Semileptonic Ξ_c baryon decays in the light cone QCD sum rules

T. M. Aliev[1,*] S. Bilmis[1,2,†] and M. Savci[1,‡]

[1]*Department of Physics, Middle East Technical University, Ankara 06800, Turkey*
[2]*TUBITAK ULAKBIM, Ankara 06510, Turkey*

Form factors of the weak $\Xi_c \to \Xi(\Lambda)$ transitions are calculated within the light cone QCD sum rules. The pollution coming from the contribution of the negative parity Ξ_c^* baryon is eliminated by considering the combinations of sum rules corresponding to the different Lorentz structures. Having obtained the form factors, the branching ratios of the $\Xi_c \to \Xi(\Lambda)\ell\nu$ decays are also calculated, and our predictions are compared with the results of other approaches as well as the measurements done by the BELLE and ALICE collaborations.

I. INTRODUCTION

The electroweak decays of the heavy flavored hadrons provide useful information about the helicity structure of the effective Hamiltonian and the matrix elements of the Cabibbo-Kobayashi-Maskawa matrix (CKM). These decays are also very promising in looking for new physics searches. For these goals, the semileptonic decays of the heavy mesons and baryons provide an ideal research area. Since the leptonic part of these transitions is well known, all probable complications can be attributed to the hadronic matrix element. The study of the semileptonic $\Xi_c \to B(B = \Xi \text{ or } \Lambda)\ell\nu$ decays that are induced by the $c \to s$ and $c \to d$ transitions are helpful for precise determination of the values of the CKM matrix elements V_{cs} and V_{cd}. Moreover, these decays can also be used to test the predictions of the heavy quark effective theory.

The form factors also play a crucial role in the theoretical analysis of the nonleptonic decays of the baryons and might also be useful for the study of *CP* violation.

Significant experimental progress on the semileptonic decays of Ξ_c baryon has been achieved recently. The Belle Collaboration reported the measurement of the branching ratios of the semileptonic decays $\Xi_c^0 \to \Xi^- \ell^+ \nu$ [1],

$$B(\Xi_c^0 \to \Xi^- e^+ \nu_e) = (1.72 \pm 0.10 \pm 0.12 \pm 0.50)\%,$$
$$B(\Xi_c^0 \to \Xi^- \mu^+ \nu_\mu) = (1.71 \pm 0.17 \pm 0.13 \pm 0.50)\%.$$

*taliev@metu.edu.tr
†sbilmis@metu.edu.tr
‡savci@metu.edu.tr

Moreover, the ALICE Collaboration has also announced the result for the branching ratio of the $\Xi_c^0 \to \Xi^- \ell\nu$ transition $[B = (1.8 \pm 0.2)\%]$ [2], which agrees with BELLE's measurement within error ranges.

The semileptonic decays of the Ξ_c baryon have been comprehensively studied in the framework of different approaches, such as the light-front formalism [3,4], relativistic quark model [5], lattice QCD [6], three-point QCD sum rules [7], and light cone QCD sum rules method [8].

In this study, we calculated the form factors and branching ratios of the semileptonic decay of Ξ_c within the light cone sum rules (LCSR) framework. It should be noted that the same channel was already studied in [8]. However, the prediction of the branching ratio obtained in that study is considerably larger than the results of the other approaches as well as experimental measurements. This study also aims to understand the source of this discrepancy. In our opinion, the reason for the discrepancy can be attributed to the fact that the interpolating current for the given heavy baryon couples not only to the ground state baryon with positive parity $J^P = \frac{1}{2}^+$ but also to a heavier baryon with negative parity $J^P = \frac{1}{2}^-$. Hence, the dispersion relation of the Ξ_c baryon gets modified when the contribution of the negative parity Ξ_c baryon is taken into account, which is 300 MeV heavier compared to the ground state Ξ_c baryon. In the light of new experimental data, we reanalyze the semileptonic decays of $\Xi_b \to \Xi(\Lambda)l\nu$ within light-cone sum rules in detail by taking into account the contributions of the $J^P = \frac{1}{2}^-$ heavy baryon.

So far, the LCSR have successfully been applied to the wide range of problems of hadronic physics, such as the nucleon electromagnetic form factor [9], form factors and strong coupling constants of the heavy baryons [10], rare $\Lambda_b \to N(N^*)\ell^+\ell^-$ decays [11], etc.

The paper is organized as follows. In Sec. II, the LCSR for the $\Xi_c \to B(\Xi \text{ or } \Lambda)$ transition form factors are derived.

In Sec. III, the numerical analysis of the transition form factors is performed. This section also contains our predictions on the decay widths of the $\Xi_c \to B\ell\nu$ transitions. Finally, we compare our results on the branching ratios with those predicted by the other approaches.

II. THE LCSR FOR THE $\Xi_c \to B$ TRANSITION FORM FACTORS

$\Xi_c \to B(\Xi \text{ or } \Lambda)$ decay is induced by the $c \to s(d)$ transition. The matrix elements induced by the vector and axial-vector transition currents are described with the help of the three-form factors,

$$\langle\Xi_c(p-q)|\bar{q}\gamma_\mu c|B(p)\rangle = \bar{u}_{\Xi_c}(p-q)\left[f_1(q^2)\gamma_\mu + i\frac{f_2(q^2)}{m_{\Xi_c}}\sigma_{\mu\nu}q^\nu + \frac{f_3(q^2)}{m_{\Xi_c}}q^\mu\right]u_B(p), \tag{1}$$

$$\langle\Xi_c(p-q)|\bar{q}\gamma_\mu\gamma_5 c|B(p)\rangle = \bar{u}_{\Xi_c}(p-q)\left[g_1(q^2)\gamma_\mu + i\frac{g_2(q^2)}{m_{\Xi_c}}\sigma_{\mu\nu}q^\nu + \frac{g_3(q^2)}{m_{\Xi_c}}q^\mu\right]\gamma_5 u_B(p). \tag{2}$$

The form factors responsible for the $\Xi_c^* \to B$ transition can be obtained from Eqs. (1) and (2) with the replacements $f_i \to \tilde{f}_i$, $g_i \to \tilde{g}_i$, inserting the Dirac matrix γ_5 after the Ξ_c baryon bispinor, and replacing Ξ_c with Ξ_c^*.

In order to derive the LCSR for the form factors, we start by considering the following correlation function(s):

$$\Pi_\mu^{V(A)}(p,q) = i\int d^4x e^{iqx}\langle 0|T\{\eta_{\Xi_c}(0)J_\mu^{V(A)}(x)\}|B(p)\rangle, \tag{3}$$

where η_{Ξ_c} is the interpolating current of the Ξ_c baryon, and $J_\mu^{V(A)} = \bar{c}\gamma_\mu q(\bar{c}\gamma_\mu\gamma_5 q)$ are the transition currents. In further calculations, we use the general form of the interpolating current Ξ_c [12],

$$\eta_{\Xi_c} = \frac{1}{\sqrt{6}}\epsilon_{abc}\{2[q^{aT}(x)Cs^b(x)]\gamma_5 c^c(x) + [q^{aT}(x)Cc^b(x)]\gamma_5 s^c(x) + [c^{aT}(x)Cs^b(x)]\gamma_5 q^c(x)$$
$$+ 2\beta[u^{aT}(x)C\gamma_5 s^b(x)]c^c(x) + \beta[q^{aT}(x)C\gamma_5 c^b(x)]s^c(x) + \beta[c^{aT}(x)C\gamma_5 s^b(x)]q^c(x)\}. \tag{4}$$

Here q is the light quark, C is the charge conjugation operator, a, b, and c are the color indices, β is an arbitrary parameter, and $\beta = -1$ corresponds to the Ioffe current.

To derive the LCSR for the transition form factors, we first calculate the hadronic part of the correlation function, which is achieved by inserting the full set of charmed-baryon states between the interpolating current η_{Ξ_c} and the transition current J_μ in Eq. (3). Thus, the hadronic part contains the contributions of the lowest positive-parity Ξ_c, as well as its negative–parity partner Ξ_c^*, i.e.,

$$\Pi_\mu^{V(A)}(p,q) = \sum_i \frac{\langle 0|\eta_{\Xi_c}(0)|\Xi_c^i(p-q,s)\rangle\langle\Xi_c^i(p-q,s)|\bar{c}\gamma_\mu q(\bar{c}\gamma_\mu\gamma_5 q)|B(p)\rangle}{m_i^2 - (p-q)^2} + \cdots, \tag{5}$$

where $\cdots$ denotes the contributions of all excited and continuum states with the quantum numbers of Ξ_c, and summation is performed over the ground and first orbital excited states. The first term on the right-hand side of Eq. (5) describes the coupling of the $\Xi_c(\Xi_c^*)$ baryon with the interpolating current η_{Ξ_c} which is defined as

$$\langle 0|\eta_{\Xi_c}(0)|\Xi_c(p-q,s)\rangle = \lambda_{\Xi_c} u_{\Xi_c}(p-q),$$
$$\langle 0|\eta_{\Xi_c^*}(0)|\Xi_c^*(p-q,s)\rangle = \lambda_{\Xi_c^*}\gamma_5 u_{\Xi_c^*}(p-q), \tag{6}$$

where $\lambda_{\Xi_c}(\lambda_{\Xi_c^*})$ is the residue of the corresponding baryon.

Using the definitions of the transition form factors for the vector transition current, and using the Dirac equation $\not{p}u_B(p) = m_B u_B(p)$ we get,

$$\Pi_\mu^V(p,q) = \frac{\lambda_{\Xi_c}}{m_{\Xi_c}^2 - (p-q)^2}\left\{ f_1(q^2)[2p_\mu + (m_{\Xi_c} - m_B)\gamma_\mu - 2q_\mu + \gamma_\mu\slashed{q}]\right.$$

$$-\frac{f_2(q^2)}{m_{\Xi_c}}[2p_\mu\slashed{q} + (m_{\Xi_c}^2 - m_B^2)\gamma_\mu + (m_{\Xi_c} + m_B)\gamma_\mu\slashed{q} - (m_{\Xi_c} + m_B)q_\mu - q_\mu\slashed{q}]$$

$$+\frac{f_3(q^2)}{m_{\Xi_c}}(m_{\Xi_c} + m_B - \slashed{q})q_\mu\bigg\}u_B(p) + \frac{\lambda_{\Xi_c^*}}{m_{\Xi_c^*}^2 - (p-q)^2}\left\{\tilde{f}_1(q^2)[-2p_\mu + (m_{\Xi_c^*} + m_B)\gamma_\mu + 2q_\mu - \gamma_\mu\slashed{q}]\right.$$

$$+\frac{\tilde{f}_2(q^2)}{m_{\Xi_c^*}}[2p_\mu\slashed{q} + (m_{\Xi_c^*}^2 - m_B^2)\gamma_\mu - (m_{\Xi_c^*} - m_B)\gamma_\mu\slashed{q} + (m_{\Xi_c^*} - m_B)q_\mu - q_\mu\slashed{q}]$$

$$+\frac{\tilde{f}_3(q^2)}{m_{\Xi_c^*}}(m_{-\Xi_c^*} - m_B + \slashed{q})q_\mu\bigg\}u_B(p). \tag{7}$$

$\Pi_\mu^A(p,q)$ can easily be obtained from $\Pi_\mu^V(p,q)$ by making the replacements $(f_i \to g_i; \tilde{f}_i \to \tilde{g}_i; m_B \to -m_B)$, and multiplying γ_5 matrix to the right end, i.e.,

$$\Pi_\mu^A(p,q) = \Pi_\mu^V(p,q)(f_i \to g_i; \tilde{f}_i \to \tilde{g}_i; m_B \to -m_B)\gamma_5.$$

We now turn our attention to the calculation of the correlation function (3) for the $\Xi_c(\Xi_c^*) \to B$ transition. We take $(p-q)^2, q^2 \ll m_{\Xi_c}^2$ to justify the expansion of the product of the two currents in the correlation function (3) near the light cone $x^2 \approx 0$, hence, the matrix element $\varepsilon^{abc}\langle 0|q_\alpha^a(0)s_\beta^b(0)s_\gamma^c(0)|B(p)\rangle$ is obtained. This matrix element is parametrized in terms of the Ξ and Λ baryon distribution amplitudes (DAs) of a different twist. The explicit expressions of the Ξ and Λ baryon DAs can be found in [13–15]. The operator product expansion is obtained by convolution of the hard-scattering amplitudes formed by the virtual c-quark propagator and the $\Xi(\Lambda)$ baryon DAs with increasing twists. In our calculations, we take into account all three particle B baryon DAs up to twist-6. However, we neglect the contributions of the four-particle (quark and gluon) DAs.

Matching the coefficient of the relevant Lorentz structures in both representations of the correlation function, we get the sum rules for the transition form factors. Finally, we perform Borel transformation over $-(p-q)^2$ in order to suppress the higher state and continuum contributions and obtain the following sum rules for the form factors of the $\bar{c}\gamma_\mu s$ transition current,

$$\Pi_1^B(p,q) = 2\lambda_{\Xi_c}f_1(q^2)e^{-m_{\Xi_c}^2/M^2} - 2\lambda_{\Xi_c^*}\tilde{f}_1(q^2)e^{-m_{\Xi_c^*}^2/M^2},$$

$$\Pi_2^B(p,q) = -2\lambda_{\Xi_c}\frac{f_2(q^2)}{m_{\Xi_c}}e^{-m_{\Xi_c}^2/M^2} + 2\lambda_{\Xi_c^*}\frac{\tilde{f}_2(q^2)}{m_{\Xi_c^*}}e^{-m_{\Xi_c^*}^2/M^2},$$

$$\Pi_3^B(p,q) = \lambda_{\Xi_c}e^{-m_{\Xi_c}^2/M^2}\left(f_1(q^2) - \frac{f_2(q^2)}{m_{\Xi_c}}(m_{\Xi_c} + m_B)\right) + \lambda_{\Xi_c^*}e^{-m_{\Xi_c^*}^2/M^2}\left(-\tilde{f}_1(q^2) - \frac{\tilde{f}_2(q^2)}{m_{\Xi_c^*}}(m_{\Xi_c^*} - m_B)\right),$$

$$\Pi_4^B(p,q) = \lambda_{\Xi_c}e^{-m_{\Xi_c}^2/M^2}\left((m_{\Xi_c} - m_B)\left(f_1(q^2) - \frac{f_2(q^2)}{m_{\Xi_c}}(m_{\Xi_c} + m_B)\right)\right)$$

$$+ \lambda_{\Xi_c^*}e^{-m_{\Xi_c^*}^2/M^2}\left((m_{\Xi_c^*} + m_B)\left(\tilde{f}_1(q^2) + \frac{\tilde{f}_2(q^2)}{m_{\Xi_c^*}}(m_{\Xi_c^*} - m_B)\right)\right),$$

$$\Pi_5^B(p,q) = \lambda_{\Xi_c}e^{-m_{\Xi_c}^2/M^2}\left(-2f_1(q^2) + \frac{(f_2(q^2) + f_3(q^2))}{m_{\Xi_c}}(m_{\Xi_c^*} + m_B)\right)$$

$$+ \lambda_{\Xi_c^*}e^{-m_{\Xi_c^*}^2/M^2}\left(2\tilde{f}_1(q^2) + \frac{(\tilde{f}_2(q^2) + \tilde{f}_3(q^2))}{m_{\Xi_c^*}}(m_{\Xi_c^*} - m_B)\right),$$

$$\Pi_6^B(p,q) = \frac{\lambda_{\Xi_c}}{m_{\Xi_c}}e^{-m_{\Xi_c}^2/M^2}(f_2(q^2) - f_3(q^2)) - \frac{\lambda_{\Xi_c^*}}{m_{\Xi_c^*}}e^{-m_{\Xi_c^*}^2/M^2}(\tilde{f}_2(q^2) - \tilde{f}_3(q^2)). \tag{8}$$

Here, $\Pi_1^B(p,q)$, $\Pi_2^B(p,q)$, $\Pi_3^B(p,q)$, $\Pi_4^B(p,q)$, $\Pi_5^B(p,q)$, and $\Pi_6^B(p,q)$ are the invariant functions for the Lorentz structures, p_μ, $p_\mu\slashed{q}$, $\gamma_\mu\slashed{q}$, γ_μ, q_μ, and $q_\mu\slashed{q}$ structures, respectively.

Note that the equations for the axial vector current $\gamma_\mu\gamma_5$ can be obtained from Eq. (8) by making the following replacements: $f_i \to -g_i$, $\tilde{f}_i \to -\tilde{g}_i$, $m_B \to -m_B$, and $\Pi_i^{(V)B} \to \Pi_i^{(A)B}$.

Solving the six equations given in (8) we obtain the LCSR for the transition form factors f_i, $\tilde{f}_i$ for vector current and g_i, and $\tilde{g}_i$ for axial vector which read as

$$f_1 = \frac{e^{m_{\Xi_c}^2/M^2}}{2\lambda_{\Xi_c}(m_{\Xi_c^*}+m_{\Xi_c})}\left\{(m_{\Xi_c}+m_B)\left[\Pi_1^{(V)B}-(m_{\Xi_c^*}-m_B)\Pi_2^{(V)B}\right]+2(m_{\Xi_c^*}-m_{\Xi_c})\Pi_3^{(V)B}+2\Pi_4^{(V)B}\right\},$$

$$f_2 = \frac{m_{\Xi_c}e^{m_{\Xi_c}^2/M^2}}{2\lambda_{\Xi_c}(m_{\Xi_c^*}+m_{\Xi_c})}\left[\Pi_1^{(V)B}-(m_{\Xi_c^*}-m_B)\Pi_2^{(V)B}-2\Pi_3^{(V)B}\right],$$

$$f_3 = \frac{m_{\Xi_c}e^{m_{\Xi_c}^2/M^2}}{2\lambda_{\Xi_c}(m_{\Xi_c^*}+m_{\Xi_c})}\left[\Pi_1^{(V)B}+2(\Pi_3^{(V)B}+\Pi_5^{(V)B})-(m_{\Xi_c^*}-m_B)(\Pi_2^{(V)B}+2\Pi_6^{(V)B})\right],$$

$$\tilde{f}_1 = \frac{-e^{m_{\Xi_c^*}^2/M^2}}{2\lambda_{\Xi_c^*}(m_{\Xi_c^*}+m_{\Xi_c})}\left\{(m_{\Xi_c^*}-m_B)\left[\Pi_1^{(V)B}+(m_{\Xi_c}+m_B)\Pi_2^{(V)B}\right]-2(m_{\Xi_c^*}-m_{\Xi_c})\Pi_3^{(V)B}-2\Pi_4^{(V)B}\right\},$$

$$\tilde{f}_2 = \frac{m_{\Xi_c^*}e^{m_{\Xi_c^*}^2/M^2}}{2\lambda_{\Xi_c^*}(m_{\Xi_c^*}+m_{\Xi_c})}\left[\Pi_1^{(V)B}+(m_{\Xi_c}+m_B)\Pi_2^{(V)B}-2\Pi_3^{(V)B}\right],$$

$$\tilde{f}_3 = \frac{m_{\Xi_c^*}e^{m_{\Xi_c^*}^2/M^2}}{2\lambda_{\Xi_c^*}(m_{\Xi_c^*}+m_{\Xi_c})}\left[\Pi_1^{(V)B}+2(\Pi_3^{(V)B}+\Pi_5^{(V)B})+(m_{\Xi_c}+m_B)(\Pi_2^{(V)B}+2\Pi_6^{(V)B})\right],$$

$$g_1 = \frac{-e^{m_{\Xi_c}^2/M^2}}{2\lambda_{\Xi_c}(m_{\Xi_c^*}+m_{\Xi_c})}\left\{(m_{\Xi_c}-m_B)\left[\Pi_1^{(A)B}-(m_{\Xi_c^*}+m_B)\Pi_2^{(A)B}\right]+2(m_{\Xi_c^*}-m_{\Xi_c})\Pi_3^{(A)B}+2\Pi_4^{(A)B}\right\},$$

$$g_2 = \frac{-m_{\Xi_c}e^{m_{\Xi_c}^2/M^2}}{2\lambda_{\Xi_c}(m_{\Xi_c^*}+m_{\Xi_c})}\left[\Pi_1^{(A)B}-(m_{\Xi_c^*}+m_B)\Pi_2^{(A)B}-2\Pi_3^{(A)B}\right],$$

$$g_3 = \frac{-m_{\Xi_c}e^{m_{\Xi_c}^2/M^2}}{2\lambda_{\Xi_c}(m_{\Xi_c^*}+m_{\Xi_c})}\left[\Pi_1^{(A)B}+2(\Pi_3^{(A)B}+\Pi_5^{(A)B})-(m_{\Xi_c^*}+m_B)(\Pi_2^{(A)B}+2\Pi_6^{(A)B})\right],$$

$$\tilde{g}_1 = \frac{e^{m_{\Xi_c^*}^2/M^2}}{2\lambda_{\Xi_c^*}(m_{\Xi_c^*}+m_{\Xi_c})}\left\{(m_{\Xi_c^*}+m_B)\left[\Pi_1^{(A)B}+(m_{\Xi_c}-m_B)\Pi_2^{(A)B}\right]-2(m_{\Xi_c^*}-m_{\Xi_c})\Pi_3^{(A)B}-2\Pi_4^{(A)B}\right\},$$

$$\tilde{g}_2 = \frac{-m_{\Xi_c^*}e^{m_{\Xi_c^*}^2/M^2}}{2\lambda_{\Xi_c^*}(m_{\Xi_c^*}+m_{\Xi_c})}\left[\Pi_1^{(A)B}+(m_{\Xi_c}-m_B)\Pi_2^{(A)B}-2\Pi_3^{(A)B}\right],$$

$$\tilde{g}_3 = \frac{-m_{\Xi_c^*}e^{m_{\Xi_c^*}^2/M^2}}{2\lambda_{\Xi_c^*}(m_{\Xi_c^*}+m_{\Xi_c})}\left[\Pi_1^{(A)B}+2(\Pi_3^{(A)B}+\Pi_5^{(A)B})+(m_{\Xi_c}-m_B)(\Pi_2^{(A)B}+2\Pi_6^{(A)B})\right]. \tag{9}$$

A few words about the theoretical calculations are in order. The correlation function with the γ_μ and $\gamma_\mu\gamma_5$ transition currents can be transformed to the following form:

$$\Pi_i^{V(A)}[(p-q)^2,q^2] = \sum_{n=1,2,3}\int_0^1 dx\frac{\rho_{\text{in}}^{V(A)}[x,(p-q)^2]}{\Delta^n},$$

where $\Delta = m_c^2 - x(p-q)^2 - \bar{x}q^2 + x\bar{x}m_{\Xi_c}^2$, and $\bar{x} = 1-x$ and $\rho_{\text{in}}^{V(A)}$ are the spectral densities of the corresponding invariant functions $\Pi_i^{V(A)}[(p-q)^2,q^2]$. Their expressions are too lengthy, hence, we do not present them here. To obtain the relevant sum rules for the form factors, Borel transformation to the dispersion integral representation and subtraction of continuum should be performed. These operations can be implemented with the help of the following replacements:

$$\int dx \frac{\rho_{i1}(x)}{\Delta} \rightarrow \int_{x_0}^{1} \frac{dx}{x} \rho_{i1}(x) e^{\frac{-s(x)}{M^2}},$$

$$\int dx \frac{\rho_{i2}(x)}{\Delta^2} \rightarrow \frac{1}{M^2} \int_{x_0}^{1} \frac{dx}{x^2} \rho_{i2}(x) e^{\frac{-s(x)}{M^2}} + \frac{\rho_{i2}(x_0) e^{\frac{-s_0}{M^2}}}{m_c^2 + x_0^2 m_{\Xi_c}^2 - q^2},$$

$$\int dx \frac{\rho_{i3}(x)}{\Delta^3} \rightarrow \frac{1}{2M^4} \int_{x_0}^{1} \frac{dx}{x^3} \rho_{i3}(x) e^{\frac{-s(x)}{M^2}}$$

$$+ \frac{1}{2M^2} \frac{\rho_{i3}(x_0) e^{\frac{-s_0}{M^2}}}{x_0 (m_c^2 + x_0^2 m_{\Xi_c}^2 - q^2)}$$

$$- \frac{1}{2} \frac{x_0^2 e^{\frac{-s_0}{M^2}}}{m_c^2 + x_0^2 m_{\Xi_c}^2 - q^2}$$

$$\times \frac{d}{dx} \left(\frac{\rho_{i3}(x)}{x(m_c^2 + x^2 m_{\Xi_c}^2 - q^2)} \right) \bigg|_{x=x_0}, \qquad (10)$$

and x_0 is the solution of the equation

$$s_0 = \frac{m_c^2 - \bar{x}q^2 + x\bar{x}m_{\Xi_c}^2}{x}.$$

The expressions of the form factors involve residues of Ξ_c and Ξ_c^* baryons. These residues can be calculated using the two-point correlation function,

$$\Pi(q^2) = i \int d^4x \, e^{iqx} \langle 0 | T\{\eta_Q(x) \bar{\eta}_Q(0)\} | 0 \rangle,$$

$$= \Pi_1^B(q^2) \slashed{q} + \Pi_2^B(q^2) I.$$

Following the standard sum rules methodology, namely, saturating the correlation function with Ξ_c and Ξ_c^*, and performing the Borel transformation and continuum subtraction, we obtain,

$$\Pi_1^B = \lambda_{\Xi_c} e^{-m_{\Xi_c}^2/M^2} + \lambda_{\Xi_c^*} e^{-m_{\Xi_c^*}^2/M^2},$$

$$\Pi_2^B = \lambda_{\Xi_c} m_{\Xi_c} e^{-m_{\Xi_c}^2/M^2} - \lambda_{\Xi_c^*} m_{\Xi_c^*} e^{-m_{\Xi_c^*}^2/M^2},$$

where $\lambda_{\Xi_c} (\lambda_{\Xi_c^*})$ and $m_{\Xi_c} (m_{\Xi_c^*})$ are the residues and the masses of the $\Xi_c(\Xi_c^*)$ baryons, respectively. Solving these equations, for the residue of the Ξ_c baryon we get,

$$\lambda_{\Xi_c} = \frac{e^{m_{\Xi_c}^2/M^2}}{m_{\Xi_c^*} + m_{\Xi_c}} (m_{\Xi_c^*} \Pi_1^B + \Pi_2^B). \qquad (11)$$

The invariant functions Π_1^B and Π_2^B are calculated in [11], which we will use in our numerical analysis.

III. NUMERICAL ANALYSIS

This section is devoted to the numerical analysis of the form factors derived in the previous section. The main input

parameters of LCSR are the DAs of the Ξ baryon, which are calculated in [13,14].

The normalization parameters of these DAs are obtained from the analysis of the two-point sum rules (see for example [13–15]), whose values are

$$f_\Xi = (9.9 \pm 0.4) \times 10^{-3} \text{ GeV}^2,$$

$$\lambda_1 = -(2.8 \pm 0.1) \times 10^{-2} \text{ GeV}^2,$$

$$\lambda_2 = (5.2 \pm 0.2) \times 10^{-2} \text{ GeV}^2,$$

$$\lambda_3 = (1.7 \pm 0.1) \times 10^{-2} \text{ GeV}^2.$$

The numerical values of other input parameters used in the calculations are presented in Table I.

The sum rules contain three auxiliary parameters, the Borel mass M^2, the continuum threshold s_0, and the parameter β in the expression of the interpolating current. According to the sum rules methodology, we should find the working regions of these parameters, where the form factors are practically insensitive to their variations.

The working interval of the Borel mass parameter is determined by demanding that both the continuum and power corrections have to be sufficiently suppressed. These requirements lead to the following working interval of the Borel mass parameter, $M^2 = (8 \pm 2) \text{ GeV}^2$. Moreover, the value of the continuum threshold is determined by requiring that the mass sum rules reproduce the measured mass of the lowest baryon mass to within 10% accuracy for definite values of the parameter β. This leads to the threshold value, $s_0 = (11 \pm 1) \text{ GeV}^2$. Finally, to find the working region of β, where $\beta = \tan\theta$, we study the dependency of mass on $\cos\theta$ at several fixed values of M^2 and s_0. We observe that the mass exhibits good stability to the variation of $\cos\theta$ in the interval $-1.0 < \cos\theta < -0.6$.

It should be emphasized that, the LCSR predictions, unfortunately, are not applicable to the whole physical region $m_\ell^2 \leq q^2 \leq [m_{\Xi_c}(m_{\Xi_c^*}) - m_\Xi]^2$. The LCSR for the form factors are reliable only up to $q^2 \leq 0.5 \text{ GeV}^2$.

TABLE I. The values of the input parameters used in our calculations.

Parameters	Value
$m_{\Xi_c^+}$	(2467.71 ± 0.23) MeV [16]
$m_{\Xi_c^0}$	(2470.44 ± 0.28) MeV [16]
$m_{\Xi_c^{*+}}$	(2791.9 ± 0.5) MeV [16]
$m_{\Xi_c^{*0}}$	(2793.9 ± 0.5) MeV [16]
m_Ξ^0	(1314.86 ± 0.20) MeV [16]
m_{Ξ^-}	(1321.71 ± 0.07) MeV [16]
$\langle \bar{q}q \rangle (1 \text{ GeV})$	$-(246^{+28}_{-19} \text{ MeV})^3$ [10]
$\bar{m}_c(\bar{m}_c)$	1.28 ± 0.03 GeV [17]

To extend this restricted domain to the full physical domain given above, we use the z-series parametrization of the form factors [18], which is given as

$$z(q^2, t_0) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}},$$

where $t_0 = q^2_{\max} = [m_{\Xi_c}(m_{\Xi_c^*}) - m_\Xi]^2$ and $t_+ = (m_{D_s} + m_K)^2$.

The following parametrization

$$f(q^2) = \frac{1}{1 - q^2/(m^f_{\text{pole}})^2}\{a^f_0 + a^f_1 z(q^2, t_0) + a^f_2[z(q^2, t_0)]^2\}, \tag{12}$$

reproduces best fits for the form factors predicted by the LCSR. The pole masses for the $\Xi_c \to \Xi(\Lambda)$ transition are

$$m_{\text{pole}} = \begin{cases} 2.112(2.010) \text{ GeV} & \text{for the form factors } f_1, f_2, \tilde{g}_1, \text{ and } \tilde{g}_2, \\ 2.535(2.423) \text{ GeV} & \text{for the form factors } g_1, g_2, \tilde{f}_1, \text{ and } \tilde{f}_2, \\ 2.317(2.300) \text{ GeV} & \text{for the form factors } f_3, \tilde{g}_3, \\ 1.969(1.870) \text{ GeV} & \text{for the form factors } g_3, \tilde{f}_3 \end{cases} \tag{13}$$

The values of the fit parameters a^f_0, a^f_1 and a^f_2 for the $\Xi_c \to \Xi$ and $\Xi_c \to \Lambda$ form factors are presented in Tables II and III, respectively.

Dependency of the form factors f_1, f_2, f_3 and g_1, g_2, and g_3 on q^2, at the fixed value of $s_0 = 10.0 \text{ GeV}^2$, and at several fixed values of the Borel mass parameter M^2 from

its working region of the $\Xi_c \to \Xi\mu\nu_\mu$ decay are presented in Figs. 1 and 2, respectively.

Having obtained the results for the form factors, we estimate the decay widths of the $\Xi_c \to B\ell\nu$ decays. The width of these decays can be calculated using helicity formalism [19]. We choose the rest frame of the Ξ_c baryon,

TABLE II. Form factors of the $\Xi_c \to \Xi$ transition.

$\Xi_c \to \Xi$	$f_i(0)$	a_0	a_1	a_2
f_1	-0.29 ± 0.05	-0.70 ± 0.08	12.11 ± 1.50	-89.50 ± 10.00
f_2	-0.12 ± 0.02	-0.55 ± 0.06	12.78 ± 1.50	-95.67 ± 11.00
f_3	-0.49 ± 0.10	-1.81 ± 0.20	37.68 ± 4.00	-263.79 ± 15.00
g_1	-0.22 ± 0.04	-0.39 ± 0.04	5.52 ± 0.60	-44.91 ± 6.00
g_2	0.45 ± 0.10	1.24 ± 0.15	-20.95 ± 2.60	132.68 ± 8.00
g_3	0.57 ± 0.12	1.26 ± 0.15	-18.27 ± 2.20	114.76 ± 12.00

TABLE III. Form factors of the $\Xi_c \to \Lambda$ transition.

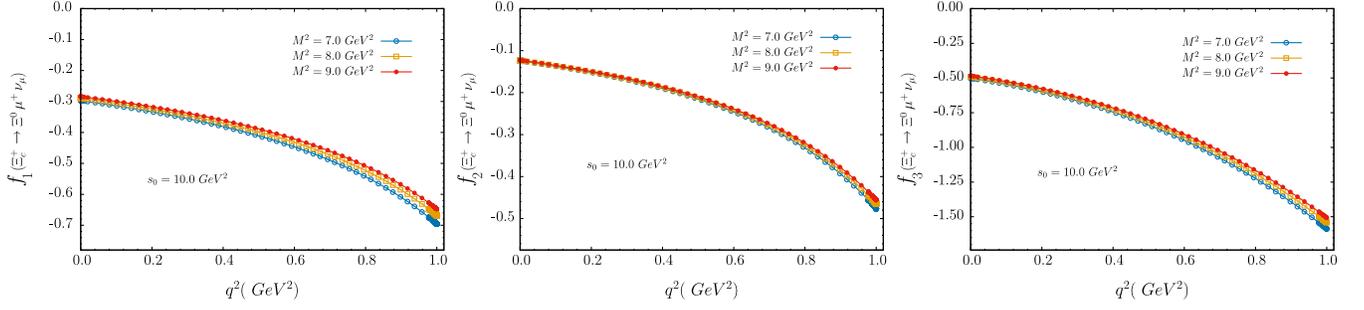| $\Xi_c \to \Lambda$	$f_i(0)$	a_0	a_1	a_2
f_1	-0.36 ± 0.06	-0.19 ± 0.05	-4.00 ± 0.80	22.68 ± 2.20
f_2	-0.18 ± 0.04	-0.21 ± 0.08	-0.49 ± 0.08	8.77 ± 1.00
f_3	0.11 ± 0.03	1.17 ± 0.12	-19.94 ± 2.50	93.66 ± 9.40
g_1	-0.12 ± 0.03	0.26 ± 0.04	-7.65 ± 0.90	38.88 ± 4.00
g_2	0.18 ± 0.04	0.38 ± 0.08	-3.10 ± 0.50	10.74 ± 1.10
g_3	-0.05 ± 0.01	-0.24 ± 0.05	4.28 ± 1.00	-22.98 ± 2.30

FIG. 1. The dependency of the form factors f_1, f_2, and f_3 for the $\Xi_c^+ \to \Xi^0 \mu^+ \nu_\mu$ transition on q^2, at $s_0 = 10$ GeV^2, and several values of the Borel mass parameter M^2.
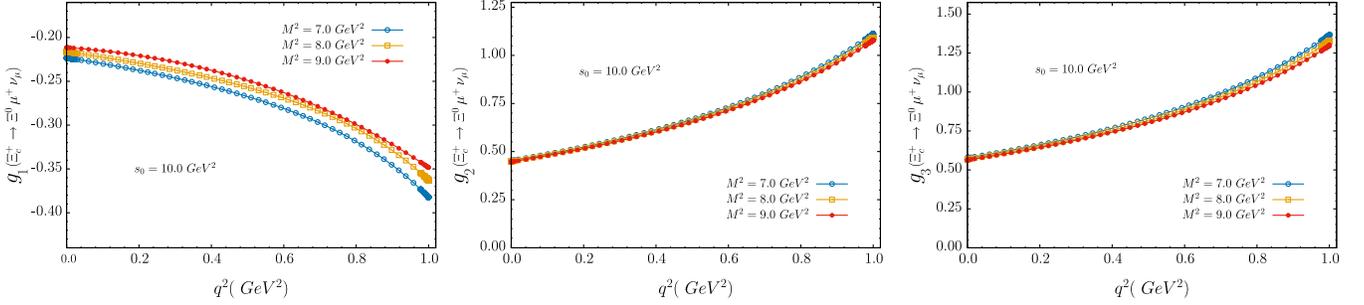

FIG. 2. The dependency of the form factors g_1, g_2, and g_3 for the $\Xi_c^+ \to \Xi^0 \mu^+ \nu_\mu$ transition on q^2, at $s_0 = 10$ GeV^2, and several values of the Borel mass parameter M^2.

where the z axis points along the $W_{\text{off-shell}}$ to calculate the helicity amplitudes, and we obtain

$$H^{V(A)}_{+\frac{1}{2},+1} = \sqrt{2Q_\mp}\left[f_1(g_1) - \frac{m_\pm}{m_{\Xi_c}}f_2(g_2)\right],$$

$$H^{V(A)}_{+\frac{1}{2},0} = \frac{\sqrt{Q_\mp}}{\sqrt{q^2}}\left[m_\pm f_1(g_1) - \frac{q^2}{m_{\Xi_c}}f_2(g_2)\right],$$

$$H^{V(A)}_{+\frac{1}{2},t} = \frac{\sqrt{Q_\pm}}{\sqrt{q^2}}\left[m_\mp f_1(g_1) - \frac{q^2}{m_{\Xi_c}}f_3(g_3)\right],$$

where $m_\pm = m_{\Xi_c} \pm m_B$, and $Q_\pm = m_\pm^2 - q^2$.

In these expressions, the first and second subindices describe the helicities of the B baryon and virtual W, correspondingly. The amplitudes for the negative values of the helicities can be obtained from the parity consideration, i.e.,

$$H^V_{-\lambda_B,-\lambda_W} = H^V_{\lambda_B,\lambda_W},$$

$$H^A_{-\lambda_B,-\lambda_W} = -H^A_{\lambda_B,\lambda_W}.$$

The total helicity amplitude is given by

$$H_{\lambda_B,\lambda_W} = H^V_{\lambda_B,\lambda_W} - H^A_{\lambda_B,\lambda_W}.$$

Using the expressions of the above-given helicity amplitudes for the differential decay widths, we obtain

$$\frac{d\Gamma(\Xi_c \to B\ell\nu)}{dq^2} = \frac{G_F^2}{(2\pi)^3}|V_{cq}|^2 \frac{\sqrt{\lambda(m_{\Xi_c}^2, m_B^2, q^2)}(q^2 - m_\ell^2)^2}{48m_{\Xi_c}^3 q^2}$$

$$\times \left\{|H_{+\frac{1}{2},+1}|^2 + |H_{-\frac{1}{2},-1}|^2\right.$$

$$+ \left(1 + \frac{m_\ell^2}{2q^2}\right)\left(|H_{+\frac{1}{2},0}|^2 + |H_{-\frac{1}{2},0}|^2\right)$$

$$+ \left. \frac{3m_\ell^2}{q^2}\left(|H_{+\frac{1}{2},t}|^2 + |H_{-\frac{1}{2},t}|^2\right)\right\},$$

where G_F is the Fermi constant, V_{cq} is the CKM matrix element ($q = s$ or d), and

$$\lambda(m_{\Xi_c}^2, m_B^2, q^2) = m_{\Xi_c}^4 + m_B^4 + q^4 - 2m_{\Xi_c}^2 m_B^2$$
$$- 2m_{\Xi_c}^2 q^2 - 2m_B^2 q^2,$$

The differential decay width for the $\Xi_c^* \to B\ell\nu$ decay can be obtained from $\Xi_c \to B\ell\nu$ decay by making the following replacements: $f_1 \to -\tilde{g}_1, f_2 \to \tilde{g}_2, g_1 \to -\tilde{f}_1, g_2 \to \tilde{f}_2$, and $m_{\Xi_c} \to m_{\Xi_c^*}$.

Using the values of the CKM matrix elements $|V_{cd}| = 0.2211 \pm 0.0700$ and $|V_{cs}| = 0.987 \pm 0.011$ [16] and the Ξ_c life time $\tau(\Xi_c^0) = (1.53 \pm 0.06) \times 10^{-13}$ s, and $\tau(\Xi_c^+) = (4.56 \pm 0.05) \times 10^{-13}$ s we can predict the branching ratios of the corresponding semileptonic decays. Our results are presented in Table IV. In this table, we also present the values

TABLE IV. The existing experimental and theoretical results on the branching ratios (in %) of the semileptonic $\Xi_c \to B\ell\nu$ decays.

Decay channel	Present work	BELLE [1]	ALICE [2]	SU(3) [3]	SU(3) [4]	RQM [5]	LATTICE [6]	3PSR [7]	LCSR [8]	LFQM [21]	LF [20]
$\Xi_c^0 \to \Xi^- e^+ \nu_e$	1.85±0.56	1.72±0.10±0.12±0.50	1.8±0.2	4.87±1.74	2.4±0.3	2.38	2.38±0.30±0.33	1.45±0.31	7.26±2.54	1.354	1.72±0.35
$\Xi_c^0 \to \Xi^- \mu^+ \nu_\mu$	1.79±0.54	1.71±0.17±0.13±0.50	1.8±0.2	...	2.4±0.3	2.31	2.29±0.29±0.31	1.45±0.31	7.15±2.50	...	...
$\Xi_c^+ \to \Xi^0 e^+ \nu_e$	5.51±1.65	...	...	$3.38^{+2.10}_{-2.26}$	9.8±1.1	9.40	7.18±0.90±0.98	...	28.6±10	5.39	5.2±1.02
$\Xi_c^+ \to \Xi^0 \mu^+ \nu_\mu$	5.34±1.61	...	...	...	9.8±1.1	9.11	6.91±0.87±0.93	...	28.2±9.9	...	...
$\Xi_c^+ \to \Lambda^0 e^+ \nu_e$	0.092±0.028	...	...	...	0.166±0.018	0.127	...	...	...	0.082	...
$\Xi_c^+ \to \Lambda^0 \mu^+ \nu_\mu$	0.089±0.027	...	...	...	...	0.124	...	...	...	...	...

of the branching ratios of the semileptonic $\Xi_c \to B\ell\nu$ decays obtained from other theoretical approaches, as well as the latest announced experimental results. From a comparison of the predictions of the different approaches, we see that our results are close to that of the ones given in [20] as well as the experimental measurements [1,2]. On the other hand, the obtained branching ratios are slightly smaller than the results presented in [4–6] but larger than the values obtained in [7,21]. However, our results are considerably different from the results obtained in [8] for the $\Xi_c \to \Xi\ell\nu$ decay, although they applied the same method as used in this work. This discrepancy can be explained as follows. The interpolating current of Ξ_c baryon interacts not only with ground state positive parity baryons $J^P = (\frac{1}{2})^+$ but also with $J^P = \frac{1}{2}^-$ negative parity baryon which was neglected in [8]. Thus, the dispersion relation of Ξ_c baryon is modified, and since the mass difference between these states is around 300 MeV, the results change considerably.

Our predictions on the branching ratios of $\Xi_c^+ \to \Lambda\ell\nu_\ell$ are also quite in agreement with the results of [5] within the error. The predictions on the branching ratios can further be improved by more precise determination of the input parameters appearing in DAs of the Ξ and Λ baryons, as well as taking into account $\mathcal{O}(\alpha_s)$ corrections.

IV. CONCLUSION

The form factors of the semileptonic $\Xi_c \to \Xi(\Lambda)\ell\nu$ decays are studied in the framework of the light cone QCD sum rules method. In order to eliminate the contamination of the negative parity Ξ_c^* baryon, the combination of the sum rules obtained from different Lorentz structures is used.

Using the obtained results on the form factors and applying the helicity formalism, we also estimated the corresponding branching ratios of the considered decays. Moreover, our results on the branching ratios are compared with the predictions of the other approaches as well as with the experimental measurements.

The branching ratios of $\Xi_c \to \Xi\ell\nu$ decays has already been studied in various models like the relativistic quark model [5], LATTICE QCD [6], 3-point sum rules [7], and light front quark models [20,21]. Our calculations within the light cone sum rule showed that the results are in good agreement with the experimental measurements done by the BELLE [1] and ALICE [2] collaborations.

The discrepancy between our finding and the results of [8] in which the same method was used can be explained by taking into account the contributions of the Ξ_c^* baryon that was neglected in [8].

Moreover, we also estimated the decay width of the CKM suppressed semileptonic $\Xi \to \Lambda l\nu$ decay within the light cone sum rules. The obtained branching ratios are close to the predictions of [5] and the magnitude of the obtained value shows that it has potential to be measured in the future experiments

[1] Y. B. Li *et al.* (Belle Collaboration), Measurements of the Branching Fractions of Semileptonic Decays $\Xi_c^0 \to \Xi^- \ell^+ \nu_\ell$ and Asymmetry Parameter of $\Xi_c^0 \to \Xi^- \pi^+$ Decay, arXiv:2103.06496 [Phys. Rev. Lett. (to be published)].

[2] S. Acharya *et al.* (ALICE Collaboration), Measurement of the cross sections of Ξ_c^0 and Ξ_c^+ baryons and branching-fraction ratio $\mathrm{BR}(\Xi_c^0 \to \Xi^- e^+ \nu_e)/\mathrm{BR}(\Xi_c^0 \to \Xi^- \pi^+)$ in pp collisions at 13 TeV, arXiv:2105.05187.

[3] C. Q. Geng, Y. K. Hsiao, C.-W. Liu, and T.-H. Tsai, Antitriplet charmed baryon decays with SU(3) flavor symmetry, Phys. Rev. D **97,** 073006 (2018).

[4] C.-Q. Geng, C.-W. Liu, T.-H. Tsai, and S.-W. Yeh, Semi-leptonic decays of anti-triplet charmed baryons, Phys. Lett. B **792,** 214 (2019).

[5] R. N. Faustov and V. O. Galkin, Semileptonic Ξ_c baryon decays in the relativistic quark model, Eur. Phys. J. C **79,** 695 (2019).

[6] Q.-A. Zhang, J. Hua, F. Huang, R. Li, Y. Li, C.-D. Lu, P. Sun, W. Sun, W. Wang, and Y.-B. Yang, $\Xi_c \to \Xi$ form factors and $\Xi_c \to \Xi \ell^+ \nu_\ell$ decay rates from lattice QCD, arXiv:2103.07064.

[7] Z.-X. Zhao, Semi-leptonic form factors of $\Xi_c \to \Xi$ in QCD sum rules, arXiv:2103.09436.

[8] K. Azizi, Y. Sarac, and H. Sundu, Light cone QCD sum rules study of the semileptonic heavy Ξ_Q and Ξ_Q' transitions to Ξ and Σ baryons, Eur. Phys. J. A **48,** 2 (2012).

[9] V. M. Braun, A. Lenz, and M. Wittmann, Nucleon form factors in QCD, Phys. Rev. D **73,** 094019 (2006).

[10] A. Khodjamirian, Ch. Klein, Th. Mannel, and Y. M. Wang, Form factors and strong couplings of heavy baryons from QCD light-cone sum rules, J. High Energy Phys. 09 (2011) 106.

[11] T. M. Aliev, K. Azizi, T. Barakat, and M. Savcı, Diagonal and transition magnetic moments of negative parity heavy baryons in QCD sum rules, Phys. Rev. D **92,** 036004 (2015).

[12] E. Bagan, M. Chabab, H. Gunter Dosch, and S. Narison, Spectra of heavy baryons from QCD spectral sum rules, Phys. Lett. B **287,** 176 (1992).

[13] Y.-L. Liu and M.-Q. Huang, Light-cone distribution amplitudes of Ξ and their applications, Phys. Rev. D **80,** 055015 (2009).

[14] Y.-L. Liu and M.-Q. Huang, Distribution amplitudes of Σ and Λ and their electromagnetic form factors, Nucl. Phys. **A821,** 80 (2009).

[15] P. Wein and A. Schäfer, Model-independent calculation of $SU(3)_f$ violation in baryon octet light-cone distribution amplitudes, J. High Energy Phys. 05 (2015) 073.

[16] P. A. Zyla *et al.* (Particle Data Group), Review of particle physics, Prog. Theor. Exp. Phys. **2020,** 083C01 (2020).

[17] K. G. Chetyrkin, J. H. Kuhn, A. Maier, P. Maierhofer, P. Marquard, M. Steinhauser, and C. Sturm, Charm and bottom quark masses: An update, Phys. Rev. D **80,** 074010 (2009).

[18] C. Bourrely, I. Caprini, and L. Lellouch, Model-independent description of $B \to \pi l \nu$ decays and a determination of $|V_{ub}|$, Phys. Rev. D **79,** 013008 (2009); Erratum, Phys. Rev. D **82,** 099902 (2010).

[19] T. Gutsche, M. A. Ivanov, J. G. Körner, V. E. Lyubovitskij, P. Santorelli, and N. Habyl, Semileptonic decay $\Lambda_b \to \Lambda_c + \tau^- + \overline{\nu_\tau}$ in the covariant confined quark model, Phys. Rev. D **91,** 074001 (2015); Erratum, Phys. Rev. D **91,** 119907 (2015).

[20] H.-W. Ke, Q.-Q. Kang, X.-H. Liu, and X.-Q. Li, The weak decays of $\Xi_c^{(')} \to \Xi$ in the light-front quark model, arXiv:2106.07013.

[21] Z.-X. Zhao, Weak decays of heavy baryons in the light-front approach, Chin. Phys. C **42,** 093101 (2018).