 \times 10^{12}$ is the total number of galaxies. The normalization constant $C(\beta)$ is determined by $\int_{10^6\ M_\odot}^{10^{11}\ M_\odot} dM \int d^3\boldsymbol{x}n(\boldsymbol{x}, M) = N_{\mathrm{galaxy}}$ as

$$C(\beta) = \left(\int_1^{10^5} dyy^{-\beta}\right)^{-1}(10^6\ M_\odot)^{\beta-1}. \quad (36)$$

Then, the density parameter (34) can be easily integrated as

$$h_0^2\Omega_{\mathrm{gw}} \sim 0.23h_0^2\xi \times 10^{-23}D(\beta), \quad (37)$$

where ξ is the dimensionless ratio, $\xi \coloneqq r_g^2/A(r_g)$, and $D(\beta)$ is defined by

$$D(\beta) = \frac{\int_1^{10^5} dy'y'^{5/2-\beta}}{\int_1^{10^5} dyy^{-\beta}}. \quad (38)$$

From Fig. 2, the energy density strongly depends on β and monotonically decreases as the value of β is increased. Now let us estimate the maximum value of the energy density.

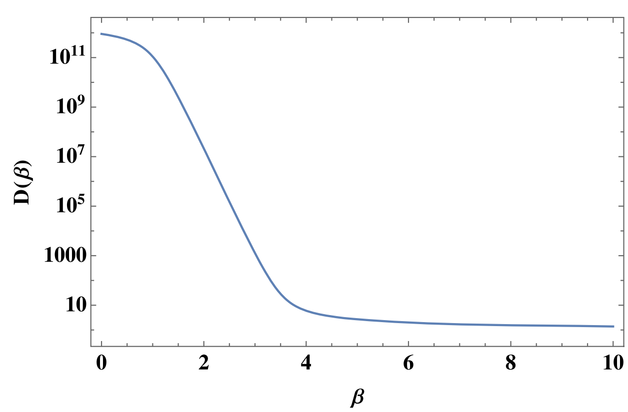

FIG. 2. Plot of $D(\beta)$.

Taking β as $0 \leq \beta \ll 1$, the value of the constant $D(\beta)$ becomes $D(\beta) \approx \frac{2(1-\beta)}{7-2\beta} \times 10^{25/2}$. Thus, we finally obtain

$$h_0^2 \Omega_{\mathrm{gw}} = h_0^2 \xi \frac{2(1-\beta)}{7-2\beta} 7.4 \times 10^{-12}. \tag{39}$$

Therefore, the order of $h_0^2 \Omega_{\mathrm{gw}}$ could be as large as 10^{-12}, and this gives rise to a motivation to invent detectors for high-frequency gravitational waves of the range around 10^{20} Hz.

VI. CONCLUSION

In this paper, we proposed a novel and robust source of high-frequency gravitational waves. We have shown that the photon spheres of supermassive black holes emit gravitational waves through photon-graviton conversion phenomenon, with the universal frequency 10^{20} Hz which depends on proton and electron masses, not on black hole mass and its magnetic field. Such gravitational waves can be observed as the stochastic gravitational wave background. We estimated the density parameter of the stochastic gravitational waves and found that it could be as large as $h_0^2 \Omega_{\mathrm{gw}} \sim 10^{-12}$.

As is evident from the formula of Eq. (34), the density parameter depends on the black hole number density $n(\boldsymbol{x}, M)$. Therefore, future observations of gravitational waves from black hole photon spheres may give a bound on $n(\boldsymbol{x}, M)$. This bound may have a significant implication for the abundance of intermediate mass black holes, the number of which is currently unknown [38].

Another important point of our work is that our prediction is based only on Einstein gravity and Maxwell electrodynamics, which have been verified to a high degree of accuracy. Therefore, high-frequency gravitational waves predicted in this paper are robust. We hope that our result motivates and boosts the development of high-frequency gravitational wave detectors for the range around 10^{20} Hz, and opens a new window to explore the Universe with gravitational waves.

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