

Custodial symmetry for muon $g - 2$

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We discuss the recent results on the muon anomalous magnetic moment in the context of new physics models with light scalars. We propose a model of one-loop contributions to $g - 2$ of a scalar and a pseudoscalar which naturally cancel in the massless limit by symmetry. This model allows us to interpolate between two possible interpretations while keeping new physics light. In the first interpretation, the results provide a strong evidence of the existence of new physics, dominated by the (positive) contribution of a CP -even scalar. In the second one, supported by the recent lattice result, the data provides a strong upper bound on new physics, specifically in the case of (negative) pseudoscalar contributions. We emphasize that tree-level signatures of the new scalars are enhanced relative to conventional explanations of the discrepancy. As a result, this model can be tested in the near future with accelerator-based experiments and possibly also at the precision frontier.

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I. INTRODUCTION

Recently, the muon $g-2$ experiment at Fermilab has announced its first preliminary result [1–3]

$$a_{\mu}^{\text{FNAL}} = 116592040(54) \times 10^{-11}, \quad (1)$$

where $a_{\mu} \equiv (g_{\mu} - 2)/2$ is the magnetic moment anomaly of the muon. Averaging it with the BNL result [4] leads to a new world average [1]

$$a_{\mu}^{\text{WA}} = 116592061(41) \times 10^{-11}. \quad (2)$$

When compared to the standard model (SM) theoretical prediction [5] $a_{\mu}^{\text{SM},R} = 116591810(43) \times 10^{-11}$,¹ one finds [1]

$$\Delta a_{\mu}^R \equiv a_{\mu}^{\text{WA}} - a_{\mu}^{\text{SM},R} = 251(59) \times 10^{-11}, \quad (3)$$

which is 4.2 standard deviations (σ) from zero. However, we can also compare a_{μ}^{WA} to the SM prediction based the leading order HVP estimated by lattice QCD calculations. Reference [11] reports $a_{\mu}^{\text{HVP-LO,lattice}} = 7075(55) \times 10^{-11}$, which translate to $a_{\mu}^{\text{SM,lattice}} = 116591952(58) \times 10^{-11}$. Then, the resulting difference between the SM and a_{μ}^{WA} is

$$\Delta a_{\mu}^{\text{lattice}} \equiv a_{\mu}^{\text{WA}} - a_{\mu}^{\text{SM,lattice}} = 109(71) \times 10^{-11}, \quad (4)$$

which is only 1.6σ from zero. The experimental error is mostly statistical while the lattice result is systematics limited. For simplicity we assume these uncorrelated uncertainties to be Gaussian-distributed and add them in quadrature.

One can interpret this result in two ways. The first interpretation is that we have a substantial evidence of new physics (NP). Most of the literature focusing on this direction is dedicated to investigating what type of SM extension could account for the difference between the experimental result and the theoretical prediction, see for example [12–15]. In this interpretation, the 2σ preferred range deduced from Eq. (3) is $133 < a_{\mu}^{\text{NP}}/10^{-11} < 369$. This interpretation is supported by the electroweak precision tests, where the same $e^+e^- \rightarrow$ hadrons data is used to calculate $\alpha(m_Z)$ [16,17]. Alternatively, one can interpret the data in view of the new lattice result of Ref. [11], which provides a bound on new states coupled to muons. Being the first result

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¹See e.g., [6–10] for details on the data-driven approach for estimating the hadronic vacuum polarization (HVP) contribution.

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at subpercent accuracy it still requires confirmation by independent lattice calculations. In this case, the allowed 2σ range from Eq. (4) yields $-33 < a_\mu^{\text{NP}}/10^{-11} < 251$. The model proposed in this letter allows us to interpolate between the above two different interpretations of the result.

We propose a model where the one-loop contribution to a_μ is naturally suppressed in the limit where the new scalar states are massless [18]. This cancellation allows the model to evade the bound of Eq. (4), especially for NP masses much lighter than the muon mass. Alternatively, a small deformation of the model could generate a large enough contribution to a_μ , thus accommodating the strong evidence of NP in Eq. (3).

Despite this one-loop suppression of a_μ , other tree-level processes, such as bremsstrahlung emission of light NP states or correction to bound-state energy levels, are unsuppressed. As a result, the relevant rates are enhanced by a factor as large as $\mathcal{O}(100)$ and become visible. Searches which probe the NP coupling to muons are therefore reviewed in this context. We also discuss implications of possible coupling of the new scalar states to electrons, assuming a naive m_e/m_μ suppression factor relative to muons.

II. THE MODEL

The basic observation behind our construction stems from the fact that the one-loop contribution to $(g-2)_\mu$ of a massless scalar, ϕ and pseudoscalar, π with sole Yukawa coupling to muons, respectively y_ϕ and y_π , are

$$a_\mu^\phi = \frac{3y_\phi^2}{16\pi^2}, \quad a_\mu^\pi = -\frac{y_\pi^2}{16\pi^2}. \quad (5)$$

We thus learn that in the massless limit a scalar can account for the central value of the anomaly provided that

$$y_\phi \approx 3.6 \times 10^{-4}. \quad (6)$$

A single pseudoscalar cannot account for the anomaly as its contribution is of opposite sign, giving a 95% CL bound on its coupling

$$y_\pi \lesssim 2.3 \times 10^{-4}. \quad (7)$$

We further observe that in a model consisting of three massless pseudoscalars π_i ($i = 1 \dots 3$) and a single massless scalar with equal Yukawa couplings, $y_\phi = y_{\pi_i}$, the one-loop contribution to a_μ vanishes [18].

We now introduce a natural model which realizes the above cancellation. Consider the two fourplet fields,

$$\varepsilon = \begin{pmatrix} \ell_1 \\ \ell_2 \\ \ell_3 \\ \ell_4 \end{pmatrix}, \quad \sigma = \begin{pmatrix} \phi \\ \pi_1 \\ \pi_2 \\ \pi_3 \end{pmatrix}, \quad (8)$$

as an extension to the SM, where ℓ_i are colorless, $\text{SU}(2)_L$ -singlet (hypercharge -1) Weyl spinors, and σ consists of

four real scalar SM singlets. For reasons that will become clear below we define the components of the σ field as one scalar, ϕ and three pseudoscalars, π_i . The Yukawa and mass terms of the fields in the Lagrangian are (using two-component spinor notations for convenience)²

$$-\mathcal{L} \supset \frac{1}{2} m_\sigma^2 \sigma^T \cdot \sigma + (\sigma^T \cdot \hat{y} \cdot \varepsilon \ell^c + m^\dagger \cdot \varepsilon \ell^c + \text{H.c.}), \quad (9)$$

where the matrix $\hat{y} \propto \langle H \rangle / \Lambda$ and the vector $m \propto y_{\text{SM}} \langle H \rangle$ are treated here as spurions, H being the SM-Higgs field, Λ the effective cutoff of the model and y_{SM} the SM lepton Yukawa coupling. The field ℓ^c is the SM left-handed (LH) lepton doublet. In the flavor-alignment limit, the linear combination of ℓ_i fields given by $m^\dagger \cdot \varepsilon$ is identified as the right-handed (RH) SM muon singlet. We implicitly assume that the other three orthogonal combinations which we denote $\psi_{1,2,3}$ acquire large Dirac masses by coupling to additional singlet fields $\psi_{1,2,3}^c$ and are therefore decoupled. In addition, we assume CP symmetry.

The Lagrangian has a global symmetry $G \equiv \text{O}(4)_\sigma \times \text{U}(4)_\varepsilon \times \text{U}(1)_{\ell^c}$ where the fields transform as

$$\sigma \rightarrow O \cdot \sigma, \quad \varepsilon \rightarrow U \cdot \varepsilon, \quad \ell^c \rightarrow e^{i\alpha} \ell^c, \quad (10)$$

and the spurions as

$$\hat{y} \rightarrow e^{-i\alpha} O \cdot \hat{y} \cdot U^\dagger, \quad m \rightarrow e^{i\alpha} U \cdot m, \quad (11)$$

with O , U and $e^{i\alpha}$ denoting $\text{O}(4)_\sigma$, $\text{U}(4)_\varepsilon$ and $\text{U}(1)_{\ell^c}$ transformations, respectively. Generically \hat{y} breaks G to a common $\text{U}(1)_\mu$ associated with the conserved muon number. Here, instead, we assume that the background value of \hat{y} is not generic but leads to the pattern of symmetry breaking

$$G \rightarrow \text{O}(4)_{\sigma+\varepsilon} \times \text{U}(1)_\mu. \quad (12)$$

Without loss of generality, the background value of m leads to

$$G \rightarrow \text{O}(4)_\sigma \times \text{U}(3)_\varepsilon \times \text{U}(1)_\mu, \quad (13)$$

where the $\text{U}(3)_\varepsilon$ symmetry corresponds to the residual freedom of transforming the three linear combinations of ℓ_i orthogonal to $m^\dagger \cdot \varepsilon$. Given the above pattern of symmetry breaking the spurions can be brought to $\langle \hat{y} \rangle = y_\ell \times \text{diag}(1, 1, 1, 1)$ and $\langle m \rangle = m_l \times (e^{i\theta_1}, e^{i\theta_2}, e^{i\theta_3}, e^{i\theta_4})^T$ where y_ℓ, m_ℓ are real and the phases $\theta_{1,2,3,4}$ are either zero or $\pi/2$ under the assumption that CP is conserved.

There are then two possibilities: either all the phases are the same or one of them (at least) is $\pi/2$. In the first case, the unbroken symmetry in the presence of both spurions is $\text{O}(3)_{\sigma+\varepsilon} \times \text{U}(1)_\mu$, since it is always possible to have

²For the sake of simplicity we only show the relevant terms in the broken electroweak-symmetry phase. We further assume alignment in the leptonic flavor space and only coupling to muons.

$\langle m \rangle \propto (1, 0, 0, 0)^T$ using an $O(4)$ transformation. In this limit only one component of σ interacts with muons (be it a scalar or pseudoscalar depends on the relative phase between the spurions) with no cancelling counterpart. In the second case the residual symmetry is only $O(2)_{\sigma+\varepsilon} \times U(1)_\mu$ since the relative phase is not removable by $O(4)$ transformations which can only yield $\langle m \rangle \propto (1, i, 0, 0)^T$. In this limit there are two states coupled to muons, one scalar and one pseudoscalar, whose interactions (and masses) are fixed by symmetries to enforce cancellation between their one-loop contributions to $(g - 2)_\mu$.

Henceforth we focus on the $O(2)$ -symmetric case. For convenience we work in a different $O(4)$ -basis where the background values of the spurions are

$$\langle \hat{y} \rangle = \frac{y_\ell}{2} \begin{pmatrix} 1 & & & \\ & i & & \\ & & i & \\ & & & i \end{pmatrix}, \quad \langle m \rangle = \frac{m_\ell}{2} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \quad (14)$$

where y_ℓ and the lepton mass m_ℓ are real and positive. In this case the combination $\ell' \equiv \frac{1}{2} \sum_{i=1}^4 \ell_i$, which couples to its SM, LH doublet, partner field ℓ^c through the Higgs field, corresponds to the SM RH muon field.

Note that the above structure is technically natural and is protected by collective breaking of the above symmetries. If only one of the spurion is present, the muon field couples at most to one scalar state. We also assume that σ has vanishing vacuum expectation value, so it does not affect the mass of the SM lepton.

The relevant part of the low-energy Lagrangian (below the $\mathcal{O}(\text{TeV})$ mass of the ψ_i 's) is

$$-\mathcal{L}_{\text{eff}} = y_\ell \phi (\ell' \ell^c + (\ell' \ell^c)^\dagger) + \sqrt{3} i y_\ell \pi (\ell' \ell^c - (\ell' \ell^c)^\dagger) + m_\ell (\ell' \ell^c + (\ell' \ell^c)^\dagger) \quad (15)$$

where

$$\pi \equiv \frac{1}{\sqrt{3}} (\pi_1 + \pi_2 + \pi_3). \quad (16)$$

The field π is the ‘‘custodian’’ field whose one-loop contribution to $g - 2$ is forced by the $O(2)$ symmetry to cancel that of ϕ . The two linear combinations orthogonal to the custodian remain noninteracting. Note that these states are stable due to the $O(2)$ symmetry and may be valid dark matter candidates. A detailed analysis is beyond the scope of the current work. We expect a small deviation from unity in the scalar-to-pseudoscalar coupling ratio y_ϕ/y_π due to different RG flow from the TeV scale, where they are equal, to the MeV scale of the measurements. However, such effects are $\mathcal{O}(y_\ell^4)$ and can be safely neglected.

Finally, one can introduce a small soft $O(4)$ breaking (yet $O(2)$ preserving) term which lifts the degeneracy between ϕ

and π masses (respectively, m_ϕ and m_π). In order to interpolate between the different limits we define

$$\Delta m \equiv m_\pi - m_\phi, \quad \Sigma m \equiv m_\pi + m_\phi, \quad (17)$$

and we denote the degenerate scalar and pseudoscalar masses by m_σ when $\Delta m = 0$.

The above construct can arise from extra-dimensional (alternatively 4-dimensional four-site) models [18] consisting of a fourplet of bulk fermions with only one zero mode identified with ℓ' , the massive orthogonal combinations being KK excitations. The structure of the spurions in Eq. (14) can be realized by imposing appropriate boundary conditions for the fields propagating in the bulk. Finally, we note that in order to keep the scalar masses light, say $m_\sigma \lesssim 10$ MeV requires new physics to appear at a scale $4\pi m_\sigma/y_\ell \sim \text{TeV}$, which is consistent with the scale of the new states in our construction.

III. PHENOMENOLOGICAL IMPLICATIONS

For $m_\sigma \ll m_\mu$, the one-loop contribution to the muon magnetic moment is suppressed and vanishes in the limit of $m_\sigma \rightarrow 0$. However, this exact cancellation is lifted by two effects. The first one arises from the finite mass of the σ field. It is straightforwardly evaluated from the one-loop functions of scalar and pseudoscalar, see, e.g., [19]. The second one is due to two-loop contributions whose ratio differs from that of the one-loop contributions in the massless limit. We estimate the two-loop correction to be of the order of a single pseudoscalar one-loop contribution times an additional suppression of $\alpha/4\pi \sim 10^{-3}$. In total, the contribution to a_μ^{NP} at the small mass limit, $m_\sigma \ll m_\mu$, can be estimated as

$$a_\mu^{\text{NP}} \approx \frac{y_\mu^2}{8\pi^2} \left(C_2 \frac{\alpha}{4\pi} - \pi \frac{m_\sigma}{m_\mu} \right), \quad (18)$$

where $C_2 \sim \mathcal{O}(1)$ with no fixed sign. In practice, the two-loop contribution is sub-leading in most of the parameter space of interest. In particular, the above estimate is at most an $\mathcal{O}(1\%)$ correction to the one-loop result for $m_\sigma = 1$ MeV in the $O(2)$ symmetric limit. For $\Delta m/\Sigma m \neq 0$, the two-loop contribution becomes important only for peculiar values of m_ϕ where the scalar and pseudoscalar contributions cancel out, see Fig. 1.

As mentioned above, the degeneracy between the scalar and pseudoscalar is lifted for $\Delta m \neq 0$. For $\Delta m > 0$, a_μ^{NP} is dominated by ϕ and positive, and can account for the central value in Eq. (3). Conversely, for $\Delta m \leq 0$, a_μ^{NP} is always negative, preventing an explanation of the central value of Eq. (3). We demonstrate this in Fig. 1, with the 2σ excluded region (in blue) from Eq. (4), in the $O(2)$ symmetric limit. The green bands account for the central value of the discrepancy in Eq. (3), for the special cases of $\Delta m/\Sigma m = 0.5$ and 0.75 .

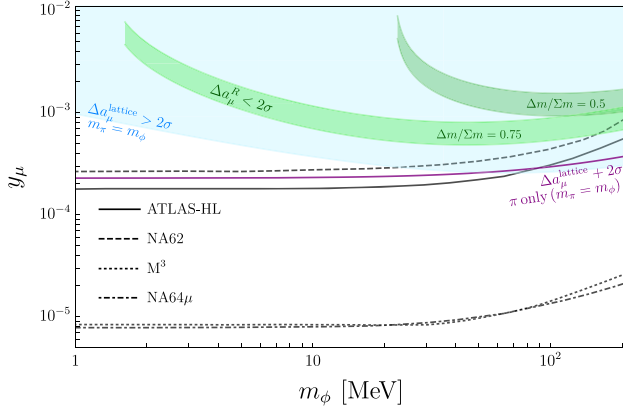


FIG. 1. Experimental sensitivity projections of tree-level based searches of muonphilic scalars. Green bands are preferred regions interpreting the $(g-2)_\mu$ result as evidence of new physics, $|\Delta a_\mu^{\text{NP}} - \Delta a_\mu^{\text{R}}| < 2\sigma^{\text{R}}$. They are cut off where the estimated two-loop contribution reaches 10% of the one-loop contribution. The blue area is the excluded region interpreting the $(g-2)_\mu$ result as a bound on new physics, $\Delta a_\mu^{\text{NP}} - \Delta a_\mu^{\text{lattice}} > 2\sigma^{\text{lattice}}$. The purple line indicates the $a_\mu^{\text{NP}} < 0$ bound on a single pseudoscalar state, $\Delta a_\mu^{\text{lattice}} - \Delta a_\mu^{\text{NP}} > 2\sigma^{\text{lattice}}$. Dashed lines represent the sensitivity of muon missing momentum proposed at ATLAS-HL, NA62, M^3 and NA64 μ experiments.

From Eq. (18), we learn that the strong bound from $(g-2)_\mu$ is weakened in our model thanks to the (partial) cancellation of the scalar and pseudoscalar contributions. This is illustrated in Fig. 1 with the $\Delta a_\mu^{\text{lattice}}$ bound on a single pseudoscalar state (purple). This region of parameter space between the purple line and the blue area is currently unexplored by $(g-2)_\mu$ but is possibly probed by tree-level searches for muon force carrier (MFC) as we show below. Among these, we find muon beam experiments [20–22], ATLAS [23], FASER ν [24], $K \rightarrow \mu\nu$ decays at NA62 [25], and B -factories [26] have promising prospects. As the cancellation is only effective for pseudoscalar/scalar masses below m_μ , it will not affect the relevant parameter space for MFC searches which focus on dimuon scalar decay as in Refs. [27,28]. In addition, we consider precision measurements such as muonium (Mu) positronium (Ps) and helium spectroscopy, and that of the electron anomalous magnetic moment.

A. Muonphilic tree-level based searches

We start by exploring the simple and minimal muonphilic case, in which the σ field only couples to muons and constitutes a new MFC. Most of the analyses provided in the various proposals for MFC searches assume a single field, usually a scalar or a vector. Here we estimate the effect of the additional pseudoscalar on the different experimental signatures and the associated projected sensitivities.

First, consider searches where a muon propagates through a fixed target emitting the MFC in a bremsstrahlung-like process. We focus on signatures in which the

MFC itself does not leave any trace in the detector either because it is long lived (which is the natural expectation for masses below the dimuon threshold) or decays predominantly to particles in a secluded sector (e.g., dark matter). Mainly two observables related solely to the muon momentum arise due to the scalar-emission: (i) missing muon momentum, see the M^3 experiment in Fermilab [21], proposed search at ATLAS [23] and NA64 μ [22,29–31] experiments (see also [32,33] for the electron case); (ii) change in the muon’s direction, a kink in its trajectory.

Observables associated with the kink are detectable with high precision in emulsion targets [34,35]. The FASER/FASER ν experiment [36–38] has an excellent spacial resolution and allows kinks detection in muon tracks at $\mathcal{O}(10^{-4}$ rad) level [39]. This is a powerful MFC probe that can cover $y_\mu \sim \mathcal{O}(10^{-3}-10^{-4})$ during the first run of FASER ν (LHC Run-3) [24].

We have estimated the corresponding production processes in the $m_\sigma \ll m_\mu$ limit, using the Weizsäcker-Williams approximation [40,41]. We find that the total cross section for a scalar is similar to that of the pseudoscalar in the limit of equal couplings. However, using the analysis of Refs. [42,43] we do observe that the pseudoscalar differential rate tends to be harder than that of the scalar, which we ignore in our recast. Thus, below we present the projected bounds using the scalar-only rates of [21–23] as a conservative estimation for the sensitivity in our model.

A different approach is taken in [25], where the MFC is probed through the $K \rightarrow \mu\nu X$ decays in NA62. When comparing the exclusive branching ratios we found (in the $m_\sigma \rightarrow 0$ limit)

$$\frac{\Gamma(K \rightarrow \mu\nu\phi)}{\Gamma(K \rightarrow \mu\nu\pi)} \sim 2, \quad (19)$$

where a missing energy cut of $E_{\text{miss}}^2 > 0.05 \text{ GeV}^2$ was imposed. (See Ref. [25] for further details.) We reach a similar conclusion regarding this search, namely for low masses the signal is dominated by the scalar and the current projected sensitivities for a single scalar constitute a conservative estimation for the projected sensitivities of our model.

These projections are presented as black lines in Fig. 1, showing that M^3 , NA62, NA64 μ and ATLAS experiments can probe unexplored parameter space, in the context of either of the two possible interpretations of the $(g-2)_\mu$ results. Note that for all these searches, an $\mathcal{O}(1)$ deformation of the projected sensitivities for a single scalar is expected from a full analysis (to be performed elsewhere).

Finally, $\mu^+\mu^-$ pairs can form a true-muonium (TM) bound state. Though not yet observed experimentally, it would serve as an important alternative probe of MFC models [44] once its properties are measured precisely.

B. Coupling to electron and muon

A possible coupling of σ to electrons would provide additional ways to probe our model. For definiteness we

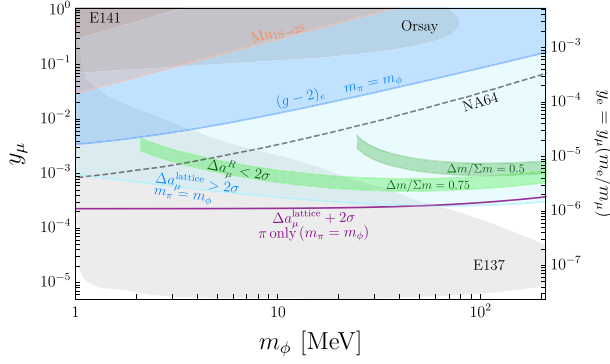


FIG. 2. Parameter space of scalars with electron and muon couplings, assuming $y_e/y_\mu = m_e/m_\mu$. The green bands are the preferred regions interpreting the $(g-2)_\mu$ result as evidence of new physics, $|\Delta a_\mu^{\text{NP}} - \Delta a_\mu^{\text{R}}| < 2\sigma^{\text{R}}$. They are cut off where the estimated two-loop contribution reaches 10% of the one-loop contribution, see Eq. (18). The light blue area is the excluded region interpreting the $(g-2)_\mu$ result as a bound on new physics, $\Delta a_\mu^{\text{NP}} - \Delta a_\mu^{\text{lattice}} > 2\sigma^{\text{lattice}}$. The dark blue area shows the excluded region from the $(g-2)_e$ measurement. The purple line indicates the $a_\mu^{\text{NP}} < 0$ bound on a single pseudoscalar state, $\Delta a_\mu^{\text{lattice}} - \Delta a_\mu^{\text{NP}} > 2\sigma^{\text{lattice}}$. The gray regions are excluded by tree-level based searches at the Orsay, E137 and E141 electron beam dump experiments, which assume a visible decay of the scalar mediator. The black dashed line represents the bounds from NA64 for an invisibly decaying scalar mediator. The orange region is excluded by the measurement of the 1S-2S transition frequency of muonium.

assume the following relation among the couplings, $y_e = y_\mu \times (m_e/m_\mu)$, motivated by, e.g., Higgs portal models [45,46]. A similar scaling can arise within the framework of minimal flavor violation [47,48].

The phenomenology of this model, associated with tree-level based searches, is similar to the model of leptonic Higgs portal [28]. At the intensity frontier, we consider bounds from the electron beam dumps E141 [49], Orsay [50] and E137 [51] and adopt the recast of Refs. [28,42,43]. As illustrated in Fig. 2, only Orsay and E137 are relevant to our model. We note that these bounds can be avoided in nonminimal models where the dominant ϕ and π decay mode is invisible. In that case, the recent experimental bounds from NA64 [52] can be used to constrain a model in which the scalars decay invisibly.

At the precision frontier, the muonium (the μ^+e^- bound state) is a dedicated candidate for testing the existence of a light force carrier between muon and electron, see [53]. The most accurate frequency measurement is for the 1S-2S transition with $\nu_{1\text{S-2S}}^{\text{Mu}}(\text{exp}) = 2455528941.0(9.8)$ MHz [54]. This is in a good agreement with the QED theory prediction $\nu_{1\text{S-2S}}^{\text{Mu}}(\text{th}) = 2455528935.4(1.4)$ MHz [54–56], whose principal uncertainty is due to the muon-to-electron mass ratio [57], currently extracted from measurements of Zeeman transitions within the muonium ground state [58]. The 1S-2S transition will be soon remeasured by the

MuMASS collaboration at PSI with $\mathcal{O}(\text{kHz})$ precision [59]. In the meantime the theory prediction is expected to improve by about one order of magnitude thanks to a new measurement of the 1S hyperfine splitting at J-PARC [60]. The Mu 1S-2S transition is mostly sensitive to spin independent force from the scalar exchange (the pseudo-scalar induce spin-dependent force, which is highly suppressed). The resulting upper bound of y_μ from the difference between the theory calculation and the experimental result is plot as the orange curve in Fig. 2. As we can see it less sensitive than other probes. We note that for $m_\sigma < 0.1$ MeV (not shown in the plot) the bounds from $(g-2)_{e,\mu}$ and from electron beam dump become weaker than the one obtained from Mu spectroscopy.

The positronium (the e^+e^- bound state) and helium atom as well as the electron magnetic moment, a_e , are sensitive to electrophilic forces. The 1S-2S transition frequency in ortho-Ps (the spin triplet configuration) has been measured with MHz accuracy [61] and agrees well with theory [62] which is limited by unknown three-loop corrections in QED. Several helium 4 transitions are measured with kHz accuracy with good agreement with theory calculation at the MHz level [63]. The sensitivity to electron-electron new physics interactions is comparable to positronium [64]. We find that the resulting bounds from precision positronium and helium spectroscopy are weaker than other bound muonium. This is because we assume that the electron coupling is suppressed by m_e/m_μ .

The magnetic moment of a free electron is measured at $\mathcal{O}(10^{-11})$ accuracy, $a_e(\text{exp}) = 0.00115965218073(28)$ [65], which requires a five-loop QED calculation [66] for a meaningful comparison to theory. The dominant theory uncertainty comes from α which is currently best extracted from a combination of fundamental constants including the Rydberg constant, the electron and Rubidium atomic masses and the ratio of the Planck constant to the Rubidium mass [67], yielding $a_e(\text{th}) = 0.001159652180252(95)$. This value of α in Ref. [67] is larger than the previous best determination based on Caesium measurements [68] by about 5.4σ . While more data is needed to understand its origin, this difference does not affect significantly $(g-2)_e$ whose uncertainty is dominated by $a_e(\text{exp})$.

IV. CONCLUSIONS

We investigated the new physics implications of the recent measurements of the muon magnetic moment in a model with new light scalars. The symmetry of the model ensures that their one-loop contributions to $(g-2)_\mu$ naturally vanishes in the massless limit. In the muonphilic case, near-future searches based on tree-level scalar emission at the luminosity frontier are able to cover most of the relevant parameter space of the model. Assuming that the new states also couple to electrons, the model is further probed by electron beam-dump experiments and muonium spectroscopy at the precision frontier.

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