# Revisit prompt  $J/\psi$  production in associated with Higgs boson via gluon fusion at the LHC

<span id="page-0-1"></span>Xue-An Pan  $\odot$ , Zhong-Ming Niu, Gang Li, Yu Zhang  $\odot$ , 2,1[,\\*](#page-0-0) Mao Song, and Jian-You Guo $\odot$ <sup>[1](https://orcid.org/0000-0003-3951-305X)</sup>

 $1$ School of Physics and Materials Science, Anhui University,

Hefei, Anhui 230039, People's Republic of China <sup>2</sup>

 $\mu$ Institutes of Physical Science and Information Technology, Anhui University,

Hefei, Anhui 230039, People's Republic of China

(Received 12 April 2021; accepted 22 June 2021; published 7 September 2021)

The production of charmonium associated with Higgs boson via gluon fusion has been investigated by Kniehl, Palisoc, and Zwirner [Phys. Rev. D 66[, 114002 \(2002\)](https://doi.org/10.1103/PhysRevD.66.114002)] in which they considered the contribution of the final Higgs boson radiation off the charm quark at tree level and found that this process is far too rare to be observable in any of the considered experiments. In this paper, the production of prompt  $J/\psi$ associated with the Higgs boson via gluon fusion at the 14 TeV LHC within the factorization formalism of nonrelativistic quantum chromodynamics is revisited. After considering the contribution from the final Higgs boson radiation off the top quark in the loop, which is more than 3 orders of magnitudes higher than the charm quark at tree level, the production of prompt  $J/\psi$  associated with the Higgs boson has great potential to be detected. The prompt  $J/\psi$  production includes the direct production and indirect production via radiative or hadronic decays of high excited charmonium states. For the direct  $J/\psi + H$  production via loop-induced gluon fusion, the  ${}^{3}S_{1}^{(8)}$  Fock state gives the dominant contribution to the cross section, which is about 95% of the total direct production. The indirect loop-induced contribution is appreciable, since the summation of  $\psi(2S) + H$ ,  $\chi_{c1} + H$  and  $\chi_{c2} + H$  is about 34% of the total cross section of prompt  $J/\psi + H$ . The indirect contribution from  $\chi_{c0} + H$  is tiny and can be neglected. With great potential to be detected, prompt  $J/\psi$  production associated with the Higgs boson can help us to further understand the mechanism of the color octet, as well as being useful to further investigate the coupling of the Higgs boson and fermions.

DOI: [10.1103/PhysRevD.104.054006](https://doi.org/10.1103/PhysRevD.104.054006)

#### I. INTRODUCTION

Since the discovery of the first heavy charmonium  $J/\psi$ in 1974 [\[1,2\]](#page-7-0), the production of heavy quarkonium has been an important topic for investigation in hadron physics. An increasing number of experiments have studied the production process of heavy quarkonium in detail. For example, the inclusive charmonium production has been measured by *BABAR* [\[3\]](#page-7-1) and Belle [\[4\]](#page-7-2), the photoproduction of  $J/\psi$  has been investigated by H1 [5–[9\]](#page-7-3) and ZEUS [\[10\]](#page-7-4), and the hadronic and polarization productions of heavy quarkonium have been studied at hadron colliders [\[11](#page-7-5)–15]. In addition, production processes of double charmonium have been detected, such as  $J/\psi + J/\psi[16–19]$ ,  $J/\psi + \chi_{c0}$  [\[20\]](#page-7-7), and  $J/\psi +$  $\eta_c$  [\[20,21\]](#page-7-7). In order to explain the production and decay of heavy quarkonium, the color-evaporation model (CEM)  $[22,23]$ , color-singlet model  $(CSM)$   $[24,25]$ , and nonrelativistic quantum chromodynamics (NRQCD) [\[26\]](#page-8-1) have been proposed. At present, NRQCD is more widely accepted and applied to the production and decay of heavy quarkonium. In NRQCD factorization, the production and decay of heavy quarkonium are factorized into the short-distance coefficients (SDCs) and long-distance matrix elements (LDMEs). The SDCs can be perturbatively calculated by using the expansion of the strongcoupling constant  $\alpha_s$ , and the LDMEs are process independent and universal, governed by nonperturbative QCD dynamics, and can be extracted from experiments. NRQCD has achieved remarkable success in explaining the puzzle of  $J/\psi$  and  $\psi(2S)$  surplus production at the Tevatron [\[27,28\]](#page-8-2); subsequently it was used extensively to explain the production and decay of heavy quarkonium, and attained many phenomenological successes (see reviews [29–[31\]](#page-8-3) for details). However, some problems in NRQCD still exist, such as it cannot provide a full description for the data of the double  $J/\psi$  measured by the CMS [\[18,32,33\],](#page-7-9) and the universality of LDMEs [\[34,35\]](#page-8-4) etc., which all indicate that NRQCD needs further investigation.

Due to the clean signature, the associated productions of heavy quarkonium and standard model (SM) bosons are [\\*](#page-0-1)

<span id="page-0-0"></span>dayu@ahu.edu.cn

ideal channels to test NRQCD, which has been attracting more and more attention. The hadronic productions of heavy quarkonium associated with  $\gamma$ ,  $W^{\pm}$ , Z, and the Higgs boson have been studied at leading order (LO) within NRQCD [36–[39\],](#page-8-5) and the next-to-leading order (NLO) QCD corrections for the production of heavy quarkonium associated with  $\gamma$ ,  $W^{\pm}$ , Z have been calculated in Refs. [\[40](#page-8-6)–44]. Furthermore, for the processes of  $J/\psi$  +  $W^{\pm}$  and  $J/\psi + Z$ , the contributions from single parton scattering (SPS) and double parton scattering (DPS) have been investigated [\[45,46\].](#page-8-7) Experimentally, the productions of  $J/\psi + Z$ , and  $J/\psi + W^{\pm}$  at hadron colliders were also detected [47–[49\].](#page-8-8) The production rate of  $J/\psi + W^{\pm}$ (measured by ATLAS [\[47\]](#page-8-8)) is an order of magnitude larger than SPS predictions from NRQCD [\[41,50\],](#page-8-9) and the gap can be filled by considering the contribution from DPS [\[46\]](#page-8-10). As for  $J/\psi + Z$ , even taking the contribution from NLO CEM  $SPS + DPS$  [\[45\]](#page-8-7) into account, the behavior of the transverse momentum distribution measured by ATLAS [\[48\]](#page-8-11) exhibits a slight difference with the theoretical prediction. As regards the production of heavy quarkonium associated with the Higgs boson, there is no positive news from either phenomenology or experiments.

In Ref. [\[36\]](#page-8-5), Kniehl *et al.* estimated the production of charmonium associated with the Higgs boson at LO based on NRQCD at  $e^+e^-, e^{\pm}p, p\bar{p}$ , and pp colliders for the first time. They considered the contribution from the partonic processes  $q + q \rightarrow Q + H$  and  $q + \bar{q} \rightarrow Q + H$ , where  $Q$  is charmonium, q is the light quark, and H is the Higgs boson, and found that the process of heavy quarkonium production associated with the Higgs boson is to be far too rare to be observable in any of the considered experiments. We repeat the calculation in Ref. [\[36\]](#page-8-5) for the direct production of  $J/\psi + H$  and  $\chi_{cJ} + H$ , and obtain consistent results of the transverse momentum and rapidity distribution. Adopting the same input parameters, we obtained the total cross section of  $J/\psi$  production associated with the Higgs boson via gluon fusion at the 14 TeV LHC to be about 0.02 fb, which is in agreement with the results shown in their other work [\[51\]](#page-8-12). One can see that, in this case, it is difficult to detect this process at the LHC. While considering the contribution from the final Higgs boson radiation off top quark via loop-induced gluon fusion, we find that the production of heavy charmonium associated with the Higgs boson can be potentially detected at the  $LHC<sup>1</sup>$ .

In this work, we will revisit the production of prompt  $J/\psi$  associated with the Higgs boson via gluon fusion at the LHC within the NRQCD factorization formalism. The final Higgs boson can be produced from two sources, including radiation off the charm quark (labeled as S1 in the following) and off the top quark via loop-induced gluon fusion (labeled as S2). Note that S1 has been considered by Kniehl et al. in Ref. [\[36\]](#page-8-5), and S2 will be considered for the first time in this work. Since the production rate from the partonic process  $q + \bar{q} \rightarrow \mathcal{Q} + H$  is tiny, we will not consider it. Besides the direct production, prompt  $J/\psi$ candidates can also be produced indirectly via radiative or hadronic decays of heavier charmonium states, such as  $\chi_{cJ} \rightarrow J/\psi + \gamma(J = 0, 1, 2)$  and  $\psi(2S) \rightarrow J/\psi + X$ . We find that the contribution of S2 is more than 3 orders of magnitudes greater than S1, and the  ${}^{3}S_{1}^{(8)} + H$  production channel plays an important role in  $J/\psi + H$  production via loop-induced gg fusion. With detectable potential at the LHC, this process can be useful to test the heavy quarkonium production mechanism and investigate the coupling of the Higgs boson and fermions.

## II. THE DETAILS OF THE CALCULATION FRAMEWORK

In this section, we introduce the details of the calculation for prompt  $J/\psi$  production in association with H via gluon fusion,  $pp \rightarrow gg \rightarrow J/\psi + H$ , at the LHC in NRQCD. The total cross section of prompt  $J/\psi + H$  production can be expressed as follows:

$$
\sigma^{\text{prompt}}(J/\psi + H) = \sigma^{\text{direct}}(J/\psi + H) + \sigma^{\text{indirect}}(\text{From } \psi(2S) + H) + \sigma^{\text{indirect}}(\text{From } \chi_{cJ} + H)
$$
  
=  $\sigma(p \, p \to gg \to J/\psi + H) + \sigma(p \, p \to gg \to \psi(2S) + H) \times \text{Br}(\psi(2S) \to J/\psi + X)$   
+  $\sum_{J=0}^{2} \sigma(p \, p \to gg \to \chi_{cJ} + H) \times \text{Br}(\chi_{cJ} \to J/\psi + \gamma),$  (1)

where  $\sigma^{\text{direct}}(J/\psi + H)$  is the cross section of the direct contribution from  $J/\psi + H$  production;  $\sigma^{\text{indirect}}(\text{From } \psi(2S)/$  $\chi_{cJ}$  + H) denotes the indirect contribution due to hadronic/ radiative decays of  $\psi(2S)/\chi_{cJ}$  from  $\psi(2S)/\chi_{cJ} + H$  production. Br $(\psi(2S) \rightarrow J/\psi + X)$  and Br $(\chi_{cJ} \rightarrow J/\psi + \gamma)$ respectively, represent the branching ratio of  $\psi(2S)$  and  $\chi_{cJ}$  decaying into  $J/\psi$ , which can be obtained from [\[52\]](#page-8-13) as

<sup>&</sup>lt;sup>1</sup>Note that the final Higgs boson can also radiate off bottom quark via loop-induced gluon fusion. The contribution of the bottom loop is about 1 order of magnitude large than that considered in Ref. [\[36\],](#page-8-5) but nearly 3 orders of magnitude smaller than that of the top loop. Therefore, we do not consider it in this work.

$$
Br(\psi(2S) \to J/\psi + X) = (61.4 \pm 0.6)\%,
$$
  
\n
$$
Br(\chi_{c0} \to J/\psi + \gamma) = (1.40 \pm 0.05)\%,
$$
  
\n
$$
Br(\chi_{c1} \to J/\psi + \gamma) = (34.3 \pm 1.0)\%,
$$
  
\n
$$
Br(\chi_{c2} \to J/\psi + \gamma) = (19.0 \pm 0.5)\%.
$$
 (2)

 $\sigma(pp \to gg \to \mathcal{Q} + H)$  denotes the cross section for the production of charmonium  $Q$  associated with H via gluon fusion, which can be factorized as

$$
\sigma(pp \to gg \to \mathcal{Q} + H) = \int dx_1 dx_2 f_{g/A}(x_1, \mu_f) f_{g/B}(x_2, \mu_f)
$$

$$
\times \sum_n \langle \mathcal{O}^{\mathcal{Q}}[n] \rangle \hat{\sigma}(g + g \to c\bar{c}[n] + H).
$$
\n(3)

Here  $f_{q/A}(x_1,\mu_f)$  and  $f_{q/B}(x_2,\mu_f)$  are the distribution functions of the gluon radiated by proton  $A$  and  $B$ respectively;  $x_{1/2}$  denotes the momentum fraction of the proton  $A/B$  momentum carried by the gluon; the summation is taken over the possible intermediate Fock states  $n = 2S+1} L_J^{(1,8)}$  of heavy  $c\bar{c}$  pairs;  $\langle \mathcal{O}^{\mathcal{Q}}[n] \rangle$  is the LDME, which describes the process for the Fock state  $c\bar{c}[n]$ evolving into an observable physical state Q by radiating soft gluons, with  $Q = J/\psi, \psi(2S), \chi_{cJ}(J = 0, 1, 2)$  in our calculation. The relation between the LDMEs of various Fock states are

<span id="page-2-0"></span>
$$
\langle O^{J/\psi,\psi(2S)}[{}^3P_J^{(8)}] \rangle = (2J+1)\langle O^{J/\psi,\psi(2S)}[{}^3P_0^{(8)}] \rangle, \n\langle O^{\chi_{cJ}}[{}^3S_1^{(8)}] \rangle = (2J+1)\langle O^{\chi_{c0}}[{}^3S_1^{(8)}] \rangle, \n\langle O^{\chi_{cJ}}[{}^3P_J^{(1)}] \rangle = (2J+1)\langle O^{\chi_{c0}}[{}^3P_0^{(1)}] \rangle.
$$
\n(4)

 $\hat{\sigma}(g + g \rightarrow c\bar{c}[n] + H)$  describes the short-distance cross section for the partonic process  $g(p_1) + g(p_2) \rightarrow$  $c\bar{c}[n](p_3) + H(p_4)$  via gluon fusion, which can be expressed as

$$
\hat{\sigma}(g+g \to c\bar{c}[n]+H) = \frac{1}{16\pi \hat{s} N_{\text{col}} N_{\text{pol}}} \int_{\hat{t}_{\text{min}}}^{\hat{t}_{\text{max}}} d\hat{t} \sum |A_{S,L}|^2.
$$
\n(5)

Here  $\hat{s}$  and  $\hat{t}$  are Mandelstam variables, which are defined as  $\hat{s} = (p_1 + p_2)^2$  and  $\hat{\tau} = (p_1 - p_3)^2$ , respectively. N<sub>col</sub>  $(N_{\text{pol}})$  refers to the color (polarization) quantum number of Fock states  $c\bar{c}[n]$ . The summation is taken over the initial state and the final state, and the bar means averaging the color and spin of the initial parton.  $A_{S,L}$  can be obtained by the covariant projection method in Ref. [\[53\].](#page-8-14) The polarization summations of the Fock state  $c\bar{c}[n]$  are

$$
\sum_{J_z} \varepsilon_{\alpha} \varepsilon_{\alpha'}^* = \Pi_{\alpha\alpha'},
$$
\n
$$
\sum_{J_z} \varepsilon_{\alpha\beta}^0 \varepsilon_{\alpha'\beta'}^{0*} = \frac{1}{3} \Pi_{\alpha\beta} \Pi_{\alpha'\beta'},
$$
\n
$$
\sum_{J_z} \varepsilon_{\alpha\beta}^1 \varepsilon_{\alpha'\beta'}^{1*} = \frac{1}{2} (\Pi_{\alpha\alpha'} \Pi_{\beta\beta'} - \Pi_{\alpha\beta'} \Pi_{\alpha'\beta}),
$$
\n
$$
\sum_{J_z} \varepsilon_{\alpha\beta}^2 \varepsilon_{\alpha'\beta'}^{2*} = \frac{1}{2} (\Pi_{\alpha\alpha'} \Pi_{\beta\beta'} + \Pi_{\alpha\beta'} \Pi_{\alpha'\beta}) - \frac{1}{3} \Pi_{\alpha\beta} \Pi_{\alpha'\beta'} \qquad (6)
$$

with

$$
\Pi_{\alpha\beta} = -g_{\alpha\beta} + \frac{p_{3\alpha}p_{3\beta}}{M_Q}.
$$
\n(7)

Here  $p_3$   $(M<sub>O</sub>)$  is the momentum (mass) of quarkonium  $Q$ .

The polarization summation of the two initial gluons for  $g + g \rightarrow c\bar{c}[n] + H$  process is expressed as follows:

$$
-g_{\mu\nu} + \frac{p_{\mu}\eta_{\nu} + p_{\nu}\eta_{\mu}}{p \cdot \eta}, \qquad (8)
$$

where  $p$  is the momentum of initial gluon, and  $\eta$  is an arbitrary lightlike vector with  $p \cdot \eta \neq 0$ . In our calculation,  $\eta$  is taken as the momentum of another initial gluon.

The Feynman diagrams for the process of  $g + g \rightarrow$  $c\bar{c}[n] + H$  from the contribution of S1 and S2 are generated with FeynArt[s\[54\],](#page-8-15) which are presented in Fig. [1](#page-3-0) and Fig. [2](#page-3-1) respectively, and we further reduce the Feynman amplitudes using FeynCalc[\[55\]](#page-8-16), Apar[t\[56,57\]](#page-8-17), and FIR[E\[58,59\]](#page-8-18). Finally, we use LoopTools and FormCalc[\[60\]](#page-8-19) to implement the numerical calculations.

#### III. NUMERICAL RESULTS

In what follows, we present numerical results for the production of prompt  $J/\psi$  associated with the Higgs boson via gg fusion from the contribution of S1 and S2 at the LHC with  $\sqrt{s}$  = 14 TeV, respectively. The masses of the charm quark, top quark and the Higgs boson are taken as  $m_c = 1.5$  GeV,  $m_t = 172$  GeV, and  $m_H = 125$  GeV. The mass of the charmonium Q is taken as  $M_{\mathcal{Q}} = 2m_c$ , and we set  $p_T^{\mathcal{Q}} > 3$  GeV for the final charmonium  $\mathcal{Q}$ , where  $p_T^{\mathcal{Q}}$  is the transverse momentum of charmonium  $\mathcal{Q}$ . In our calculation, the factorization scale and PDFs are chosen as  $\mu_f = \sqrt{(p_T^{\mathcal{Q}})^2 + M_{\mathcal{Q}}^2}$  and CTEQ6L1[\[61\],](#page-8-20) respectively. The LDMEs of  $J/\psi$  is chosen from [\[62,63\]](#page-8-21) as

> $\langle \mathcal{O}^{J/\psi} [{}^1S_0^{(8)}] \rangle = 1 \times 10^{-2} \text{ GeV}^3,$  $\langle \mathcal{O}^{J/\psi} [{}^3P_0^{(8)}] \rangle = 11.25 \times 10^{-3} \text{ GeV}^5,$  $\langle \mathcal{O}^{J/\psi} [{}^3S_1^{(8)}] \rangle = 1.12 \times 10^{-2} \text{ GeV}^3.$  (9)

<span id="page-3-0"></span>

<span id="page-3-1"></span>FIG. 1. Feynman diagrams for the partonic process  $g + g \to c\bar{c}[n] + H$  via gg fusion with the final Higgs boson radiation off charm quark.



FIG. 2. Feynman diagrams for the partonic process  $g + g \to c\bar{c}[n] + H$  via gg fusion with the final Higgs boson radiation off top quark in the loop.

The LDMEs of  $\psi(2S)$  and  $\chi_{c0}$  are taken from [64–[66\]](#page-8-22) as and

$$
\langle \mathcal{O}^{\psi'}[{}^{1}S_{0}^{(8)}] \rangle = 5 \times 10^{-3} \text{ GeV}^{3},
$$
  
\n
$$
\langle \mathcal{O}^{\psi'}[{}^{3}P_{0}^{(8)}] \rangle = 3.214 \times 10^{-3} \text{ GeV}^{5},
$$
  
\n
$$
\langle \mathcal{O}^{\psi'}[{}^{3}S_{1}^{(8)}] \rangle = 2 \times 10^{-3} \text{ GeV}^{3},
$$
  
\n
$$
\langle \mathcal{O}^{\psi}^{\psi}[{}^{3}S_{1}^{(8)}] \rangle = 2 \times 10^{-3} \text{ GeV}^{3},
$$
  
\n
$$
\langle \mathcal{O}^{\psi}^{\psi}[{}^{3}S_{1}^{(8)}] \rangle = \frac{3N_{c}}{2\pi} \times 0.075 \text{ GeV}^{5},
$$
  
\n
$$
\langle \mathcal{O}^{\psi}[{}^{3}S_{1}^{(8)}] \rangle = \frac{3N_{c}}{2\pi} \times 0.075 \text{ GeV}^{5},
$$
  
\n
$$
\langle \mathcal{O}^{\psi}[{}^{3}S_{1}^{(8)}] \rangle = \frac{3N_{c}}{2\pi} \times 0.075 \text{ GeV}^{5},
$$
  
\n
$$
\langle \mathcal{O}^{\psi}[{}^{3}S_{1}^{(8)}] \rangle = \frac{3N_{c}}{2\pi} \times 0.075 \text{ GeV}^{5},
$$
 (11)

<span id="page-4-0"></span>TABLE I. The cross section of charmonium  $Q$  production in associated with the Higgs boson via gg fusion from the contrition of S1 (in unit of ab) and S2 (in unit of fb) at the 14 TeV LHC.

	Fock state		${}^{1}S_{0}^{(8)}$ ${}^{3}P_{I}^{(8)}$	${}^{3}P_{I}^{(1)}$	${}^{3}S_1^{(8)}$	Total
S1	$\sigma(pp \to gg \to J/\psi + H)$	0.97	2.64		4.84	8.45
	$\sigma(p \, p \rightarrow gg \rightarrow \psi(2S) + H)$	0.49	0.76		0.80	2.05
	$\sigma(p \, p \rightarrow gg \rightarrow \chi_{c0} + H)$			0.62	0.95	1.57
	$\sigma(p p \rightarrow gg \rightarrow \chi_{c1} + H)$			3.81	2.85	6.66
	$\sigma(p \, p \rightarrow qq \rightarrow \chi_{c2} + H)$			9.02	4.75	13.77
	S2 $\sigma(pp \to gg \to J/\psi + H)$	0.38	0.71		44.25	45.34
	$\sigma(p \, p \rightarrow qq \rightarrow \psi(2S) + H)$	0.19	0.20		7.90	8.29
	$\sigma(p \, p \rightarrow gg \rightarrow \chi_{c0} + H)$			0.69	8.69	9.36
	$\sigma(p \, p \rightarrow qq \rightarrow \chi_{c1} + H)$	$\overline{\phantom{0}}$		1.75	26.08	27.82
	$\sigma(p p \rightarrow gg \rightarrow \chi_{c2} + H)$			119	43.47	44.64

with  $N_c = 3$ . In accordance with Eq. [\(4\)](#page-2-0) we can acquire the LDMEs of  $\chi_{c1}$  and  $\chi_{c2}$ . In the calculation, we have considered the conventions of different theories for LDMEs of the color-singlet states.

In Table [I,](#page-4-0) we present the cross section of the charmonium  $J/\psi$ ,  $\psi(2S)$  and  $\chi_{cJ}(J = 0, 1, 2)$  production associated with the Higgs boson via gg fusion (from the contribution of S1 and S2 at the 14 TeV LHC) within the NRQCD framework. We find that the contribution of S2 is more than 3 orders of magnitudes larger than S1 in the production of charmonium associated with the Higgs boson. The Fock state  ${}^{3}S_{1}^{(8)}$  plays a major role in the direct production of charmonium associated with the Higgs boson via loop-induced gg fusion, from which the contribution can account for more than 90% to the total direct production cross section. For the direct production of  $J/\psi + H$ and  $\chi_{cJ}$  + H via loop-induced gg fusion, the cross section can reach about 45.34 fb and 81.82 fb respectively, which can provide abundant and fascinating investigations of phenomenology at the LHC.

We present the transverse momentum  $(p_T^Q)$  and rapidity  $(y<sub>Q</sub>)$  distributions of the final charmonium  $Q$  for the process  $pp \rightarrow gg \rightarrow \mathcal{Q} + H$  via loop-induced production at the 14 TeV LHC in Fig. [3](#page-4-1). It can be found that, the  $p_T$ distributions for the considered direct Q production all decrease rapidly with the increment of  $p_T^{\mathcal{Q}}$ . The  $p_T$ distributions of the direct  $\chi_{c2}$  and  $J/\psi$  production are almost the same in the whole  $p_T$  region, because the Fock state  ${}^{3}S_{1}^{(8)}$  dominates the contributions and LDME values of the  ${}^{3}S_{1}^{(8)}$  for  $J/\psi$  and  $\chi_{c2}$  are quite close. Due to same reason, rapidity distributions of  $J/\psi$  and  $\chi_{c2}$  are also almost consistent.

In order to illustrate the transverse momentum  $p_T$  and rapidity y distributions from each Fock state, we take direct  $J/\psi$  production associated with the Higgs boson as an example and present the contribution from Fock states  ${}^{1}S_{0}^{(8)}$ ,<br> ${}^{3}P_{0}^{(8)}$  and  ${}^{3}S_{0}^{(8)}$  for *L*(win Fig. 4, separately. The graduation  $P_J^{(8)}$  and  ${}^{3}S_1^{(8)}$  for  $J/\psi$  in Fig. [4](#page-5-0), separately. The production channel of  ${}^{3}S_{1}^{(1)} + H$  is forbidden due to charge-conjugation invariance. We can see that the contribution from each Fock state decreases rapidly with the increment of  $p_T^{J/\psi}$ , but  ${}^3S_1^{(8)}$ decreases slower than  ${}^{1}S_{0}^{(8)}$  and  ${}^{3}P_{J}^{(8)}$ . In the range of

<span id="page-4-1"></span>

FIG. 3. The distributions of  $p_T^{\mathcal{Q}}$  (left) and  $y_{\mathcal{Q}}$  (right) for the process  $pp \to gg \to \mathcal{Q} + H$  via loop-induced at the 14 TeV LHC. The black solid, blue dashed, purple dotted, olive short dashed and orange dashed-dotted-dotted lines denote the final charmonium  $Q$  are  $J/\psi$ ,  $\psi$ (2S),  $\chi$ <sub>c0</sub>,  $\chi$ <sub>c1</sub> and  $\chi$ <sub>c2</sub>, respectively.

<span id="page-5-0"></span>

FIG. 4. The distributions of  $p_T^{J/\psi}$  (left) and  $y_{J/\psi}$  (right) for direct production of the  $J/\psi$  associated with the Higgs boson via loopinduced gg fusion at the 14 TeV LHC. The blue dashed-dotted-dotted, cyan dashed-dotted, orange dashed and black solid lines are for  ${}^{1}S_{0}^{(8)}, {}^{3}P_{J}^{(8)}, {}^{3}S_{1}^{(8)}$  and total, respectively.

3 GeV  $\leq p_T^{J/\psi} \leq 15$  GeV, the differential cross section of  $J/\psi$  associated with Higgs boson production lies in the range of [0.62,3.85] fb/GeV. From Fig. [4](#page-5-0), we can find that the contribution from  ${}^{3}S_{1}^{(8)}$  Fock state is always dominant, especially at large  $p_T$ ; the contribution due to the other Fock states can be negligible. It implies that this process will help to extract the LDMEs of  ${}^{3}S_{1}^{(8)}$  Fock state if it is detected in future experiments.

We respectively present the total cross section for the prompt  $J/\psi$  associated with the Higgs boson via gg fusion from the contribution of S1 and S2 at the 14 TeV LHC in Table [II.](#page-5-1) The contributions due to direct and indirect production are also shown respectively. From Table [II](#page-5-1), we can see that for S1, even considering the direct and indirect contributions, the cross section of prompt  $J/\psi + H$ production is still tiny compared to S2. As for the contribution of S2, we can find that direct production is dominant, and can reach about 66% of the prompt production. Indirect production due to excited charmonium decays is not negligible as its rate can account for about 34%. In indirect production, the contributions of  $\psi(2S) + H$ ,  $\chi_{c1} + H$  and  $\chi_{c2} + H$  are sizable, and are about 22%, 41% and 36% of the indirect contribution, respectively. The indirect contribution from  $\chi_{c0} + H$  is less than 0.2% of the prompt production, because the decay branching ratio  $Br(\chi_{c0} \to J/\psi + \gamma)$  is tiny. Considering the contribution from direct and indirect productions, the total cross section of prompt  $J/\psi + H$  production can reach 68.58 fb.

We further consider the final Higgs boson decaying into  $b\bar{b}$ . For the *b*-jets, applying the cuts of  $p_T^b > 20 \text{ GeV}$ ,  $|\eta_b|$  < 2.5, and considering the branching ratio  $Br(H \rightarrow b\bar{b}) = 58.14\%$ [\[67\]](#page-8-23) and b-tagging efficiency  $\epsilon_b = 77\%$  [\[68\]](#page-8-24), we can get the cross section from S2 as 11.12 (5.7) fb for the direct (indirect) production of  $J/\psi$ associated with the Higgs boson via gg fusion. Therefore, the cross section of prompt  $J/\psi + H$  with the Higgs boson decaying into  $b\bar{b}$  can reach about 18 fb even after considering the kinematical constraints on the Higgs boson in experiments. This implies that this process may be detected in future experiments, which will help us to further understand the color-octet mechanism.

<span id="page-5-1"></span>TABLE II. The cross sections for the prompt  $J/\psi$  associated with the Higgs boson via gg fusion including the contribution of S1 (in unit of ab) and S2 (in unit of fb) from direct and indirect production at the 14 TeV LHC, respectively.

Source	From $\psi(2S) + H$	From $\chi_{c0} + H$	From $\chi_{c1} + H$	From $\chi_{c2} + H$	Indirect	Direct	Prompt
$\sigma(S1)$	1.26	0.02	2.28	2.62	6.18	8.45	14.63
$\sigma(S2)$	5.09		9.54	8.48	23.24	45.34	68.58

<span id="page-6-0"></span>

FIG. 5. The distributions of  $p_T^{J/\psi}$  (left) and  $y_{J/\psi}$  (right) for direct and indirect contribution of the  $J/\psi$  associated with the Higgs boson via loop-induced gluon fusion at the 14 TeV LHC. The black solid, red solid, blue dashed, purple dotted, orange dashed-dotted and royal short dashed lines are contributions of from direct, indirect,  $\psi(2S)$ ,  $\chi_{c0}$ ,  $\chi_{c1}$  and  $\chi_{c2}$ .

The distributions of  $p_T^{J/\psi}$  (left) and  $y_{J/\psi}$  (right) for direct and indirect productions of the  $J/\psi$  associated with the Higgs boson via loop-induced gluon fusion at the 14 TeV LHC are shown in Fig [5](#page-6-0). It can be seen that the behavior of the  $p_T^{J/\psi}$  and  $y_{J/\psi}$  distribution for the direct and indirect productions are similar. The curves of the  $J/\psi + H$  direct production are always at the top in the whole plotted region of  $p_T^{J/\psi}$  and  $y_{J/\psi}$ . The indirect contributions from  $\psi(2S)$ ,  $\chi_{c1}$  and  $\chi_{c2}$  are considerable, although the contribution from  $\chi_{c0}$  is much less than from other charmonia because of the small decay branching ratio. For the  $y_{J/\psi}$  distributions, the convex curves of the direct and indirect contributions reach their peaks when  $y_{J/\psi}=0$ .

As we know, in the study of heavy quarkonium production and decay processes, LDMEs is considered universal. The LDMEs of the color octet are extracted from experiments, which lead to uncertainty. In order to study the uncertainty of the results caused by the LDMEs, we present the cross sections of the direct  $J/\psi$  associated with Higgs boson production via gg fusion from the contribution of S2 at the 14 TeV LHC with three other different sets of LDMEs in Table [III.](#page-6-1) Set 1 and Set 2 are extracted by fitting the  $p_T$  distributions from CDF using PDFs for MRST98L[O\[69\]](#page-8-25) and MRS(R2)[\[70\]](#page-8-26), respectively. Set 3 is extracted from the hadroproduction of  $\eta_c$  at the LHC[\[71\]](#page-8-27). From Table [III,](#page-6-1) we can see that the cross section of  $J/\psi + H$  varies between 46%–131% compared with the result with the default set of LDMEs. Moreover, we take the production channel of  $J/\psi({}^3S_1^{(8)})$  as an example to study the uncertainty due to PDFs. In Table [IV,](#page-7-10) we respectively show the cross sections of  $J/\psi(^3S_1^{(8)}) + H$ via gg fusion from the contribution of S2 at the 14 TeV LHC with three other different PDFs. From Table [IV,](#page-7-10) we can find that the results are slightly affected by the different

<span id="page-6-1"></span>TABLE III. The cross sections (fb) of the direct  $J/\psi$  associated with the Higgs boson production via gg fusion from the contribution of S2 at the 14 TeV LHC with four different sets of LDMEs, in which Set 1 and Set 2 are extracted by fitting the  $p_T$  distributions from CDF using PDFs for MRST98LO[\[69\]](#page-8-25) and MRS(R2[\)\[70\]](#page-8-26), respectively. Set 3 is extracted from the hadroproduction of  $\eta_c$  at the LHC[\[71\].](#page-8-27)

	Set 1	Set 2	Set 3	Default set
${}<\mathcal{O}^{J/\psi}[^1S_0^{[8]}] > (GeV^3)$	$4.35 \times 10^{-2}$	$5.45 \times 10^{-2}$	$0.78 \times 10^{-2}$	$1 \times 10^{-2}$
${}<\mathcal{O}^{J/\psi}[^3P_0^{[8]}] > (GeV^5)$	$2.879 \times 10^{-2}$	$3.50 \times 10^{-2}$	$4.3515 \times 10^{-2}$	$1.125 \times 10^{-2}$
${}<\mathcal{O}^{J/\psi}[^3S_1^{[8]}] > (GeV^3)$	$4.4 \times 10^{-3}$	$1.4 \times 10^{-2}$	$1.057 \times 10^{-2}$	$1.12 \times 10^{-2}$
$\sigma(pp \to gg \to J/\psi + H)(\text{fb})$	20.85	59.59	44.81	45.34

<span id="page-7-10"></span>TABLE IV. The cross sections of the  $J/\psi({}^3S_1^{(8)}) + H$  production channel via gg fusion from the contribution of S2 at the 14 TeV LHC with four different PDFs, respectively.

<b>PDFs</b>			MSHT20lo_as130[72] NNPDF30lo_as_0130[73] NNPDF31_lo_pch_as_0130[74] Default	
$\sigma(p \, p \rightarrow gg \rightarrow J/\psi(^3S_1^{(8)}) + H)(\text{fb})$	45.01	43.50	42.65	44.25

PDFs. In addition to the LDMEs and PDFs mentioned above, the numerical uncertainty in our calculation is also affected by the factorization scale and quark mass.

### IV. SUMMARY AND DISCUSSION

In this paper, we revisit the production of prompt  $J/\psi$ associated with the Higgs boson via gg fusion, including the contribution of S1 and S2 at the 14 TeV LHC within the framework of NRQCD. We consider the direct contribution from  $J/\psi + H$  and indirect contribution from  $\psi(2S) + H$ and  $\chi_{cJ} + H$ . From our calculations, we find that the contribution of S2 is more than 3 orders of magnitudes larger than the contribution of S1 for the production of charmonium associated with the Higgs boson. Unlike the contribution of S1, which is to be far too rare to be observable, the contribution of S2 has great potential to be detected. We not only give the total cross section of prompt  $J/\psi + H$ , but also present the distribution of the transverse momentum and rapidity for the final  $J/\psi$  in the contribution of S2. We find that the total cross section of prompt

 $J/\psi + H$  can reach about 70 fb and the <sup>3</sup>S<sup>(8)</sup> Fock state gives the dominant contribution in all production channels. In the process of prompt  $J/\psi + H$  production via loopinduced gluon fusion, the direct contribution is a majority, while the total indirect contribution is also considerable (accounting for 34% of the prompt  $J/\psi + H$ ). The productions of  $\psi(2S) + H$ ,  $\chi_{c1} + H$  and  $\chi_{c2} + H$  provide 22%, 41% and 36% of the total indirect contribution, respectively. Since the decay channel of  $J/\psi$  is clean, it is possible to detect the production of  $J/\psi$  associated with the Higgs boson at the LHC in the future, which will help us to further understand the mechanism of the color octet, as well as helping us to further investigate the coupling of the Higgs boson and fermions.

#### ACKNOWLEDGMENTS

This work was supported in part by the National Natural Science Foundation of China (Grants No. 11805001, No. 11935001, and No. 11875070).

- <span id="page-7-0"></span>[1] J. J. Aubert et al. (E598 Collaboration), [Phys. Rev. Lett.](https://doi.org/10.1103/PhysRevLett.33.1404) 33, [1404 \(1974\)](https://doi.org/10.1103/PhysRevLett.33.1404).
- [2] J. E. Augustin et al. (SLAC-SP-017 Collaboration), [Phys.](https://doi.org/10.1103/PhysRevLett.33.1406) Rev. Lett. 33[, 1406 \(1974\)](https://doi.org/10.1103/PhysRevLett.33.1406).
- <span id="page-7-1"></span>[3] B. Aubert et al. (BABAR Collaboration), [Phys. Rev. Lett.](https://doi.org/10.1103/PhysRevLett.87.162002) 87, [162002 \(2001\).](https://doi.org/10.1103/PhysRevLett.87.162002)
- <span id="page-7-2"></span>[4] K. Abe et al. (Belle Collaboration), [Phys. Rev. Lett.](https://doi.org/10.1103/PhysRevLett.88.052001) 88, [052001 \(2002\).](https://doi.org/10.1103/PhysRevLett.88.052001)
- <span id="page-7-3"></span>[5] S. Aid et al. (H1 Collaboration), [Nucl. Phys.](https://doi.org/10.1016/0550-3213(96)00274-X) **B472**, 3 [\(1996\).](https://doi.org/10.1016/0550-3213(96)00274-X)
- [6] S. Aid et al. (H1 Collaboration), [Nucl. Phys.](https://doi.org/10.1016/0550-3213(96)00192-7) **B468**, 3 [\(1996\);](https://doi.org/10.1016/0550-3213(96)00192-7) B548[, 639\(E\) \(1999\).](https://doi.org/10.1016/S0550-3213(98)00849-9)
- [7] C. Adloff et al. (H1 Collaboration), [Phys. Lett. B](https://doi.org/10.1016/S0370-2693(00)00530-X) 483, 23 [\(2000\).](https://doi.org/10.1016/S0370-2693(00)00530-X)
- [8] A. Aktas et al. (H1 Collaboration), [Eur. Phys. J. C](https://doi.org/10.1140/epjc/s2006-02519-5) 46, 585 [\(2006\).](https://doi.org/10.1140/epjc/s2006-02519-5)
- [9] C. Alexa et al. (H1 Collaboration), [Eur. Phys. J. C](https://doi.org/10.1140/epjc/s10052-013-2466-y) 73, 2466 [\(2013\).](https://doi.org/10.1140/epjc/s10052-013-2466-y)
- <span id="page-7-4"></span>[10] S. Chekanov et al. (ZEUS Collaboration), [Eur. Phys. J. C](https://doi.org/10.1007/s10052-002-0953-7) 24[, 345 \(2002\).](https://doi.org/10.1007/s10052-002-0953-7)
- <span id="page-7-5"></span>[11] V. Khachatryan et al. (CMS Collaboration), [Phys. Rev. D](https://doi.org/10.1103/PhysRevD.83.112004) 83[, 112004 \(2011\).](https://doi.org/10.1103/PhysRevD.83.112004)
- [12] R. Aaij et al. (LHCb Collaboration), [Eur. Phys. J. C](https://doi.org/10.1140/epjc/s10052-014-2872-9) 74, [2872 \(2014\)](https://doi.org/10.1140/epjc/s10052-014-2872-9).
- [13] T. Aaltonen et al. (CDF Collaboration), [Phys. Rev. Lett.](https://doi.org/10.1103/PhysRevLett.108.151802) 108[, 151802 \(2012\).](https://doi.org/10.1103/PhysRevLett.108.151802)
- [14] A. Abulencia et al. (CDF Collaboration), [Phys. Rev. Lett.](https://doi.org/10.1103/PhysRevLett.99.132001) 99[, 132001 \(2007\).](https://doi.org/10.1103/PhysRevLett.99.132001)
- [15] R. Aaij et al. (LHCb Collaboration), [Eur. Phys. J. C](https://doi.org/10.1140/epjc/s10052-013-2631-3) 73, [2631 \(2013\)](https://doi.org/10.1140/epjc/s10052-013-2631-3).
- <span id="page-7-6"></span>[16] K. Abe et al. (Belle Collaboration), [Phys. Rev. D](https://doi.org/10.1103/PhysRevD.70.071102) 70, [071102 \(2004\).](https://doi.org/10.1103/PhysRevD.70.071102)
- [17] R. Aaij et al. (LHCb Collaboration), [Phys. Lett. B](https://doi.org/10.1016/j.physletb.2011.12.015) 707, 52 [\(2012\).](https://doi.org/10.1016/j.physletb.2011.12.015)
- <span id="page-7-9"></span>[18] V. Khachatryan et al. (CMS Collaboration), [J. High Energy](https://doi.org/10.1007/JHEP09(2014)094) [Phys. 09 \(2014\) 094.](https://doi.org/10.1007/JHEP09(2014)094)
- [19] V. M. Abazov et al. (D0 Collaboration), [Phys. Rev. D](https://doi.org/10.1103/PhysRevD.90.111101) 90, [111101 \(2014\).](https://doi.org/10.1103/PhysRevD.90.111101)
- <span id="page-7-7"></span>[20] B. Aubert et al. (BABAR Collaboration), [Phys. Rev. D](https://doi.org/10.1103/PhysRevD.72.031101) 72, [031101 \(2005\).](https://doi.org/10.1103/PhysRevD.72.031101)
- [21] K. Abe et al. (Belle Collaboration), [Phys. Rev. Lett.](https://doi.org/10.1103/PhysRevLett.89.142001) 89, [142001 \(2002\).](https://doi.org/10.1103/PhysRevLett.89.142001)
- <span id="page-7-8"></span>[22] V. D. Barger, W. Y. Keung, and R. J. N. Phillips, [Phys. Lett.](https://doi.org/10.1016/0370-2693(80)90444-X) 91B[, 253 \(1980\)](https://doi.org/10.1016/0370-2693(80)90444-X).
- [23] V. D. Barger, W. Y. Keung, and R. J. N. Phillips, [Z. Phys. C](https://doi.org/10.1007/BF01588844) 6[, 169 \(1980\).](https://doi.org/10.1007/BF01588844)
- <span id="page-8-0"></span>[24] M. Ablikim et al. (BESIII Collaboration), [Phys. Rev. Lett.](https://doi.org/10.1103/PhysRevLett.125.052004) 125[, 052004 \(2020\).](https://doi.org/10.1103/PhysRevLett.125.052004)
- [25] E. L. Berger and D. L. Jones, Phys. Rev. D 23[, 1521 \(1981\).](https://doi.org/10.1103/PhysRevD.23.1521)
- <span id="page-8-1"></span>[26] G. T. Bodwin, E. Braaten, and G. P. Lepage, [Phys. Rev. D](https://doi.org/10.1103/PhysRevD.51.1125) 51[, 1125 \(1995\)](https://doi.org/10.1103/PhysRevD.51.1125); 55[, 5853\(E\) \(1997\).](https://doi.org/10.1103/PhysRevD.55.5853)
- <span id="page-8-2"></span>[27] E. Braaten and S. Fleming, [Phys. Rev. Lett.](https://doi.org/10.1103/PhysRevLett.74.3327) 74, 3327 (1995).
- [28] P. L. Cho and A. K. Leibovich, *[Phys. Rev. D](https://doi.org/10.1103/PhysRevD.53.150)* **53**, 150 (1996).
- <span id="page-8-3"></span>[29] J. P. Lansberg, [Int. J. Mod. Phys. A](https://doi.org/10.1142/S0217751X06033180) 21, 3857 (2006).
- [30] J. P. Lansberg, [Phys. Rep.](https://doi.org/10.1016/j.physrep.2020.08.007) **889**, 1 (2020).
- [31] A. Usachov, [arXiv:1910.08796](https://arXiv.org/abs/1910.08796).
- [32] L. P. Sun, H. Han, and K. T. Chao, [Phys. Rev. D](https://doi.org/10.1103/PhysRevD.94.074033) 94, 074033 [\(2016\).](https://doi.org/10.1103/PhysRevD.94.074033)
- [33] J. P. Lansberg and H. S. Shao, [Phys. Rev. Lett.](https://doi.org/10.1103/PhysRevLett.111.122001) 111, 122001 [\(2013\).](https://doi.org/10.1103/PhysRevLett.111.122001)
- <span id="page-8-4"></span>[34] Y. Q. Ma, K. Wang, and K. T. Chao, [Phys. Rev. Lett.](https://doi.org/10.1103/PhysRevLett.106.042002) 106, [042002 \(2011\).](https://doi.org/10.1103/PhysRevLett.106.042002)
- [35] Y. J. Zhang, Y. Q. Ma, K. Wang, and K. T. Chao, [Phys. Rev.](https://doi.org/10.1103/PhysRevD.81.034015) D **81**[, 034015 \(2010\)](https://doi.org/10.1103/PhysRevD.81.034015).
- <span id="page-8-5"></span>[36] B. A. Kniehl, C. P. Palisoc, and L. Zwirner, [Phys. Rev. D](https://doi.org/10.1103/PhysRevD.66.114002) 66, [114002 \(2002\).](https://doi.org/10.1103/PhysRevD.66.114002)
- [37] D. P. Roy and K. Sridhar, [Phys. Lett. B](https://doi.org/10.1016/0370-2693(95)80023-Q) 341, 413 (1995).
- [38] C. S. Kim, J. Lee, and H. S. Song, [Phys. Rev. D](https://doi.org/10.1103/PhysRevD.55.5429) 55, 5429 [\(1997\).](https://doi.org/10.1103/PhysRevD.55.5429)
- [39] E. Braaten, J. Lee, and S. Fleming, [Phys. Rev. D](https://doi.org/10.1103/PhysRevD.60.091501) 60, 091501 [\(1999\).](https://doi.org/10.1103/PhysRevD.60.091501)
- <span id="page-8-6"></span>[40] R. Li and J. X. Wang, [Phys. Lett. B](https://doi.org/10.1016/j.physletb.2008.12.050) 672, 51 (2009).
- <span id="page-8-9"></span>[41] G. Li, M. Song, R. Y. Zhang, and W. G. Ma, [Phys. Rev. D](https://doi.org/10.1103/PhysRevD.83.014001) 83[, 014001 \(2011\).](https://doi.org/10.1103/PhysRevD.83.014001)
- [42] M. Song, W. G. Ma, G. Li, R. Y. Zhang, and L. Guo, [J. High](https://doi.org/10.1007/JHEP02(2011)071) [Energy Phys. 02 \(2011\) 071;](https://doi.org/10.1007/JHEP02(2011)071) [12 \(2012\) 010\(E\).](https://doi.org/10.1007/JHEP12(2012)010)
- [43] G. Li, W. G. Ma, M. Song, R. Y. Zhang, and J. Y. Guo, [J. High Energy Phys. 01 \(](https://doi.org/10.1007/JHEP01(2013)034)2013) 034.
- [44] M. Song, G. Li, W. G. Ma, R. Y. Zhang, L. Guo, and J. Y. Guo, [Chin. Phys. Lett.](https://doi.org/10.1088/0256-307X/30/9/091201) 30, 091201 (2013).
- <span id="page-8-7"></span>[45] J. P. Lansberg and H. S. Shao, [J. High Energy Phys. 10](https://doi.org/10.1007/JHEP10(2016)153) [\(2016\) 153.](https://doi.org/10.1007/JHEP10(2016)153)
- <span id="page-8-10"></span>[46] J. P. Lansberg, H. S. Shao, and N. Yamanaka, [Phys. Lett. B](https://doi.org/10.1016/j.physletb.2018.04.020) 781[, 485 \(2018\)](https://doi.org/10.1016/j.physletb.2018.04.020).
- <span id="page-8-8"></span>[47] G. Aad et al. (ATLAS Collaboration), [J. High Energy Phys.](https://doi.org/10.1007/JHEP04(2014)172) [04 \(2014\) 172.](https://doi.org/10.1007/JHEP04(2014)172)
- <span id="page-8-11"></span>[48] G. Aad et al. (ATLAS Collaboration), [Eur. Phys. J. C](https://doi.org/10.1140/epjc/s10052-015-3406-9) 75, [229 \(2015\)](https://doi.org/10.1140/epjc/s10052-015-3406-9).
- [49] M. Aaboud et al. (ATLAS Collaboration), [J. High Energy](https://doi.org/10.1007/JHEP01(2020)095) [Phys. 01 \(2020\) 095.](https://doi.org/10.1007/JHEP01(2020)095)
- [50] J. P. Lansberg and C. Lorce, [Phys. Lett. B](https://doi.org/10.1016/j.physletb.2013.07.059) 726, 218 (2013); 738[, 529\(E\) \(2014\)](https://doi.org/10.1016/j.physletb.2014.10.015).
- <span id="page-8-12"></span>[51] B. A. Kniehl, C. P. Palisoc, and L. Zwirner, [Phys. Rev. D](https://doi.org/10.1103/PhysRevD.69.115005) 69, [115005 \(2004\).](https://doi.org/10.1103/PhysRevD.69.115005)
- <span id="page-8-13"></span>[52] M. Tanabashi et al. (Particle Data Group), [Phys. Rev. D](https://doi.org/10.1103/PhysRevD.98.030001) 98, [030001 \(2018\).](https://doi.org/10.1103/PhysRevD.98.030001)
- <span id="page-8-14"></span>[53] A. Petrelli, M. Cacciari, M. Greco, F. Maltoni, and M. L. Mangano, Nucl. Phys. B514[, 245 \(1998\)](https://doi.org/10.1016/S0550-3213(97)00801-8).
- <span id="page-8-15"></span>[54] T. Hahn, [Comput. Phys. Commun.](https://doi.org/10.1016/S0010-4655(01)00290-9) 140, 418 (2001).
- <span id="page-8-16"></span>[55] V. Shtabovenko, R. Mertig, and F. Orellana, [Comput. Phys.](https://doi.org/10.1016/j.cpc.2016.06.008) Commun. 207[, 432 \(2016\)](https://doi.org/10.1016/j.cpc.2016.06.008).
- <span id="page-8-17"></span>[56] F. Feng, [Comput. Phys. Commun.](https://doi.org/10.1016/j.cpc.2012.03.025) 183, 2158 (2012).
- [57] F. Feng, [J. Phys. Conf. Ser.](https://doi.org/10.1088/1742-6596/523/1/012041) 523, 012041 (2014).
- <span id="page-8-18"></span>[58] A. V. Smirnov, [J. High Energy Phys. 10 \(2008\) 107.](https://doi.org/10.1088/1126-6708/2008/10/107)
- [59] J. Fleischer, F. Jegerlehner, and O. V. Tarasov, [Nucl. Phys.](https://doi.org/10.1016/S0550-3213(99)00678-1) B566[, 423 \(2000\).](https://doi.org/10.1016/S0550-3213(99)00678-1)
- <span id="page-8-19"></span>[60] T. Hahn and M. Perez-Victoria, [Comput. Phys. Commun.](https://doi.org/10.1016/S0010-4655(98)00173-8) 118[, 153 \(1999\)](https://doi.org/10.1016/S0010-4655(98)00173-8).
- <span id="page-8-20"></span>[61] J. Pumplin, D. R. Stump, J. Huston, H. L. Lai, P. M. Nadolsky, and W. K. Tung, [J. High Energy Phys. 07 \(2002\) 012.](https://doi.org/10.1088/1126-6708/2002/07/012)
- <span id="page-8-21"></span>[62] S. Fleming and T. Mehen, Phys. Rev. D 57[, 1846 \(1998\)](https://doi.org/10.1103/PhysRevD.57.1846).
- [63] B. A. Kniehl and L. Zwirner, Nucl. Phys. **B621**[, 337 \(2002\).](https://doi.org/10.1016/S0550-3213(01)00564-8)
- <span id="page-8-22"></span>[64] B. A. Kniehl and C. P. Palisoc, [Eur. Phys. J. C](https://doi.org/10.1140/epjc/s10052-006-0044-2) 48, 451 [\(2006\).](https://doi.org/10.1140/epjc/s10052-006-0044-2)
- [65] E.J. Eichten and C. Quigg, Phys. Rev. D **52**[, 1726 \(1995\).](https://doi.org/10.1103/PhysRevD.52.1726)
- [66] Y. Q. Ma, K. Wang, and K. T. Chao, [Phys. Rev. D](https://doi.org/10.1103/PhysRevD.83.111503) 83, [111503 \(2011\).](https://doi.org/10.1103/PhysRevD.83.111503)
- <span id="page-8-23"></span>[67] S. Dawson, C. Englert, and T. Plehn, [Phys. Rep.](https://doi.org/10.1016/j.physrep.2019.05.001) 816, 1 [\(2019\).](https://doi.org/10.1016/j.physrep.2019.05.001)
- <span id="page-8-24"></span>[68] G. Li, X. A. Pan, M. Song, and Y. Zhang, [Phys. Rev. D](https://doi.org/10.1103/PhysRevD.100.074019) 100, [074019 \(2019\).](https://doi.org/10.1103/PhysRevD.100.074019)
- <span id="page-8-25"></span>[69] E. Braaten, B. A. Kniehl, and J. Lee, [Phys. Rev. D](https://doi.org/10.1103/PhysRevD.62.094005) 62, [094005 \(2000\).](https://doi.org/10.1103/PhysRevD.62.094005)
- <span id="page-8-26"></span>[70] M. Beneke and M. Krämer, Phys. Rev. D 55[, R5269 \(1997\).](https://doi.org/10.1103/PhysRevD.55.R5269)
- <span id="page-8-27"></span>[71] Z. Sun, X. G. Wu, and H. F. Zhang, [Phys. Rev. D](https://doi.org/10.1103/PhysRevD.92.074021) 92, [074021 \(2015\).](https://doi.org/10.1103/PhysRevD.92.074021)
- <span id="page-8-28"></span>[72] S. Bailey, T. Cridge, L. A. Harland-Lang, A. D. Martin, and R. S. Thorne, [Eur. Phys. J. C](https://doi.org/10.1140/epjc/s10052-021-09057-0) 81, 341 (2021).
- <span id="page-8-29"></span>[73] R. D. Ball et al. (NNPDF Collaboration), [J. High Energy](https://doi.org/10.1007/JHEP04(2015)040) [Phys. 04 \(2015\) 040.](https://doi.org/10.1007/JHEP04(2015)040)
- <span id="page-8-30"></span>[74] R. D. Ball et al. (NNPDF Collaboration), [Eur. Phys. J. C](https://doi.org/10.1140/epjc/s10052-017-5199-5) 77, [663 \(2017\)](https://doi.org/10.1140/epjc/s10052-017-5199-5).