Radiation of arions by electromagnetic field of rotating magnetic dipole

V. I. Denisov and B. D. Garmaev
Physics Department, Moscow State University, Moscow 119991, Russia

I. P. Denisova[†]
Moscow Aviation Institute (National Research University), Volokolamskoe Highway 4, Moscow 125993, Russia

(Received 28 January 2021; accepted 6 August 2021; published 14 September 2021)

The electromagnetic source of arions, as well as axions, is a scalar product of the magnetic field induction and the electric field intensity. For electromagnetic waves, this product can be nonzero only in the near zone. Pulsars and magnetars are natural sources of this type. Based on these considerations, we calculate the generation of arions by coherent electromagnetic field of rotating magnetic dipole of pulsars and magnetars. It is shown that the radiation of arion waves occurs at the frequency of magnetic dipole rotation. This radiation has a maximum when the angle between the rotation axis and the magnetic dipole moment of the neutron star is $\pi/4$ and it is completely absent, when the magnetic dipole moment is perpendicular or parallel to this axis. A formula for the angular distribution of arion radiation is constructed; and on its basis, it is shown that the radiation is maximal in the plane which is perpendicular to the axis of rotation.

DOI: 10.1103/PhysRevD.104.055018

I. INTRODUCTION

Many extensions of the Standard Model predict the existence of light Goldstone bosons, which in the scientific literature are called axionlike particles. The density of the Lagrange function for an axionlike particle which is interacting with an electromagnetic field can be written as

$$L = \frac{1}{2} g^{nm} \frac{\partial a}{\partial x^n} \frac{\partial a}{\partial x^m} - \frac{g_{a\gamma}}{4} F_{nm} \tilde{F}^{nm} a - \frac{1}{2} m_a^2 a^2,$$

where a is the pseudoscalar field of the axionlike particle, F_{nm} is the electromagnetic field tensor, m_a is the axion mass, $g_{a\gamma}$ is the axion-photon coupling constant (with dimension of inverse energy), $\tilde{F}^{nm} = e^{nmik} F_{ik}/2$ and e^{nmik} is the axial, absolutely antisymmetric Levi-Chivita tensor, moreover $e^{0123} = +1$.

One of axionlike particles, is the arion α, which was introduced in 1982, in works [1–3] by Prof. A. A. Anselm and his coauthors. The arion is a strictly massless pseudoscalar Goldstone particle corresponding to the spontaneous breaking of an exact symmetry.

According to papers [1–3], the density of the Lagrange function for arions interacting with the electromagnetic field, can be expressed

$$L = \frac{1}{2} g^{nm} \frac{\partial \alpha}{\partial x^n} \frac{\partial \alpha}{\partial x^m} - \frac{g}{4} F_{nm} \tilde{F}^{nm} \alpha, \qquad (1)$$

where α is the arionic field, g is the arion-photon coupling constant.

Nowadays, the search for axionlike particles is being currently under way not only in the experiments at accelerators [4–6] and in laboratory optical experiments [7,8], but also in astrophysical conditions [9–15]. However, they have not been successful yet. Therefore, it is necessary to continue theoretical research to find out the experimental situations where the radiation of arions and axions reach its maximum possible value.

In pseudo-Euclidean space-time, the equation for the arion field, obtained from the density of the Lagrange function (1) has the form

$$\Delta\alpha(\mathbf{r}, t) - \frac{1}{c^2} \frac{\partial^2 \alpha(\mathbf{r}, t)}{\partial t^2} = \frac{g}{4} F_{nm} \tilde{F}^{nm} = -g(\mathbf{E} \cdot \mathbf{B}), \qquad (2)$$

where $\mathbf{E}$ and $\mathbf{B}$ are the intensity of the electric field and the induction of the magnetic field, respectively, that create the arion field.

According to Eq. (2), the electromagnetic source of the arion field is a scalar product $(\mathbf{E} \cdot \mathbf{B})$. Since in the wave zone for any electromagnetic waves this product is equal to zero, a noticeable radiation of arions is possible only from the near zone of electromagnetic waves, where this pseudoinvariant is not zero. Therefore, effective electromagnetic

[†]Corresponding author
denisovaip@mati.ru

generators of arions are coherent electromagnetic waves in which the fields **E** and **B** in the nonwave zone are not orthogonal to each other.

Pulsars and magnetars are natural sources of this type. A typical pulsar or magnetar is a neutron star with a radius of about 10 km, rotating around an axis that does not coincide with the vector of the magnetic dipole moment of the star. The rotation period of these stars usually ranges from 1.6 milliseconds (PSR J1810 + 1744 [15]) to ten seconds (MG J1647-4552 [16]).

Pulsars and magnetars have strong magnetic fields, that are comparable and even exceed the Schwinger field $B_q = 4.41\times10^{13}$ gauss. So, for example, the induction of a magnetic dipole field on the surface of a pulsar can reach 10^{13} gauss [15], and on the surface of a magnetar up to values of 2×10^{15} gauss [16].

Therefore, the scalar product $(\mathbf{E}\cdot\mathbf{B})$ for the magnetic dipole radiation of pulsars and magnetars in the near zone, takes on values that can hardly be created in other electromagnetic processes.

Let us study the possibilities of arion generation by low frequency magnetic dipole radiation of pulsars and magnetars.

II. CALCULATION OF ARION RADIATION BY MAGNETIC DIPOLE WAVES OF PULSARS AND MAGNETARS

Consider a pulsar or magnetar of radius R_s with a magnetic dipole moment **m** rotating with a frequency of ω around an axis making an angle β with a vector **m**.

Due to the rotation of the vector **m**, magnetic dipole radiation of electromagnetic waves is being generated. According to work [17], this radiation vectors **B** and **E**, can be written in the form

$$\mathbf{B}(\mathbf{r},\tau) = \frac{3(\mathbf{m}(\tau)\cdot\mathbf{r})\mathbf{r} - r^2\mathbf{m}(\tau)}{r^5} - \frac{\dot{\mathbf{m}}(\tau)}{cr^2} + \frac{3(\dot{\mathbf{m}}(\tau)\cdot\mathbf{r})\mathbf{r}}{cr^4} + \frac{(\ddot{\mathbf{m}}(\tau)\cdot\mathbf{r})\mathbf{r} - r^2\ddot{\mathbf{m}}(\tau)}{c^2r^3},$$
$$\mathbf{E}(\mathbf{r},\tau) = \frac{(\mathbf{r}\times\dot{\mathbf{m}}(\tau))}{cr^3} + \frac{(\mathbf{r}\times\ddot{\mathbf{m}}(\tau))}{c^2r^2}, \quad (3)$$

where the dot above the vector means the derivative on retarded time $\tau = t - r/c$, and the pulsar magnetic dipole moment in the following task has the components

$$\mathbf{m}(\tau) = |\mathbf{m}|(\cos(\omega\tau)\sin\beta, \qquad \sin(\omega\tau)\sin\beta, \cos\beta). \quad (4)$$

Since for pulsars and magnetars the condition $\omega R_s \ll c$, are met, they are in the near zone of their own magnetic dipole radiation.

Using Eq. (3), we calculate the scalar product $(\mathbf{B}\cdot\mathbf{E})$

$$(\mathbf{B}\cdot\mathbf{E}) = \frac{(\mathbf{r}\cdot(\mathbf{m}\times\dot{\mathbf{m}}))}{cr^6} + \frac{(\mathbf{r}\cdot(\mathbf{m}\times\ddot{\mathbf{m}}))}{c^2r^5}.$$

Substituting expression (4) here, and keeping only the time-dependent part, we get

$$(\mathbf{B}\cdot\mathbf{E}) = -\frac{km^2}{2r^6}\{x\cos(kr-\omega t) - y\sin(kr-\omega t) + kr[x\sin(kr-\omega t) + y\cos(kr-\omega t)]\}\sin 2\beta$$
$$= -\frac{km^2}{2r^5}\{kr\sin(\varphi + kr - \omega t) + \cos(\varphi + kr - \omega t)\}\sin\theta\sin 2\beta, \quad (5)$$

where $k = \omega/c$.

Equation (2), taking into account Eq. (5), can be rewritten in a complex form

$$\Delta\alpha(\mathbf{r},t) - \frac{1}{c^2}\frac{\partial^2\alpha(\mathbf{r},t)}{\partial t^2} = \frac{gkm^2}{2r^5}(1+ikr)\sin\theta\sin(2\beta) \times\exp[-i(kr+\varphi-\omega t)], \quad (6)$$

assuming that after solving it, we take only the real part.

Since Eq. (6) describes strictly massless axionlike particles radiation, we can write the retarded solution of this equation in the following form:

$$\alpha(\mathbf{r},t) = \frac{gkm^2}{8\pi}\sin 2\beta \times\int_V \frac{dV'(1+ikr')}{r'^5|\mathbf{r}-\mathbf{r}'|}P_1^1(\cos\theta') \times\exp[-i(kr'+\varphi'-\omega t + k|\mathbf{r}-\mathbf{r}'|)], \quad (7)$$

where $P_1^1(\cos\theta') = -\sin\theta'$.

This solution can be applied in the case when the mass of axionlike particles m_a is not zero, but meets the condition $k \gg m_a$. In other cases (when $k\sim m_a$) the problem of the axionlike particle radiation by the electromagnetic field of a rotating magnetic dipole, should be solved regardless of the problem of the arions radiation.

Let us use the fact that according to the Gegenbauer theorem [18] at $r > r'$ the expansion is valid

$$\frac{\exp\{-ik|\mathbf{r}-\mathbf{r}'|\}}{|\mathbf{r}-\mathbf{r}'|} = -\frac{\pi i}{2\sqrt{rr'}}\sum_{n=0}^{\infty}(2n+1)J_{n+1/2}(kr') \times H^{(2)}_{n+1/2}(kr)P_n(\cos\gamma),$$

and at $r < r'$, a similar expansion

$$\frac{\exp\{-ik|\mathbf{r}-\mathbf{r}'|\}}{|\mathbf{r}-\mathbf{r}'|} = -\frac{\pi i}{2\sqrt{rr'}}\sum_{n=0}^{\infty}(2n+1)J_{n+1/2}(kr) \times H^{(2)}_{n+1/2}(kr')P_n(\cos\gamma),$$

where

$$\cos\gamma = \frac{(\mathbf{r}'\cdot\mathbf{r})}{|\mathbf{r}'||\mathbf{r}|} = \cos\theta\cos\theta' + \sin\theta\sin\theta'\cos(\varphi'-\varphi).$$

Substituting these expansions into Eq. (7) for the arion field outside the star ($r > R_s$), we get

$$\begin{aligned}\alpha(\mathbf{r},t) = &-\frac{igk^{7/2}m^2}{16\sqrt{r}}\exp[i\omega t]\sin(2\beta) \\ &\times\sum_{n=0}^{\infty}(2n+1)\int_0^{\pi}\sin\theta' P_1^1(\cos\theta')d\theta' \\ &\times\int_0^{2\pi}d\varphi' e^{-i\varphi'}P_n(\cos\gamma) \\ &\times\left\{H^{(2)}_{n+1/2}(kr)\int_{kR_s}^{kr}\frac{(1+i\eta)d\eta}{\eta^{7/2}}J_{n+1/2}(\eta)e^{-i\eta}\right. \\ &\left.+J_{n+1/2}(kr)\int_{kr}^{\infty}\frac{(1+i\eta)d\eta}{\eta^{7/2}}H^{(2)}_{n+1/2}(\eta)e^{-i\eta}\right\}.\end{aligned} \tag{8}$$

Function $P_n(\cos\gamma)$ can be [19] expanded as a series in spherical functions

$$\begin{aligned}P_n(\cos\gamma) = &P_n(\cos\theta)P_n(\cos\theta') \\ &+2\sum_{m=1}^{n}\frac{(n-m)!}{(n+m)!}P_n^m(\cos\theta)P_n^m(\cos\theta') \\ &\times\cos m(\varphi'-\varphi).\end{aligned}$$

Substituting this series into Eq. (8) and integrating it over r', angles θ' and φ', we arrive at the relation

$$\begin{aligned}\alpha(\mathbf{r},t) = &-Re\frac{i\pi gk^{7/2}m^2}{4\sqrt{r}}\exp[i(\omega t-\varphi)]\sin\theta\sin(2\beta) \\ &\times\{H^{(2)}_{3/2}(kr)[f_1(kr)-f_1(kR_s)]-J_{3/2}(kr)f_2(kr)\},\end{aligned} \tag{9}$$

where

$$f_1(z) = \frac{\sqrt{2}}{8\sqrt{\pi}z^4}\{[2z-i]e^{-2iz}+i[2z^2+1]\},$$

$$f_2(z) = \frac{\sqrt{2}}{4\sqrt{\pi}z^4}[2z-i]e^{-2iz}.$$

This is the exact solution to Eq. (6).

It should be noted that in addition to low-frequency radiation arising from the rotation of the magnetic dipole moment, pulsars and magnetars are sources of high-frequency radiation, which cover a wide range of frequencies; from radio, through optical, x-ray, and up to gamma range [15,16]. However, this radiation is incoherent and the calculation above, is inapplicable for these frequencies.

III. THE RADIATION PATTERN OF ARION WAVES

For further investigation of the results of the generation of arion radiation by the electromagnetic field of a rotating magnetic dipole, we only need the wave part of Eq. (9), which decreases at $kr \gg 1$ as $1/r$. In addition, we take into account that for most pulsars and magnetars $kR_s \ll 1$, and keeping in the resulting expression only the asymptotically main term in the expansions with respect to this small parameter. Then, keeping the real part in Eq. (9) and discarding the nonwave terms we get

$$\alpha(\mathbf{r},t) = -\frac{gk^2m^2}{6R_s r}\sin(2\beta)\sin\theta\sin(\omega t - kr - \varphi). \tag{10}$$

Let us investigate the obtained solution (10) and first of all study the angular distribution of the arising arionic radiation or, as they sometimes say, its radiation pattern. By definition [20], the amount of energy dI, emitted by the source per unit time through the solid angle $d\Omega = \sin\theta d\theta d\varphi$, is given by the formula

$$\frac{dI}{d\Omega} = \lim_{r\to\infty} r(\mathbf{W}\cdot\mathbf{r}),$$

where $\mathbf{W}$ is the energy flux density vector associated with the components of the energy-momentum tensor T^{nm} by the relation $W^\mu = cT^{0\mu}$.

The energy-momentum tensor of the field, described by the function $q(\vec{r},t)$ according to the monograph [20], has the form

$$T_i^k = q_{,i}\frac{\partial\mathcal{L}}{\partial q_{,k}} - \delta_i^k\mathcal{L},$$

where $q_{,i} = \partial q/\partial x^i$.

For the free arionic field $q(\mathbf{r},t) = \alpha(\mathbf{r},t)$, so

$$T^{ik} = g^{\mathrm{in}}g^{km}\frac{\partial\alpha}{\partial x^n}\frac{\partial\alpha}{\partial x^m} - \frac{1}{2}g^{ik}\frac{\partial\alpha}{\partial x^n}\frac{\partial\alpha}{\partial x^m}g^{nm}.$$

Then the arion radiation pattern will be determined by the expression

$$\frac{dI}{d\Omega} = -\lim_{r\to\infty} r(\mathbf{r}\cdot\nabla\alpha)\frac{\partial\alpha}{\partial t}.$$

Substituting here Eq. (10) and averaging the resulting relation over period $T = 2\pi/\omega$ of the wave, we have

$$\frac{dI}{d\Omega} = \frac{cg^2k^6m^4}{72R_s^2}\sin^2\theta\sin^2(2\beta). \tag{11}$$

Multiplying this expression by $\sin\theta d\theta d\varphi$ and integrating it over the angles θ and φ, we obtain the total intensity I—the amount of energy of the arionic waves emitted in all directions by a rotating magnetic dipole per time unit

$$I = \frac{\pi c g^2 k^6 m^4}{27R_s^2}\sin^2(2\beta). \tag{12}$$

Let us express the dipole moment of a pulsar or magnetar $\mathbf{m}$ through the maximal magnitude of the magnetic field on the surface of the star B_s. In order of magnitude, we have $|\mathbf{m}| = B_s R_s^3/2$. Then formulas (11) and (12) take the form

$$\frac{dI}{d\Omega} = \frac{cg^2k^6B_s^4R_s^{10}}{1152}\sin^2(2\beta)\sin^2\theta.$$
$$I = \frac{\pi c g^2k^6B_s^4R_s^{10}}{432}\sin^2(2\beta). \tag{13}$$

It follows from Eqs. (12) and (13) that the arion radiation produced by the rotation of the magnetic dipole moment of pulsars and magnetars, like any physical field, carries non-negative energy.

IV. CONCLUSION

The calculation showed that arion radiation occurs when the magnetic dipole moment rotates. The radiation of arions occurs at the rotation frequency of the magnetic dipole moment of the neutron star.

It follows from the expressions (13), the generation of arions by rotating magnetic dipole radiation is maximal in the case when the angle β between the magnetic dipole and the axis of its rotation is $\pi/4$. If this angle is $\pi/2$, then arion radiation does not occur. It should be noted that in the recently discovered "Magnificent Seven" magnetars [21,22], arionic radiation is either absent or strongly suppressed, since they have the angle between the magnetic dipole moment and the rotation axis close to $\pi/2$.

The radiation pattern of the arion radiation has a maximum in the plane which is perpendicular to the axis of rotation ($\theta = \pi/2$).

Currently, the value of the electromagnetic and arion fields interaction constant g is unknown. Let us roughly estimate the value of this interaction constant. We achieve this in the following way. Due to the rotation, pulsars and magnetars emit magnetic-dipole electromagnetic waves [17], quadruple gravitational waves [23] and, as we have seen, also are able to generate arions. The energy loss for this radiation should occur due to kinetic energy of the star rotation. Therefore, the rotation period of the star P should increase, and its time derivative $\dot{P}$ must be non-negative: $\dot{P} > 0$. These values are determined quite easily from the results of observations for pulsed radiation from pulsars and magnetars. Using these observational data, astrophysicists (see [16,23] and the literature cited there) calculated the quantity $\dot{E}$ for magnetars, which was called spin-down luminosity. It should be noted, that this is not a measured luminosity, it is the measured loss rate of rotational energy of magnetars.

According to Table 2 in [16], different magnetars have different spin-down luminosity that can vary from $\dot{E} = 2.1 \times 10^{35}$ erg s^{-1}, (magnetar 1E 1547.0-5408) to 2.1×10^{29} erg s^{-1}, (magnetar SGR 0418 + 5729).

Since a part of $\dot{E}$ is originated due to arion radiation, we can write

$$I < |\dot{E}|. \tag{14}$$

Using the second relation of the Eq. (13), and supposing that $\sin^2(2\beta) = 1$, from this inequality, we obtain that

$$g^2 < \frac{432|\dot{E}|}{\pi c k^6 B_s^4 R_s^{10}}. \tag{15}$$

Let us find out which form inequality (15) takes in case of magnetar SGR 1806-20, which has the maximum value of the magnetic field on the surface of the star, among the other known [16] magnetars.

Equation (15) was derived by us into the Gaussian system of units of measurement. Therefore, we must substitute $c = 3 \times 10^{10}$ cm/s and $B_s = 2 \times 10^{15}$ gauss into it.

According to [16], the spin-down luminosity of magnetar SGR 1806-20 is equal to $|\dot{E}| = 4.5 \times 10^{34}$ erg/s.

The period of rotation of this magnetar is equal to $P = 7.5$ s, therefore $k = 0.28 \times 10^{-10}$ cm^{-1}. Its radius is unknown.

However, following [24], if we admit that $R_s = 11$ km $= 1.1 \times 10^6$ cm, then the inequality (15) should be written as

$$g^2 < \frac{432|\dot{E}|}{\pi c k^6 B_s^4 R_s^{10}} = 1.0 \times 10^{-32}\,\frac{\text{cm}}{\text{erg}}. \tag{16}$$

Let us rewrite this formula in the natural system of units. If we take into account that $1erg = 624$ GeV, and $1cm = 0.5 \times 10^{14}$ GeV^{-1}, then from the Eq. (16) we get $g^2 < 8.2 \times 10^{-22}$ GeV^{-2}.

From here we obtain the estimate for the arion-photon coupling constant

$$g < 0.29 \times 10^{-10}\text{Gev}^{-1}. \tag{17}$$

For axions, the strongest constraints on the axion-photon coupling constant $g_{a\gamma}$ have also emerged from astrophysical considerations.

On the basis of the observed duration of stationary burning helium stars, in works [25,26] the restriction was obtained

$$g_{a\gamma} < 1 \times 10^{-10}\ \mathrm{GeV}^{-1}. \tag{18}$$

Another estimate for the coupling constant $g_{a\gamma}$ is obtained by studying supernova outbursts.

The analysis of events during the Supernova SN1987A, carried out in [27], gave in the low mass region a new upper limit on the photon-axion coupling constant

$$g_{a\gamma} \leq 5.3 \times 10^{-12}\ \mathrm{GeV}^{-1}. \tag{19}$$

The most recent results [28] of the CAST experiment set an upper limit

$$g_{a\gamma} < 0.66 \times 10^{-10} \mathrm{Gev}^{-1}, \tag{20}$$

for all axions with masses below 0.02 eV.

Comparing constraint (17) on the arion-photon coupling constant g with constraints (18), (19), and (20) on the axion-photon coupling constant $g_{a\gamma}$ it can be stated that they coincide in order of magnitude.

[1] A. A. Anselm and N. G. Uraltsev, A second massless axion?, Phys. Lett. **114B**, 39 (1982).

[2] A. A. Anselm, Possible new long-range interaction and methods for detecting it, Pis'ma Zh. Eksp. Teor Fiz. **36**, 46 (1982) [JETP Lett. **36**, 55 (1982)].

[3] A. A. Anselm and N. G. Uraltsev, Long range "arion" field in the radio frequency band, Phys. Lett. **116B**, 161 (1982).

[4] L. Maiani, R. Petronzio, and E. Zavattini, Effects of nearly massless, spin-zero particles on light propagation in a magnetic field, Phys. Lett. B **175**, 359 (1986).

[5] R. Ballou, G. Deferne, M. Finger *et al.*, New exclusion limits on scalar and pseudoscalar axionlike particles from light shining through a wall, Phys. Rev. D **92**, 092002 (2015).

[6] G. L. Klimchitskaya, P. Kuusk, and V. M. Mostepanenko, Constraints on non-Newtonian gravity and axionlike particles from measuring the Casimir force in nanometer separation range, Phys. Rev. D **101**, 056013 (2020).

[7] D. Banerjee *et al.* (NA64 Collaboration), Search for Axion-like and Scalar Particles with the NA64 Experiment, Phys. Rev. Lett. **125**, 081801 (2020).

[8] T. Inada, T. Yamazaki, T. Namba *et al.*, Search for Two-Photon Interaction with Axionlike Particles Using High-Repetition Pulsed Magnets and Synchrotron X Rays, Phys. Rev. Lett. **118**, 071803 (2017).

[9] A. A. Anselm, Experimental test for arion-photon oscillations in a homogeneous constant magnetic field, Phys. Rev. D **37**, 2001 (1988).

[10] D. Harari and P. Sikivie, Effects of a Nambu-Goldstone boson on the polarization of radio galaxies and the cosmic microwave background, Phys. Lett. B **289**, 67 (1992).

[11] Yu. N. Gnedin and S. V. Krasnikov, Polarimetric effects associated with the detection of Goldstone bosons in stars and galaxies, Sov. Phys. JETP **75**, 1729 (1992).

[12] G. Raffelt and L. Stodolsky, Mixing of the photon with low-mass particles, Phys. Rev. D **37**, 1237 (1988).

[13] S. V. Krasnikov, New Astrophysical Constraints on the Light Pseudoscalar-Photon Coupling, Phys. Rev. Lett. **76**, 2633 (1996).

[14] I. P. Denisova, On a mathematical problem in the theory of Goldstone bosons, Russ. Math. **64**, 73 (2020).

[15] R. N. Manchester, G. B. Hobbs, A. Teoh, and M. Hobbs, The Australia telescope national facility pulsar catalogue, Astron. J. **129**, 1993 (2005).

[16] S. A. Olausen and V. M. Kaspi, The McGill magnetar catalog, Astrophys. J. Suppl. Ser. **212**, 6 (2014).

[17] V. I. Denisov, I. P. Denisova, A. B. Pimenov, and V. A. Sokolov, Rapidly rotating pulsar radiation in vacuum non-linear electrodynamics, Eur. Phys. J. C **76**, 612 (2016).

[18] G. N. Watson, *A Treatise on the Theory of Bessel Functions*, 2nd ed. (Cambridge University Press, Cambridge, England, 1944).

[19] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*, 5th ed. (Academic Press, New York, 1994).

[20] L. D. Landau and E. M. Lifshitz, *The Classical Theory of Fields* (Butterworth-Heinemann, Washington, DC, 1975).

[21] R. Turolla, Isolated neutron stars: The challenge of simplicity, Astrophysics and Space Science Library **357**, 141 (2009).

[22] R. P. Mignani, V. Testa, D. González Caniulef, R. Taverna, R. Turolla, S. Zane, and K. Wu, Evidence for vacuum birefringence from the first optical polarimetry measurement of the isolated neutron star RX J1856.5-3754, Mon. Not. R. Astron. Soc. **465**, 492 (2017).

[23] C. Palomba, Pulsars ellipticity revised, Astron. Astrophys. **354**, 163 (2000).

[24] C. D. Capano, I. Tews, S. M. Brown, B. Margalit, S. De, S. Kumar, D. A. Brown, B. Krishnan, and S. Reddy, Stringent constraints on neutron-star radii from multimessenger observations and nuclear theory, Nat. Astron. **4**, 625 (2020).

[25] G. Raffelt and D. Dearbom, Bounds on hadronic axions from stellar evolution, Phys. Rev. D **36**, 2211 (1987).

[26] G. Raffelt, Astrophysical methods to constrain axions and other novel particle phenomena, Phys. Rep. **198**, 1 (1990).

[27] A. Payez, C. Evoli, T. Fischer, M. Giannotti, A. Mirizzi, and A. Ringwald, Revisiting the SN1987A gamma-ray limit on ultralight axion-like particles, J. Cosmol. Astropart. Phys. 02 (2015) 006.

[28] V. Anastassopoulos *et al.* (CAST Collaboration), New CAST limit on the axion-photon interaction, Nat. Phys. **13**, 584 (2017).