Twist-3 gluon fragmentation contribution to polarized hyperon production in unpolarized proton-proton collision

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Understanding the origin and mechanism of the transverse polarization of hyperons produced in unpolarized proton-proton collision, $pp \to \Lambda^{\uparrow}X$, has been one of the longstanding issues in high-energy spin physics. In the framework of the collinear factorization applicable to large-p_T hadron productions, this phenomenon is a twist-3 observable which is caused by multiparton correlations either in the initial protons or in the process of fragmentation into the hyperon. We derive the twist-3 gluon fragmentation function (FF) contribution to this process in the leading order (LO) with respect to the QCD coupling constant. Combined with the known results for the contribution from the twist-3 distribution function and the twist-3 quark FF, this completes the LO twist-3 cross section. We also found that the model-independent relations among the twist-3 gluon FFs based on the QCD equation of motion and the Lorentz invariance property of the correlation functions guarantee the color gauge invariance and the frame independence of the cross section.

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I. INTRODUCTION

It has been known that the hyperons produced in unpolarized proton-proton collisions are polarized perpendicularly to the scattering plane, $pp \to \Lambda^{\uparrow}X$[1] [1–11]. The observed polarizations show a tendency that they become larger in the forward rapidity region, where the asymmetry is as large as 30%. Hyperon polarization was also observed in other reactions such as $\gamma p \to \Lambda^{\uparrow}X$ [12,13], quasireal photoproduction of Λ's in lepton scattering [14,15], and electron-positron collisions, $e^+e^- \to \Lambda^{\uparrow}X$ [16,17]. These transverse polarizations in unpolarized collisions are the examples of the transverse single spin asymmetries (SSAs), where only one particle participating in the scattering process is polarized. Another well-known SSA is the asymmetry with regard to the initial (transverse) spin such as $p^{\uparrow}p \to hX$ ($h = \pi, K, \eta$, jet, etc.) [18–23] and $ep^{\uparrow} \to ehX$ [24,25]. For the last several decades, many efforts have been made to understand the origin and mechanism for these large SSAs, since perturbative QCD at twist-2 level gives almost zero SSAs [26].

For a high-energy collision in which particles with large transverse momentum are produced, the cross section can be computed in the framework of the collinear factorization of perturbative QCD. In this framework, SSAs appear as twist-3 observables to which nonperturbative multiparton correlation functions contribute instead of collinear twist-2 parton distribution functions (PDFs) and/or fragmentation functions (FFs). Through the studies of SSAs, the technique of calculating the twist-3 cross section has made much progress and has been applied to many relevant processes in the leading order (LO) with respect to the QCD coupling. For example, the complete LO cross section for $p^{\uparrow}p \to hX$ ($h = \pi, D, \gamma, \gamma^*$) has been derived [27–40], and the Relativistic Heavy Ion Collider data have been analyzed and interpreted, which suggests the main source of the asymmetry is the twist-3 fragmentation contribution [41,42].

In this paper, we study $pp \to \Lambda^{\uparrow}X$ in the collinear twist-3 factorization. Two kinds of twist-3 cross sections contribute to this process: (i) twist-3 unpolarized PDF in one of the initial proton convoluted with the twist-2

[1]Here, Λ collectively denotes spin-1/2 hyperons such as Λ Σ, Ξ, etc.

"transversity" FF for the final hyperon and the twist-2 unpolarized PDF in another proton and (ii) twist-3 FFs for the polarized hyperon convoluted with the twist-2 unpolarized PDFs in the initial protons. The complete LO cross section for i was derived in Refs. [43–45]. The second one (ii) can be further classified into two, depending on whether the twist-3 FF is of ii-a) quark-gluon correlation type or of ii-b) gluon correlation type. The complete LO cross section for ii-a was derived in Ref. [46], while ii-b has not been studied so far. In this paper, we focus on the ii-b contribution and derive the corresponding cross section, which completes the LO twist-3 cross section for this process. (A short version of the present work was presented in Refs. [47,48].) We develop a formalism for deriving the gauge- and frame-independent contribution to this twist-3 cross section from the purely gluonic FFs and present the result for $pp \to \Lambda^{\uparrow} X$.

Besides the intrinsic importance of the formal development, we emphasize the phenomenological relevance of the twist-3 gluon FFs in $pp \to \Lambda^{\uparrow} X$ in comparison to A_N in $p^{\uparrow} p \to \pi X$. In the latter case, the twist-3 quark FFs for the unpolarized final pion are considered to be the main source of the asymmetry. Since both asymmetries show a similar tendency, i.e., increase in the forward direction, one may expect the twist-3 fragmentation contribution to also be important for $pp \to \Lambda^{\uparrow} X$. Owing to the chiral-odd nature of the twist-3 quark FFs for $p^{\uparrow} p \to \pi X$, there are no counter FFs in the gluon sector for this process. On the other hand, for $pp \to \Lambda^{\uparrow} X$, the chiral-even twist-3 quark FFs accompany the twist-3 gluon FFs, which could play an important role in the asymmetry, given that the gluons are ample in the collision environment and the effect of their correlations leading to hyperon polarizations could be sizable. In addition, since these chiral-even quark and gluon twist-3 FFs mix under renormalization, inclusion of the twist-3 gluon FFs becomes necessary even in LO when one includes correct scale dependence of these twist-3 FFs. Therefore, the present work will be important for a phenomenology as well.

The remainder of this paper is organized as follows. In Sec. II, we introduce the complete set of the gluonic FFs for spin-$1/2$ hadron up to twist 3 defined from correlators of two- and three-gluon field strengths, which are necessary to derive the twist-3 cross section. We also recall from Ref. [49] the exact relations among those FFs based on the QCD equation of motion and the Lorentz invariance, which play a crucial role in guaranteeing the gauge and Lorentz invariance of the cross section.[2] In Sec. III, we develop a formalism to derive the twist-3 gluon FF contribution to the twist-3 cross section and present the corresponding LO cross section for $pp \to \Lambda^{\uparrow} X$. We will discuss how the Lorentz invariance of the twist-3 cross section is realized, using the relations introduced in Sec. II. Gauge invariance of the cross section is discussed in Appendix B. Section IV is devoted to a brief summary. In other Appendixes, we discuss some technical aspects of the actual calculations.

II. GLUON FRAGMENTATION FUNCTIONS

A. Three types of twist-3 gluon fragmentation functions

In this section, we introduce twist-3 gluon FFs for a spin-$1/2$ baryon relevant to $pp \to \Lambda^{\uparrow} X$ [49,53,54] and summarize their basic properties derived in Ref. [49]. They are classified into three types; *intrinsic*, *kinematical*, and *dynamical* FFs. The intrinsic twist-3 gluon FFs are defined as the Fourier transform of the light-cone correlator of the gluon's field strength $F_a^{\mu\nu}$,

$$\begin{aligned}\hat{\Gamma}^{\alpha\beta}(z) &= \frac{1}{N^2-1}\sum_X \int \frac{d\lambda}{2\pi} e^{-i\frac{\lambda}{z}} \langle 0|([\infty w, 0]F^{w\beta}(0))_a|h(P_h,S_h)X\rangle\langle h(P_h,S_h)X|(F^{w\alpha}(\lambda w)[\lambda w,\infty w])_a|0\rangle \\ &= -g_\perp^{\alpha\beta}\hat{G}(z) - iM_h\epsilon^{P_h w\alpha\beta}(S_h\cdot w)\Delta\hat{G}(z) - iM_h\epsilon^{P_h w S_\perp [\alpha} w^{\beta]}\Delta\hat{G}_{3T}(z) + M_h\epsilon^{P_h w S_\perp \{\alpha} w^{\beta\}}\Delta\hat{G}_{3\bar{T}}(z), \end{aligned} \quad (1)$$

where $N=3$ is the number of colors, $|h(P_h,S_h)\rangle$ is the baryon state with the 4-momentum P_h ($P_h^2 = M_h^2$) and the spin vector S_h ($S_h^2 = -M_h^2$), and $[\lambda w, \infty w]$ is the gauge link in the adjoint representation connecting λw and ∞w. For the transversely polarized baryon, we use the spin vector $S_\perp$ normalized as $S_\perp^2 = -1$. In the twist-3 accuracy, P_h can be regarded as lightlike. For a baryon with large momentum, $P_h \simeq (|\vec{P}_h|, \vec{P}_h)$, another lightlike vector w is defined as $w = 1/(2|\vec{P}_h|^2)(|\vec{P}_h|, -\vec{P}_h)$, which satisfies $P_h\cdot w = 1$. In (1), we use the notation $F^{w\beta} \equiv F^{\mu\beta}w_\mu$, and $\{\}$ ($[]$) implies the symmetrization (antisymmetrization) of Lorentz indices, i.e., for arbitrary 4-vectors a^α and b^β, $a^{\{\alpha}b^{\beta\}} \equiv a^\alpha b^\beta + a^\beta b^\alpha$ and $a^{[\alpha}b^{\beta]} \equiv a^\alpha b^\beta - a^\beta b^\alpha$. $\hat{G}(z)$ and $\Delta\hat{G}(z)$ are twist 2, and $\Delta\hat{G}_{3T}(z)$ and $\Delta\hat{G}_{3\bar{T}}(z)$ are twist 3. We also note $\Delta\hat{G}_{3\bar{T}}(z)$ is naively T odd, contributing to SSAs. Each function in (1) has a support on $0 < z < 1$.

The kinematical FFs contain the transverse derivative of the correlation functions of the field strengths,

[2] The importance of these relations for the frame independence of the twist-3 cross sections has been realized for the twist-3 quark distribution functions and FFs [46,50–52].

$$\hat{\Gamma}_{\partial}^{\alpha\beta\gamma}(z) = \frac{1}{N^2-1}\sum_X \int \frac{d\lambda}{2\pi} e^{-i\frac{\lambda}{z}} \langle 0|([\infty w, 0]F^{w\beta}(0))_a|h(P_h, S_\perp)X\rangle\langle h(P_h, S_\perp)X|(F^{w\alpha}(\lambda w)[\lambda w, \infty w])_a|0\rangle \overleftarrow{\partial}^{\gamma}$$
$$= -i\frac{M_h}{2} g_\perp^{\alpha\beta}\epsilon^{P_h w S_\perp \gamma}\hat{G}_T^{(1)}(z) + \frac{M_h}{2}\epsilon^{P_h w\alpha\beta}S_\perp^\gamma \Delta\hat{G}_T^{(1)}(z) - i\frac{M_h}{8}(\epsilon^{P_h w S_\perp\{\alpha}g_\perp^{\beta\}\gamma} + \epsilon^{P_h w\gamma\{\alpha}S_\perp^{\beta\}})\Delta\hat{H}_T^{(1)}(z), \tag{2}$$

where each function is defined to be real. The kinematical FFs are related to the k_T^2/M_h^2-moment of the transverse-momentum-dependent FFs [53]. Each function in (2) has a support on $0 < z < 1$.

To define the dynamical FFs, we introduce the light-cone correlation functions of three field strengths,

$$\hat{\Gamma}_{F,abc}^{\alpha\beta\gamma}\left(\frac{1}{z_1},\frac{1}{z_2}\right) = \sum_X \int \frac{d\lambda}{2\pi}\int\frac{d\mu}{2\pi} e^{-i\frac{\lambda}{z_1}} e^{-i\mu(\frac{1}{z_2}-\frac{1}{z_1})}\langle 0|F_b^{w\beta}(0)|h(P_h, S_\perp)X\rangle\langle h(P_h, S_\perp)X|F_a^{w\alpha}(\lambda w)gF_c^{w\gamma}(\mu w)|0\rangle, \tag{3}$$

where the gauge link operators are suppressed for simplicity. The color indices of this correlator can be expanded in terms of the antisymmetric and symmetric structure constants of color SU(N), $-if^{abc}$ and d^{abc}, as

$$\hat{\Gamma}_{F,abc}^{\alpha\beta\gamma}\left(\frac{1}{z_1},\frac{1}{z_2}\right) = \frac{if^{abc}}{N}\hat{\Gamma}_{FA}^{\alpha\beta\gamma}\left(\frac{1}{z_1},\frac{1}{z_2}\right) + d^{abc}\frac{N}{N^2-4}\hat{\Gamma}_{FS}^{\alpha\beta\gamma}\left(\frac{1}{z_1},\frac{1}{z_2}\right). \tag{4}$$

The dynamical FFs can be defined from $\hat{\Gamma}_{FA}^{\alpha\beta\gamma}(\frac{1}{z'},\frac{1}{z})$ and $\hat{\Gamma}_{FS}^{\alpha\beta\gamma}(\frac{1}{z'},\frac{1}{z})$:

$$\hat{\Gamma}_{FA}^{\alpha\beta\gamma}\left(\frac{1}{z_1},\frac{1}{z_2}\right) = \frac{-if_{abc}}{N^2-1}\sum_X \int \frac{d\lambda}{2\pi}\int\frac{d\mu}{2\pi} e^{-i\frac{\lambda}{z_1}} e^{-i\mu(\frac{1}{z_2}-\frac{1}{z_1})}\langle 0|F_b^{w\beta}(0)|h(P_h, S_\perp)X\rangle\langle h(P_h, S_\perp)X|F_a^{w\alpha}(\lambda w)gF_c^{w\gamma}(\mu w)|0\rangle$$
$$= -M_h\left(\hat{N}_1\left(\frac{1}{z_1},\frac{1}{z_2}\right)g_\perp^{\alpha\gamma}\epsilon^{P_h w S_\perp\beta} + \hat{N}_2\left(\frac{1}{z_1},\frac{1}{z_2}\right)g_\perp^{\beta\gamma}\epsilon^{P_h w S_\perp\alpha} - \hat{N}_2\left(\frac{1}{z_2}-\frac{1}{z_1},\frac{1}{z_2}\right)g_\perp^{\alpha\beta}\epsilon^{P_h w S_\perp\gamma}\right), \tag{5}$$

$$\hat{\Gamma}_{FS}^{\alpha\beta\gamma}\left(\frac{1}{z_1},\frac{1}{z_2}\right) = \frac{d_{abc}}{N^2-1}\sum_X \int \frac{d\lambda}{2\pi}\int\frac{d\mu}{2\pi} e^{-i\frac{\lambda}{z_1}} e^{-i\mu(\frac{1}{z_2}-\frac{1}{z_1})}\langle 0|F_b^{w\beta}(0)|h(P_h, S_\perp)X\rangle\langle h(P_h, S_\perp)X|F_a^{w\alpha}(\lambda w)gF_c^{w\gamma}(\mu w)|0\rangle$$
$$= -M_h\left(\hat{O}_1\left(\frac{1}{z_1},\frac{1}{z_2}\right)g_\perp^{\alpha\gamma}\epsilon^{P_h w S_\perp\beta} + \hat{O}_2\left(\frac{1}{z_1},\frac{1}{z_2}\right)g_\perp^{\beta\gamma}\epsilon^{P_h w S_\perp\alpha} + \hat{O}_2\left(\frac{1}{z_2}-\frac{1}{z_1},\frac{1}{z_2}\right)g_\perp^{\alpha\beta}\epsilon^{P_h w S_\perp\gamma}\right). \tag{6}$$

Correlation functions (5) and (6), respectively, define two independent set of the *complex* functions $\{\hat{N}_1, \hat{N}_2\}$ and $\{\hat{O}_1, \hat{O}_2\}$ due to the exchange symmetry of the field strengths. Functions $\hat{N}_1$ and $\hat{O}_1$ satisfy the relations

$$\hat{N}_1\left(\frac{1}{z_1},\frac{1}{z_2}\right) = -\hat{N}_1\left(\frac{1}{z_2}-\frac{1}{z_1},\frac{1}{z_2}\right),$$
$$\hat{O}_1\left(\frac{1}{z_1},\frac{1}{z_2}\right) = \hat{O}_1\left(\frac{1}{z_2}-\frac{1}{z_1},\frac{1}{z_2}\right). \tag{7}$$

The real parts of these four FFs are T even, and the imaginary parts are T odd, the latter being the sources of SSAs. $\hat{N}_{1,2}(\frac{1}{z_1},\frac{1}{z_2})$ and $\hat{O}_{1,2}(\frac{1}{z_1},\frac{1}{z_2})$ have a support on $\frac{1}{z_2} > 1$ and $\frac{1}{z_2} > \frac{1}{z_1} > 0$.

For the derivation of the twist-3 gluon FF contribution to $pp\to\Lambda^\uparrow X$, one also needs another dynamical FF,

$$\tilde{\Delta}_{ij}^{\alpha}\left(\frac{1}{z_1},\frac{1}{z_2}\right) = \frac{1}{N}\sum_X \int \frac{d\lambda}{2\pi}\int\frac{d\mu}{2\pi} e^{-i\frac{\lambda}{z_1}} e^{-i\mu(\frac{1}{z_2}-\frac{1}{z_1})}\langle 0|F_a^{w\alpha}(\mu w)|hX\rangle\langle hX|\bar{\psi}_j(\lambda w)t^a\psi_i(0)|0\rangle$$
$$= M_h\left[e^{\alpha P_h w S_\perp}(\not{P}_h)_{ij}\tilde{D}_{FT}\left(\frac{1}{z_1},\frac{1}{z_2}\right) + iS_\perp^\alpha(\gamma_5\not{P}_h)_{ij}\tilde{G}_{FT}\left(\frac{1}{z_1},\frac{1}{z_2}\right)\right], \tag{8}$$

where we explicitly wrote spinor indices i, j. $\tilde{D}_{FT}$ and $\tilde{G}_{FT}$ are also complex functions, with the T-even real part and the T-odd imaginary part. They have a support on $\frac{1}{z_1} > 0$, $\frac{1}{z_2} < 0$ and $\frac{1}{z_1} - \frac{1}{z_2} > 1$. As was shown in Ref. [49], the constraint relations for the twist-3 gluon FFs involve these quark-gluon correlation functions. We collectively call the functions in (5), (6), and (8) dynamical twist-3 FFs.

B. Constraint relations among the twist-3 gluon FFs

Three types of the twist-3 gluon FFs defined in the previous subsection are not independent from each other but are related by the QCD equation-of-motion (EOM) and the Lorentz invariance properties of the correlation functions (LIRs). In Ref. [49], the complete set of those relations is derived. Here, we recall those relations, which, as we will see in the next section, play a crucial role in guaranteeing the Lorentz and gauge invariance of the twist-3 cross sections for $pp \to \Lambda^\uparrow X$. Here, we quote from Ref. [49] the relevant relations.

First, the intrinsic FFs $\Delta\hat{G}_{3\bar{T}}(z)$ can be written in terms of the kinematical FFs and the dynamical ones as (see Eq. (50) of Ref. [49]),

$$\frac{1}{z}\Delta\hat{G}_{3\bar{T}}(z) = -\Im\tilde{D}_{FT}(z) + \int_0^{1/z} d\left(\frac{1}{z'}\right)\left(\frac{1}{\frac{1}{z}-\frac{1}{z'}}\right)\Im\left\{2\hat{N}_1\left(\frac{1}{z'},\frac{1}{z}\right) + \hat{N}_2\left(\frac{1}{z'},\frac{1}{z}\right) - \hat{N}_2\left(\frac{1}{z}-\frac{1}{z'},\frac{1}{z}\right)\right\} + \frac{1}{2}(\hat{G}_T^{(1)}(z) + \Delta\hat{H}_T^{(1)}(z)), \tag{9}$$

where $\tilde{D}_{FT}(z)$ is defined as

$$\tilde{D}_{FT}(z) \equiv \frac{2}{C_F}\int_0^{1/z} d\left(\frac{1}{z_1}\right)\tilde{D}_{FT}\left(\frac{1}{z_1},\frac{1}{z_1}-\frac{1}{z}\right), \tag{10}$$

with $C_F = \frac{N^2-1}{2N}$ and $\mathfrak{J}\tilde{D}_{FT}$ indicates the imaginary part of $\tilde{D}_{FT}$. The kinematical FFs can be expressed in terms of the dynamical ones as (see Eqs. (74) and (75) of Ref. [49]):

$$\hat{G}_T^{(1)}(z) = -\frac{2}{z^2}\int_1^{1/z} d\left(\frac{1}{z_2}\right) z_2^3\Im\tilde{D}_{FT}(z_2) + \frac{4}{z^2}\int_1^{1/z} d\left(\frac{1}{z_2}\right) z_2^3\int_0^{1/z_2} d\left(\frac{1}{z_1}\right)\frac{1}{1/z_2-1/z_1}\Im\left[\hat{N}_1\left(\frac{1}{z_1},\frac{1}{z_2}\right) - \hat{N}_2\left(\frac{1}{z_2}-\frac{1}{z_1},\frac{1}{z_2}\right)\right] + \frac{2}{z^2}\int_1^{1/z} d\left(\frac{1}{z_2}\right) z_2^2\int_0^{1/z_2} d\left(\frac{1}{z_1}\right)\frac{1}{(1/z_2-1/z_1)^2}\Im\left[\hat{N}_1\left(\frac{1}{z_1},\frac{1}{z_2}\right) + \hat{N}_2\left(\frac{1}{z_1},\frac{1}{z_2}\right) - 2\hat{N}_2\left(\frac{1}{z_2}-\frac{1}{z_1},\frac{1}{z_2}\right)\right], \tag{11}$$

and

$$\Delta\hat{H}_T^{(1)}(z) = -\frac{4}{z^4}\int_1^{1/z} d\left(\frac{1}{z_2}\right) z_2^5\Im\tilde{D}_{FT}(z_2) + \frac{8}{z^4}\int_1^{1/z} d\left(\frac{1}{z_2}\right) z_2^5\int_0^{1/z_2} d\left(\frac{1}{z_1}\right)\frac{1}{1/z_2-1/z_1}\Im\left[\hat{N}_1\left(\frac{1}{z_1},\frac{1}{z_2}\right) + \hat{N}_2\left(\frac{1}{z_1},\frac{1}{z_2}\right)\right] + \frac{4}{z^4}\int_1^{1/z} d\left(\frac{1}{z_2}\right) z_2^4\int_0^{1/z_2} d\left(\frac{1}{z_1}\right)\frac{1}{(1/z_2-1/z_1)^2}\Im\left[\hat{N}_1\left(\frac{1}{z_1},\frac{1}{z_2}\right) + \hat{N}_2\left(\frac{1}{z_1},\frac{1}{z_2}\right)\right]. \tag{12}$$

From (9), (11), and (12), the intrinsic FF $\Delta\hat{G}_{3\bar{T}}(z)$ is also written by the dynamical ones. For the derivation of the twist-3 cross section for $pp \to \Lambda^\uparrow X$, one needs derivatives of the kinematic FFs. From (11) and (12), we can obtain those derivatives in terms of the kinematical FFs themselves and the dynamical ones as

$$\frac{1}{z}\frac{\partial}{\partial(1/z)}[\hat{G}_T^{(1)}(z)] = -2\Im\tilde{D}_{FT}(z) + 2\hat{G}_T^{(1)}(z) + 4\int_0^{1/z} d\left(\frac{1}{z'}\right)\frac{1}{\frac{1}{z}-\frac{1}{z'}}\Im\left\{\hat{N}_1\left(\frac{1}{z'},\frac{1}{z}\right) - \hat{N}_2\left(\frac{1}{z}-\frac{1}{z'},\frac{1}{z}\right)\right\} + \frac{2}{z}\int_0^{1/z} d\left(\frac{1}{z'}\right)\frac{1}{(\frac{1}{z}-\frac{1}{z'})^2}\Im\left\{\hat{N}_1\left(\frac{1}{z'},\frac{1}{z}\right) + \hat{N}_2\left(\frac{1}{z'},\frac{1}{z}\right) - 2\hat{N}_2\left(\frac{1}{z}-\frac{1}{z'},\frac{1}{z}\right)\right\}, \tag{13}$$

and

$$\frac{1}{z}\frac{\partial}{\partial(1/z)}[\Delta\hat{H}_T^{(1)}(z)] = -4\Im\tilde{D}_{FT}(z) + 4\Delta\hat{H}_T^{(1)}(z) + 8\int_0^{1/z} d\left(\frac{1}{z'}\right)\frac{1}{\frac{1}{z}-\frac{1}{z'}}\Im\left\{\hat{N}_1\left(\frac{1}{z'},\frac{1}{z}\right) + \hat{N}_2\left(\frac{1}{z'},\frac{1}{z}\right)\right\} + \frac{4}{z}\int_0^{1/z} d\left(\frac{1}{z'}\right)\frac{1}{(\frac{1}{z}-\frac{1}{z'})^2}\Im\left\{\hat{N}_1\left(\frac{1}{z'},\frac{1}{z}\right) + \hat{N}_2\left(\frac{1}{z'},\frac{1}{z}\right)\right\}. \tag{14}$$

In the next section, we will use (9), (13), and (14) to write the twist-3 gluon FF contribution to $pp \to \Lambda^\uparrow X$ in a frame-independent form.

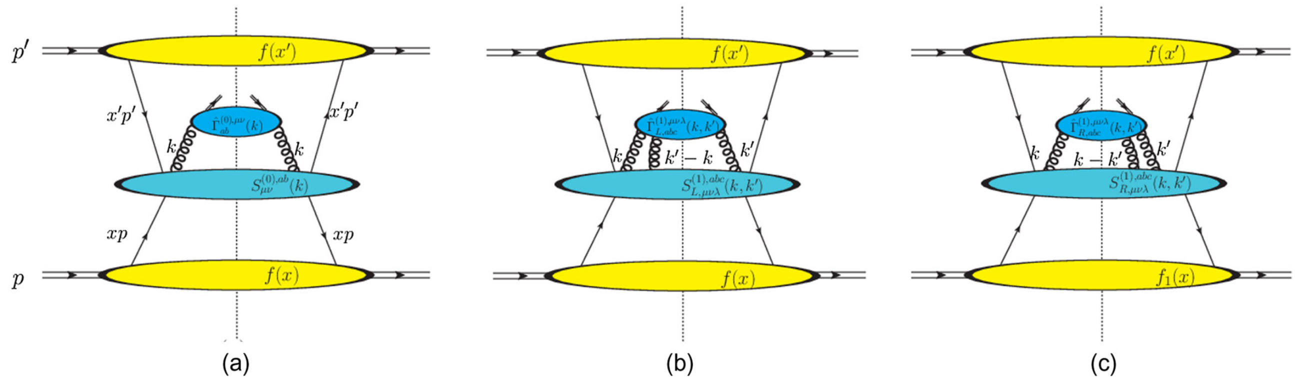

FIG. 1. Diagrams representing the twist-3 gluon fragmentation contribution to $pp \to \Lambda^{\uparrow}X$.

III. TWIST-3 GLUON FRAGMENTATION CONTRIBUTION TO $pp \to \Lambda^{\uparrow}X$

In this section, we develop a formalism for calculating the twist-3 gluon fragmentation contribution to

$$p(p) + p(p') \to \Lambda^{\uparrow}(P_h, S_\perp) + X, \tag{15}$$

where p, p', and P_h are the momenta of the particles and $S_\perp$ is the transverse spin vector of $\Lambda^{\uparrow}$. We work in Feynman gauge so that one can check the appearance of the gauge-invariant FFs explicitly. As in the case of the twist-3 quark FF [46,55], naively T-odd FFs give rise to the cross section as a nonpole contribution. The twist-3 FF contribution to (15) can be written as

$$E_h \frac{d\sigma(p, p', P_h; S_\perp)}{d^3P_h} = \frac{1}{16\pi^2 S_E} \int \frac{dx}{x} f_1(x) \int \frac{dx'}{x'} f_1(x') W_{q,g}(xp, x'p', P_h/z, S_\perp), \tag{16}$$

where $S_E = (p + p')^2$ is the center-of-mass energy squared; $E_h = \sqrt{M_h^2 + \vec{P}_h^2}$ is the energy of the hyperon; x, x' are the momentum fractions of the partons coming out of the initial nucleons; and $f_1(x)$ is the unpolarized quark or gluon distributions in the nucleon. W_q (W_g) is the hadronic tensor representing the partonic hard scattering followed by the fragmentation of a quark (gluon) into the final $\Lambda^{\uparrow}$. In (16), summation over all possible channels is implied. The LO cross section for W_q was derived in Ref. [46]. Here, we focus on the twist-3 gluon fragmentation contribution W_g in (16), which is diagrammatically shown in Fig. 1. W_g consists of three terms corresponding to Figs. 1(a)–1(c),

$$\begin{aligned} W_g(xp, x'p', P_h; S_\perp) &\equiv W_g^{(a)} + W_g^{(b)} + W_g^{(c)} \\ &= \int \frac{d^4k}{(2\pi)^4} [\Gamma_{ab}^{(0),\mu\nu}(k) S_{\mu\nu}^{(0),ab}(k)] + \frac{1}{2} \int \frac{d^4k}{(2\pi)^4} \int \frac{d^4k'}{(2\pi)^4} [\Gamma_{L,abc}^{(1),\mu\nu\lambda}(k, k') S_{L,\mu\nu\lambda}^{(1),abc}(k, k') \\ &\quad + \Gamma_{R,abc}^{(1),\mu\nu\lambda}(k, k') S_{R,\mu\nu\lambda}^{(1),abc}(k, k')], \end{aligned} \tag{17}$$

where the factor $1/2$ in front of $W_g^{(b)}$ and $W_g^{(c)}$ takes into account the exchange symmetry of the gluon fields in the fragmentation matrix elements. The 4-momenta k and k' are those of the partons fragmenting into the final Λ. $S_{\mu\nu}^{(0),ab}(k)$, $S_{L,\mu\nu\lambda}^{(1),abc}(k, k')$, and $S_{R,\mu\nu\lambda}^{(1),abc}(k, k')$ are the partonic hard scattering parts, and $\hat{\Gamma}_{ab}^{(0),\mu\nu}(k)$, $\hat{\Gamma}_{L,abc}^{(1),\mu\nu\lambda}(k, k')$, and $\hat{\Gamma}_{R,abc}^{(1),\mu\nu\lambda}(k, k')$ are the corresponding hadronic matrix elements representing fragmentation of partons into h ($h = \Lambda^{\uparrow}$). The upper indices (0) and (1) represent the number of extra gluon lines compared with the lowest-order gluon fragmentation contribution to the cross section. Hadronic matrix elements are defined as

$$\Gamma_{ab}^{(0),\mu\nu}(k) = \sum_X \int d^4\xi e^{-ik\xi} \langle 0|A_b^\nu(0)|hX\rangle \langle hX|A_a^\mu(\xi)|0\rangle, \tag{18}$$

$$\Gamma^{(1),\mu\nu\lambda}_{L,abc}(k,k') = \sum_X \int d^4\xi \int d^4\eta e^{-ik\xi} e^{-i(k'-k)\eta} \langle 0|A^\nu_b(0)|hX\rangle\langle hX|A^\mu_a(\xi) g A^\lambda_c(\eta)|0\rangle, \tag{19}$$

$$\Gamma^{(1),\mu\nu\lambda}_{R,abc}(k,k') = \sum_X \int d^4\xi \int d^4\eta e^{-ik\xi} e^{-i(k'-k)\eta} \langle 0|A^\nu_b(0) g A^\lambda_c(\eta)|hX\rangle\langle hX|A^\mu_a(\xi)|0\rangle, \tag{20}$$

where the gauge coupling g associated with the attachment of the extra gluon line to the hard part is included in $\hat{\Gamma}^{(1)}_L(k,k')$ and $\hat{\Gamma}^{(1)}_R(k,k')$. Therefore, the hard parts $S^{(0)}$, $S^{(1)}_L$, and $S^{(1)}_R$ are of $O(g^4)$ in the LO calculation. From Hermiticity, one has $\hat{\Gamma}^{(1),\mu\nu\lambda}_{L,abc}(k,k')^\star = \hat{\Gamma}^{(1)\nu\mu\lambda}_{R,bac}(k',k)$ and $S^{(1),abc}_{L,\mu\nu\lambda}(k,k')^\star = S^{(1),bac}_{R,\nu\mu\lambda}(k',k)$, which guarantees W_g in (17) is real. One can extract the twist-3 effect from (17) by applying the collinear expansion. The collinear expansion of $S^{(0)}$, $S^{(1)}_L$, and $S^{(1)}_R$ with respect to the parton momenta k^μ and k'^μ around the parent hadron's momentum P^μ_h reads

$$S^{(0),ab}_{\mu\nu}(k) = S^{(0),ab}_{\mu\nu}(z) + \frac{\partial S^{(0),ab}_{\mu\nu}(k)}{\partial k^\alpha}\bigg|_{\rm c.l.} \Omega^\alpha{}_\sigma k^\sigma + \frac{1}{2}\frac{\partial^2 S^{(0),ab}_{\mu\nu}(k)}{\partial k^\alpha \partial k^\beta}\bigg|_{\rm c.l.} \Omega^\alpha_\sigma k^\sigma \Omega^\beta_\rho k^\rho + \frac{1}{6}\frac{\partial^3 S^{(0),ab}_{\mu\nu}(k)}{\partial k^\alpha \partial k^\beta \partial k^\gamma}\bigg|_{\rm c.l.} \Omega^\alpha{}_\sigma k^\sigma \Omega^\beta{}_\rho k^\rho \Omega^\gamma_\tau k^\tau + \cdots, \tag{21}$$

and

$$\begin{aligned} S^{(1),abc}_{L,\mu\nu\lambda}(k,k') =& S^{(1),abc}_{L,\mu\nu\lambda}(z,z') + \frac{\partial S^{(1),abc}_{L,\mu\nu\lambda}(k,k')}{\partial k^\alpha}\bigg|_{\rm c.l.} \Omega^\alpha{}_\sigma k^\sigma + \frac{\partial S^{(1),abc}_{L,\mu\nu\lambda}(k,k')}{\partial k'^\alpha}\bigg|_{\rm c.l.} \Omega^\alpha{}_\sigma k'^\sigma + \frac{1}{2}\frac{\partial^2 S^{(1),abc}_{L,\mu\nu\lambda}(k,k')}{\partial k^\alpha \partial k^\beta}\bigg|_{\rm c.l.} \Omega^\alpha{}_\sigma k^\sigma \Omega^\beta{}_\rho k^\rho \\ &+ \frac{1}{2}\frac{\partial^2 S^{(1),abc}_{L,\mu\nu\lambda}(k,k')}{\partial k'^\alpha \partial k'^\beta}\bigg|_{\rm c.l.} \Omega^\alpha{}_\sigma k'^\sigma \Omega^\beta{}_\rho k'^\rho + \frac{\partial^2 S^{(1),abc}_{L,\mu\nu\lambda}(k,k')}{\partial k^\alpha \partial k'^\beta}\bigg|_{\rm c.l.} \Omega^\alpha{}_\sigma k^\sigma \Omega^\beta{}_\rho k'^\rho + \frac{1}{6}\frac{\partial^3 S^{(1),abc}_{L,\mu\nu\lambda}(k,k')}{\partial k^\alpha \partial k^\beta \partial k^\gamma}\bigg|_{\rm c.l.} \Omega^\alpha{}_\sigma k^\sigma \Omega^\beta{}_\rho k^\rho \Omega^\gamma{}_\tau k^\tau \\ &+ \frac{1}{6}\frac{\partial^3 S^{(1),abc}_{L,\mu\nu\lambda}(k,k')}{\partial k'^\alpha \partial k'^\beta \partial k'^\gamma}\bigg|_{\rm c.l.} \Omega^\alpha{}_\sigma k'^\sigma \Omega^\beta{}_\rho k'^\rho \Omega^\gamma{}_\tau k'^\tau + \frac{1}{2}\frac{\partial^3 S^{(1),abc}_{L,\mu\nu\lambda}(k,k')}{\partial k^\alpha \partial k^\beta \partial k'^\gamma}\bigg|_{\rm c.l.} \Omega^\alpha{}_\sigma k^\sigma \Omega^\beta{}_\rho k^\rho \Omega^\gamma{}_\tau k'^\tau \\ &+ \frac{1}{2}\frac{\partial^3 S^{(1),abc}_{L,\mu\nu\lambda}(k,k')}{\partial k^\alpha \partial k'^\beta \partial k'^\gamma}\bigg|_{\rm c.l.} \Omega^\alpha{}_\sigma k^\sigma \Omega^\beta{}_\rho k'^\rho \Omega^\gamma{}_\tau k'^\tau + \cdots, \end{aligned} \tag{22}$$

where $\Omega^\alpha{}_\mu \equiv g^\alpha{}_\mu - P^\alpha{}_h w_\mu$; $S^{(0),ab}_{\mu\nu}(z) \equiv S^{(0),ab}_{\mu\nu}(P_h/z)$; $S^{(1),abc}_{L,\mu\nu\lambda}(z,z') \equiv S^{(1),abc}_{L,\mu\nu\lambda}(P_h/z, P_h/z')$; "$|_{\rm c.l.}$" indicates taking the collinear limit, i.e., $k \to P_h/z$ and $k' \to P_h/z'$; and $\cdots$ denotes the contribution of twist 4 or higher. The collinear expansion for $S^{(1)}_R$ can be performed similarly. We also decompose the gauge field A^μ as

$$A^\mu = (A\cdot w)P^\mu_h + \Omega^\mu{}_\nu A^\nu. \tag{23}$$

Inserting this decomposition into the expression for $\hat{\Gamma}^{(0)}$, $\hat{\Gamma}^{(1)}_L$, and $\hat{\Gamma}^{(1)}_R$, one obtains

$$\Gamma^{(0),\mu\nu}_{ab} = P^\mu_h P^\nu_h \Gamma^{(0),ww}_{ab} + P^\mu_h \Omega^\nu{}_\rho \Gamma^{(0),w\rho}_{ab} + P^\nu_h \Omega^\mu{}_\sigma \Gamma^{(0),\sigma w}_{ab} + \Omega^\mu{}_\sigma \Omega^\nu{}_\rho \Gamma^{(0),\sigma\rho}_{ab}, \tag{24}$$

$$\begin{aligned} \Gamma^{(1),\mu\nu\lambda}_{L,abc} =& P^\mu_h P^\nu_h P^\lambda_h \Gamma^{(1),www}_{L,abc} + P^\mu_h P^\nu_h \Omega^\lambda{}_\tau \Gamma^{(1),ww\tau}_{L,abc} + P^\mu_h P^\lambda_h \Omega^\nu{}_\rho \Gamma^{(1),w\rho w}_{L,abc} + P^\nu_h P^\lambda_h \Omega^\mu{}_\sigma \Gamma^{(1),\sigma ww}_{L,abc} \\ &+ P^\mu_h \Omega^\nu{}_\rho \Omega^\lambda{}_\tau \Gamma^{(1),w\rho\tau}_{L,abc} + P^\nu_h \Omega^\mu{}_\sigma \Omega^\lambda{}_\tau \Gamma^{(1),\sigma w\tau}_{L,abc} + P^\lambda_h \Omega^\mu{}_\sigma \Omega^\nu{}_\rho \Gamma^{(1),\sigma\rho w}_{L,abc} + \Omega^\mu{}_\sigma \Omega^\nu{}_\rho \Omega^\lambda{}_\tau \Gamma^{(1),\sigma\rho\tau}_{L,abc}, \end{aligned} \tag{25}$$

and likewise for $\Gamma^{(1),\mu\nu\lambda}_{R,abc}$. Inserting the above expansion into (17) and keeping the terms with two or three $\Omega^\mu{}_\nu$s in the product of the hard parts $S^{(0)}, S^{(1)}_L, S^{(1)}_R$ and the fragmentation matrix elements $\hat{\Gamma}^{(0)}, \hat{\Gamma}^{(1)}_L, \hat{\Gamma}^{(1)}_R$, one can obtain the twist-2 and -3 contributions to the cross section. To get a gauge-invariant cross section, one needs to fully utilize the following Ward identities for the hard parts (see Appendix A):

$$k^\mu S^{ab}_{\mu\nu}(k) = k^\nu S^{ab}_{\mu\nu}(k) = 0, \tag{26}$$

$$(k'-k)^\lambda S^{abc}_{L\mu\nu\lambda}(k,k') = \frac{-if^{abc}}{N^2-1} S_{\mu\nu}(k') + G^{abc}_{\mu\nu}(k,k'), \tag{27}$$

$$k^\mu S^{abc}_{L\mu\nu\lambda}(k,k') = \frac{if^{abc}}{N^2-1} S_{\lambda\nu}(k') + G^{cab}_{\lambda\nu}(k'-k,k'), \tag{28}$$

$$k'^{\nu} S^{abc}_{L\mu\nu\lambda}(k,k') = 0, \tag{29}$$

$$(k'-k)^{\lambda} S^{abc}_{R\mu\nu\lambda}(k,k') = \frac{if^{abc}}{N^2-1} S_{\mu\nu}(k) + (G^{bac}_{\nu\mu}(k',k))^{\star}, \tag{30}$$

$$k'^{\nu} S^{abc}_{R\mu\nu\lambda}(k,k') = \frac{if^{abc}}{N^2-1} S_{\mu\lambda}(k) + (G^{cab}_{\lambda\mu}(k-k',k))^{\star}, \tag{31}$$

$$k^{\mu} S^{abc}_{R\mu\nu\lambda}(k,k') = 0. \tag{32}$$

Here and below, we suppress the upper indices (0) and (1) from the hard parts $S^{(0)}$, $S^{(1)}_L$, and $S^{(1)}_R$ for simplicity and $S^{\mu\nu}(k) \equiv S^{\mu\nu,ab}(k)\delta_{ab}$. The G terms appear due to the off-shell-ness of the parton momenta entering the fragmentation matrix elements. We present the actual forms of those "ghost terms" in Appendix A. Here, we only mention that they are proportional to f^{abc} and satisfy the relation

$$k^{\mu} G^{abc}_{\mu\nu}(k,k') = k'^{\nu} G^{abc}_{\mu\nu}(k,k') = 0. \tag{33}$$

We have found that the ghost terms do not contribute to the twist-3 cross section.[3] We thus discard them in the following. To get the twist-3 cross section in the gauge-invariant form, we use the collinear limit of these identities as well as the first, second, and third derivatives with respect to k and k' of these Ward identities in the collinear limit.

After very lengthy calculation, one eventually obtains the twist-2 and -3 contributions from Figs. 1(a)–1(c) to W_g in the following form:

$$\begin{aligned} W^{(a)}_g &= \Omega^{\mu}{}_{\alpha}\Omega^{\nu}{}_{\beta}\sum_X \int d\left(\frac{1}{z}\right) z^2 (\Gamma^{(0)\alpha\beta}(z) - \Gamma^{(1)\alpha\beta}_{AA}(z) - \Gamma^{(1)\alpha\beta}_{AAR}(z)) S_{\mu\nu}(z) \\ &\quad - i\Omega^{\mu}{}_{\alpha}\Omega^{\nu}{}_{\beta}\Omega^{\lambda}{}_{\gamma}\sum_X \int d\left(\frac{1}{z}\right) z^2 (\Gamma^{(0)\alpha\beta\gamma}_{\partial}(z) - \Gamma^{(1)\alpha\beta\gamma}_{AA\partial}(z) - \Gamma^{(1)\alpha\beta\gamma}_{AAR\partial}(z)) \frac{\partial S_{\mu\nu}(k)}{\partial k^{\lambda}}\bigg|_{\text{c.l.}}, \end{aligned} \tag{34}$$

$$\begin{aligned} W^{(b)}_g &= \Omega^{\mu}{}_{\alpha}\Omega^{\nu}{}_{\beta}\sum_X \int d\left(\frac{1}{z}\right) z^2 \left(\Gamma^{(1)\alpha\beta}_{[A]}(z) + \Gamma^{(1)\alpha\beta}_{AA}(z)\right) S_{\mu\nu}(z) \\ &\quad - i\Omega^{\mu}{}_{\alpha}\Omega^{\nu}{}_{\beta}\Omega^{\lambda}{}_{\gamma}\sum_X \int d\left(\frac{1}{z}\right) z^2 (\Gamma^{(1)\alpha\beta\gamma}_{\partial[F]}(z) + \Gamma^{(1)\alpha\beta\gamma}_{\partial[\partial A]}(z) + \Gamma^{(1)\alpha\beta\gamma}_{\partial[A]}(z) + \Gamma^{(1)\alpha\beta\gamma}_{AA\partial}(z)) \frac{\partial S_{\mu\nu}(k)}{\partial k^{\lambda}}\bigg|_{\text{c.l.}} \\ &\quad + \frac{i}{2}\Omega^{\mu}{}_{\alpha}\Omega^{\nu}{}_{\beta}\Omega^{\lambda}{}_{\gamma}\sum_X \int d\left(\frac{1}{z}\right) \int d\left(\frac{1}{z'}\right) \frac{zz'}{1/z' - 1/z} \Gamma^{(1)\alpha\beta\gamma}_{F,abc}\left(\frac{1}{z},\frac{1}{z'}\right) S^{L,abc}_{\mu\nu\lambda}(z,z'), \end{aligned} \tag{35}$$

$$\begin{aligned} W^{(c)}_g &= \Omega^{\mu}{}_{\alpha}\Omega^{\nu}{}_{\beta} \int d\left(\frac{1}{z}\right) z^2 (\Gamma^{(1)\alpha\beta}_{[AR]}(z) + \Gamma^{(1)\alpha\beta}_{AAR}(z)) S_{\mu\nu}(z) \\ &\quad - i\Omega^{\mu}{}_{\alpha}\Omega^{\nu}{}_{\beta}\Omega^{\lambda}{}_{\gamma}\sum_X \int d\left(\frac{1}{z}\right) z^2 (\Gamma^{(1)\alpha\beta\gamma}_{\partial[A\partial]}(z) + \Gamma^{(1)\alpha\beta\gamma}_{AAR\partial}(z)) \frac{\partial S_{\mu\nu}(k)}{\partial k^{\lambda}}\bigg|_{\text{c.l.}} \\ &\quad + \frac{i}{2}\Omega^{\mu}{}_{\alpha}\Omega^{\nu}{}_{\beta}\Omega^{\lambda}{}_{\gamma}\sum_X \int d\left(\frac{1}{z}\right) \int d\left(\frac{1}{z'}\right) \frac{zz'}{1/z' - 1/z} \Gamma^{(1)\alpha\beta\gamma}_{FR,abc}\left(\frac{1}{z},\frac{1}{z'}\right) S^{R,abc}_{\mu\nu\lambda}(z,z'). \end{aligned} \tag{36}$$

Each fragmentation matrix elements Γ appearing in the above expression is defined as follows:

$$\Gamma^{(0)\alpha\beta}(z) = \frac{\delta^{ab}}{N^2-1}\sum_X \int \frac{d\lambda}{2\pi} e^{-i\frac{\lambda}{z}} \langle 0|F^{\beta w}_b(0)|hX\rangle\langle hX|F^{\alpha w}_a(\lambda w)|0\rangle, \tag{37}$$

$$\Gamma^{(0)\alpha\beta\gamma}_{\partial}(z) = \frac{\delta^{ab}}{N^2-1}\sum_X \int \frac{d\lambda}{2\pi} e^{-i\frac{\lambda}{z}} \langle 0|F^{\beta w}_b(0)|hX\rangle\langle hX|F^{\alpha w}_a(\lambda w)\overleftarrow{\partial}^{\gamma}|0\rangle, \tag{38}$$

[3] This was also the case for the twist-3 quark FF contribution to $ep^{\uparrow} \to ehX$ [55] and the three-gluon distribution function contribution to $\vec{p}p^{\uparrow} \to DX$ [56].

$$\Gamma_{AA}^{(1)\alpha\beta}(z) = \frac{1}{N^2-1}\sum_X \int \frac{d\lambda}{2\pi} e^{-i\frac{\lambda}{z}} \langle 0|F_a^{\beta w}(0)|hX\rangle\langle hX|gf_{abc}A_b^\alpha(\lambda w)A_c^w(\lambda w)|0\rangle, \tag{39}$$

$$\Gamma_{AAR}^{(1)\alpha\beta}(z) = \frac{1}{N^2-1}\sum_X \int \frac{d\lambda}{2\pi} e^{-i\frac{\lambda}{z}} \langle 0|gf_{abc}A_b^\beta(0)A_c^w(0)|hX\rangle\langle hX|F_a^{\alpha w}(\lambda w)|0\rangle, \tag{40}$$

$$\Gamma_{AA\partial}^{(1)\alpha\beta\gamma}(z) = \frac{1}{N^2-1}\sum_X \int \frac{d\lambda}{2\pi} e^{-i\frac{\lambda}{z}} \langle 0|F_a^{\beta w}(0)|hX\rangle\langle hX|gf_{abc}A_b^\alpha(\lambda w)A_c^w(\lambda w)\overleftarrow{\partial}^\gamma|0\rangle, \tag{41}$$

$$\Gamma_{AAR\partial}^{(1)\alpha\beta\gamma}(z) = \frac{1}{N^2-1}\sum_X \int \frac{d\lambda}{2\pi} e^{-i\frac{\lambda}{z}} \langle 0|gf_{abc}A_b^\beta(0)A_c^w(0)|hX\rangle\langle hX|F_a^{\alpha w}(\lambda w)\overleftarrow{\partial}^\gamma|0\rangle, \tag{42}$$

$$\Gamma_{[A]}^{(1)\alpha\beta}(z) = \frac{(-if^{abc})}{N^2-1}\sum_X \int \frac{d\lambda}{2\pi} e^{-i\frac{\lambda}{z}} \langle 0|F_b^{\beta w}(0)|hX\rangle\langle hX|F_a^{\alpha w}(\lambda w)ig\int_\infty^\lambda d\mu A_c^w(\mu w)|0\rangle, \tag{43}$$

$$\Gamma_{\partial[F]}^{(1)\alpha\beta\gamma}(z) = \frac{(-if^{abc})}{N^2-1}\sum_X \int \frac{d\lambda}{2\pi} e^{-i\frac{\lambda}{z}} \langle 0|F_b^{\beta w}(0)|hX\rangle\langle hX|F_a^{\alpha w}(\lambda w)ig\int_\infty^\lambda d\mu F_c^{\gamma w}(\mu w)|0\rangle, \tag{44}$$

$$\Gamma_{\partial[\partial A]}^{(1)\alpha\beta\gamma}(z) = \frac{(-if^{abc})}{N^2-1}\sum_X \int \frac{d\lambda}{2\pi} e^{-i\frac{\lambda}{z}} \langle 0|F_b^{\beta w}(0)|hX\rangle\langle hX|F_a^{\alpha w}(\lambda w)\overleftarrow{\partial}^\gamma ig\int_\infty^\lambda d\mu A_c^w(\mu w)|0\rangle, \tag{45}$$

$$\Gamma_{\partial[A]}^{(1)\alpha\beta\gamma}(z) = \frac{(-if^{abc})}{N^2-1}\sum_X \int \frac{d\lambda}{2\pi} e^{-i\frac{\lambda}{z}} \langle 0|F_b^{\beta w}(0)|hX\rangle\langle hX|F_a^{\alpha w}(\lambda w)igA_c^\gamma(\lambda w)|0\rangle, \tag{46}$$

$$\Gamma_{F,abc}^{(1)\alpha\beta\gamma}\left(\frac{1}{z},\frac{1}{z'}\right) = \int \frac{d\lambda}{2\pi}\int \frac{d\mu}{2\pi} e^{-i\frac{\lambda}{z}} e^{-i\mu(\frac{1}{z'}-\frac{1}{z})} \langle 0|F_b^{\beta w}(0)|hX\rangle\langle hX|F_a^{\alpha w}(\lambda w)gF_c^{\gamma w}(\mu w)|0\rangle, \tag{47}$$

$$\Gamma_{[AR]}^{(1)\alpha\beta}(z) = \frac{(-if^{abc})}{N^2-1}\sum_X \int \frac{d\lambda}{2\pi} e^{-i\frac{\lambda}{z}} \langle 0|ig\int_0^\infty d\mu A_c^w(\mu w)F_b^{\beta w}(0)|hX\rangle\langle hX|F_a^{\alpha w}(\lambda w)|0\rangle, \tag{48}$$

$$\Gamma_{\partial[A\partial]}^{(1)\alpha\beta\gamma}(z) = \frac{(-if^{abc})}{N^2-1}\sum_X \int \frac{d\lambda}{2\pi} e^{-i\frac{\lambda}{z}} \langle 0|ig\int_0^\infty d\mu A_c^w(\mu w)F_b^{\beta w}(0)|hX\rangle\langle hX|F_a^{\alpha w}(\lambda w)\overleftarrow{\partial}^\gamma|0\rangle, \tag{49}$$

$$\Gamma_{FR,abc}^{(1)\alpha\beta\gamma}\left(\frac{1}{z},\frac{1}{z'}\right) = \sum_X \int \frac{d\lambda}{2\pi}\int \frac{d\mu}{2\pi} e^{-i\frac{\lambda}{z}} e^{-i\mu(\frac{1}{z'}-\frac{1}{z})} \langle 0|gF_c^{\gamma w}(\mu w)F_b^{\beta w}(0)|hX\rangle\langle hX|F_a^{\alpha w}(\lambda w)|0\rangle. \tag{50}$$

In the LO calculation of Figs. 1(a)–1(c), only $O(1)$ and $O(g)$ contributions from the hadronic matrix elements are produced. We thus note that, in (34)–(36), one can identify the correlation functions of the field strength $F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gf^{abc}A_\mu^b A_\nu^c$ to this accuracy. The first term in $W_g^{(a)}$ is the $O(1)$ contribution from $\hat{\Gamma}(z)$ in (1), ignoring $O(g^2)$ terms $\sim\langle 0|gf^{abc}A^bA^c|hX\rangle\langle hX|gf^{ab'c'}A^{b'}A^{c'}|0\rangle$. The first terms in $W_g^{(b)}$ and $W_g^{(c)}$ are the $O(g)$ terms arising from the expansion of the gauge link and the $O(g)$ part of the field strength in $\hat{\Gamma}(z)$. These terms contain both twist-2 and intrinsic twist-3 FFs. Likewise, the second term in $W_g^{(a)}$ is the $O(1)$ contribution from the kinematical twist-3 FFs $\hat{\Gamma}_\partial(z)$ in (2). The second terms in $W_g^{(b)}$ and $W_g^{(c)}$ are the $O(g)$ terms arising from the expansion of the gauge link and the $O(g)$ part of the field strength in $\hat{\Gamma}_\partial(z)$. The third term in $W_g^{(b)}$ is the $O(g)$ contribution from $\hat{\Gamma}_F$ defined in (5) and (6). Likewise, the third term in $W_g^{(c)}$ is associated with $\hat{\Gamma}_{FR} \sim (\hat{\Gamma}_F)^\star$. This way, we have obtained the sum of $W_g^{(a)}$, $W_g^{(b)}$, and $W_g^{(c)}$ in the color-gauge-invariant form in terms of the intrinsic, kinematical, and dynamical FFs. Inserting these expressions into (16), one can eventually express the twist-3 gluon FF contribution to the cross section as

$$E_h \frac{d\sigma(p,p',P_h;S_\perp)}{d^3P_h} = \frac{1}{16\pi^2 S_E}\int_0^1 \frac{dx}{x} f_1(x)\int_0^1 \frac{dx'}{x'} f_1(x')\left[\Omega^\mu{}_\alpha \Omega^\nu{}_\beta \int_0^1 dz\,\mathrm{Tr}[\hat{\Gamma}^{\alpha\beta}(z)S_{\mu\nu}(P_h/z)]\right.$$
$$- i\Omega^\mu{}_\alpha\Omega^\nu{}_\beta\Omega^\lambda{}_\gamma\int_0^1 dz\,\mathrm{Tr}\left[\hat{\Gamma}_\partial^{\alpha\beta\gamma}(z)\frac{\partial S_{\mu\nu}(k)}{\partial k^\lambda}\bigg|_{\rm c.l.}\right] + \Re\left\{i\Omega^\mu{}_\alpha\Omega^\nu{}_\beta\Omega^\lambda{}_\gamma\int_0^1\frac{dz}{z}\int_z^\infty\frac{dz'}{z'}\left(\frac{1}{1/z-1/z'}\right)\right.$$
$$\left.\left.\times\,\mathrm{Tr}\left[\left(-\frac{if^{abc}}{N}\hat{\Gamma}_{FA}^{\alpha\beta\gamma}\left(\frac{1}{z'},\frac{1}{z}\right)+d^{abc}\frac{N}{N^2-4}\hat{\Gamma}_{FS}^{\alpha\beta\gamma}\left(\frac{1}{z'},\frac{1}{z}\right)\right)S^L_{\mu\nu\lambda,abc}(z',z)\right]\right\}\right], \tag{51}$$

where we have used the expression (4) and $|_{\rm c.l.}$ implies the collinear limit, $k\to P_h/z$. In writing down the contribution from the dynamical FFs in (51), we have interchanged the role of the variables, z and z', from (35) and (36) for later convenience.

Next, we substitute (1), (2), (5), and (6) into (51). We recall that the $q\bar{q}g$-type FFs (8) are related to the purely gluonic FFs, as was shown in Sec. II B. Therefore, we consider the contribution shown in Fig. 2 together. Note also that each hard part contains the factor $\delta((xp+x'p'-k)^2)\sim\delta((xp+x'p'-P_h/z)^2)$ corresponding to the on-shell condition for the final unobserved parton, and its derivative with respect to k causes the derivative of the kinematical FFs by partial integration with respect to $1/z$. Separating various contributions based on the z' dependence of the hard cross sections for the dynamical FFs (see Appendix C), we can write the cross section as

$$E_h\frac{d\sigma(p,p',P_h;S_\perp)}{d^3P_h} = \frac{M_h\alpha_s^2}{S_E}\int_0^1\frac{dx}{x}f_1(x)\int_0^1\frac{dx'}{x'}f_1(x')\int_0^1 dz\,\delta((xp+x'p'-P_h/z)^2)$$
$$\times\left\{\frac{\Delta\hat{G}_{3\bar{T}}(z)}{z}\hat{H}_{\rm int}+\left(\hat{G}_T^{(1)}(z)\hat{H}_{NDG}+\frac{1}{z}\frac{\partial\hat{G}_T^{(1)}(z)}{\partial(1/z)}\hat{H}_{DG}\right)+\left(\Delta\hat{H}_T^{(1)}(z)\hat{H}_{NDH}+\frac{1}{z}\frac{\partial\Delta\hat{H}_T^{(1)}(z)}{\partial(1/z)}\hat{H}_{DH}\right)\right.$$
$$+\int_0^{1/z}d\left(\frac{1}{z'}\right)\left[\sum_{i=1}^3\Im\hat{N}_i\left(\frac{1}{1/z-1/z'}\hat{H}_1^{N_i}+\frac{1}{z}\left(\frac{1}{1/z-1/z'}\right)^2\hat{H}_2^{N_i}+z'\hat{H}_3^{N_i}+\frac{z'^2}{z}\hat{H}_4^{N_i}\right)\right.$$
$$\left.+\sum_{i=1}^3\Im\hat{O}_i\left(\frac{1}{1/z-1/z'}\hat{H}_1^{O_i}+\frac{1}{z}\left(\frac{1}{1/z-1/z'}\right)^2\hat{H}_2^{O_i}+z'\hat{H}_3^{O_i}+\frac{z'^2}{z}\hat{H}_4^{O_i}\right)\right]$$
$$+\int_0^{1/z}d\left(\frac{1}{z'}\right)\frac{2}{C_F}\left[\Im\tilde{D}_F\left(\frac{1}{z'},\frac{1}{z'}-\frac{1}{z}\right)\left(\hat{H}_{DF1}+\frac{1}{z}\frac{1}{1/z-1/z'}\hat{H}_{DF2}+\frac{z'}{z}\hat{H}_{DF3}\right)\right.$$
$$\left.\left.+\Im\tilde{G}_F\left(\frac{1}{z'},\frac{1}{z'}-\frac{1}{z}\right)\left(\hat{H}_{GF1}+\frac{1}{z}\frac{1}{1/z-1/z'}\hat{H}_{GF2}+\frac{z'}{z}\hat{H}_{GF3}\right)\right]\right\}, \tag{52}$$

where $\hat{H}_{\rm int}$, $\hat{H}_{NDG}$, $\hat{H}_{DG}$, etc., represent the partonic hard cross sections for each FF (after separating the z' dependence for dynamical FFs), and they are the functions of the Mandelstam variables in the parton level, $s=(xp+x'p')^2=2xx'p\cdot p'$, $t=(xp-P_h/z)^2=-2(x/z)p\cdot P_h$, and $u=(x'p'-P_h/z)^2=-2(x'/z)p'\cdot P_h$, multiplied by kinematic factors with $\epsilon^{pP_hwS_\perp}$ and $\epsilon^{p'P_hwS_\perp}$. [See Eq. (54) below as an example.] In (52), we have used the shorthand notation, $\hat{N}_3\equiv-\hat{N}_2(\frac{1}{z}-\frac{1}{z'},\frac{1}{z})$ and $\hat{O}_3\equiv\hat{O}_2(\frac{1}{z}-\frac{1}{z'},\frac{1}{z})$. A change of the variable $\frac{1}{z'}\leftrightarrow\frac{1}{z}-\frac{1}{z'}$ in the z' and z'^2/z terms in the contribution from $\hat{N}_i$ and $\hat{O}_i$ leads to the following form, owing to the exchange symmetry (7) of $\hat{N}_1$ and $\hat{O}_1$:

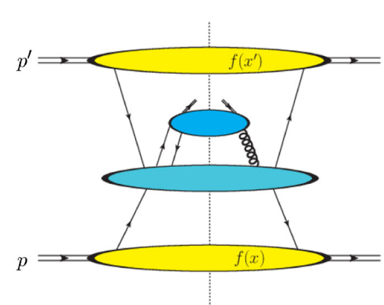

FIG. 2. Diagram for the $q\bar{q}g$-type correlation function in $pp\to\Lambda^\uparrow X$. The mirror diagram also contributes.

$$\begin{aligned}
E_h\frac{d\sigma(p,p',P_h;S_\perp)}{d^3P_h}=&\frac{M_h\alpha_s^2}{S_E}\int_0^1\frac{dx}{x}f_1(x)\int_0^1\frac{dx'}{x'}f_1(x')\int_0^1 dz\delta(s+t+u)\\
&\times\Bigg\{\frac{\Delta\hat{G}_{3\bar{T}}(z)}{z}\hat{H}_{\rm int}+\hat{G}_T^{(1)}(z)\hat{H}_{NDG}+\frac{1}{z}\frac{\partial\hat{G}_T^{(1)}(z)}{\partial(1/z)}\hat{H}_{DG}+\Delta\hat{H}_T^{(1)}(z)\hat{H}_{NDH}+\frac{1}{z}\frac{\partial\Delta\hat{H}_T^{(1)}(z)}{\partial(1/z)}\hat{H}_{DH}\\
&+\int_0^{1/z}d\left(\frac{1}{z'}\right)\Bigg[\frac{1}{1/z-1/z'}\Im\Bigg\{\hat{N}_1\left(\frac{1}{z'},\frac{1}{z}\right)(\hat{H}_1^{N_1}-\hat{H}_3^{N_1})+\hat{N}_2\left(\frac{1}{z'},\frac{1}{z}\right)(\hat{H}_1^{N_2}-\hat{H}_3^{N_3})\\
&-\hat{N}_2\left(\frac{1}{z}-\frac{1}{z'},\frac{1}{z}\right)(\hat{H}_1^{N_3}-\hat{H}_3^{N_2})\Bigg\}\\
&+\frac{1}{z}\left(\frac{1}{1/z-1/z'}\right)^2\Im\Bigg\{\hat{N}_1\left(\frac{1}{z'},\frac{1}{z}\right)(\hat{H}_2^{N_1}-\hat{H}_4^{N_1})+\hat{N}_2\left(\frac{1}{z'},\frac{1}{z}\right)(\hat{H}_2^{N_2}-\hat{H}_4^{N_3})-\hat{N}_2\left(\frac{1}{z}-\frac{1}{z'},\frac{1}{z}\right)(\hat{H}_2^{N_3}-\hat{H}_4^{N_2})\Bigg\}\\
&+\frac{1}{1/z-1/z'}\Im\Bigg\{\hat{O}_1\left(\frac{1}{z'},\frac{1}{z}\right)(\hat{H}_1^{O_1}+\hat{H}_3^{O_1})+\hat{O}_2\left(\frac{1}{z'},\frac{1}{z}\right)(\hat{H}_1^{O_2}+\hat{H}_3^{O_3})+\hat{O}_2\left(\frac{1}{z}-\frac{1}{z'},\frac{1}{z}\right)(\hat{H}_1^{O_3}+\hat{H}_3^{O_2})\Bigg\}\\
&+\frac{1}{z}\left(\frac{1}{1/z-1/z'}\right)^2\Im\Bigg\{\hat{O}_1\left(\frac{1}{z'},\frac{1}{z}\right)(\hat{H}_2^{O_1}+\hat{H}_4^{O_1})+\hat{O}_2\left(\frac{1}{z'},\frac{1}{z}\right)(\hat{H}_2^{O_2}+\hat{H}_4^{O_3})\\
&+\hat{O}_2\left(\frac{1}{z}-\frac{1}{z'},\frac{1}{z}\right)(\hat{H}_2^{O_3}+\hat{H}_4^{O_2})\Bigg\}\Bigg]\\
&+\int_0^{1/z}d\left(\frac{1}{z'}\right)\frac{2}{C_F}\Bigg[\Im\tilde{D}_F\left(\frac{1}{z'},\frac{1}{z'}-\frac{1}{z}\right)\left(\hat{H}_{DF1}+\frac{1}{z}\frac{1}{1/z-1/z'}\hat{H}_{DF2}+\frac{z'}{z}\hat{H}_{DF3}\right)\\
&+\Im\tilde{G}_F\left(\frac{1}{z'},\frac{1}{z'}-\frac{1}{z}\right)\left(\hat{H}_{GF1}+\frac{1}{z}\frac{1}{1/z-1/z'}\hat{H}_{GF2}+\frac{z'}{z}\hat{H}_{GF3}\right)\Bigg]\Bigg\}. \qquad (53)
\end{aligned}$$

Here, we remind the reader that the gauge invariance and the frame independence of the cross section (53) are realized in a very nontrivial manner. We show that those properties are guaranteed by the EOM relation and the LIRs introduced in the previous section. The gauge invariance is satisfied by the EOM relation (9), which is discussed in detail in Appendix B. Here, we demonstrate how the frame independence of the cross section is achieved.

To make clear the issue of frame dependence, we pick up the hard cross sections $\hat{H}_2^{N_1}-\hat{H}_4^{N_1}$, $\hat{H}_{DG}$, and $\hat{H}_{DH}$ in (53) in the $qg\to gq$ channel, as an example. They can be computed to be

$$\begin{aligned}
\hat{H}_2^{N_1}-\hat{H}_4^{N_1}&=\frac{C_F}{N}((2t+u)x\epsilon^{pP_hwS_\perp}+tx'\epsilon^{p'P_hwS_\perp})\left(\frac{s^2+t^2}{s^2t^2}\right)-\frac{1}{2}(u(2t+u)x\epsilon^{pP_hwS_\perp}+t(2t+3u)x'\epsilon^{p'P_hwS_\perp})\frac{s^2+t^2}{stu^3},\\
\hat{H}_{DG}&=-\frac{C_F}{N}(x\epsilon^{pP_hwS_\perp}+x'\epsilon^{p'P_hwS_\perp})\left(\frac{s^2+t^2}{s^2t}\right)+(x\epsilon^{pP_hwS_\perp}+x'\epsilon^{p'P_hwS_\perp})\left(\frac{s^2+t^2}{su^2}\right),\\
\hat{H}_{DH}&=0. \qquad (54)
\end{aligned}$$

We note that each cross section contains the lightlike vector w^μ. On the other hand, the physical cross section should be able to be represented in terms of the vectors p, p' P_h, and $S_\perp$ in a Lorentz-invariant form. Since w^μ is defined from P_h^μ, its actual form depends on the frame. One can express the vector w in terms of p, p', and P_h as[52]

$$w^\mu=\alpha\frac{p^\mu}{p\cdot P_h}+(1-\alpha)\frac{p^\mu}{p'\cdot P_h}+\left\{-\alpha(1-\alpha)\frac{p\cdot p'}{p\cdot P_hp'\cdot P_h}+\beta^2p\cdot p'p\cdot P_hp'\cdot P_h\right\}P_h^\mu+\beta\epsilon^{\mu pp'P_h}, \qquad (55)$$

which satisfies $P_h\cdot w=1$ and $w^2=0$. The values of α and β specify the frame we choose, and the above form of the hard cross sections leads to α- and β-dependent cross sections. However, use of the EOM relation (9) and the LIRs (13) and (14) leads to the cross section independent from α and β as will be seen below. In the twist-3 cross section (53), we eliminate the intrinsic FF $\Delta\hat{G}_{3\bar{T}}(z)/z$ and the derivative of the two kinematical FFs by using those relations. Then, the resulting cross section is written in terms of the (nonderivative) kinematical FFs and the dynamical FFs. If we pick up the hard cross section for

$$\frac{1}{z}\int_0^{1/z}d\left(\frac{1}{z'}\right)\frac{1}{(1/z-1/z')^2}\Im\hat{N}_1\left(\frac{1}{z'},\frac{1}{z}\right), \qquad (56)$$

we have the combination

$$\hat{H}_2^{N_1} - \hat{H}_4^{N_1} + 2\hat{H}_{DG} + 4\hat{H}_{DH} = \mathcal{E}\hat{\sigma}_{DN1}, \tag{57}$$

where

$$\hat{\sigma}_{DN1} \equiv -\frac{C_F}{N}\left(\frac{1}{s^2}+\frac{1}{t^2}\right) - \frac{(2t-u)(s^2+t^2)}{2stu^3}, \tag{58}$$

$$\mathcal{E} \equiv x' t \epsilon^{p' P_h w S_\perp} - x u \epsilon^{p P_h w S_\perp} = -2\frac{xx'}{z}\epsilon^{p p' P_h S_\perp}. \tag{59}$$

One should note that the kinematical factor $\mathcal{E}$ appearing in this combination is free from α and β, which is written after the last equal sign in (59). We have found that all the coefficient hard cross sections for all FFs with the same z' dependence define the frame-independent cross section with the common kinematic factor $\mathcal{E}$. This shows that the frame dependence has been removed from the twist-3 cross section thanks to the EOM relations and the LIRs. This way, we can define the following set of frame-independent hard cross sections [in addition to (57)]:

$$\mathcal{E}\hat{\sigma}_G = \hat{H}_{NDG} + \frac{1}{2}\hat{H}_{\rm int} + 2\hat{H}_{DG}, \tag{60}$$

$$\mathcal{E}\hat{\sigma}_H = \hat{H}_{NDH} + \frac{1}{2}\hat{H}_{\rm int} + 4\hat{H}_{DH}, \tag{61}$$

$$\mathcal{E}\hat{\sigma}_{N1} = \hat{H}_1^{N_1} - \hat{H}_3^{N_1} + 2\hat{H}_{\rm int} + 4\hat{H}_{DG,} + 8\hat{H}_{DH}, \tag{62}$$

$$\mathcal{E}\hat{\sigma}_{N2} = \hat{H}_1^{N_2} - \hat{H}_3^{N_3} + \hat{H}_{\rm int} + 8\hat{H}_{DH}, \tag{63}$$

$$\mathcal{E}\hat{\sigma}_{N3} = -\hat{H}_1^{N_3} + \hat{H}_3^{N_2} - \hat{H}_{\rm int} - 4\hat{H}_{DG}, \tag{64}$$

$$\mathcal{E}\hat{\sigma}_{DN2} = \hat{H}_2^{N_2} - \hat{H}_4^{N_3} + 2\hat{H}_{DG} + 4\hat{H}_{DH}, \tag{65}$$

$$\mathcal{E}\hat{\sigma}_{DN3} = -\hat{H}_2^{N_3} + \hat{H}_4^{N_2} - 4\hat{H}_{DG}, \tag{66}$$

$$\mathcal{E}\hat{\sigma}_{O1} = \hat{H}_1^{O_1} + \hat{H}_3^{O_1}, \tag{67}$$

$$\mathcal{E}\hat{\sigma}_{O2} = \hat{H}_1^{O_2} + \hat{H}_3^{O_3}, \tag{68}$$

$$\mathcal{E}\hat{\sigma}_{O3} = \hat{H}_1^{O_3} + \hat{H}_3^{O_2}, \tag{69}$$

$$\mathcal{E}\hat{\sigma}_{DO1} = \hat{H}_2^{O_1} + \hat{H}_4^{O_1}, \tag{70}$$

$$\mathcal{E}\hat{\sigma}_{DO2} = \hat{H}_2^{O_2} + \hat{H}_4^{O_3}, \tag{71}$$

$$\mathcal{E}\hat{\sigma}_{DO3} = \hat{H}_2^{O_3} + \hat{H}_4^{O_2}, \tag{72}$$

$$\mathcal{E}\hat{\sigma}_{DF1} = \hat{H}_{DF1} - \hat{H}_{\rm int} - 2\hat{H}_{DG} - 4\hat{H}_{DH}, \tag{73}$$

$$\mathcal{E}\hat{\sigma}_{DF2} = \hat{H}_{DF2}, \tag{74}$$

$$\mathcal{E}\hat{\sigma}_{DF3} = \hat{H}_{DF3}, \tag{75}$$

$$\mathcal{E}\hat{\sigma}_{GF1} = \hat{H}_{GF1}, \tag{76}$$

$$\mathcal{E}\hat{\sigma}_{GF2} = \hat{H}_{GF2}, \tag{77}$$

$$\mathcal{E}\hat{\sigma}_{GF3} = \hat{H}_{GF3}. \tag{78}$$

With these hard cross sections, the manifestly frame-independent twist-3 cross section is given by

$$\begin{aligned}
E_h\frac{d\sigma(p,p',P_h;S_\perp)}{d^3P_h} =& \frac{M_h\alpha_s^2}{S_E}\int_0^1\frac{dx}{x}f_1(x)\int_0^1\frac{dx'}{x'}f_1(x')\int_0^1 dz\delta(s+t+u)\left(-2\frac{xx'}{z}\epsilon^{pp'P_hS_\perp}\right)\\
&\times\Bigg\{\hat{G}_T^{(1)}(z)\hat{\sigma}_G + \Delta\hat{H}_T^{(1)}(z)\hat{\sigma}_H + \int_0^{1/z}d\left(\frac{1}{z'}\right)\left[\frac{1}{1/z-1/z'}\mathfrak{I}\left(\hat{N}_1\left(\frac{1}{z'},\frac{1}{z}\right)\hat{\sigma}_{N1} + \hat{N}_2\left(\frac{1}{z'},\frac{1}{z}\right)\hat{\sigma}_{N2}\right.\right.\\
&\left.+\hat{N}_2\left(\frac{1}{z}-\frac{1}{z'},\frac{1}{z}\right)\hat{\sigma}_{N3}\right) + \frac{1}{z}\left(\frac{1}{1/z-1/z'}\right)^2\mathfrak{I}\left(\hat{N}_1\left(\frac{1}{z'},\frac{1}{z}\right)\hat{\sigma}_{DN1} + \hat{N}_2\left(\frac{1}{z'},\frac{1}{z}\right)\hat{\sigma}_{DN2} + \hat{N}_2\left(\frac{1}{z}-\frac{1}{z'},\frac{1}{z}\right)\hat{\sigma}_{DN3}\right)\\
&+\frac{1}{1/z-1/z'}\mathfrak{I}\left(\hat{O}_1\left(\frac{1}{z'},\frac{1}{z}\right)\hat{\sigma}_{O1} + \hat{O}_2\left(\frac{1}{z'},\frac{1}{z}\right)\hat{\sigma}_{O2} + \hat{O}_2\left(\frac{1}{z}-\frac{1}{z'},\frac{1}{z}\right)\hat{\sigma}_{O3}\right)\\
&\left.+\frac{1}{z}\left(\frac{1}{1/z-1/z'}\right)^2\mathfrak{I}\left(\hat{O}_1\left(\frac{1}{z'},\frac{1}{z}\right)\hat{\sigma}_{DO1} + \hat{O}_2\left(\frac{1}{z'},\frac{1}{z}\right)\hat{\sigma}_{DO2} + \hat{O}_2\left(\frac{1}{z}-\frac{1}{z'},\frac{1}{z}\right)\hat{\sigma}_{DO3}\right)\right]\\
&+\int_0^{1/z}d\left(\frac{1}{z'}\right)\frac{2}{C_F}\left[\mathfrak{I}\tilde{D}_F\left(\frac{1}{z'},\frac{1}{z'}-\frac{1}{z}\right)\left(\hat{\sigma}_{DF1} + \frac{1}{z}\frac{1}{1/z-1/z'}\hat{\sigma}_{DF2} + \frac{z'}{z}\hat{\sigma}_{DF3}\right)\right.\\
&\left.+\mathfrak{I}\tilde{G}_F\left(\frac{1}{z'},\frac{1}{z'}-\frac{1}{z}\right)\left(\hat{\sigma}_{GF1} + \frac{1}{z}\frac{1}{1/z-1/z'}\hat{\sigma}_{GF2} + \frac{z'}{z}\hat{\sigma}_{GF3}\right)\right]\Bigg\}.
\end{aligned} \tag{79}$$

Equation (79) is the final result for the twist-3 gluon FF contribution to $pp \to \Lambda^{\uparrow} X$.

Below, we give the LO Feynman diagrams for the hard part in each channel and present the results for hard cross sections, using the partonic Mandelstam variables, s, t, and u:

(i) $qg \to gq$ channel (Figs. 3 and 4):

$$\begin{aligned}
&\hat{\sigma}_G = -\frac{C_F}{N}\left(\frac{1}{s^2}-\frac{1}{t^2}\right) - \frac{2(s-t)}{(s+t)^3}, \qquad \hat{\sigma}_H = 0,\\
&\hat{\sigma}_{N1} = -\frac{C_F}{N}\frac{4}{s^2} - \frac{4t^3+10t^2u+4tu^2+u^3}{tu^3(t+u)}, \qquad \hat{\sigma}_{N2} = 0, \qquad \hat{\sigma}_{N3} = -\hat{\sigma}_{N1},\\
&\hat{\sigma}_{DN1} = -\frac{C_F}{N}\left(\frac{1}{s^2}+\frac{1}{t^2}\right) + \frac{(2t-u)(s^2+t^2)}{2tu^3(t+u)}, \qquad \hat{\sigma}_{DN2} = \hat{\sigma}_{DN1}, \qquad \hat{\sigma}_{DN3} = -2\hat{\sigma}_{DN1},\\
&\hat{\sigma}_{O1} = 0, \qquad \hat{\sigma}_{O2} = -\frac{C_F}{N}\left(\frac{2}{s^2}+\frac{2}{t^2}\right) + \frac{3(s^2+t^2)}{st(s+t)^2}, \qquad \hat{\sigma}_{O3} = \hat{\sigma}_{O2},\\
&\hat{\sigma}_{DO1} = -\frac{C_F}{N}\left(\frac{1}{s^2}+\frac{1}{t^2}\right) + \frac{3(s^2+t^2)}{2st(s+t)^2}, \qquad \hat{\sigma}_{DO2} = \hat{\sigma}_{DO1}, \qquad \hat{\sigma}_{DO3} = 2\hat{\sigma}_{DO1},\\
&\hat{\sigma}_{DF1} = \frac{C_F}{N}\frac{2}{s^2} + \frac{4t^3+10t^2u+4tu^2+u^3}{2tu^3(t+u)},\\
&\hat{\sigma}_{DF2} = \frac{C_F}{N}\frac{(s^2+t^2)}{st(s+t)^2} + \frac{(s+2t)(s^2+t^2)}{4st(s+t)^2u} + \frac{1}{N}\frac{s}{2t(s+t)^2} - \frac{C_F}{N^2}\frac{1}{2t^2},\\
&\hat{\sigma}_{DF3} = -\frac{C_F}{N}\frac{(s^2+t^2)}{st(s+t)^2} + \frac{(2s+t)(s^2+t^2)}{4st(s+t)^3} - \frac{1}{N}\frac{t}{2s(s+t)^2} + \frac{C_F}{N^2}\frac{1}{2s^2},\\
&\hat{\sigma}_{GF1} = 0, \qquad \hat{\sigma}_{GF2} = -\hat{\sigma}_{DF2}, \qquad \hat{\sigma}_{GF3} = \hat{\sigma}_{DF3}.
\end{aligned} \tag{80}$$

(ii) $q\bar{q} \to gg$ channel (Figs. 5 and 6):

$$\begin{aligned}
&\hat{\sigma}_G = -\frac{C_F}{N^2}\left(\frac{1}{t^2}-\frac{1}{u^2}\right) - C_F\frac{(t-u)(t^2+u^2)(t^2+4tu+u^2)}{t^2u^2(t+u)^3}, \qquad \hat{\sigma}_H = 0,\\
&\hat{\sigma}_{N1} = \frac{C_F}{N^2}\frac{4(t-u)}{tu(t+u)} - C_F\frac{6(t-u)(t^2+u^2)}{tu(t+u)^3}, \qquad \hat{\sigma}_{N2} = 0, \qquad \hat{\sigma}_{N3} = -\hat{\sigma}_{N1},\\
&\hat{\sigma}_{DN1} = -\frac{C_F}{N^2}\frac{(t-u)(t^2+u^2)}{t^2u^2(t+u)} + C_F\frac{(t^2+u^2)(t^3-u^3)}{t^2u^2(t+u)^3}, \qquad \hat{\sigma}_{DN2} = \hat{\sigma}_{DN1}, \qquad \hat{\sigma}_{DN3} = -2\hat{\sigma}_{DN1},\\
&\hat{\sigma}_{O1} = 0, \qquad \hat{\sigma}_{O2} = \frac{C_F}{N^2}\left(\frac{2}{t^2}+\frac{2}{u^2}\right) - C_F\frac{2(t^2+u^2)(t^2-tu+u^2)}{t^2u^2(t+u)^2}, \qquad \hat{\sigma}_{O3} = \hat{\sigma}_{O2},\\
&\hat{\sigma}_{DO1} = \frac{C_F}{N^2}\left(\frac{1}{t^2}+\frac{1}{u^2}\right) - C_F\frac{(t^2+u^2)(t^2-tu+u^2)}{t^2u^2(t+u)^2}, \qquad \hat{\sigma}_{DO2} = \hat{\sigma}_{DO1}, \qquad \hat{\sigma}_{DO3} = 2\hat{\sigma}_{DO1},\\
&\hat{\sigma}_{DF1} = -\frac{C_F}{N^2}\frac{2(t-u)}{tu(t+u)} + C_F\frac{3(t-u)(t^2+u^2)}{tu(t+u)^3},\\
&\hat{\sigma}_{DF2} = -\frac{C_F}{N^2}\frac{(t^2+u^2)}{tu(t+u)^2} + C_F\frac{(t^2+u^2)}{2t(t+u)^3} - \frac{C_F}{N}\frac{(t^2+u^2)}{2t^2(t+u)^2} + \frac{C_F}{N^3}\frac{1}{2t^2},\\
&\hat{\sigma}_{DF3} = \frac{C_F}{N^2}\frac{(t^2+u^2)}{tu(t+u)^2} - C_F\frac{(t^2+u^2)}{2u(t+u)^3} + \frac{C_F}{N}\frac{(t^2+u^2)}{2u^2(t+u)^2} - \frac{C_F}{N^3}\frac{1}{2u^2},\\
&\hat{\sigma}_{GF1} = 0, \qquad \hat{\sigma}_{GF2} = -\hat{\sigma}_{DF2}, \qquad \hat{\sigma}_{GF3} = \hat{\sigma}_{DF3}.
\end{aligned} \tag{81}$$

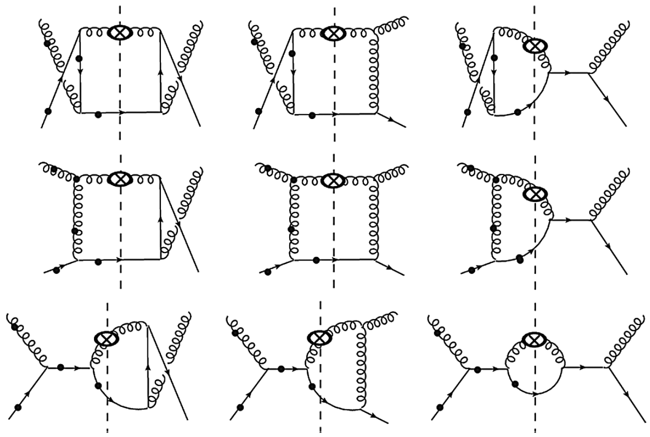

FIG. 3. Diagrams in the $qg \to gq$ channel for the hard part $S(k)$ and $S_L(z', z)$ in Eq. (51). For $S_L(z', z)$, an extra gluon line connecting ⊗ and each of the black dots should be added for each diagram.

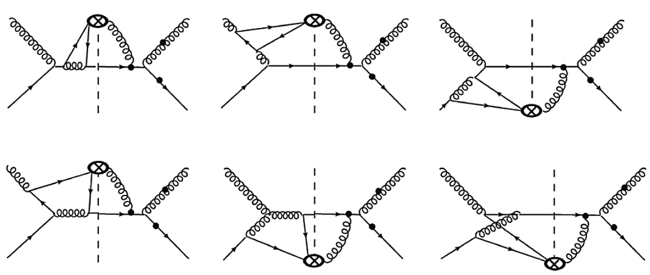

FIG. 4. Diagrams for the hard part in Fig. 2. For the upper middle diagram, a quark loop with the reversed arrow also needs to be included.

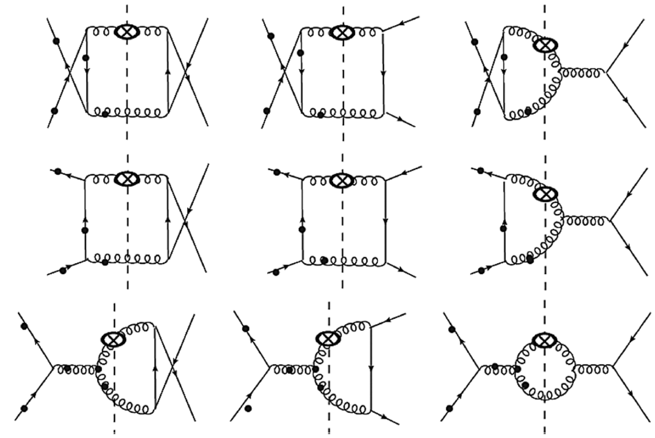

FIG. 5. Diagrams in the $q\bar{q} \to gg$ channel for the hard parts $S(k)$ and $S_L(z', z)$ in Eq. (51). For $S_L(z', z)$, an extra gluon line connecting ⊗ and each of the black dots should be added for each diagram.

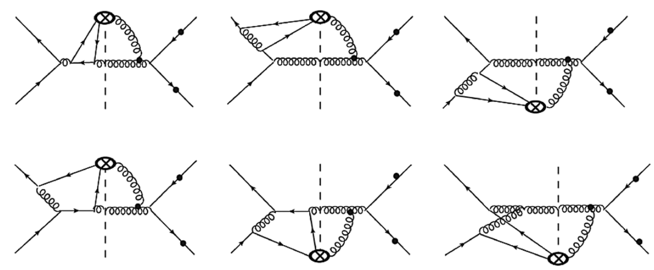

FIG. 6. Diagrams for the hard part in Fig. 2. For the upper left diagram, a quark loop with the reversed arrow also needs to be included.

(iii) $gg \to gg$ channel (Figs. 7 and 8):

$$
\begin{aligned}
&\hat{\sigma}_G = -\frac{N}{C_F}\frac{2(s^2+st+t^2)^2(s-t)(2s+t)(s+2t)}{s^3t^3(s+t)^3}, \qquad \hat{\sigma}_H = 0,\\
&\hat{\sigma}_{N1} = -\frac{N}{C_F}\frac{2(t^2+tu+u^2)^2(t-u)(2t^2+7tu+2u^2)}{t^3u^3(t+u)^3}, \qquad \hat{\sigma}_{N2} = 0, \qquad \hat{\sigma}_{N3} = -\hat{\sigma}_{N1},\\
&\hat{\sigma}_{DN1} = \frac{N}{C_F}\frac{(t^2+tu+u^2)^2(t-u)(2t^2+3tu+2u^2)}{t^3u^3(t+u)^3}, \qquad \hat{\sigma}_{DN2} = \hat{\sigma}_{DN1}, \qquad \hat{\sigma}_{DN3} = -2\hat{\sigma}_{DN1},\\
&\hat{\sigma}_{DF1} = \frac{N}{C_F}\frac{(t^2+tu+u^2)^2(t-u)(2t^2+7tu+2u^2)}{t^3u^3(t+u)^3},\\
&\hat{\sigma}_{DF2} = -\frac{N}{C_F}\frac{(t^2+tu+u^2)^2(t-u)(2t+u)(t+2u)}{4t^3u^3(t+u)^3},\\
&\hat{\sigma}_{DF3} = -\frac{N}{C_F}\frac{(t^2+tu+u^2)^2(t-u)(2t+u)(t+2u)}{4t^3u^3(t+u)^3},\\
&\hat{\sigma}_{GF1} = 0, \qquad \hat{\sigma}_{GF2} = -\hat{\sigma}_{DF2}, \qquad \hat{\sigma}_{GF3} = \hat{\sigma}_{DF3}.
\end{aligned}
\tag{82}
$$

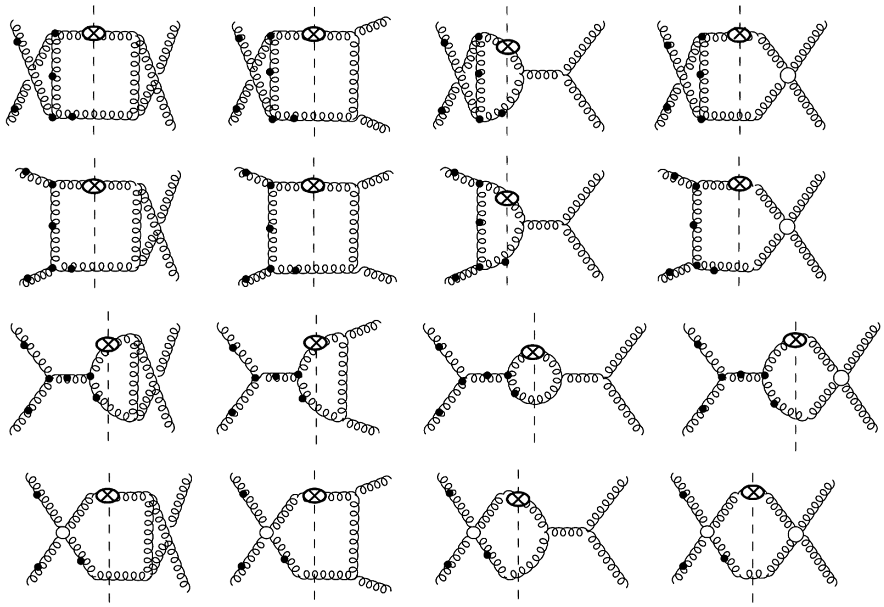

FIG. 7. Diagrams in the $gg \to gg$ channel for the hard parts $S(k)$ and $S_L(z', z)$ in Eq. (51). For $S_L(z', z)$, an extra gluon line connecting $\otimes$ and each of the black dots should be added for each diagram. A white circle represents a four-gluon vertex, making clear the difference from the attachment of the extra gluon line.

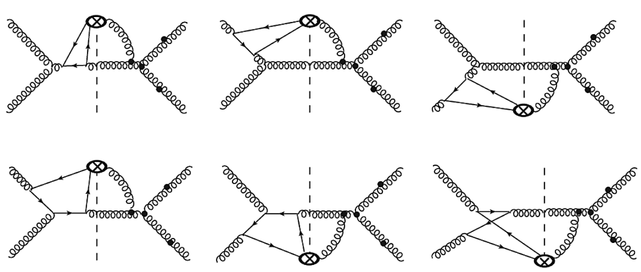

FIG. 8. Diagrams contributing to the hard part in Fig. 2 in the $gg \to gg$ channel. For all diagrams, a quark loop with the reversed arrow also needs to be included.

IV. CONCLUSION

In this paper, we have studied the transverse polarization of a spin-$1/2$ hyperon produced in the unpolarized proton-proton collision, $pp \to \Lambda^{\uparrow} X$, within the framework of the collinear twist-3 factorization, which is relevant for the large-p_T hyperon production. We focused on the contribution from the twist-3 gluon FFs, which had never been studied in previous studies. To this end, we have developed a formalism to include all effects associated with the twist-3 gluon FFs. The twist-3 cross section receives contributions from three types of the gluon FFs, i.e., intrinsic, kinematical, and dynamical (purely gluonic and quark-antiquark-gluon type) ones. Applying the formalism, we have calculated the LO cross section for $pp \to \Lambda^{\uparrow} X$. This completes the LO twist-3 cross section combined with the known results for the other contributions from the twist-3 distribution in the unpolarized proton and the twist-3 quark FFs for the hyperon. Using the EOM relation and the LIRs for the twist-3 gluon FFs, we have shown that the derived cross section satisfies the color gauge invariance and the frame independence. Since the formalism developed here is a general one, it can be applied to other processes to which the twist-3 gluon FFs contribute.

ACKNOWLEDGMENTS

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APPENDIX A: WARD IDENTITIES FOR THE GLUON FRAGMENTATION CHANNELS

1. Derivation of the Ward identities

To get a twist-3 gluon fragmentation contribution to the cross section in a gauge-invariant form, one needs to fully utilize the Ward identities for the hard parts (26)–(32). Here, we present their derivation.

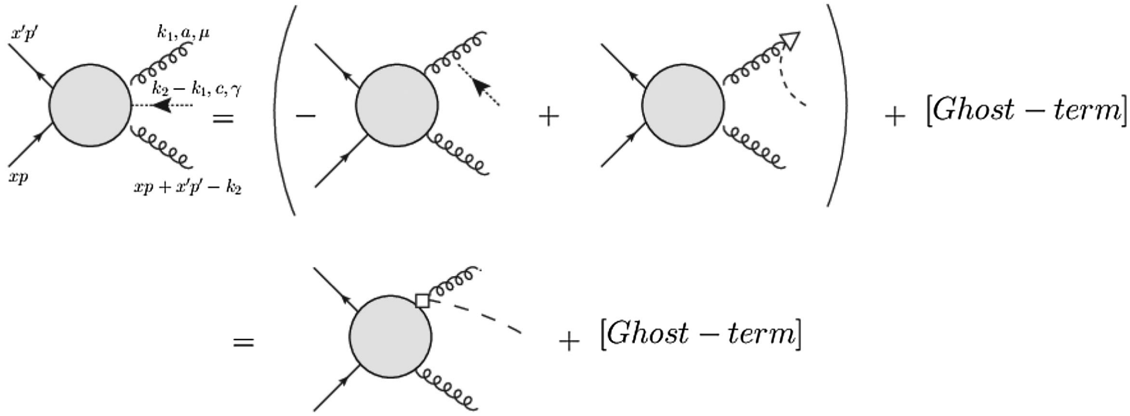

FIG. 9. Ward identity (A1) for the $q\bar{q} \to gg$ channel. The figure shows the left side of the cut, representing the attachment of the scalar polarized gluon (dotted line) with the momentum $k_2 - k_1$. Note that the hard part represented on the lhs does not contain the diagram in which the scalar polarized gluon is directly attached to the gluon line fragmenting into the final hadron; hence, it appears in the first term of the rhs. For the meaning of the notations in the figure, see Appendix A 2.

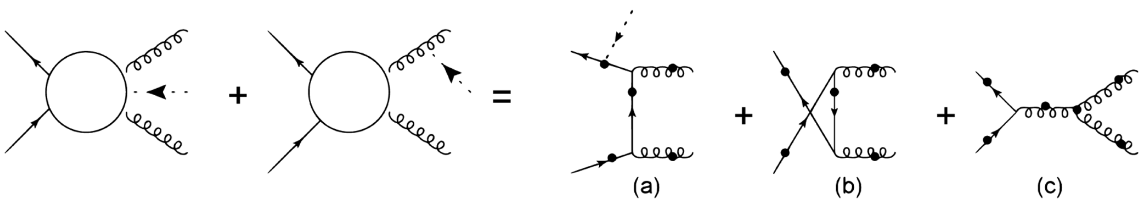

FIG. 10. Attachment of the scalar polarized gluon of momentum $k_2 - k_1$ to $qq \to gg$ diagrams. On the rhs, it is implied that the scalar polarized gluon is attached one of the black dots in all possible ways.

a. $q\bar{q} \to gg$ channel

Ward identities in this channel read

$$(k_2 - k_1)_\lambda S_L^{\mu\nu\lambda,abc}(k_1,k_2) = \frac{-if^{abc}}{N^2-1} S^{\mu\nu}(k_2) + G^{\mu\nu,abc}_{q\bar{q}\to gg}(k_1,k_2), \tag{A1}$$

$$k_{1\mu} S_L^{\mu\nu\lambda,abc}(k_1,k_2) = \frac{if^{abc}}{N^2-1} S^{\lambda\nu}(k_2) + G^{\lambda\nu,cab}_{q\bar{q}\to gg}(k_2-k_1,k_2), \tag{A2}$$

$$k_{2\nu} S_L^{\mu\nu\lambda,abc}(k_1,k_2) = 0, \tag{A3}$$

$$(k_2 - k_1)_\lambda S_R^{\mu\nu\lambda,abc}(k_1,k_2) = \frac{if^{abc}}{N^2-1} S^{\mu\nu}(k_1) + (G^{\nu\mu,bac}_{q\bar{q}\to gg}(k_2,k_1))^\star, \tag{A4}$$

$$k_{2\nu} S_R^{\mu\nu\lambda,abc}(k_1,k_2) = \frac{if^{abc}}{N^2-1} S^{\mu\lambda}(k_1) + (G^{\lambda\mu,cab}_{q\bar{q}\to gg}(k_1-k_2,k_1))^\star, \tag{A5}$$

$$k_{1\mu} S_R^{\mu\nu\lambda,abc}(k_1,k_2) = 0, \tag{A6}$$

where $S_{L(R)}^{\mu\nu\lambda,abc}(k_1,k_2)$ represents a hard part for a three-gluon correlation functions which has two gluon legs in the left (right) of the final-state cut and $S^{\mu\nu}(k) \equiv S^{\mu\nu}_{ab}(k)\delta^{ab}$ represents a hard part for a two-gluon correlation functions. G terms in the right-hand side of these equations are ghost terms. Figure 9 shows the Ward identity (A1), which states that the attachment of the scalar polarized gluon with the momentum $k_2 - k_1$ to the hard part $S^{abc}_{L\mu\nu\lambda}(k_1,k_2)$ can be decomposed into the two-body hard part and the ghost terms. To identify the ghost terms, we consider the diagrams in Fig. 10. The first term on the lhs of the figure represents the hard part $S^{abc}_{L\mu\nu\lambda}(k_1,k_2)$, and the second term represents the attachment of the scalar polarized gluon to the gluon line fragmenting into $\Lambda^\uparrow$. The LO diagrams can be classified into three types as shown in the rhs of this figure. Using the tree level Ward identities (See Appendix A 2), each diagram in Fig. 10 can be decomposed into several pieces, and some of them cancel each other, owing to the on-shell condition of the external lines (see Appendix A 3). Taking these facts into account, we rearrange each term on the rhs of Fig. 10.

First, diagrams in Fig. 10(a) can be rewritten as in Fig. 11.

Similarly, Fig. 10(b) can be rewritten as in Fig. 12.

Likewise, Fig. 10(c) can be rewritten as in Fig. 13.

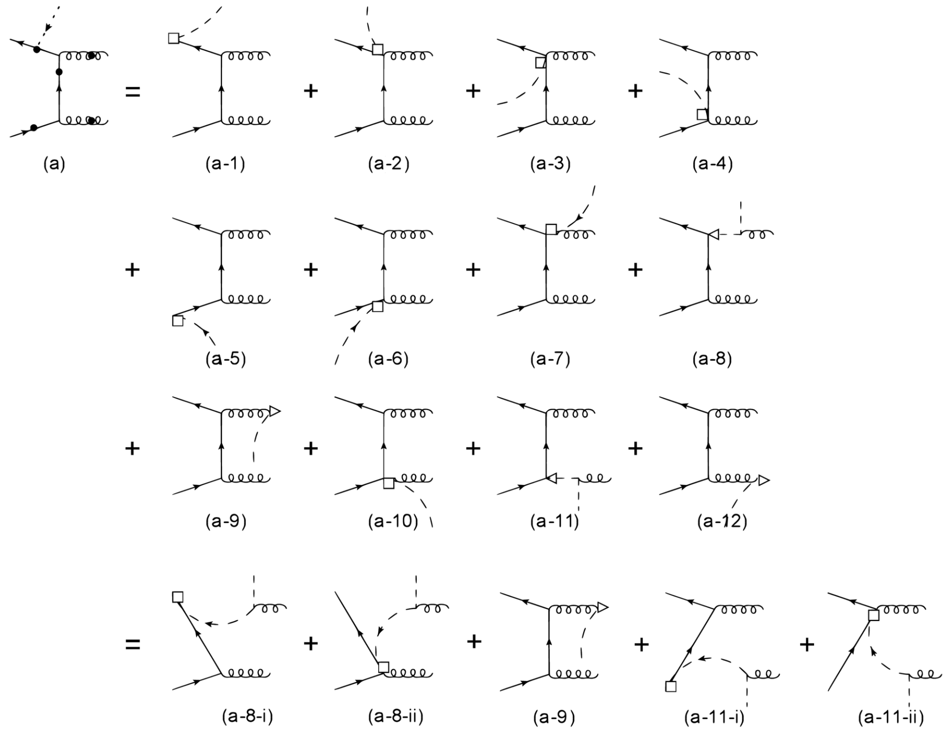

FIG. 11. Decomposition of Fig. 10(a) and the resulting terms due to the cancellation among diagrams. Diagrams (a-1), (a-5), and (a-12) vanish, owing to the on-shell condition. Combination of (a-2), (a-3), and (a-7) cancels that of (a-4), (a-6), and (a-10), owing to (A18).

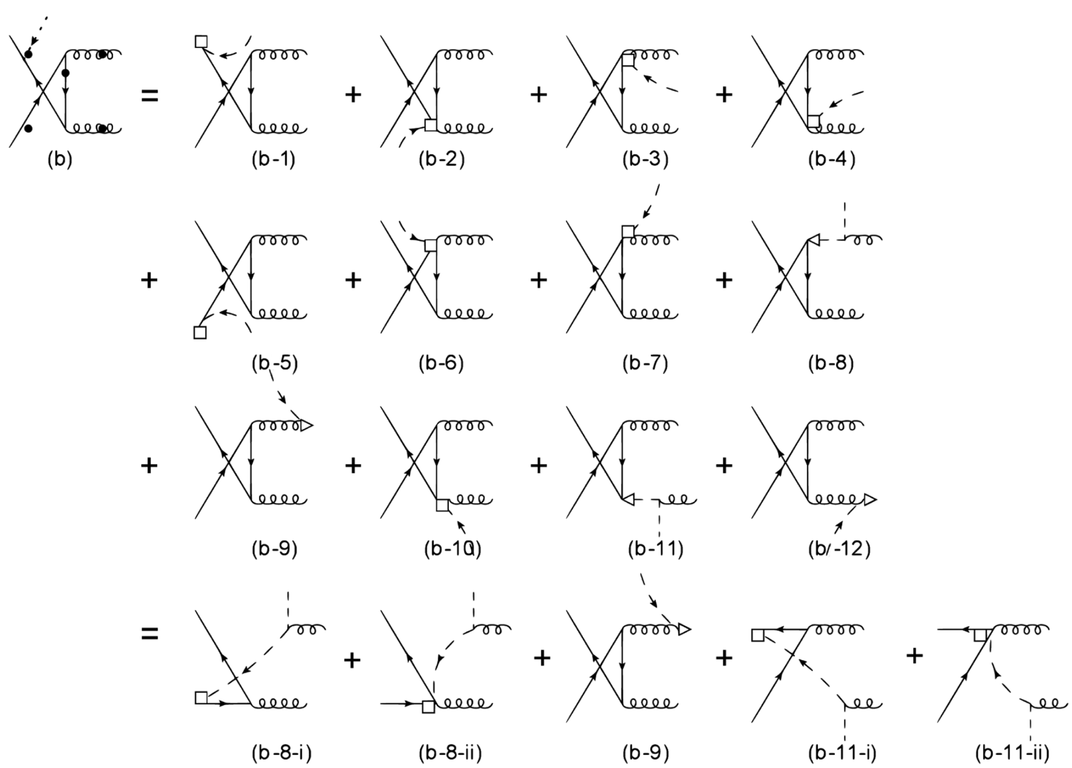

FIG. 12. Decomposition of Fig. 10(b) and the resulting terms due to the cancellation among diagrams. Diagrams (b-1), (b-5) and (b-12) vanish, owing to the on-shell condition. Combination of (b-2), (b-4), and (b-10) cancels that of (b-3), (b-6), and (b-7), owing to (A18).

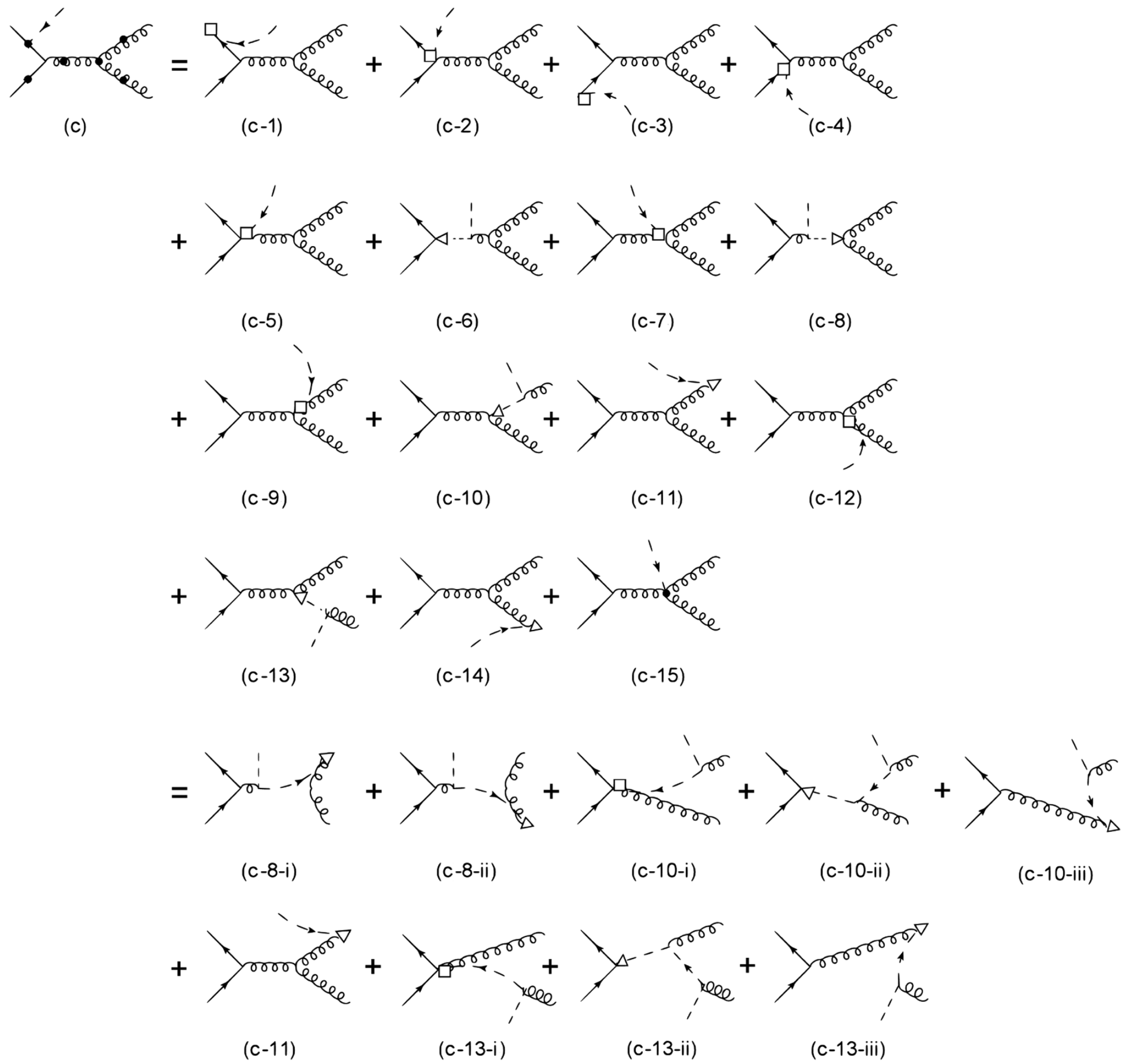

FIG. 13. Decomposition of Fig. 10(c) and the resulting terms due to the cancellation among the diagrams. Contributions from (c1), (c-3), (c-6), and (c-1)4 vanish due to the on-shell conditions. Diagrams (c-2), (c-4), and (c-5) cancel each other by (A18). Diagrams (c-7), (c-9), (c-12), and (c-15) cancel each other by (A19).

Using the on-shell condition, we find that diagrams (a-8-i), (a-11-i), (b-8-i), (b-11-i), and (c-8-ii) vanish, and diagrams (c-10-ii), (c-10-iii), and (c-13-ii) also vanish. In addition, contribution from (a-8-ii), (b-8-ii), and (c-10-i) cancels among them, and likewise for the combination of (a-11-ii), (b-11-ii), and (c-13-i) by (A18). Therefore, besides (a-9), (b-9), and (c-11) (see Fig. 9), remaining contributions (c-8-i) and (c-13-iii) become the ghost term in (A1). The ghost term takes the following structure in the $q\bar{q} \to gg$ channel,[4]

$$G^{\mu\nu,abc}_{q\bar{q}\to gg}(k_1,k_2) = -\left\{(-if^{cef})(-if^{fda})d^{\mu\sigma}(k_1)\frac{(k_2-k_1)_\rho}{(xp+x'p'-k_1)^2} + (-if^{aef})(-if^{fdc})d^{\mu}_{\rho}(k_1)\frac{(k_2-k_1)^\sigma}{(xp+x'p'-(k_2-k_1))^2}\right\}$$
$$\times H^d_\sigma(xp,x'p')\frac{1}{(xp+x'p')^2}P^{\rho\gamma}(xp+x'p'-k_2)\times(M^{\nu,be}_\gamma(k_2))^\star, \tag{A7}$$

[4]In these Appendixes, we follow the convention of Ref. [57] for the Feynman rule.

where $d^{\mu\sigma}(k_1) \equiv k_1^2 g^{\mu\sigma} - k_1^\mu k_1^\sigma$, $H_\sigma^d(xp, x'p')$ represents the quark-line part in the left of the cut, which is connected to the right of the cut $(M_\gamma^{\nu,be}(k_2))^\star$, and the gluon's polarization tensor is given by

$$P^{\rho\gamma}(xp + x'p' - k_2) \equiv -g^{\rho\gamma} + \frac{(xp + x'p' - k_2)^\rho k_1^\gamma + (xp + x'p' - k_2)^\gamma k_1^\rho}{(xp + x'p' - k_2) \cdot k_1}. \tag{A8}$$

Diagrammatically, the ghost term (A7) can be written as in Fig. 14. This completes the derivation of the Ward identity in the $q\bar{q} \to gg$ channel.

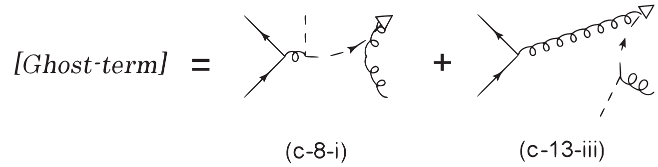

FIG. 14. Diagrams for the ghost term (A7) in the $q\bar{q} \to gg$ channel.

b. $qg \to gq$ channel

Ward identities in this channel are given by (A1)–(A6), except that the ghost terms $G_{q\bar{q}\to gg}^{\mu\nu,abc}(k_1, k_2)$ are replaced by $G_{qg\to gq}^{\mu\nu,abc}(k_1, k_2)$, which takes the structure

$$G_{qg\to gq}^{\mu\nu,abc}(k_1,k_2) = -\left\{(-if^{cef})(-if^{fda})d^{\mu\alpha}(k_1)\frac{(k_2-k_1)^\lambda}{(x'p'-k_1)^2} + (-if^{aef})(-if^{fdc})d^{\mu\lambda}(k_1)\frac{(k_2-k_1)^\alpha}{(x'p'-(k_2-k_1))^2}\right\} \times H'^e_\lambda(xp, xp+x'p'-k_2)\frac{1}{(x'p'-k_2)^2}(M_\alpha^{\nu,bd}(k_2))^\star, \tag{A9}$$

where $H'^e_\lambda(xp, xp + x'p' - k_2)$ is the quark-line part connected to the left of the cut $(M_\alpha^{\nu,bd}(k_2))^\star$.

c. $gg \to gg$ channel

Ward identities in this channel are given by (A1)–(A6), in which $G_{q\bar{q}\to gg}^{\mu\nu,abc}(k_1, k_2)$ is replaced by $G_{gg\to gg}^{\mu\nu,abc}(k_1, k_2)$,

$$G_{gg\to gg}^{\mu\nu,abc}(k_1,k_2) \equiv G_{1gg\to gg}^{\mu\nu,abc}(k_1,k_2) + G_{2gg\to gg}^{\mu\nu,abc}(k_1,k_2) + G_{3gg\to gg}^{\mu\nu,abc}(k_1,k_2) + G_{4gg\to gg}^{\mu\nu,abc}(k_1,k_2), \tag{A10}$$

where $G_{1gg\to gg}$ through $G_{4gg\to gg}$ are the ghost terms which occur from four types of diagrams shown in Fig. 15 for the hard scattering amplitudes in this channel. Each term in (A10) takes the structures

$$G_{1gg\to gg}^{\mu\nu,abc}(k_1,k_2) = (-if^{cdg})\frac{(k_2-k_1)^\alpha}{(x'p'-(k_2-k_1))^2}\left\{(-if^{gfh})(-if^{hea})d^{\mu\rho}(k_1)\frac{(x'p'-(k_2-k_1))^\beta}{(xp+x'p'-(k_2-k_1))^2} + (-if^{geh})(-if^{hfa})d^{\mu\beta}(k_1)\frac{(x'p'-(k_2-k_1))_\rho}{(xp-k_1)^2} - (-if^{gha})V_{\sigma\rho}^{\beta,hfe}d^{\mu\sigma}(k_1)\frac{1}{(x'p'-k_2)^2}\right\} \times P^{\rho\gamma}(xp+x'p'-k_2) \times (M_{\nu\gamma,\alpha\beta}^{be,df}(k_2))^\star, \tag{A11}$$

$$G_{2gg\to gg}^{\mu\nu,abc}(k_1,k_2) = (-if^{cfg})\frac{(k_2-k_1)^\beta}{(xp-(k_2-k_1))^2}\left\{(-if^{gdh})(-if^{hea})d^{\mu\rho}(k_1)\frac{(xp-(k_2-k_1))^\alpha}{(xp+x'p'-(k_2-k_1))^2} - (-if^{gha})V_{\sigma\rho}^{\alpha,hde}d^{\mu\sigma}(k_1)\frac{1}{(xp-k_2)^2} + (-if^{geh})(-if^{hda})d^{\mu\alpha}(k_1)\frac{(xp-(k_2-k_1))_\rho}{(x'p'-k_1)^2}\right\} \times P^{\rho\gamma}(xp+x'p'-k_2) \times (M_{\nu\gamma,\alpha\beta}^{be,df}(k_2))^\star, \tag{A12}$$

$$G_{3gg\to gg}^{\mu\nu,abc}(k_1,k_2) = (-if^{cgh})(k_2-k_1)^\sigma\left\{(-if^{hea})V_\sigma^{\alpha\beta,dfg}d^{\mu\rho}(k_1)\frac{1}{(xp+x'p'-(k_2-k_1))^2}\frac{1}{(xp+x'p')^2} + (-if^{gfa})V_\sigma^{\alpha\rho,hde}d^{\mu\beta}(k_1)\frac{1}{(xp-k_1)^2}\frac{1}{(xp-k_2)^2} + (-if^{gda})V_\sigma^{\beta\rho,hfe}d^{\mu\alpha}(k_1)\frac{1}{(x'p'-k_2)^2}\frac{1}{(x'p'-k_1)^2}\right\} \times P_\rho^\gamma(xp+x'p'-k_2) \times (M_{\nu\gamma,\alpha\beta}^{be,df}(k_2))^\star, \tag{A13}$$

and

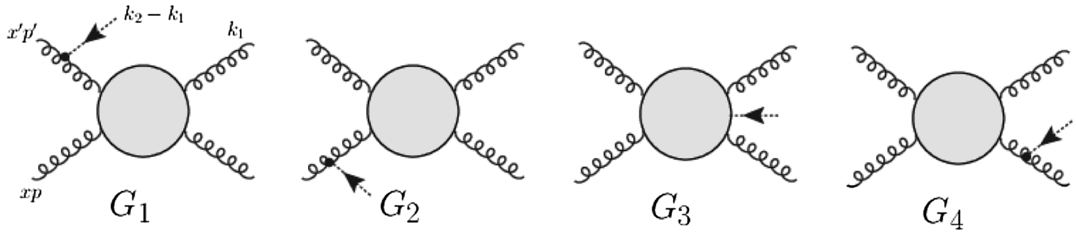

FIG. 15. Diagrammatic representation of the ghost terms in the $gg \to gg$ channel. Depending on the position of attachment of the scalar polarized gluon, diagrams can be classified into four types. Figure G_3 represents diagrams in which the scalar polarized gluon is attached to an internal propagator.

$$G^{\mu\nu,abc}_{4gg\to gg}(k_1,k_2) = (-if^{ceg})\frac{(k_2-k_1)_\rho}{(xp+x'p'-k_1)^2}\bigg\{-(-if^{gha})V^{\alpha\beta,dfh}_{\sigma}d^{\mu\sigma}(k_1)\frac{1}{(xp+x'p')^2} + (-if^{gdh})(-if^{hfa})d^{\mu\beta}(k_1)\frac{(k_1-xp)^\alpha}{(xp-k_1)^2} + (-if^{gfh})(-if^{hda})d^{\mu\alpha}(k_1)\frac{(k_1-x'p')^\beta}{(x'p'-k_1)^2}\bigg\}P^{\rho\gamma}(xp+x'p'-k_2)\times(M^{be,df}_{\nu\gamma,\alpha\beta}(k_2))^\star, \tag{A14}$$

where $V^{\beta,hfe}_{\sigma\rho}$ represents an appropriate three-gluon vertex in each channel.

2. Decomposition of vertices

Attachment of the scalar polarized gluon to a quark or gluon line can be decomposed as follows:

(i) quark-gluon vertex (Fig. 16):

$$k^\alpha \cdot (\gamma_\alpha)_{ij}T^a = (\not p + \not k)_{ij}T^a - (\not p)_{ij}T^a, \tag{A15}$$

where T^a is the generator of color SU(N) group;

(ii) three-gluon vertex (Fig. 17):

$$k^\alpha \cdot V^{abc}_{\alpha\beta\gamma}(k,p,-p-k) = F^{abc}d_{\beta\gamma}(p+k) + F^{acb}d_{\beta\gamma}(p), \tag{A16}$$

where $V^{abc}_{\alpha\beta\gamma}(p_1,p_2,p_3) \equiv F^{abc}[(p_1-p_2)_\gamma g_{\alpha\beta} + (p_2-p_3)_\alpha g_{\beta\gamma} + (p_3-p_1)_\beta g_{\alpha\gamma}]$ and $F^{abc} \equiv -if^{abc}$.

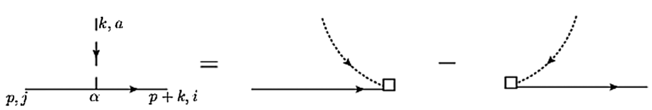

FIG. 16. Attachment of a scalar polarized gluon to a quark-gluon vertex corresponding to (A15).

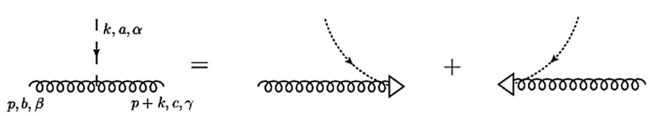

FIG. 17. Attachment of a scalar polarized gluon through a three-gluon vertex corresponding to (A16).

Furthermore, $d^{\beta\gamma}$ in the above equations is decomposed as (Fig. 18)

$$F^{acb}d^{\beta\gamma}(p) = p^2 g^{\beta\gamma}F^{acb} + (-p^\beta p^\gamma)F^{acb}. \tag{A17}$$

3. Cancellation among vertices

After decomposition of vertices, the following cancellation holds among vertices:

(i) quark-gluon vertex (Fig. 19):

$$[-(T^aT^b)_{lm} + (T^bT^a)_{lm} + F^{alb}(T^l)_{lm}](\gamma^\alpha)_{ij} = 0; \tag{A18}$$

(ii) purely gluon vertex (Fig. 20):

$$(-P_X)^\sigma \cdot W^{abcd}_{\alpha\beta\gamma\sigma} + F^{ald}V^{lbc}_{\alpha\beta\gamma}(-p_2-p_3,p_2,p_3) + F^{bld}V^{alc}_{\alpha\beta\gamma}(p_1,-p_1-p_3,p_3) + F^{cld}V^{abl}_{\alpha\beta\gamma}(p_1,p_2,-p_1-p_2) = 0, \tag{A19}$$

where $P_X \equiv p_1 + p_2 + p_3$.

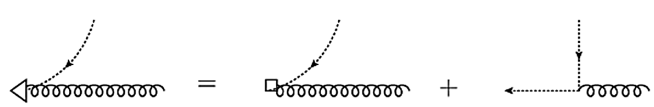

FIG. 18. Decomposition of $d^{\beta\gamma}$ in (A17).

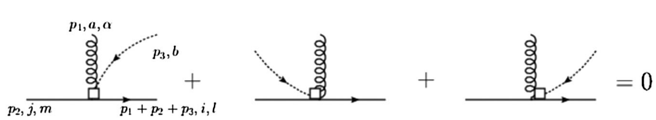

FIG. 19. Cancelation for quark-gluon vertices (A18).

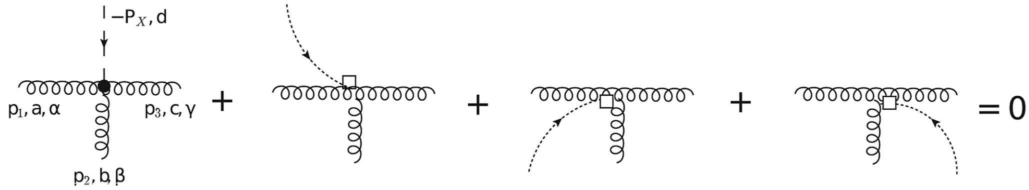

FIG. 20. Cancellation among three- and four-gluon vertices (A19).

APPENDIX B: COLOR GAUGE INVARIANCE OF THE TWIST-3 CROSS SECTION

Here, we prove that the twist-3 cross section in (51) supplemented by the quark-antiquark-gluon contribution shown in Fig. 2 is color gauge invariant, owing to the EOM relation (9). We illustrate this property for the $qg \to gq$ channel. The proof for other channels is essentially the same. The cross section in this channel can be written as

$$E_h \frac{d\sigma(p,p',P_h;S_\perp)}{d^3P_h} = \frac{1}{16\pi^2 S_E}\int \frac{dx}{x} f_1(x)\int \frac{dx'}{x'}G(x')\int dz\left(-\frac{1}{2}g_\perp^{\rho\sigma}(p')\right)W_{\rho\sigma}(xp,x'p,P_h/z), \tag{B1}$$

where $G(x')$ is the unpolarized gluon distribution in the proton with momentum p' and the Lorentz indices ρ and σ are contracted with $g_\perp^{\rho\sigma}(p') \equiv g^{\rho\sigma} - p'^\rho n'^\sigma - p'^\sigma n'^\rho$, where n' is the usual lightlike vector satisfying $p'\cdot n' = 1$ to extract the contribution from $G(x')$. From (51), we can read off $W_{\rho\sigma}(xp,x'p',P_h/z)$ as

$$\begin{aligned} W_{\rho\sigma}(xp,x'p',P_h/z) ={}& \Omega^\mu{}_\alpha\Omega^\nu{}_\beta \mathrm{Tr}[\hat\Gamma^{\alpha\beta}(z)S_{\mu\nu,\rho\sigma}(P_h/z)] - i\Omega^\mu{}_\alpha\Omega^\nu{}_\beta\Omega^\lambda{}_\gamma \mathrm{Tr}\left[\hat\Gamma_\partial^{\alpha\beta\gamma}(z)\frac{\partial S_{\mu\nu,\rho\sigma}(k)}{\partial k^\lambda}\bigg|_{\mathrm{c.l.}}\right] \\ &+ \Re\left\{i\Omega^\mu{}_\alpha\Omega^\nu{}_\beta\Omega^\lambda{}_\gamma \frac{1}{z}\int\frac{dz'}{z'}\left(\frac{1}{1/z-1/z'}\right)\right. \\ &\left.\times \mathrm{Tr}\left[\left(-\frac{if^{abc}}{N}\hat\Gamma_{FA}^{\alpha\beta\gamma}\left(\frac{1}{z'},\frac{1}{z}\right) + d^{abc}\frac{N}{N^2-4}\hat\Gamma_{FS}^{\alpha\beta\gamma}\left(\frac{1}{z'},\frac{1}{z}\right)\right)S^{L,abc}_{\mu\nu\lambda,\rho\sigma}(z',z)\right]\right\} \\ &+ w^{q\bar qg}_{\rho\sigma}(xp,x'p',P_h/z), \end{aligned} \tag{B2}$$

where the hard part $S^{ab}_{\mu\nu,\rho\sigma}(k)$ is related to the hard part $S^{ab}_{\mu\nu}(k)$ in (51) by $(-\frac{1}{2}g_\perp^{\rho\sigma}(p'))S^{ab}_{\mu\nu,\rho\sigma}(k) = S^{ab}_{\mu\nu}(k)$ and likewise for $S^{abc}_{\mu\nu\lambda,\rho\sigma}(k)$, and $w^{q\bar qg}_{\rho\sigma}(xp,x'p',P_h/z)$ represents the contribution from Fig. 2. Then, the color gauge invariance of the cross section implies

$$x'p'^\rho W_{\rho\sigma}(xp,x'p',P_h/z) = x'p'^\sigma W_{\rho\sigma}(xp,x'p',P_h/z) = 0. \tag{B3}$$

To show (B3), we use the EOM relation (9) and eliminate the intrinsic twist-3 FF $\Delta\hat G_{3\bar T}(z)$ in the first term of the rhs of (B2). Then, $W_{\rho\sigma}$ can be decomposed into three pieces,

$$\begin{aligned} &W_{\rho\sigma}(xp,x'p',P_h/z) \\ &\quad = W^{(\mathrm{i})}_{\rho\sigma}(xp,x'p',P_h/z) + W^{(\mathrm{ii})}_{\rho\sigma}(xp,x'p',P_h/z) \\ &\qquad + W^{(\mathrm{iii})}_{\rho\sigma}(xp,x'p',P_h/z), \end{aligned} \tag{B4}$$

where $W^{(\mathrm{i})}_{\rho\sigma}$ represents the contribution from the dynamical three-body correlation function in (5) and (6), $W^{(\mathrm{ii})}_{\rho\sigma}$ represents the one from the dynamical $q\bar q g$-correlation function in (8), and $W^{(\mathrm{iii})}_{\rho\sigma}$ represents the one from the kinematical FFs in (2). To show (B3), it suffices to prove that each term of (B4) separately satisfies (B3).

1. Contribution from dynamical FFs: $W^{(\mathrm{i})}_{\rho\sigma}$

We first show $W^{(\mathrm{i})}_{\rho\sigma}$ satisfies (B3). Relevant diagrams are shown in Figs. 21(a)–21(c). Here, Fig. 21(b) is meant to contain diagrams including those of Fig. 21(c), which is not a part of partonic cross sections for the dynamical FFs. Therefore, after using the EOM relations, we get the following combination of the hard cross section,

$$W^{(\mathrm{i})}_{\rho\sigma} = W^{(a)}_{\rho\sigma} + (W^{(b)L}_{\rho\sigma} - W^{(c)L}_{\rho\sigma}) + (W^{(b)R}_{\rho\sigma} - W^{(c)R}_{\rho\sigma}), \tag{B5}$$

where the hard part with index L indicates diagrams in Fig. 21(b) and 21(c) and that with R indicates their Hermitian conjugate diagrams. We define the amplitude $M^{ad}_{\rho,\alpha}$ for $qg \to gq$ scattering and $M^{acd}_{\rho,\alpha\gamma}$ for $qg \to ggq$ scattering as shown in Fig. 21. With the physical

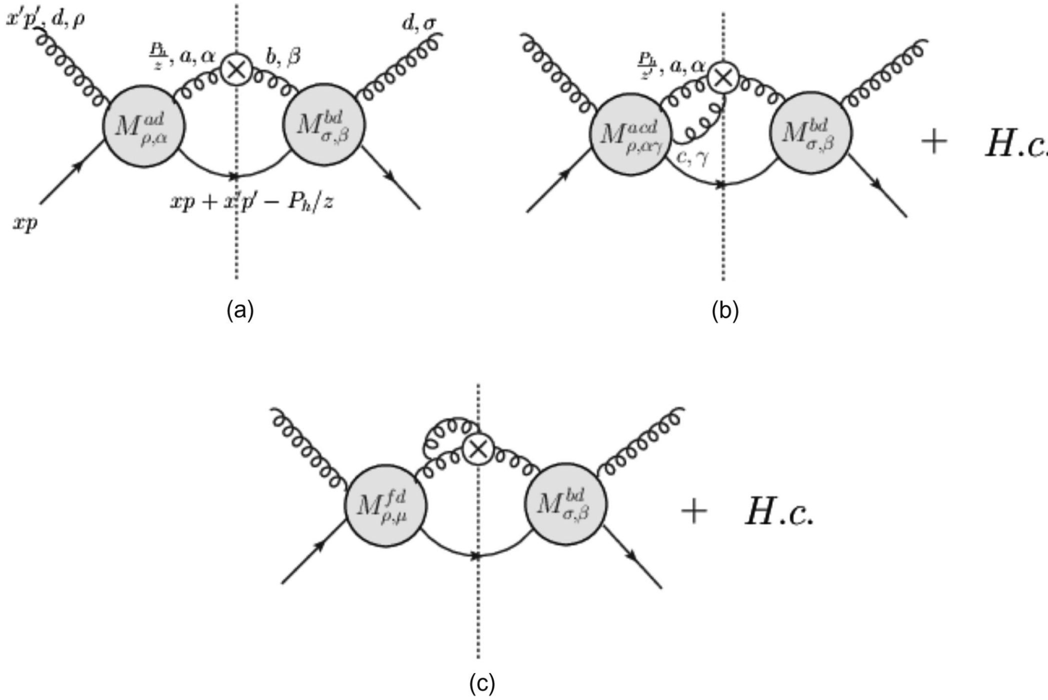

FIG. 21. Diagrams contributing to $W^{(\mathrm{i})}_{\rho\sigma}$. $M^{ad}_{\rho,\alpha}$, and $M^{acd}_{\rho,\alpha\gamma}$, respectively, represent the $qg \to gq$ and $qg \to ggq$ scattering amplitudes, which constitute the hard cross section as shown in the figure.

polarizations for the Lorentz indices α and γ, these amplitudes satisfy the following relations:

$$x' p'^{\rho} M^{ad}_{\rho,\alpha} = 0, \qquad x' p'^{\rho} M^{acd}_{\rho,\alpha\gamma} = 0. \tag{B6}$$

This implies

$$x' p'^{\rho} W^{(b)L}_{\rho\sigma} = 0, \quad x' p'^{\rho} W^{(b)R}_{\rho\sigma} = 0, \quad x' p'^{\rho} W^{(c)R}_{\rho\sigma} = 0. \tag{B7}$$

We thus have

$$x' p'^{\rho} W^{(\mathrm{i})}_{\rho\sigma} = x' p'^{\rho} W^{(a)}_{\rho\sigma} - x' p'^{\rho} W^{(c)L}_{\rho\sigma}. \tag{B8}$$

The first term on the rhs of (B8) reads

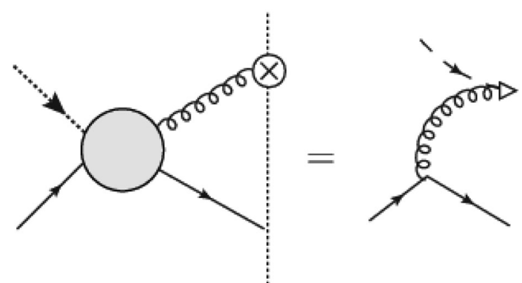

FIG. 22. Diagrammatic representation for $x' p'^{\rho} W^{(a)}_{\rho\sigma}$.

$$x' p'^{\rho} W^{(a)}_{\rho\sigma} = \frac{F^{ade}}{(x'p' - P_h/z)^2} d^{\alpha\rho}(P_h/z) \times \mathrm{Tr}[\not{P}_X \gamma_\rho \not{p} T^e M^{\star,ad}_{\sigma,\beta}][-w^\alpha \epsilon^{\beta P_h w S_\perp} - w^\beta \epsilon^{\alpha P_h w S_\perp}] \Delta\hat{G}_{3\bar{T}}(z)|_{3-\mathrm{gluon}}$$
$$= \frac{1}{z} \epsilon^{\beta P_h w S_\perp} \frac{F^{ade}}{(x'p' - P_h/z)^2} \times \mathrm{Tr}[\not{P}_X \not{P}_h \not{p} T^e M^{\star,ad}_{\sigma,\beta}] \frac{\Delta\hat{G}_{3\bar{T}}(z)}{z}\bigg|_{3-\mathrm{gluon}}, \tag{B9}$$

where $P_X = xp + x'p' - P_h/z$, $F^{abc} \equiv -if^{abc}$, and $d^{\mu\sigma}(k) \equiv k^2 g^{\mu\sigma} - k^\mu k^\sigma$ and we have denoted the $\hat{N}_i$ contribution to $\Delta\hat{G}_{3\bar{T}}(z)$ on the rhs of (9) by $\Delta\hat{G}_{3\bar{T}}(z)|_{3-\mathrm{gluon}}$. This result is shown in Fig. 22.

Next, we consider the second term on the rhs of (B8). Since its hard part is proportional to f^{abc}, the contribution from $\hat{\Gamma}^{\alpha\beta\gamma}_{FS}$ drops. The contribution from $\hat{\Gamma}^{\alpha\beta\gamma}_{AS}$ can be written as

$$\begin{aligned}
x' p'^{\rho} W^{(c)L}_{\rho\sigma} \sim & -\frac{1}{2} z \epsilon^{\beta P_h w S_\perp} \frac{F^{ade}}{(x'p'-P_h/z)^2} \int d\left(\frac{1}{z'}\right) \frac{z'}{z^2} \frac{1}{1-z/z'} \left(1-2\frac{z}{z'}\right) \mathrm{Tr}[\not{P}_X \not{P}_h \not{w} T^e M^{\star,ad}_{\sigma,\beta}] \\
& \times \left[-\hat{N}_2\left(\frac{1}{z}-\frac{1}{z'},\frac{1}{z}\right) + \hat{N}_2\left(\frac{1}{z'},\frac{1}{z}\right) + 2\hat{N}_1\left(\frac{1}{z'},\frac{1}{z}\right)\right] \\
= & -\frac{1}{2} z \epsilon^{\beta P_h w S_\perp} \frac{F^{ade}}{(x'p'-P_h/z)^2} \mathrm{Tr}[\not{P}_X \not{P}_h \not{w} T^e M^{\star,ad}_{\sigma,\beta}] \\
& \times \int d\left(\frac{1}{z'}\right) \frac{1}{z} \left\{ -\frac{1}{1-z/z'} + \frac{z'}{z} \right\} \left[-\hat{N}_2\left(\frac{1}{z}-\frac{1}{z'},\frac{1}{z}\right) + \hat{N}_2\left(\frac{1}{z'},\frac{1}{z}\right) + 2\hat{N}_1\left(\frac{1}{z'},\frac{1}{z}\right)\right] \\
= & -\frac{1}{2} z \epsilon^{\beta P_h w S_\perp} \frac{F^{ade}}{(x'p'-P_h/z)^2} \mathrm{Tr}[\not{P}_X \not{P}_h \not{w} T^e M^{\star,ad}_{\sigma,\beta}] \\
& \times \int d\left(\frac{1}{z'}\right) \frac{1}{z} \left(-2 \times \frac{1}{1-z/z'} \right) \left[-\hat{N}_2\left(\frac{1}{z}-\frac{1}{z'},\frac{1}{z}\right) + \hat{N}_2\left(\frac{1}{z'},\frac{1}{z}\right) + 2\hat{N}_1\left(\frac{1}{z'},\frac{1}{z}\right)\right] \\
= & \frac{1}{z} \epsilon^{\beta P_h w S_\perp} \frac{F^{ade}}{(x'p'-P_h/z)^2} \mathrm{Tr}[\not{P}_X \not{P}_h \not{w} T^e M^{\star,ad}_{\sigma,\beta}] \\
& \times \int d\left(\frac{1}{z'}\right) \frac{1}{1/z-1/z'} \left[-\hat{N}_2\left(\frac{1}{z}-\frac{1}{z'},\frac{1}{z}\right) + \hat{N}_2\left(\frac{1}{z'},\frac{1}{z}\right) + 2\hat{N}_1\left(\frac{1}{z'},\frac{1}{z}\right)\right], \qquad \text{(B10)}
\end{aligned}$$

which can be diagrammatically written as Fig. 23. Using the EOM relation (9), one finds (B9) is equal to (B10), which implies (B8)=0. This completes the proof for the relation $x'p'^{\rho}W^{(\mathrm{i})}_{\rho\sigma}(xp, x'p', P_h/z) = 0$.

2. Contribution from quark-antiquark-gluon FF: $W^{(\mathrm{ii})}_{\rho\sigma}$

Next, we show $W^{(\mathrm{ii})}_{\rho\sigma}$ shown in Fig. 24 satisfies (B3). Figure 24(b') is defined to include the contribution of the type Fig. 24(c'), which is not a part of the hard cross section for a quark-antiquark-gluon contribution. Accordingly the corresponding hard cross section is written as

$$W^{(\mathrm{iii})}_{\rho\sigma} = W^{(a)}_{\rho\sigma} + (W^{(b)'L}_{\rho\sigma} - W^{(c)'L}_{\rho\sigma}) + (W^{(b)'R}_{\rho\sigma} - W^{(c)'R}_{\rho\sigma}). \qquad \text{(B11)}$$

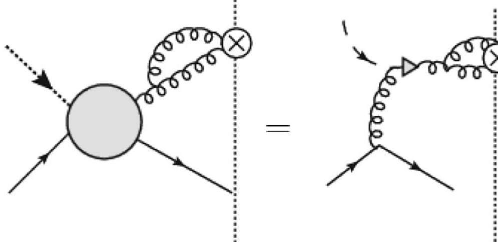

FIG. 23. Diagrammatic representation for $x'p'^{\rho}W^{(c)L}_{\rho\sigma}$ in the $qg \to gq$ channel.

Taking into account the relation (B6), we have

$$x'p'^{\rho}W^{(b)'L}_{\rho\sigma} = 0, \quad x'p'^{\rho}W^{(b)'R}_{\rho\sigma} = 0, \quad x'p'^{\rho}W^{(c)'R}_{\rho\sigma} = 0. \qquad \text{(B12)}$$

Therefore, we obtain

$$x'p'^{\rho}W^{(\mathrm{iii})}_{\rho\sigma} = x'p'^{\rho}W^{(a)}_{\rho\sigma} - x'p'^{\rho}W^{(c)'L}_{\rho\sigma}. \qquad \text{(B13)}$$

The second term on the rhs of this equation can be written as

$$\begin{aligned}
x'p'^{\rho}W^{(c)'L}_{\rho\sigma} \sim & -\frac{1}{z}\frac{2}{C_F} \epsilon^{\beta P_h w S_\perp} \frac{F^{ade}}{(x'p'-P_h/z)^2} \\
& \times \mathrm{Tr}[\not{P}_X \not{P}_h \not{w} T^e M^{\star,ad}_{\sigma,\beta}] \tilde{D}_{FT}, \qquad \text{(B14)}
\end{aligned}$$

which is diagrammatically written as Fig. 25. We again find the coefficient of $(2/C_F)\tilde{D}_{FT}$ in (B14) is equal to the coefficient of $\Delta\hat{G}_{3\bar{T}}(z)/z|_{3-\mathrm{gluon}}$ in (B9), which shows (B13) vanishes, owing to the EOM relation (9).

3. Contribution from kinematical FFs: $W^{(\mathrm{iii})}_{\rho\sigma}$

Using the explicit forms of the intrinsic and kinematical FFs in (1) and (2), $W^{(\mathrm{iii})}_{\rho\sigma}(xp, x'p', P_h/z)$ can be written as

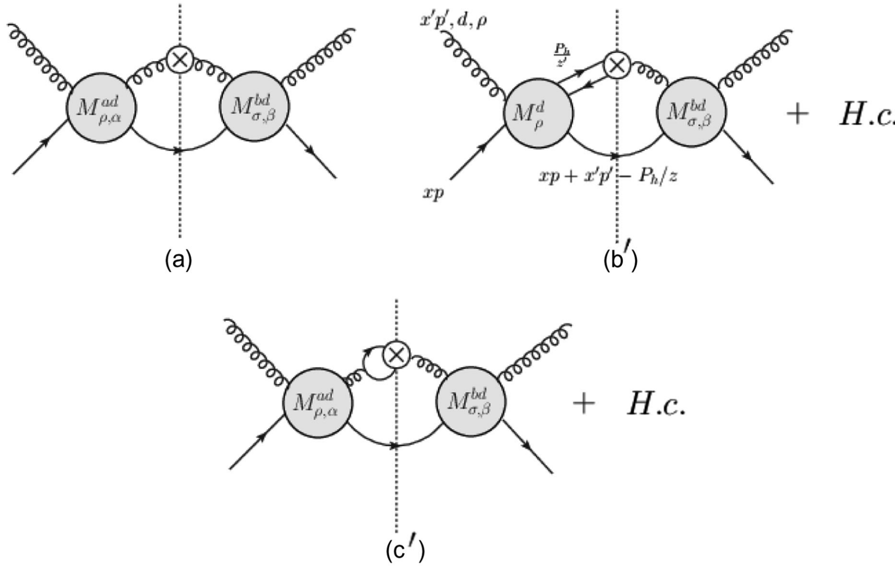

FIG. 24. Diagrams contributing to $W^{(\mathrm{iii})}_{\rho\sigma}$. M^d_ρ represents the $gq \to q\bar{q}q$ scattering amplitude.

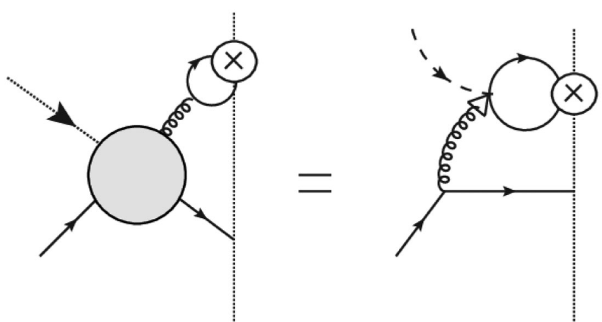

FIG. 25. Diagrammatic representation of $x'p'^\rho W^{(c)'L}_{\rho\sigma}$.

$$
\begin{aligned}
&W^{(\mathrm{iii})}_{\rho\sigma}(xp,x'p',P_h/z)\\
&=\frac{M_h}{2}\mathrm{Tr}[(\epsilon^{P_h w S_\perp \mu}w^\nu+\epsilon^{P_h w S_\perp \nu}w^\mu)S_{\mu\nu,\rho\sigma}(P_h/z)]\\
&\quad\times z(\hat{G}_T^{(1)}(z)+\Delta\hat{H}_T^{(1)}(z))\\
&\quad-\frac{M_h}{2}\mathrm{Tr}\left[g_\perp^{\mu\nu}\epsilon^{P_h w S_\perp \lambda}\frac{\partial S_{\mu\nu,\rho\sigma}(k)}{\partial k^\lambda}\bigg|_{\mathrm{c.l.}}\right]\hat{G}_T^{(1)}(z)\\
&\quad-\frac{M_h}{8}\mathrm{Tr}\left[(\epsilon^{P_h w S_\perp \{\mu}g_\perp^{\nu\}\lambda}+\epsilon^{P_h w \lambda\{\mu}S_\perp^{\nu\}})\frac{\partial S_{\mu\nu,\rho\sigma}(k)}{\partial k^\lambda}\bigg|_{\mathrm{c.l.}}\right]\\
&\quad\times\Delta\hat{H}_T^{(1)}(z). \qquad\qquad \text{(B15)}
\end{aligned}
$$

with $g_\perp^{\mu\nu}=g^{\mu\nu}-P_h^\mu w^\nu-P_h^\nu w^\mu$. To prove $x'p'^\rho W^{(iii)}_{\rho\sigma}(xp, x'p', P_h/z)=0$, one needs to show the coefficients of $\hat{G}_T^{(1)}(z)$ and $\Delta\hat{H}_T^{(1)}(z)$ in (B15) satisfy this property. To this end, we first note that one can set k on shell, $k=(k^+,k^-,\vec{k}_\perp)=(k^+,\frac{\vec{k}_\perp^2}{2k^+},\vec{k}_\perp)$, in $S_{\mu\nu,\rho\sigma}(k)$ by regarding k^- as a dependent variable, since we take the collinear limit $k\to P_h/z$ after taking the derivative. We also introduce the following tensors for an on-shell k,

$$A_1^{\mu\nu\lambda}(k)=\epsilon^{P_h w S_\perp \lambda}g_\perp^{\mu\nu}(k), \qquad \text{(B16)}$$

$$
\begin{aligned}
A_2^{\mu\nu\lambda}(k)=\frac{1}{k\cdot w}&(\epsilon^{kwS_\perp\mu}g_\perp^{\nu\lambda}(k)+\epsilon^{kwS_\perp\nu}g_\perp^{\mu\lambda}(k)\\
&+\epsilon^{kw\lambda\mu}S_\perp^\nu(k)+\epsilon^{kw\lambda\nu}S_\perp^\mu(k)), \qquad \text{(B17)}
\end{aligned}
$$

where

$$g_\perp^{\mu\nu}(k)=g^{\mu\nu}-\frac{k^\mu w^\nu+k^\nu w^\mu}{k\cdot w}, \qquad S_\perp^\mu(k)=S_\perp^\mu-\frac{k\cdot S_\perp}{k\cdot w}w^\mu. \qquad \text{(B18)}$$

Since $g_\perp^{\mu\nu}(k)k_\mu=0$ and $S_\perp^\mu(k)k_\mu=0$, $A_{1,2}^{\mu\nu\lambda}(k)k_\mu=A_{1,2}^{\mu\nu\lambda}(k)k_\nu=0$. $A_{1,2}^{\mu\nu\lambda}(k)$ also satisfy the following relations:

$$A_1^{\mu\nu\lambda}(P_h/z)=\epsilon^{P_h w S_\perp \lambda}g_\perp^{\mu\nu}, \qquad \text{(B19)}$$

$$A_2^{\mu\nu\lambda}(P_h/z)=\epsilon^{P_h w S_\perp \{\mu}g_\perp^{\nu\}\lambda}+\epsilon^{P_h w\lambda\{\mu}S_\perp^{\nu\}}, \qquad \text{(B20)}$$

$$\frac{\partial A_1^{\mu\nu\lambda}(k)}{\partial k^\lambda}\bigg|_{k=P_h/z}=-z(\epsilon^{P_h w S_\perp\mu}w^\nu+\epsilon^{P_h w S_\perp\nu}w^\mu), \qquad \text{(B21)}$$

$$\frac{\partial A_2^{\mu\nu\lambda}(k)}{\partial k^\lambda}\bigg|_{k=P_h/z}=-4z(\epsilon^{P_h w S_\perp\mu}w^\nu+\epsilon^{P_h w S_\perp\nu}w^\mu). \qquad \text{(B22)}$$

From these relations, we have

$$A_{1,2}^{\mu\nu\lambda}(k)S_{\mu\nu,\rho\sigma}(k)x'p'^\rho=0. \qquad \text{(B23)}$$

This is because the Lorentz indices μ and ν provide physical polarizations for the on-shell gluon with momentum k. By taking the derivative of (B23) with respect to k^λ and setting $k \to P_h/z$, it is easy to see $x'p'^\rho W^{(\mathrm{iii})}_{\rho\sigma}(xp, x'p', P_h/z) = x'p'^\sigma W^{(\mathrm{iii})}_{\rho\sigma}(xp, x'p', P_h/z) = 0$, which completes the proof.

APPENDIX C: SEPARATION OF THE THREE-BODY PARTONIC CROSS SECTIONS BASED ON z' DEPENDENCE

Here, we discuss separation of the three-body cross section (52) based on z' dependence. Inserting (5) into (51), we write the cross section for $\Im\hat{N}_i$ as

$$E_h \frac{d\sigma(p,p',P_h;S_\perp)}{d^3P_h} \sim \int \left(\frac{1}{z}\right) z^2 \int d\left(\frac{1}{z'}\right) \delta((xp + x'p' - P_h/z)^2) \times \frac{z'}{z} P\left(\frac{1}{1/z - 1/z'}\right) \sum_{i=1}^{3} \Im\hat{N}_i \hat{\sigma}\left(\frac{1}{z'}, \frac{1}{z}\right), \tag{C1}$$

where $\hat{\sigma}(\frac{1}{z'}, \frac{1}{z})$ is a partonic hard cross section defined by

$$\Re\left[-i\Omega^\mu_\alpha \Omega^\nu_\beta \Omega^\lambda_\gamma \frac{if^{abc}}{N} \hat{\Gamma}^{\alpha\beta\gamma}_{FA}\left(\frac{1}{z'}, \frac{1}{z}\right) S^L_{\mu\nu\lambda,abc}(z', z))\right] = \delta((xp + x'p' - P_h/z)^2) \sum_{i=1}^{3} \Im\hat{N}_i \hat{\sigma}\left(\frac{1}{z'}, \frac{1}{z}\right), \tag{C2}$$

to which diagrams shown in Fig. 26 contribute in the $qg \to gq$ channel.

The amplitude in Fig. 26 has two propagators with z' dependence (① and ②). For these two propagators, z' dependence is written as follows:

$$① \frac{x\not{p} - \not{P}_h/z'}{(xp - P_h/z')^2} = \underbrace{-\frac{\not{P}_h/z}{\hat{t}}}_{①-\mathrm{I}} + \frac{z'}{z} \underbrace{\frac{x\not{p}}{\hat{t}}}_{①-\mathrm{II}}, \tag{C3}$$

and

$$② \frac{x\not{p} + x'\not{p}' - \not{P}_h/z'}{(xp + x'p' - P_h/z')^2} = \underbrace{\frac{\not{P}_h/z}{\hat{s}}}_{②-\mathrm{I}} + \frac{1}{1 - z/z'} \underbrace{\frac{x\not{p} + x'\not{p}' - \not{P}_h/z}{\hat{s}}}_{②-\mathrm{II}}. \tag{C4}$$

In these two equations, we call each term on the rhs ①-I, ①-II, ②-I, and ②-II as shown in the figure. Then, depending on the combination of the propagators, one can separate the z' dependence of the cross section as

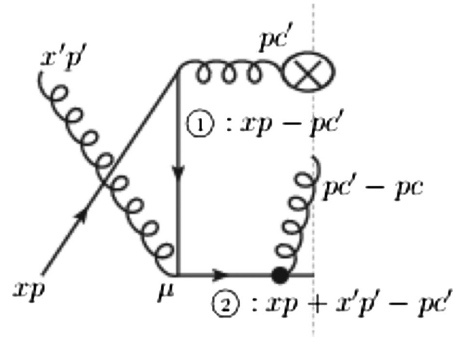

FIG. 26. An example of diagrams which contribute to $qg \to gq$ channel. Notation $pc \equiv P_h/z$ and $pc' \equiv P_h/z'$ is used.

$$E_h \frac{d\sigma(p,p',P_h;S_\perp)}{d^3P_h} \sim \int \left(\frac{1}{z}\right) z^2 \int d\left(\frac{1}{z'}\right) \delta((xp + x'p' - P_h/z)^2) \times \frac{z'}{z} P\left(\frac{1}{1/z - 1/z'}\right) \sum_{i=1}^{3} \Im\hat{N}_i \left[\hat{\sigma}_{p1} + \frac{1}{1 - z/z'}\hat{\sigma}_{p2} + \frac{z'}{z}\hat{\sigma}_{p3}\right], \tag{C5}$$

where each partonic cross section is defined as

$$\hat{\sigma}_{p1} \sim \cdots \mathrm{Tr}[\cdots(② - \mathrm{I})\gamma^\mu(① - \mathrm{I})\cdots]\cdots, \tag{C6}$$

$$\hat{\sigma}_{p2} \sim \cdots \mathrm{Tr}[\cdots(② - \mathrm{II})\gamma^\mu(① - \mathrm{I})\cdots] + \cdots \mathrm{Tr}[\cdots(② - \mathrm{II})\gamma^\mu(① - \mathrm{II})\cdots]\cdots, \tag{C7}$$

$$\hat{\sigma}_{p3} \sim \cdots \mathrm{Tr}[\cdots(② - \mathrm{I})\gamma^\mu(① - \mathrm{II})\cdots] + \cdots \mathrm{Tr}[\cdots(② - \mathrm{II})\gamma^\mu(① - \mathrm{II})\cdots],\cdots \tag{C8}$$

and we used the relation $z'/z \times 1/(1 - z/z') = z'/z + 1/(1 - z/z')$.

Taking into account of the overall factor $z'/z \times 1/(1/z - 1/z')$ and rearranging the z' dependence, one obtains the cross section in the form

$$E_h \frac{d\sigma(p,p',P_h;S_\perp)}{d^3P_h} \sim \int \left(\frac{1}{z}\right) z^2 \int d\left(\frac{1}{z'}\right) \delta((xp + x'p' - P_h/z)^2) \times \sum_{i=1}^{3} \Im\hat{N}_i \left[\frac{1}{1/z - 1/z'}\hat{\sigma}_1 + \frac{1}{z}\frac{1}{(1/z - 1/z')^2}\hat{\sigma}_2 + z'\hat{\sigma}_3 + \frac{z'^2}{z}\hat{\sigma}_4\right], \tag{C9}$$

where

$$\hat{\sigma}_1 = \hat{\sigma}_{p1} + \hat{\sigma}_{p2} + \hat{\sigma}_{p3}, \quad \text{(C10)}$$

$$\hat{\sigma}_2 = \hat{\sigma}_{p2}, \quad \text{(C11)}$$

$$\hat{\sigma}_3 = \hat{\sigma}_{p1} + \hat{\sigma}_{p2} + \hat{\sigma}_{p3}, \quad \text{(C12)}$$

$$\hat{\sigma}_4 = \hat{\sigma}_{p3}. \quad \text{(C13)}$$

This form is the expression used in (52). One can thus separate different z' dependence of the cross section.

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