Magnetic moments and transition magnetic moments of P_c and P_{cs} states

Ming-Wei Li,[1,2] Zhan-Wei Liu[1,2,4,5,*] Zhi-Feng Sun,[1,4,5,†] and Rui Chen[3]

[1]*School of Physical Science and Technology, Lanzhou University, Lanzhou 730000, China*
[2]*Cuiying Honors College, Lanzhou University, Lanzhou 730000, China*
[3]*Center of High Energy Physics, Peking University, Beijing 100871, China*
[4]*Research Center for Hadron and CSR Physics, Lanzhou University and Institute of Modern Physics of CAS,
Lanzhou 730000, China*
[5]*Lanzhou Center for Theoretical Physics, Key Laboratory of Theoretical Physics of Gansu Province,
and Frontiers Science Center for Rare Isotopes, Lanzhou University, Lanzhou, Gansu 730000, China*

We study the magnetic moments and transition magnetic moments of P_c and P_{cs} states in the molecular picture. We first revisit the magnetic moments of $P_c(4312)$, $P_c(4440)$, and $P_c(4457)$ as the S- wave molecular states without coupled channel effects. The coupled channel effects and the D- wave contributions are then investigated carefully. The coupled channel effects contribute to the change of 0.1∼0.4 nuclear magneton μ_N for most cases while the D wave only induces the variation of less than 0.03 μ_N. In addition, we obtain the transition magnetic moments between different P_c states and the related electromagnetic decay widths of $P_c' \to P_c\gamma$. The magnetic moments of $P_{cs}(4459)$ are much different for the assumption of spin being $1/2$ or $3/2$. The study of electromagnetic properties will help us disclose further the structure of these unconventional states.

I. INTRODUCTION

In 2015, LHCb collaboration announced their discovery of $P_c(4380)^+$ and $P_c(4450)^+$ in $\Lambda_b^0 \to J/\psi K^- p$ decay with the significance of more than nine standard deviations [1]. However, due to the limitation of the data, LHCb cannot distinguish all the particles at that time. In 2019, LHCb analyzed the data of Run 1 together with Run 2, and discovered three resonances named $P_c(4312)^+$, $P_c(4440)^+$, and $P_c(4457)^+$, respectively [2]. Note that the peak of the $P_c(4450)^+$ splits into two structures, i.e., $P_c(4440)^+$ and $P_c(4457)^+$, which attributes to the improvement of the precision of the measurement. The new fit of LHCb can neither confirm nor contradict the existence of the $P_c(4380)^+$.

In 2020, LHCb collaboration performed an amplitude analysis of the $\Xi_b^- \to J/\psi \Lambda K^-$ decay using the data of both Run 1 and Run 2. They observed a structure named $P_{cs}(4459)^0$ with the mass and width $4458.8 \pm 2.9^{+4.7}_{-1.1}$ MeV and $17.3 \pm 6.5^{+8.0}_{-5.7}$ MeV, respectively [3].

After the P_c states were discovered, many groups tried to explain their structures within the molecular states assumption [4–16]. Meanwhile, other explanations were also tried, such as hadro-charmonium [17], compact pentaquark states [18–25], virtual states [26], and triangle singularities [27]. The triangle mechanism is applied to investigate the decays of the P_c states [10,28,29]. For reviews see Refs. [30–34]. $P_{cs}(4459)^0$ is interpreted as a hadronic molecular state [34–43], a bound state consisting of two diquarks and an antiquark [44], or of a diquark and a triquark [45]. In addition, the production, decay properties, spin, and other properties are discussed in Refs. [46–49].

Before the discovery of the P_c states by LHCb, several theoretical groups had studied the molecular structure composed of a charmed baryon and an anticharmed meson [50–59]. The P_{cs} state was predicted in Refs. [51,60–63], and was suggested to search for in $\Xi_b^- \to J/\psi \Lambda K^-$ decays.

As pointed out by LHCb, the mass of the $P_c(4312)^+$ is close to the threshold of $\bar{D}\Sigma_c$, while the ones of the $P_c(4440)^+$ and the $P_c(4457)^+$ are both close to the threshold of $\bar{D}^*\Sigma_c$. As a consequence, these states can be the candidates for the molecular states. The fact that the mass splitting of the $P_c(4440)^+/P_c(4457)^+$ and the $P_c(4312)^+$ is close to that of the D^* and the D also supports this hypothesis. Since the mass of $P_{cs}(4459)^0$ is about 19 MeV below the $\bar{D}^*\Xi_c$ threshold, it is arguably

*Corresponding author.
liuzhanwei@lzu.edu.cn
†Corresponding author.
sunzf@lzu.edu.cn

considered as a hadronic molecular state [34–43]. We investigate the electromagnetic properties of P_c and P_{cs} using the molecular picture in this work.

The electromagnetic properties of hadrons are very important for the study of the strong interaction and hadron structure. The electromagnetic form factors of nucleon octet and Δ decuplet are affected by the πN strong interaction and the baryon structure, and they have been widely investigated in various approaches [64–68]. We have studied the loop corrections to the magnetic moments and transition magnetic moments for hadrons within chiral perturbation theory [69–75].

The magnetic moments of the hidden-charmed penta-quark states are studied with the molecule, diquark-triquark, and other configurations within the quark model [76]. The magnetic moments of the P_c and P_{cs} states are investigated using QCD sum rules [77–80]. The magnetic moments of P_c states and couplings with conventional baryon and photon are discussed within the quark model in Ref. [81]. The study of the P_c and P_{cs} magnetic moments would help us understand the structure and search for it in the photoproduction process.

As is well known, the deuteron is a typical molecular state of neutron and proton. If assuming the dominance of the S wave, one expects its magnetic moment is close to $\mu_p + \mu_n = 0.879 \ \mu_N$. The difference from the experiment measurement $\mu_D = 0.857406 \pm 0.000001 \ \mu_N$ can be corrected by the D- wave contribution [82]. Thus one would guess that the magnetic moment of an S-wave dominant molecular should be related to the linear combination of the daughter particle magnetic moments. If P_c and P_{cs} are also typical molecular states, their magnetic moments would also be mainly related to those of conventional charmed mesons and baryons.

Although the short lives of P_c and P_{cs} states make the measurement of the magnetic moments difficult currently, much accumulation data at experiment in the future may make it possible. $\Delta(1232)$ is also very short lived, but its magnetic moments can still be extracted from the experimental data through the $\gamma N \to \Delta \to \Delta\gamma \to \pi N \gamma$ process [83,84]. Moreover, the magnetic moment is a much well defined observable compared to the new proposed observables related to the complicated hadron collider experiments, and thus it can be studied with different approaches and checking the model consistence. Lattice QCD can also extract the magnetic moments [85,86].

We study both the magnetic moments and transition magnetic moments of P_c and P_{cs} states in the molecular picture in this work. In Sec. II, we provide the magnetic moments of P_c and P_{cs} states, and carefully study both the coupled channel effects and the D- wave contributions. In Sec. III, we show the transition magnetic moments, and the partial decay widths $P'_c \to P_c \gamma$, and they are closely related. A short summary follows in Sec. IV.

II. MAGNETIC MOMENTS OF P_c AND P_{cs} STATES

In this section we provide the magnetic moments of P_c and P_{cs} states in the molecular picture. Before doing that, we first revisit how to obtain the magnetic moment for a conventional hadron within the quark model.

We use a baryon $|B\rangle$ with $J = 1/2$ as an example. The magnetic moment is related to the matrix element of the electromagnetic current J_μ, that is, $\langle B|J_\mu|B\rangle$. Usually the matrix element is constrained by the gauge invariance, P-parity conservation, Lorentz covariance, etc. Thus it can be parametrized in terms of few form factors. In the nonrelativistic limit

$$
\begin{aligned}
&\langle B(p)|J_\mu|B(p')\rangle \\
&= e\bar{u}_B \left\{ v_\mu G_E(q^2) - \frac{[S_\mu, S_\nu]q^\nu}{M_B} G_M(q^2) \right\} u_B,
\end{aligned} \tag{1}
$$

where the static velocity $v_\nu = (1, \vec{0})$, the spin matrix $S_\mu = \frac{i}{2}\gamma_5 \sigma_{\mu\nu} v^\nu$, and the difference of baryon momenta $q = p' - p$ is carried by the electromagnetic current.

To see the connection between the magnetic moment and the form factor, we calculate the interacting energy among the baryon and the photon:

$$
\begin{aligned}
\langle \mathcal{L}_{\text{QED}} \rangle &= \langle B(p)\gamma(q)|J_\mu A^\mu|B(p')\rangle \\
&= e\bar{u}_B \left\{ v \cdot \epsilon_\gamma^* G_E(q^2) - \frac{[S_\mu, S_\nu]q^\nu \epsilon_\gamma^{*\mu}}{M_B} G_M(q^2) \right\} u_B \\
&\stackrel{\text{small q}}{\approx} e\epsilon_\gamma^{*0} G_E(0) - \frac{eG_M(0)\vec{S}}{M_B} \cdot (i\vec{q} \times \vec{\epsilon}_\gamma^*).
\end{aligned} \tag{2}
$$

The second term is related to the energy in magnetic field $\vec{\mathcal{B}}$,

$$
\begin{aligned}
\langle B\gamma|J_\mu A^\mu|B\rangle_{\vec{\mathcal{B}}} &\equiv -\frac{eG_M(0)\vec{S}}{M_B} \cdot (i\vec{q} \times \vec{\epsilon}_\gamma^*) \\
&= -\frac{eG_M(0)\vec{S}}{M_B} \cdot \vec{\mathcal{B}}.
\end{aligned} \tag{3}
$$

Comparing it with the magnetic moment energy $-\vec{\mu}_B \cdot \vec{\mathcal{B}}$, one obtains

$$
\vec{\mu}_B = \frac{eG_M(0)\vec{S}}{M_B}. \tag{4}
$$

We can obtain the magnetic moment by directly calculating the matrix element at quark level within the constituent quark model. Take $\mu_{\Sigma_c^{++}}$ as an example. One can give the wave function of Σ_c^{++} with $S_z = +\frac{1}{2}$ in flavor-spin space where the z axis is chosen along the $\vec{\mathcal{B}}$ direction for convenience,

$$|\Sigma_c^{++}\rangle = \sqrt{\frac{2}{3}}|u_\uparrow u_\uparrow c_\downarrow\rangle - \frac{1}{\sqrt{6}}|u_\uparrow u_\downarrow c_\uparrow\rangle - \frac{1}{\sqrt{6}}|u_\downarrow u_\uparrow c_\uparrow\rangle, \quad (5)$$

and the electromagnetic current becomes $J_\mu = Q_u \bar{u}\gamma_\mu u + Q_d \bar{d}\gamma_\mu d + \cdots$ explicitly. From the $\langle \Sigma_c^{++}\gamma |J_\mu A^\mu|\Sigma_c^{++}\rangle_{\vec{\mathcal{B}}}$ at the quark level,

$$\langle \Sigma_c^{++}\gamma |J_\mu A^\mu|\Sigma_c^{++}\rangle_{\vec{\mathcal{B}}}$$
$$= \frac{2}{3}\langle u_\uparrow u_\uparrow c_\downarrow \gamma |J_\mu A^\mu|u_\uparrow u_\uparrow c_\downarrow\rangle_{\vec{\mathcal{B}}} + \frac{1}{6}\langle u_\uparrow u_\downarrow c_\uparrow \gamma |J_\mu A^\mu|u_\uparrow u_\downarrow c_\uparrow\rangle_{\vec{\mathcal{B}}}$$
$$+ \frac{1}{6}\langle u_\downarrow u_\uparrow c_\uparrow \gamma |J_\mu A^\mu|u_\downarrow u_\uparrow c_\uparrow\rangle_{\vec{\mathcal{B}}}$$
$$= -\frac{2}{3}[\vec{\mu}_{u\uparrow} + \vec{\mu}_{u\uparrow} + \vec{\mu}_{c\downarrow}]\cdot\vec{\mathcal{B}} - \frac{1}{6}[\vec{\mu}_{u\uparrow} + \vec{\mu}_{u\downarrow} + \vec{\mu}_{c\uparrow}]\cdot\vec{\mathcal{B}}$$
$$- \frac{1}{6}[\vec{\mu}_{u\downarrow} + \vec{\mu}_{u\uparrow} + \vec{\mu}_{c\uparrow}]\cdot\vec{\mathcal{B}}, \quad (6)$$

we obtain

$$\mu_{\Sigma_c^{++}} = \frac{4}{3}\mu_u - \frac{1}{3}\mu_c. \quad (7)$$

Here, we neglect the anomaly magnetic moments of quarks, $\mu_q = -\mu_{\bar{q}} \approx \frac{Q_q}{2M_q}$.

We list the magnetic moments of conventional hadrons within the quark model in Table I. Similarly, from the $\langle B\gamma |J_\mu A^\mu|B'\rangle_{\vec{\mathcal{B}}}$ at the quark level, we can extract the transition magnetic moment $\mu_{B'\to B}$, which is shown in Table II. These expressions will be used for the magnetic moments of P_c states.

In this work, the same framework is always applied to obtain the magnetic moment μ_ψ and the transition magnetic moment $\mu_{\psi'\to\psi}$. The only tiny difference lies in whether the wave functions of the initial and final states are the same or not. Thus, the calculations are extremely similar for μ_ψ and $\mu_{\psi'\to\psi}$. Throughout this article we usually illustrate how to obtain the magnetic moments with different

TABLE I. Magnetic moments of conventional hadrons.

	$I(J^P)$	Magnetic moments
$\bar{D}^{*0}$	$\frac{1}{2}(1^-)$	$\mu_{\bar{c}} + \mu_u$
$\bar{D}^{*-}$		$\mu_{\bar{c}} + \mu_d$
Ξ_c^0	$\frac{1}{2}(\frac{1}{2}^+)$	μ_c
Ξ_c^+		μ_c
Σ_c^{++}	$1(\frac{1}{2}^+)$	$\frac{4}{3}\mu_u - \frac{1}{3}\mu_c$
Σ_c^+		$\frac{2}{3}\mu_u + \frac{2}{3}\mu_d - \frac{1}{3}\mu_c$
Σ_c^0		$\frac{4}{3}\mu_d - \frac{1}{3}\mu_c$
Σ_c^{*++}	$1(\frac{3}{2}^+)$	$2\mu_u + \mu_c$
Σ_c^{*+}		$\mu_u + \mu_d + \mu_c$
Σ_c^{*0}		$2\mu_d + \mu_c$

TABLE II. Transition magnetic moments between hadrons.

	Transition magnetic moments
$\bar{D}^{*0} \to \bar{D}^0$	$\mu_u - \mu_{\bar{c}}$
$\bar{D}^{*-} \to \bar{D}^-$	$\mu_d - \mu_{\bar{c}}$
$\Sigma_c^{*++} \to \Sigma_c^{++}$	$\frac{2\sqrt{2}}{3}(\mu_u - \mu_c)$
$\Sigma_c^{*+} \to \Sigma_c^+$	$\frac{\sqrt{2}}{3}(\mu_u + \mu_d - 2\mu_c)$
$\Sigma_c^{*0} \to \Sigma_c^0$	$\frac{2\sqrt{2}}{3}(\mu_d - \mu_c)$

scenarios in much detail and will not repeat for the transition magnetic moments.

A. Magnetic moments without coupled channel effects

We consider P_c states as pure molecular states without flavor mixing and do not include the coupled channel effects in this subsection. That is, we assume $P_c(4312)$ is the $\bar{D}\Sigma_c$ molecular state with $I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$, and $P_c(4440)$ and $P_c(4457)$ are the $\bar{D}^*\Sigma_c$ molecular states with $I(J^P)$ being $\frac{1}{2}(\frac{1}{2}^-)$ and $\frac{1}{2}(\frac{3}{2}^-)$, respectively. We show their wave functions in flavor-spin space in Table III.

By calculating the matrix element $\langle P_c\gamma |J_\mu A^\mu|P_c\rangle_{\vec{\mathcal{B}}}$ at the hadronic level with the wave functions of P_c states and the magnetic moments of conventional hadrons, we can extract μ_{P_c}. We list the expressions of them in the second column of Table IV.

To provide the numerical results, we use the masses of the constituent quarks as in Ref. [87]:

$$m_u = m_d = 0.336 \text{ GeV}, \qquad m_s = 0.450 \text{ GeV},$$
$$m_c = 1.680 \text{ GeV}. \quad (8)$$

TABLE III. The possible wave functions of $P_c(4312)$, $P_c(4440)$, and $P_c(4457)$ in flavor $\otimes$ spin space without considering coupled channel effects.

	Flavor-spin wave functions	$I(J^P)$
$P_c(4312)^+$	$\left[\sqrt{\frac{2}{3}}\bar{D}^-\Sigma_c^{++} - \frac{1}{\sqrt{3}}\bar{D}^0\Sigma_c^+\right] \otimes [|0,0\rangle|\frac{1}{2},\frac{1}{2}\rangle]$	$\frac{1}{2}(\frac{1}{2}^-)$
$P_c(4312)^0$	$\left[-\sqrt{\frac{2}{3}}\bar{D}^0\Sigma_c^0 + \frac{1}{\sqrt{3}}\bar{D}^-\Sigma_c^+\right] \otimes [|0,0\rangle|\frac{1}{2},\frac{1}{2}\rangle]$	
$P_c(4440)^+$	$\left[\sqrt{\frac{2}{3}}\bar{D}^{*-}\Sigma_c^{++} - \frac{1}{\sqrt{3}}\bar{D}^{*0}\Sigma_c^+\right]$ $\otimes \left[\sqrt{\frac{2}{3}}|1,1\rangle|\frac{1}{2},-\frac{1}{2}\rangle - \frac{1}{\sqrt{3}}|1,0\rangle|\frac{1}{2},\frac{1}{2}\rangle\right]$	$\frac{1}{2}(\frac{1}{2}^-)$
$P_c(4440)^0$	$\left[-\sqrt{\frac{2}{3}}\bar{D}^{*0}\Sigma_c^0 + \frac{1}{\sqrt{3}}\bar{D}^{*-}\Sigma_c^+\right]$ $\otimes \left[\sqrt{\frac{2}{3}}|1,1\rangle|\frac{1}{2},-\frac{1}{2}\rangle - \frac{1}{\sqrt{3}}|1,0\rangle|\frac{1}{2},\frac{1}{2}\rangle\right]$	
$P_c(4457)^+$	$\left[\sqrt{\frac{2}{3}}\bar{D}^{*-}\Sigma_c^{++} - \frac{1}{\sqrt{3}}\bar{D}^{*0}\Sigma_c^+\right] \otimes [|1,1\rangle|\frac{1}{2},\frac{1}{2}\rangle]$	$\frac{1}{2}(\frac{3}{2}^-)$
$P_c(4457)^0$	$\left[-\sqrt{\frac{2}{3}}\bar{D}^{*0}\Sigma_c^0 + \frac{1}{\sqrt{3}}\bar{D}^{*-}\Sigma_c^+\right] \otimes [|1,1\rangle|\frac{1}{2},\frac{1}{2}\rangle]$	

TABLE IV. The magnetic moment of P_c states for different scenarios in units of the nucleon magneton $\mu_N = \frac{e}{2M_p}$. The scenarios A, B, and C are corresponding to the results in Sec. II A (with neither coupled channel effects nor D- wave contribution), Sec. II B (with coupled channel effects but without D- wave contribution), and Sec. II C (with both coupled channel and D- wave effects), respectively.

Scenario	A		B	C
$P_c(4312)^+$	$\frac{2}{3}\mu_{\Sigma_c^{++}} + \frac{1}{3}\mu_{\Sigma_c^+}$		1.737	1.626	1.624
$P_c(4312)^0$	$\frac{2}{3}\mu_{\Sigma_c^0} + \frac{1}{3}\mu_{\Sigma_c^+}$		-0.744	-0.290	-0.289
$P_c(4440)^+$	$\frac{4}{9}\mu_{\bar{D}^{*-}} - \frac{2}{9}\mu_{\Sigma_c^{++}} + \frac{2}{9}\mu_{\bar{D}^{*0}} - \frac{1}{9}\mu_{\Sigma_c^+}$		-0.827	-0.974	-0.979
$P_c(4440)^0$	$\frac{4}{9}\mu_{\bar{D}^{*0}} - \frac{2}{9}\mu_{\Sigma_c^0} + \frac{2}{9}\mu_{\bar{D}^{*-}} - \frac{1}{9}\mu_{\Sigma_c^+}$		0.620	0.810	0.811
$P_c(4457)^+$	$\frac{2}{3}\mu_{\Sigma_c^{++}} + \frac{2}{3}\mu_{\bar{D}^{*-}} + \frac{1}{3}\mu_{\Sigma_c^+} + \frac{1}{3}\mu_{\bar{D}^{*0}}$		1.365	1.120	1.145
$P_c(4457)^0$	$\frac{2}{3}\mu_{\Sigma_c^0} + \frac{2}{3}\mu_{\bar{D}^{*0}} + \frac{1}{3}\mu_{\Sigma_c^+} + \frac{1}{3}\mu_{\bar{D}^{*-}}$		-0.186	0.106	0.100

We show the numerical results in the third column of Table IV.

From Table IV, the magnetic moment signs of $P_c(4312)^+$ and $P_c(4457)^+$ [$P_c(4312)^0$ and $P_c(4457)^0$] are the same as that of proton [neutron], while that of $P_c(4440)^+$ [$P_c(4440)^0$] is opposite to that of proton [neutron]. This property may help us to distinguish the structures of two charged P_c^+ states with $J^P = \frac{1}{2}^-$, that is, the one with positive magnetic moment measured in the future is more probably $\bar{D}\Sigma$ rather than $\bar{D}^*\Sigma$ molecule with $J^P = \frac{1}{2}^-$.

B. Magnetic moments with coupled channel effects

In Ref. [4], we studied these P_c states as hidden charmed molecular states within the one-boson exchange (OBE) model. The coupled channel effects and D- wave contributions have been investigated. We have considered the interactions among the channels $\bar{D}\Sigma_c$, $\bar{D}^*\Sigma_c$, $\bar{D}\Sigma_c^*$, and $\bar{D}^*\Sigma_c^*$, which are induced from the π, η, ρ, ω, σ exchanges. Our results demonstrate explicitly that these P_c states correspond to the loosely bound states made of an anti-charmed meson and a charmed baryon.

With the framework in Ref. [4], we further study the magnetic moments with considering the coupled channel effect and D- wave contributions. Taking $P_c(4457)$ as an example, we show how we study the coupled channel effects for the magnetic moments. The wave function of $P_c(4457)$ is

$$|P_c(4457)\rangle \sim \left|\bar{D}^*\Sigma_c, I = \frac{1}{2}, J = \frac{3}{2}\right\rangle \otimes Y_{00}(\Omega)R_{S1}(r)$$

$$+ \left|\bar{D}^*\Sigma_c^*, I = \frac{1}{2}, J = \frac{3}{2}\right\rangle \otimes Y_{00}(\Omega)R_{S2}(r)$$

$$+ D\text{-wave contribution}, \qquad (9)$$

where $R_{Si}(r)$ are the radial wave functions of the corresponding channel in S wave, and

$$\int dr\, r^2 \left(|R_{S1}|^2 + |R_{S2}|^2 + |R_{D\,\text{wave}}|^2\right) = 1. \quad (10)$$

Since the D-wave component is small in the OBE model ($\int dr\, r^2 |R_{S1}|^2 + |R_{S2}|^2 = 95\%$ [4]), we neglect the contribution of D wave to the magnetic moment and mainly focus on the effects of coupled channels in this subsection.

Because the S wave does not contribute the orbital momentum magnetic moment, $\mu_{P_c(4457)}$ has three contributions:

$$\mu_{P_c(4457)} = \mu_{S1}\int dr\, r^2 |R_{S1}|^2 + \mu_{S2}\int dr\, r^2 |R_{S2}|^2$$

$$+ \mu_{S1\to S2}\int dr\, r^2 (R_{S1}R_{S2}^\dagger + R_{S1}^\dagger R_{S2}), \qquad (11)$$

where μ_{S1} and μ_{S2} are the magnetic moments from the contributions of the first and second term in Eq. (9), and $\mu_{S1\to S2}$ is the transition magnetic moment between the two components. These can be extracted with the similar method in Sec. II A.

With the radial wave functions in Ref. [4], we can obtain the magnetic moment of $P_c(4457)$ with the coupled channel effect:

$$\mu_{P_c(4457)^+} = 1.120\,\mu_N, \qquad \mu_{P_c(4457)^0} = 0.106\,\mu_N. \quad (12)$$

Similarly, we can also obtain those for $P_c(4312)$ and $P_c(4440)$, and put the numerical results with coupled channel effects in the fourth column of Table IV. By comparing the numerical results between the third and fourth columns, we can see that the coupled channel effects can change the magnetic moments by less than 0.3 μ_N for $P_c(4312)^+$, $P_c(4440)^+$, $P_c(4440)^0$, $P_c(4457)^+$, and $P_c(4457)^0$. $\mu_{P_c(4312)^0}$ varies by about 0.5 μ_N which is a littler bigger than other cases. $\mu_{P_c(4457)^0}$ is very small, and the coupled channel effects would make the sign of $\mu_{P_c(4457)^0}$ change.

C. Contributions from D waves

First we write the wave function of $P_c(4457)$ with the contents of the D waves explicitly:

$$|P_c(4457)\rangle \sim \left|\bar{D}^*\Sigma_c, I = \frac{1}{2}, J = \frac{3}{2}\right\rangle \otimes Y_{00}(\Omega)R_{S1}(r) + \left|\bar{D}^*\Sigma_c^*, I = \frac{1}{2}, J = \frac{3}{2}\right\rangle \otimes Y_{00}(\Omega)R_{S2}(r)$$

$$+ \left|\bar{D}^*\Sigma_c, I = \frac{1}{2}\right\rangle \otimes |^2D_{3/2}\rangle \times R_{D1}(r) + \left|\bar{D}^*\Sigma_c^*, I = \frac{1}{2}\right\rangle \otimes |^2D'_{3/2}\rangle \times R_{D2}(r)$$

$$+ \left|\bar{D}^*\Sigma_c, I = \frac{1}{2}\right\rangle \otimes |^4D_{3/2}\rangle \times R_{D3}(r) + \left|\bar{D}^*\Sigma_c^*, I = \frac{1}{2}\right\rangle \otimes |^4D'_{3/2}\rangle \times R_{D4}(r)$$

$$+ \left|\bar{D}^*\Sigma_c^*, I = \frac{1}{2}\right\rangle \otimes |^6D'_{3/2}\rangle \times R_{D5}(r), \tag{13}$$

where $|^{2S+1}L_J\rangle$ can be constructed with the help of the Clebsch-Gordan coefficients, for example, for an upward $P_c(4457)$,

$$|^2D_{3/2}\rangle = \sqrt{\frac{4}{5}}\left|S_{\bar{D}^*\Sigma_c} = \frac{1}{2}, -\frac{1}{2}\right\rangle \times Y_{2,2} - \sqrt{\frac{1}{5}}\left|S_{\bar{D}^*\Sigma_c}\right.$$

$$= \frac{1}{2}, +\frac{1}{2}\right\rangle \times Y_{2,1}. \tag{14}$$

Now the magnetic moment of $P_c(4457)$ can be expressed as

$$\mu_{P_c(4457)} = \sum_i \mu_{Si} \int dr\, r^2 |R_{Si}|^2$$

$$+ \sum_{i \neq j} \mu_{Si \to Sj} \int dr\, r^2 R_{Si} R_{Sj}^{\dagger}$$

$$+ \sum_i \mu_{Di} \int dr\, r^2 |R_{Di}|^2$$

$$+ \sum_{i \neq j} \mu_{Di \to Dj} \int dr\, r^2 R_{Di} R_{Dj}^{\dagger}. \tag{15}$$

Please notice there is no transition magnetic moment between S and D waves since there is a selection rule $\Delta L = 0$ for the magnetic dipole transition due to the conservation of angular momentum and parity.

The magnetic moments μ_{Di} and $\mu_{Di \to Dj}$ from D-wave contributions contain two parts. One is from spin contributions, which can be obtained as in previous subsections. The other is contributed from the orbital angular momenta, and can be expressed as

$$\vec{\mu}_{\alpha_1\alpha_2}^L = \mu_{\alpha_1\alpha_2}^{\text{orbital}}\vec{L}, \tag{16}$$

where

$$\mu_{\alpha_1\alpha_2}^{\text{orbital}} \equiv \frac{M_{\alpha_1}}{M_{\alpha_1} + M_{\alpha_2}}\frac{Q_{\alpha_2}}{2M_{\alpha_2}} + \frac{M_{\alpha_2}}{M_{\alpha_1} + M_{\alpha_2}}\frac{Q_{\alpha_1}}{2M_{\alpha_1}}. \tag{17}$$

Now we can obtain expressions for the magnetic moments μ_{Di} and $\mu_{Di \to Dj}$ from D-wave contributions. Take μ_{D1} as an example. μ_{D1} is contributed by the third line of Eq. (13). With the help of Eq. (14),

$$\mu_{D1} = \frac{4}{5}(-\mu_{S'} + 2\mu_{\bar{D}^*\Sigma_c}^{\text{orbital}}) + \frac{1}{5}(\mu_{S'} + \mu_{\bar{D}^*\Sigma_c}^{\text{orbital}}), \tag{18}$$

where $\mu_{S'}$ is short for the magnetic moment of the S-wave state $|\bar{D}^*\Sigma_c, I = \frac{1}{2}, J = \frac{1}{2}\rangle$ which only has the spin contribution.

Now we can provide the magnetic moment of $P_c(4457)$ with coupled channel effects and contributions of D waves:

$$\mu_{P_c(4457)^+} = 1.153\mu_N, \qquad \mu_{P_c(4457)^0} = 0.093\mu_N. \tag{19}$$

$\mu_{P_c(4312)}$ and $\mu_{P_c(4440)}$ can be similarly obtained. We arrange the numerical results with both coupled channel effects and D-wave contributions in the last column of Table IV. From the table, we can see that the D-wave contribution is small. The magnetic moments of $P_c(4312)$ are almost unchanged after the D waves are included. The D-wave contributions to $P_c(4440)$ and $P_c(4457)$ are relatively bigger because they contain a few more D-wave components.

D. Magnetic moments of P_{cs} states

If P_{cs} is the S-wave $\bar{D}^*\Xi_c$ molecular states with $I(J^P) = 0(\frac{1}{2}^-)$, the magnetic moments should be

$$\mu_{P_{cs}}^{1/2^-} = \frac{1}{3}\mu_{\bar{D}^{*0}} + \frac{1}{3}\mu_{\bar{D}^{*-}} - \frac{1}{6}\mu_{\Xi_c^0} - \frac{1}{6}\mu_{\Xi_c^+} = -0.062\mu_N. \tag{20}$$

If with $I(J^P) = 0(\frac{3}{2}^-)$,

$$\mu_{P_{cs}}^{3/2^-} = \frac{1}{2}\mu_{\bar{D}^{*0}} + \frac{1}{2}\mu_{\bar{D}^{*-}} + \frac{1}{2}\mu_{\Xi_c^0} + \frac{1}{2}\mu_{\Xi_c^+} = 0.465\mu_N. \tag{21}$$

TABLE V. Transition magnetic moments in units of μ_N and electromagnetic decay widths in units of keV for the P_c states.

Scenario	Without coupled channel and D -wave effects			With both effects	
	$\mu_{P'_c \to P_c}$		$\Gamma_{P'_c \to P_c \gamma}$	$\mu_{P'_c \to P_c}$	$\Gamma_{P'_c \to P_c \gamma}$
$P_c(4440)^+ \to P_c(4312)^+$	$-\frac{\sqrt{3}}{9}(2\mu_{\bar{D}^{*-} \to \bar{D}^-} + \mu_{\bar{D}^{*0} \to \bar{D}^0})$		-0.215	0.769	-0.205	0.699
$P_c(4440)^0 \to P_c(4312)^0$	$-\frac{\sqrt{3}}{9}(2\mu_{\bar{D}^{*0} \to \bar{D}^0} + \mu_{\bar{D}^{*-} \to \bar{D}^-})$		-0.752	9.423	-0.658	7.205
$P_c(4457)^+ \to P_c(4312)^+$	$\frac{\sqrt{6}}{9}(2\mu_{\bar{D}^{*-} \to \bar{D}^-} + \mu_{\bar{D}^{*0} \to \bar{D}^0})$		0.304	1.112	0.381	1.743
$P_c(4457)^0 \to P_c(4312)^0$	$\frac{\sqrt{6}}{9}(2\mu_{\bar{D}^{*0} \to \bar{D}^0} + \mu_{\bar{D}^{*-} \to \bar{D}^-})$		1.064	13.621	0.700	5.897
$P_c(4457)^+ \to P_c(4440)^+$	$\frac{2\sqrt{2}}{9}(\mu_{\bar{D}^{*-}} - 2\mu_{\Sigma_c^{++}}) + \frac{\sqrt{2}}{9}(\mu_{\bar{D}^{*0}} - 2\mu_{\Sigma_c^+})$		-1.813	0.0666	-0.984	0.0196
$P_c(4457)^0 \to P_c(4440)^0$	$\frac{2\sqrt{2}}{9}(\mu_{\bar{D}^{*0}} - 2\mu_{\Sigma_c^0}) + \frac{\sqrt{2}}{9}(\mu_{\bar{D}^{*-}} - 2\mu_{\Sigma_c^+})$		0.965	0.0189	0.538	0.0059

If the magnetic moment of P_{cs} is measured smaller than $0.1\mu_N$ in experiment in the future, P_{cs} would be the state with $J^P = \frac{1}{2}^-$ rather than $J^P = \frac{3}{2}^-$.

III. TRANSITION MAGNETIC MOMENTS AND THE ELECTROMAGNETIC DECAY WIDTHS OF P_c STATES

With the similar approach in Sec. II, we can obtain the transition magnetic moments between different P_c states. We have checked that the D -wave contribution is very tiny, which is the same as that in Sec. II. We show the results of two different scenarios in Table V. One is without considering coupled channel effects and D wave as in Sec. II A. The other is with coupled channel effects and D wave as in Sec. II C. We show the results in Table V. The coupled channel effects make $\mu_{P_c(4457)^+ \to P_c(4440)^+}$ change by about $0.9\mu_N$.

Furthermore, we can study the decay width $P'_c \to P_c \gamma$ with the values of the transition magnetic moments. Of course, $P'_c \to P_c \gamma$ contains two kinds of contribution: magnetic dipole and electric quadrupole transitions. However, the quadrupole contribution would be suppressed by a factor $[(M_{P'_c} - M_{P_c})/M_{P_c}]^2$ compared with the magnetic dipole decay width. Therefore, we neglect the electric quadrupole contributions.

The decay width is related to the transition magnetic moments as in Ref. [88]:

$$\Gamma_{P'_c \to P_c \gamma} = \alpha_{\text{EM}} \frac{2}{2J+1} \frac{E_\gamma^3}{m_N^2} \left(\frac{\mu_{P'_c \to P_c}}{\mu_N}\right)^2, \qquad (22)$$

where α_{EM} is electromagnetic fine-structure constant, J is initial baryon spin, and E_γ is the energy of photon. The decay width is also listed in Table V.

From Table V, $\Gamma(P_c(4457) \to P_c(4440)\gamma)$ is very small, which is because the mass difference is tiny and thus the kinetic phase space is suppressed. The decay widths of the charged channels $P_c(4440)^+ \to P_c(4312)^+\gamma$ and $P_c(4457)^+ \to P_c(4312)^+\gamma$ are about 1 keV.

IV. SUMMARY

Inspired by the recently observed $P_c(4312)^+$, $P_c(4440)^+$, and $P_c(4457)^+$ as well as the $P_{cs}(4459)^0$, we calculate the magnetic moments, the transition magnetic moments, and the electromagnetic transition decay widths of these particles. Firstly we consider these particles as pure molecular states without flavor mixing. Secondly we take into account the coupled channel effects and the D -wave contribution step by step.

The decay width of $P_c(4457) \to P_c(4312)$ is also studied in the molecular picture in Ref. [29]. A hadronic triangle loop is introduced between the initial and final states. The results rely on the $P_c - \bar{D}^{(*)} - \Sigma^{(*)}$ couplings which can be extracted from the binding energy of P_c states. They estimated the decay width of $P_c(4457) \to P_c(4312)\gamma$ is around 1.5 keV, which is consistent with the results in this work.

The magnetic moments of P_c states are studied within the quark model in Ref. [76]. Without considering the coupled channel effects and D -wave contribution, the results for the negative-parity molecules are $\mu_{P_c(4312)^+} = 1.760\mu_N, \mu_{P_c(4440)^+} = -0.856\mu_N$, and $\mu_{P_c(4457)^+} = 1.357\mu_N$ [76], which are very close to ours.

With $P_c(4312)$ as a $\bar{D}\Sigma_c$ molecular state, $\mu_{P_c(4312)^+} = 1.75^{+0.15}_{-0.11}\mu_N$ is given by QCD sum rules [80]. Özdem uses light-cone QCD sum rules and obtains $\mu_{P_c(4312)^+} = 1.98 \pm 0.75\mu_N$ with the molecule interpolating current while $\mu_{P_c(4312)^+} = 0.40 \pm 0.15\mu_N$ with the diquark-diquark-antiquark configuration [78], which clearly shows the magnetic moment strongly depends on the structure of the hadron. It seems that the QCD sum rule results with the molecule assumptions are closer to that without considering the coupled channel effects and D -wave contribution in this work. We expect $\mu_{P_c(4312)^+}$ might also decrease a little after studying the mixing effects of different meson-baryon interpolating currents within QCD sum rule.

The authors in Refs. [77,79] also obtain $\mu_{P_c(4380)^+}$, $\mu_{P_c(4457)^+}$, $\mu_{P_c(4440)^+}$, and $\mu_{P_{cs}(4459)^0}$ with the molecule

assumption within QCD sum rules. However, we cannot directly compare their results with ours because they construct P_c/P_{cs} molecules with different meson-baryon combinations from this work. In Ref. [81], the magnetic moments of four pentaquark states with $J^P = \frac{3}{2}^-$ are about 1.1–3.1 μ_N, but all masses of these four pentaquark states are lower than those of the observed P_c states.

We find that the coupled channel effects and the D-wave inclusions contribute to about $0.1 \sim 0.4$ μ_N for most cases and less than 0.03 μ_N, respectively, which is not unexpected since these two elements are not dominant in forming the molecular structures of the corresponding states. The D-wave contributions of P_c states are very similar to that of the deuteron. Because the spin of $P_{cs}(4459)^0$ has not been experimentally known yet due to lack of data, the magnetic moments of $P_{cs}(4459)^0$ are obtained with the spin of $1/2$ or $3/2$.

The P_c and P_{cs} electromagnetic properties are closely related to their structure and the $\bar{D}^{(*)} - \Sigma_c^{(*)}/\Xi_c^{(*)}$ strong interactions, and thus the study will help us understand the nonperturbative behaviors of QCD from a different aspect. Hopefully, our investigation may attract the lattice QCD simulations and experiment plans in future.

[1] R. Aaij *et al.* (LHCb Collaboration), Observation of $J/\psi p$ Resonances Consistent with Pentaquark States in $\Lambda_b^0 \to J/\psi K^- p$ Decays, Phys. Rev. Lett. **115**, 072001 (2015).

[2] R. Aaij *et al.* (LHCb Collaboration), Observation of a Narrow Pentaquark State, $P_c(4312)^+$, and of Two-Peak Structure of the $P_c(4450)^+$, Phys. Rev. Lett. **122**, 222001 (2019).

[3] R. Aaij *et al.* (LHCb Collaboration), Evidence of a $J/\psi \Lambda$ structure and observation of excited Ξ^- states in the $\Xi_b^- \to J/\psi \Lambda K^-$ decay, Sci. Bull. **66**, 1278 (2021).

[4] R. Chen, Z.-F. Sun, X. Liu, and S.-L. Zhu, Strong LHCb evidence supporting the existence of the hidden-charm molecular pentaquarks, Phys. Rev. D **100**, 011502 (2019).

[5] C. W. Xiao, J. Nieves, and E. Oset, Heavy quark spin symmetric molecular states from $\bar{D}^{(*)}\Sigma_c^{(*)}$ and other coupled channels in the light of the recent LHCb pentaquarks, Phys. Rev. D **100**, 014021 (2019).

[6] Y. Yamaguchi, H. García-Tecocoatzi, A. Giachino, A. Hosaka, E. Santopinto, S. Takeuchi, and M. Takizawa, P_c pentaquarks with chiral tensor and quark dynamics, Phys. Rev. D **101**, 091502 (2020).

[7] M.-Z. Liu, T.-W. Wu, M. S. Sánchez, M. P. Valderrama, L.-S. Geng, and J.-J. Xie, Spin-parities of the $P_c(4440)$ and $P_c(4457)$ in the one-boson-exchange model, Phys. Rev. D **103**, 054004 (2021).

[8] M. Pavon Valderrama, One pion exchange and the quantum numbers of the P$_c$(4440) and P$_c$(4457) pentaquarks, Phys. Rev. D **100**, 094028 (2019).

[9] M.-L. Du, V. Baru, F.-K. Guo, C. Hanhart, U.-G. Meißner, J. A. Oller, and Q. Wang, Interpretation of the LHCb P_c States as Hadronic Molecules and Hints of a Narrow $P_c(4380)$, Phys. Rev. Lett. **124**, 072001 (2020).

[10] C.-J. Xiao, Y. Huang, Y.-B. Dong, L.-S. Geng, and D.-Y. Chen, Exploring the molecular scenario of Pc(4312), Pc(4440), and Pc(4457), Phys. Rev. D **100**, 014022 (2019).

[11] S. Sakai, H.-J. Jing, and F.-K. Guo, Decays of P_c into $J/\psi N$ and $\eta_c N$ with heavy quark spin symmetry, Phys. Rev. D **100**, 074007 (2019).

[12] L. Meng, B. Wang, G.-J. Wang, and S.-L. Zhu, The hidden charm pentaquark states and $\Sigma_c \bar{D}^{(*)}$ interaction in chiral perturbation theory, Phys. Rev. D **100**, 014031 (2019).

[13] T. J. Burns and E. S. Swanson, Molecular interpretation of the $P_c(4440)$ and $P_c(4457)$ states, Phys. Rev. D **100**, 114033 (2019).

[14] Q. Wu and D.-Y. Chen, Production of P_c states from Λ_b decay, Phys. Rev. D **100**, 114002 (2019).

[15] K. Azizi, Y. Sarac, and H. Sundu, Properties of the $P_c(4312)$ pentaquark and its bottom partner, Chin. Phys. C **45**, 053103 (2021).

[16] K. Phumphan, W. Ruangyoo, C.-C. Chen, A. Limphirat, and Y. Yan, P_c resonances in molecular picture, arXiv:2105.03150.

[17] M. I. Eides, V. Y. Petrov, and M. V. Polyakov, New LHCb pentaquarks as hadrocharmonium states, Mod. Phys. Lett. A **35**, 2050151 (2020).

[18] A. Ali and A. Y. Parkhomenko, Interpretation of the narrow $J/\psi p$ Peaks in $\Lambda_b \to J/\psi p K^-$ decay in the compact diquark model, Phys. Lett. B **793**, 365 (2019).

[19] H. Mutuk, Neural network study of hidden-charm pentaquark resonances, Chin. Phys. C **43**, 093103 (2019).

[20] Z.-G. Wang, Analysis of the $P_c(4312)$, $P_c(4440)$, $P_c(4457)$ and related hidden-charm pentaquark states with QCD sum rules, Int. J. Mod. Phys. A **35**, 2050003 (2020).

[21] J.-B. Cheng and Y.-R. Liu, $P_c(4457)^+$, $P_c(4440)^+$, and $P_c(4312)^+$: Molecules or compact pentaquarks?, Phys. Rev. D **100**, 054002 (2019).

[22] X.-Z. Weng, X.-L. Chen, W.-Z. Deng, and S.-L. Zhu, Hidden-charm pentaquarks and P_c states, Phys. Rev. D **100**, 016014 (2019).

[23] R. Zhu, X. Liu, H. Huang, and C.-F. Qiao, Analyzing doubly heavy tetra- and penta-quark states by variational method, Phys. Lett. B **797**, 134869 (2019).

[24] A. Pimikov, H.-J. Lee, and P. Zhang, Hidden charm pentaquarks with color-octet substructure in QCD Sum Rules, Phys. Rev. D **101**, 014002 (2020).

[25] W. Ruangyoo, K. Phumphan, C.-C. Chen, A. Limphirat, and Y. Yan, P_c resonances in the compact pentaquark picture, arXiv:2105.14249.

[26] C. Fernández-Ramírez, A. Pilloni, M. Albaladejo, A. Jackura, V. Mathieu, M. Mikhasenko, J. A. Silva-Castro, and A. P. Szczepaniak (JPAC Collaboration), Interpretation of the LHCb $P_c(4312)^+$ Signal, Phys. Rev. Lett. **123**, 092001 (2019).

[27] S. X. Nakamura, $P_c(4312)^+$, $P_c(4380)^+$, and $P_c(4457)^+$ as double triangle cusps, Phys. Rev. D **103**, L111503 (2021).

[28] Y.-H. Lin and B.-S. Zou, Strong decays of the latest LHCb pentaquark candidates in hadronic molecule pictures, Phys. Rev. D **100**, 056005 (2019).

[29] X.-Z. Ling, J.-X. Lu, M.-Z. Liu, and L.-S. Geng, $P_c(4457) \rightarrow P_c(4312)\pi/\gamma$ in the molecular picture, arXiv:2106.12250.

[30] Y.-R. Liu, H.-X. Chen, W. Chen, X. Liu, and S.-L. Zhu, Pentaquark and tetraquark states, Prog. Part. Nucl. Phys. **107**, 237 (2019).

[31] N. Brambilla, S. Eidelman, C. Hanhart, A. Nefediev, C.-P. Shen, C. E. Thomas, A. Vairo, and C.-Z. Yuan, The XYZ states: Experimental and theoretical status and perspectives, Phys. Rep. **873**, 1 (2020).

[32] F.-K. Guo, X.-H. Liu, and S. Sakai, Threshold cusps and triangle singularities in hadronic reactions, Prog. Part. Nucl. Phys. **112**, 103757 (2020).

[33] G. Yang, J. Ping, and J. Segovia, Tetra- and penta-quark structures in the constituent quark model, Symmetry **12**, 1869 (2020).

[34] X.-K. Dong, F.-K. Guo, and B.-S. Zou, A survey of heavy-antiheavy hadronic molecules, Progr. Phys. **41**, 65 (2021).

[35] R. Chen, Can the newly reported $P_{cs}(4459)$ be a strange hidden-charm $\Xi_c\bar{D}^*$ molecular pentaquark?, Phys. Rev. D **103**, 054007 (2021).

[36] H.-X. Chen, W. Chen, X. Liu, and X.-H. Liu, Establishing the first hidden-charm pentaquark with strangeness, Eur. Phys. J. C **81**, 409 (2021).

[37] C. W. Xiao, J. J. Wu, and B. S. Zou, Molecular nature of $P_{cs}(4459)$ and its heavy quark spin partners, Phys. Rev. D **103**, 054016 (2021).

[38] J.-X. Lu, M.-Z. Liu, R.-X. Shi, and L.-S. Geng, Understanding $P_{cs}(4459)$ as a hadronic molecule in the $\Xi_b^- \rightarrow J/\psi\Lambda K^-$ decay, arXiv:2104.10303.

[39] Z.-G. Wang and Q. Xin, Analysis of the hidden-charm pentaquark molecular states with strangeness and without strangeness via the QCD sum rules, arXiv:2103.08239.

[40] R. Chen, Strong decays of the newly $P_{cs}(4459)$ as a strange hidden-charm $\Xi_c\bar{D}^*$ molecule, Eur. Phys. J. C **81**, 122 (2021).

[41] H.-X. Chen, Covalent hadronic molecules induced by shared light quarks, arXiv:2105.09193.

[42] J.-T. Zhu, L.-Q. Song, and J. He, $P_{cs}(4459)$ and other possible molecular states from $\Xi_c^{(*)}\bar{D}^{(*)}$ and $\Xi_c'\bar{D}^{(*)}$ interactions, Phys. Rev. D **103**, 074007 (2021).

[43] F.-Z. Peng, M.-J. Yan, M. Sánchez Sánchez, and M. P. Valderrama, The $P_{cs}(4459)$ pentaquark from a combined effective field theory and phenomenological perspectives, arXiv:2011.01915.

[44] Z.-G. Wang, Analysis of the $P_{cs}(4459)$ as the hidden-charm pentaquark state with QCD sum rules, Int. J. Mod. Phys. A **36**, 2150071 (2021).

[45] J. F. Giron, R. F. Lebed, and S. R. Martinez, Spectrum of hidden-charm, open-strange exotics in the dynamical diquark model, arXiv:2106.05883.

[46] S. Clymton, H.-J. Kim, and H.-C. Kim, Production of hidden-charm strange pentaquarks P_{cs} from the $K^- p \rightarrow J/\psi\Lambda$ reaction, Phys. Rev. D **104**, 014023 (2021).

[47] K. Azizi, Y. Sarac, and H. Sundu, Investigation of $P_{cs}(4459)^0$ pentaquark via its strong decay to $\Lambda J/\Psi$, Phys. Rev. D **103**, 094033 (2021).

[48] Q. Wu, D.-Y. Chen, and R. Ji, Production of $P_{cs}(4459)$ from Ξ_b decay, Chin. Phys. Lett. **38**, 071301 (2021).

[49] M.-Z. Liu, Y.-W. Pan, and L.-S. Geng, Can discovery of hidden charm strange pentaquark states help determine the spins of $P_c(4440)$ and $P_c(4457)$?, Phys. Rev. D **103**, 034003 (2021).

[50] Z.-C. Yang, Z.-F. Sun, J. He, X. Liu, and S.-L. Zhu, The possible hidden-charm molecular baryons composed of anti-charmed meson and charmed baryon, Chin. Phys. C **36**, 6 (2012).

[51] J.-J. Wu, R. Molina, E. Oset, and B. S. Zou, Prediction of Narrow N^* and Λ^* Resonances with Hidden Charm Above 4 GeV, Phys. Rev. Lett. **105**, 232001 (2010).

[52] C. W. Xiao, J. Nieves, and E. Oset, Combining heavy quark spin and local hidden gauge symmetries in the dynamical generation of hidden charm baryons, Phys. Rev. D **88**, 056012 (2013).

[53] Y. Huang, J. He, H.-F. Zhang, and X.-R. Chen, Discovery potential of hidden charm baryon resonances via photo-production, J. Phys. G **41**, 115004 (2014).

[54] X.-Y. Wang and X.-R. Chen, The production of hidden charm baryon N*(4261) from $\pi^- p \rightarrow \eta_c n$ reaction, Europhys. Lett. **109**, 41001 (2015).

[55] E. J. Garzon and J.-J. Xie, Effects of a N$_{c\bar{c}}^*$ resonance with hidden charm in the $\pi p D\Sigma_c^+$ reaction near threshold, Phys. Rev. C **92**, 035201 (2015).

[56] X.-Y. Wang and X.-R. Chen, Production of the superheavy baryon $\Lambda_{c\bar{c}}^*$ (4209) in kaon-induced reaction, Eur. Phys. J. A **51**, 85 (2015).

[57] W. L. Wang, F. Huang, Z. Y. Zhang, and B. S. Zou, $\Sigma_c\bar{D}$ and $\Lambda_c\bar{D}$ states in a chiral quark model, Phys. Rev. C **84**, 015203 (2011).

[58] J.-J. Wu, T.-S. H. Lee, and B. S. Zou, Nucleon resonances with hidden charm in coupled-channel models, Phys. Rev. C **85**, 044002 (2012).

[59] S. G. Yuan, K. W. Wei, J. He, H. S. Xu, and B. S. Zou, Study of $qqqc\bar{c}$ five quark system with three kinds of quark-quark hyperfine interaction, Eur. Phys. J. A **48,** 61 (2012).

[60] R. Chen, J. He, and X. Liu, Possible strange hidden-charm pentaquarks from $\Sigma_c^{(*)}\bar{D}_s^*$ and $\Xi_c^{(',*)}\bar{D}^*$ interactions, Chin. Phys. C **41,** 103105 (2017).

[61] E. Santopinto and A. Giachino, Compact pentaquark structures, Phys. Rev. D **96,** 014014 (2017).

[62] C.-W. Shen, J.-J. Wu, and B.-S. Zou, Decay behaviors of possible $\Lambda_{c\bar{c}}$ states in hadronic molecule pictures, Phys. Rev. D **100,** 056006 (2019).

[63] C. W. Xiao, J. Nieves, and E. Oset, Prediction of hidden charm strange molecular baryon states with heavy quark spin symmetry, Phys. Lett. B **799,** 135051 (2019).

[64] P. E. Shanahan, R. Horsley, Y. Nakamura, D. Pleiter, P. E. L. Rakow, G. Schierholz, H. Stüben, A. W. Thomas, R. D. Young, and J. M. Zanotti, Electric form factors of the octet baryons from lattice QCD and chiral extrapolation, Phys. Rev. D **90,** 034502 (2014).

[65] P. Wang, D. B. Leinweber, A. W. Thomas, and R. D. Young, Chiral extrapolation of octet-baryon charge radii, Phys. Rev. D **79,** 094001 (2009).

[66] T. Fuchs, J. Gegelia, and S. Scherer, Electromagnetic form-factors of the nucleon in relativistic baryon chiral perturbation theory, J. Phys. G **30,** 1407 (2004).

[67] S. J. Puglia, M. J. Ramsey-Musolf, and S.-L. Zhu, Octet baryon charge radii, chiral symmetry and decuplet intermediate states, Phys. Rev. D **63,** 034014 (2001).

[68] D. B. Leinweber, R. M. Woloshyn, and T. Draper, Electromagnetic structure of octet baryons, Phys. Rev. D **43,** 1659 (1991).

[69] H.-S. Li, Z.-W. Liu, X.-L. Chen, W.-Z. Deng, and S.-L. Zhu, Decuplet to octet baryon transitions in chiral perturbation theory, Eur. Phys. J. C **79,** 66 (2019).

[70] H.-S. Li, L. Meng, Z.-W. Liu, and S.-L. Zhu, Radiative decays of the doubly charmed baryons in chiral perturbation theory, Phys. Lett. B **777,** 169 (2018).

[71] L. Meng, G.-J. Wang, C.-Z. Leng, Z.-W. Liu, and S.-L. Zhu, Magnetic moments of the spin-$\frac{3}{2}$ singly heavy baryons, Phys. Rev. D **98,** 094013 (2018).

[72] G.-J. Wang, L. Meng, H.-S. Li, Z.-W. Liu, and S.-L. Zhu, Magnetic moments of the spin-$\frac{1}{2}$ singly charmed baryons in chiral perturbation theory, Phys. Rev. D **98,** 054026 (2018).

[73] H.-S. Li, Z.-W. Liu, X.-L. Chen, W.-Z. Deng, and S.-L. Zhu, Magnetic moments and electromagnetic form factors of the decuplet baryons in chiral perturbation theory, Phys. Rev. D **95,** 076001 (2017).

[74] H.-S. Li, L. Meng, Z.-W. Liu, and S.-L. Zhu, Magnetic moments of the doubly charmed and bottom baryons, Phys. Rev. D **96,** 076011 (2017).

[75] L. Meng, H.-S. Li, Z.-W. Liu, and S.-L. Zhu, Magnetic moments of the spin-$\frac{3}{2}$ doubly heavy baryons, Eur. Phys. J. C **77,** 869 (2017).

[76] G.-J. Wang, R. Chen, L. Ma, X. Liu, and S.-L. Zhu, Magnetic moments of the hidden-charm pentaquark states, Phys. Rev. D **94,** 094018 (2016).

[77] U. Özdem and K. Azizi, Electromagnetic multipole moments of the $P_c^+(4380)$ pentaquark in light-cone QCD, Eur. Phys. J. C **78,** 379 (2018).

[78] U. Özdem, Electromagnetic properties of the P_c (4312) pentaquark state, Chin. Phys. C **45,** 023119 (2021).

[79] U. Özdem, Magnetic dipole moments of the hidden-charm pentaquark states: $P_c(4440)$, $P_c(4457)$, and $P_{cs}(4459)$, Eur. Phys. J. C **81,** 277 (2021).

[80] Y.-J. Xu, Y.-L. Liu, and M.-Q. Huang, The magnetic moment of $P_c(4312)$ as a $\bar{D}\Sigma_c$ molecular state, Eur. Phys. J. C **81,** 421 (2021).

[81] E. Ortiz-Pacheco, R. Bijker, and C. Fernández-Ramírez, Hidden charm pentaquarks: Mass spectrum, magnetic moments, and photocouplings, J. Phys. G **46,** 065104 (2019).

[82] N. Honzawa and S. Ishida, Electromagnetic static moments of deuteron in the Bethe-Salpeter formalism, Phys. Rev. C **45,** 47 (1992).

[83] V. Pascalutsa and M. Vanderhaeghen, Magnetic Moment of the Delta(1232)-Resonance in Chiral Effective Field Theory, Phys. Rev. Lett. **94,** 102003 (2005).

[84] V. Pascalutsa and M. Vanderhaeghen, Chiral effective-field theory in the Delta(1232) region. II. Radiative pion photo-production, Phys. Rev. D **77,** 014027 (2008).

[85] K. U. Can, G. Erkol, B. Isildak, M. Oka, and T. T. Takahashi, Electromagnetic structure of charmed baryons in Lattice QCD, J. High Energy Phys. 05 (2014) 125.

[86] D. B. Leinweber, T. Draper, and R. M. Woloshyn, Decuplet baryon structure from lattice QCD, Phys. Rev. D **46,** 3067 (1992).

[87] S. Kumar, R. Dhir, and R. C. Verma, Magnetic moments of charm baryons using effective mass and screened charge of quarks, J. Phys. G **31,** 141 (2005).

[88] J. Dey, V. Shevchenko, P. Volkovitsky, and M. Dey, Radiative decays of S wave charmed baryons, Phys. Lett. B **337,** 185 (1994).