$(g-2)_\mu$ in the 2HDM and slightly beyond: An updated view

P. M. Ferreira,[1,2] B. L. Gonçalves,[3,2] F. R. Joaquim,[3] and Marc Sher[4]

[1]*Instituto Superior de Engenharia de Lisboa, Instituto Politécnico de Lisboa, 1959-007 Lisboa, Portugal*
[2]*Centro de Física Teórica e Computacional, Faculdade de Ciências, Universidade de Lisboa, Campo Grande, Edifício C8, 1749-016 Lisboa, Portugal*
[3]*Departamento de Física and CFTP, Instituto Superior Técnico, Universidade de Lisboa, 1049-001 Lisboa, Portugal*
[4]*High Energy Theory Group, William & Mary, Williamsburg, Virginia 23187, USA*

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The recent measurement of the muon $g-2$ anomaly continues to defy a Standard Model explanation but can be accommodated within the framework of two-Higgs doublet models, although the pseudoscalar mass must be fairly light. If one further includes extra fermion content in the form of a generation of vectorlike leptons, the allowed parameter range that explains the anomaly is even further extended, and clashes with B-decay constraints may be avoided. We show how the muon magnetic moment anomaly can be fit within these models, under the assumption that the vectorlike leptons do not mix with the muon. We update previous analyses and include all theoretical and experimental constraints, including searches for extra scalars. It is shown that the inclusion of vectorlike fermions allows the lepton-specific and muon-specific models to perform much better in fitting the muon's $g-2$. However, these fits do require the Yukawa coupling between the Higgs and the vectorlike leptons to be large, causing potential problems with perturbativity and unitarity, and thus, models in which the vectorlike leptons mix with the muon may be preferred.

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I. INTRODUCTION

Recently, the Muon g-2 Collaboration at Fermilab reported new results [1] from run 1 of their experiment measuring the anomalous magnetic moment of the muon a_μ. Prior to this announcement, the discrepancy between the experimental measurement $a_\mu^{\rm exp}$ [2] and the Standard Model (SM) theoretical prediction $a_\mu^{\rm SM}$ [3–6] was

$$\Delta a_\mu^{\rm exp} = a_\mu^{\rm exp} - a_\mu^{\rm SM} = (279 \pm 76) \times 10^{-11} \quad (3.7\sigma), \quad (1)$$

while the new combined result is [1]

$$\Delta a_\mu^{\rm exp} = (251 \pm 59) \times 10^{-11} \quad (4.2\sigma). \quad (2)$$

There are hundreds of papers with new physics explanations for the $(g-2)_\mu$ anomaly such as supersymmetric models, left-right symmetric models, scotogenic models, 331 models, $L_\mu - L_\tau$ models, seesaw models, and the Zee-Babu model, as well as two-Higgs doublet models (2HDMs); an extensive review can be found in Ref. [7]. In this paper, we focus on 2HDMs (for a review, see Ref. [8]) and discuss the implications of the new result from the Muon g-2 Collaboration.

In the 2HDM, it is possible to ensure that tree-level flavor-changing neutral currents mediated by scalars do not exist by imposing a discrete Z_2 symmetry on the model. There are four such versions of the 2HDM, referred to as type-I, type-II, type-X (sometimes called lepton-specific), and type-Y (sometimes called flipped) models. In the type-II and type-X models, the coupling of the muon to the heavy Higgs bosons is enhanced by a factor of $\tan\beta$, which is the ratio of the two vacuum expectation values, but in type-II models, the $Q = -1/3$ quark couplings also get the same enhancement, leading to possible problems with radiative B-meson decays. In fact, explaining the $(g-2)_\mu$ anomaly without affecting B decays is one of the motivations for studies of the type-X model. On the other hand, in the type-I and type-Y models, the couplings of the muon to heavy Higgs bosons are suppressed by $\tan\beta$, and thus, these models are not favored for explaining the $(g-2)_\mu$ discrepancy. For simplicity, we only consider models without tree-level flavor-changing neutral currents and with CP conservation in the Higgs sector.

One of the earlier $(g-2)_\mu$ studies in 2HDMs following the discovery of the light Higgs boson was the work of Broggio *et al.* [9]. They restricted their analysis to the "alignment limit" [$\cos(\beta-\alpha) = 0$] in which the tree-level couplings of the lighest 2HDM scalar are identical to those of the SM Higgs. As noted above, they found that only the type-II and type-X models could account for the $(g-2)_\mu$ discrepancy. In both models, a very light pseudoscalar Higgs mass (m_A) is required, typically below 100 GeV, as is a relatively large $\tan\beta$, typically greater than 60. Another analysis of the type-X 2HDM with low-mass pseudoscalars is that of [10], where it was shown that m_A could be as low as 10 GeV. With values for m_A as in these references, unitarity and electroweak precision constraints then force the charged Higgs mass (m_{H^+}) to be less than about 200 GeV. This is a problem within the type II model, since radiative B decays, Δm_{B_s}, and the hadronic $Z \to \bar{b}b$ branching ratio force $m_{H^+} \gtrsim 600$ GeV [11]. As a result, the type-X model is favored. A subsequent analysis in Ref. [12] focusing explicitly on the type-X model considered all experimental constraints. It was noted that bounds from leptonic τ decay restricted the parameter space further and that the discrepancy in $(g-2)_\mu$ could be explained at the 2σ level with m_A between 10 and 30 GeV, m_{H^+} between 200 and 350 GeV, and $\tan\beta$ between 30 and 50. Shortly thereafter, Ref. [13] included more recent data from lepton universality tests and found bounds that were somewhat weaker but in general agreement with [12]. In the same token, studies have been performed in Refs. [14,15], and a more recent work including limits from Higgs decays to AA [16] also found similar restrictions on the parameter space.

Another attempt to explain the $(g-2)_\mu$ discrepancy involved adding vectorlike leptons (VLL). It was shown [17,18] that this alternative works if the VLLs mix with the muon. However, in the SM, this not only alters the Higgs dimuon branching ratio but also affects excessively the diphoton branching ratio and is thus phenomenologically unacceptable. Recently, the addition of VLLs to the type-II and type-X models was considered [19], and it was shown that the parameter space can be significantly expanded even without mixing the VLLs with muons, since the VLLs contribute to Δa_μ in two-loop Barr-Zee diagrams. In this case, the parameter space of the type-X model is substantially widened, and the type-II model is not completely excluded. Even more recently, Dermisek *et al.* [20] explored the type-II model with VLLs but now allowed mixing with the muon. Here, the effects on the muon coupling to the weak vector and scalar bosons must be considered. They showed that the extra Higgs bosons and VLLs could be extremely heavy and still explain the $(g-2)_\mu$ discrepancy without major effects on the dimuon SM Higgs decay. A more detailed and comprehensive paper by Dermisek *et al.* has just appeared [21]. None of the above papers deal with the smaller discrepancy for the $(g-2)$ of the electron. A model explaining both discrepancies was proposed by Chun and Mondal [22] in which VLLs mix with the muon and the electron. It turns out that also in such case a light (<100 GeV) pseudoscalar and large $\tan\beta$ are needed.

The belief that only the standard four 2HDMs can avoid flavor-changing neutral currents at tree level was shown to be incorrect by Abe *et al.* [23]. They proposed a muon-specific (μSpec) model in which one doublet couples to the muon and the other couples to all other fermions. This was implemented with one of the usual Z_2 symmetries, complemented with a muon number conservation symmetry [which is a global $U(1)$, although they referred to it as an overall Z_4 symmetry]. This symmetry cannot be implemented in the quark sector since it eliminates mixing in the Cabibbo-Kobayashi-Maskawa matrix, but neutrino mixing can easily be produced in a heavy Majorana sector. They studied $(g-2)_\mu$ in the model and found that the discrepancy could be solved with very large $\tan\beta$ values of approximately 1000 (but they also showed that perturbation theory was still valid). A study of the model by PF and MS [24] showed that this was fine-tuned and, leaving aside any explanation of the $(g-2)_\mu$ discrepancy, studied other properties of the model, including substantial effects on the dimuon decay of the Higgs.

In the above, we have only considered 2HDMs in which a symmetry eliminates tree-level flavor-changing neutral currents. An alternate approach is to simply assume that the Yukawa coupling matrices of the two doublets are proportional. This is the aligned 2HDM [25,26] (A2HDM). The conventional 2HDMs are special cases of the A2HDM. Early treatments of $(g-2)$ in this model were discussed in Refs. [15,27]. Since the parameter space is larger, one has an expanded range of allowed pseudoscalar masses (although the pseudoscalar mass must still be relatively light) and can accommodate smaller $\tan\beta$. Notice that $\tan\beta$ is defined in the A2HDM in the usual manner (the ratio of doublet vevs), but the scalar-fermion couplings do not depend on it in the same way as in the flavor conserving 2HDMs; however, in the latter models, the up, down, and lepton Yukawa couplings have correlated $\tan\beta$ dependencies, in the A2HDM, those couplings are essentially independent, which clearly increases the allowed parameter space. A much more detailed two-loop computation in the A2HDM was in Ref. [28], and a complete phenomenological analysis, focusing on the effects of a light pseudoscalar, was in Ref. [29]. The papers of [27,29] also studied the particularly interesting decay of the Higgs into AA, which is important if the pseudoscalar is below 62 GeV. In this paper, we focus on the models in which a symmetry forces tree-level FCNC to vanish but mention the A2HDM in the conclusions.

Finally, one must take into account that there are many alternative attempts to explain $(g-2)_\mu$, even in the context of 2HDMs. For instance, the model of [30] has a lepton-specific inert doublet with μ Yukawa couplings having an opposite sign to those of e and τ; models with tree-level FCNC were considered in this context in [31,32]. In [33], the impact of a fourth generation of vectorlike fermions and an extra scalar was studied. Contributions from additional gauge scalar singlets were investigated in [34,35]. An extremely light (sub-GeV) scalar was considered in [36]. A 2HDM study of $(g-2)_\mu$ considering the effects of warped space was undertaken in [37], and a 2HDM complemented with a fourth generation of fermions and a Z' gauge boson from an extra $U(1)_X$ symmetry was considered in [38]. As for non-2HDM explanations, one has, for instance, an addition of a leptophilic scalar to the SM in [39]. UV complete models with a $L_\mu - L_\tau$ symmetry were considered in [40]. A simultaneous study of dark matter and $(g-2)_\mu$ with extra scalars and vectorlike fermions was undertaken in [41]. A generic analysis of radiative leptonic-mass generation and its impact on $(g-2)_\mu$ can be found in [42]; and the interplay between new physics contributions and SM effective field theory for $(g-2)_\mu$ is discussed in [43]. On a different strand, the experimental implications of the muon anomalous magnetic moment at a future muon collider were studied in [44–46].

All of the above analyses relied on the old experimental result [2], and one would expect the new result from the Muon g-2 Collaboration [1] to have a small effect on those studies. In this paper, we update some of the 2HDM results to include the new combined experimental value for $\Delta a_\mu^{\rm exp}$ given in (2). After discussing the impact of the new measurement in the context of the type-X and type-II models, we focus on the 2HDM extended with VLLs which do not mix with the muon. Adding such mixing increases the number of parameters, and many of the relevant formulas are in the recent work of Dermisek *et al.* [21]. In addition, we also look at the μSpec model, which can suffer substantial changes in the Higgs dimuon decay, and analyze the $(g-2)_\mu$ discrepancy when we add VLLs.

II. TYPE-II, TYPE-X, AND μSpec 2HDMs

In the 2HDM, the scalar-fermion Yukawa interactions can be expressed in the compact form

$$\mathcal{L} = \frac{\sqrt{2}}{v} H^+ \{\bar{u}[\xi_u M_u^\dagger V P_L - \xi_d V M_d P_R] d - \xi_\ell \bar{\nu} M_\ell P_R \ell\} - \frac{1}{v}\sum_{k,f} y_f^k S_k^0 \bar{f} M_f P_R f + \text{H.c.}, \tag{3}$$

where $S_k^0 = \{h, H, A\}$ are the neutral-scalar mass eigenstates (h is the SM Higgs with mass $m_h = 125.1$ GeV [47]), and $f = u, d, \ell$ denote any SM charged-fermion type with mass matrix M_f. Following the notation of Ref. [25], the couplings $y_{f,\ell}^k$ are

$$y_{d,\ell}^k = \mathcal{R}_{k1} + (\mathcal{R}_{k2} + i\mathcal{R}_{k3})\xi_{d,\ell},$$
$$y_u^k = \mathcal{R}_{k1} + (\mathcal{R}_{k2} - i\mathcal{R}_{k3})\xi_u^*, \tag{4}$$

where $\mathcal{R}$ is the orthogonal matrix which relates the scalar weak states with the mass eigenstates S_k^0. For a CP-conserving potential, A does not mix with h and H, implying $\mathcal{R}_{3j} = \mathcal{R}_{j3} = 0$ for $j \neq 3$ and $\mathcal{R}_{33} = 1$. On the other hand, $h - H$ mixing is determined by $\mathcal{R}_{11} = -\mathcal{R}_{22} = \sin(\beta - \alpha)$ and $\mathcal{R}_{12} = \mathcal{R}_{21} = \cos(\beta - \alpha)$, with $\beta - \alpha = \pi/2$ in the alignment limit. For the type-II, type-X, and μSpec 2HDM, the ξ_f couplings are

$$\begin{aligned} \text{Type II}:\ & \xi_{d,\ell} = -\tan\beta, & \xi_u = \cot\beta, \\ \text{Type X}:\ & \xi_\ell = -\tan\beta, & \xi_{u,d} = \cot\beta, \\ \mu\text{Spec}:\ & \xi_\mu = -\tan\beta, & \xi_{u,d,\tau,e} = \cot\beta. \end{aligned} \tag{5}$$

In the alignment limit, the tree-level couplings of the SM-like scalar state, h, are therefore identical to those of the SM Higgs boson; the agreement of observed Higgs production and decays to that of the SM makes this assumption quite reasonable.

In the early post-Higgs discovery study of the $(g-2)_\mu$ anomaly of Ref. [9], the type-II and type-X models were studied in the alignment limit. The relevant free parameters in the model are $\tan\beta$, the pseudoscalar mass m_A, the charged Higgs mass m_{H^+}, the heavy scalar mass, m_H, and the Z_2 soft-breaking parameter m_{12}^2. Broggio *et al.* considered constraints on these parameters due to electroweak precision tests, vacuum stability, and perturbativity. These imposed bounds on the heavy Higgs masses, and they showed that for a pseudoscalar mass of less than 100 GeV, the charged Higgs mass could not exceed 200 GeV. This was not substantially affected by the value of $\tan\beta$ or by deviations from the $\cos(\beta-\alpha) = 0$ assumption. They then calculated the constraints from the $(g-2)_\mu$ results (they also included constraints from the $g-2$ of the electron, but these are very weak).

We now revisit these findings in light of the new Muon g-2 Collaboration result considering all one-loop and two-loop Barr-Zee (BZ) contributions to Δa_μ as given in Ref. [15]. In Fig. 1, we show the allowed regions in the m_A and $\tan\beta$ plane corresponding to the Δa_μ intervals in Eq. (2) at the 1σ, 2σ, and 3σ levels (green, yellow, and gray shaded regions) for the type-X (top panel) and type-II (bottom panel) cases. The solid, dashed, and dash-dotted contours delimit the analogue regions when the old result in Eq. (2) is considered. As noted earlier, radiative B decays favor $m_{H^+} > 600$ GeV in the type-II model [11]. Thus, we take $m_{H^+} = 600$ GeV as the reference value in the bottom panel of Fig. 1, while for type X a lighter H^+ is considered (changing m_{H^+} does not have a significant impact on Δa_μ). As pointed out in the Introduction, such small values of m_A in the type-II model are in conflict with unitarity and electroweak precision constraints. On the other hand, type X is still allowed. The results show that, as expected, the impact of the new $(g-2)_\mu$ result is marginal. The μSpec 2HDM requires extreme fine-tuning and values of $\tan\beta$ of $O(1000)$ in order to accommodate the $(g-2)_\mu$ discrepancy, and we do not show a plot in this case. It is

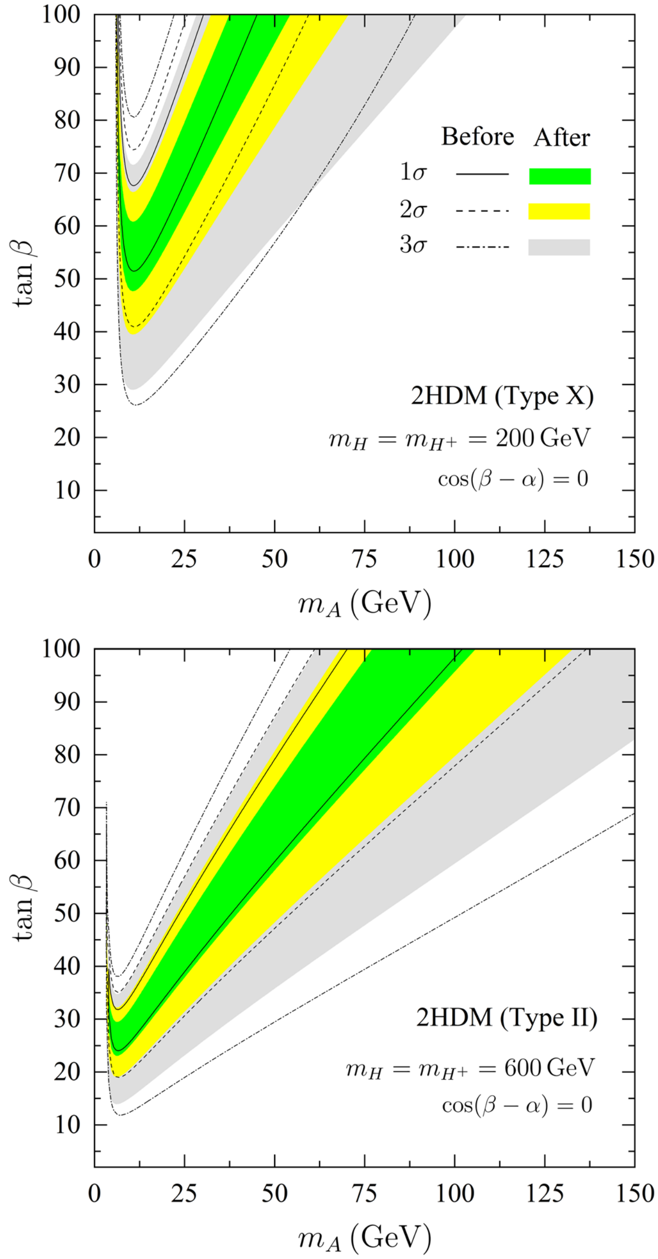

FIG. 1. Δa_μ allowed regions in the $(m_A, \tan\beta)$ plane for the type-X (top) and type-II (bottom) 2HDMs. The green, yellow, and gray shaded regions were obtained taking the new Δa_μ interval given in Eq. (2) at 1, 2 and 3σ, respectively. For comparison, we also show the lines which delimit the same regions when the old result in Eq. (1) is considered.

worth mentioning that for μSpec the BZ contributions to Δa_μ are not as significant as they are for type II and type X since both the b-quark and τ-lepton couplings are not $\tan\beta$ enhanced.

As a check, by keeping the fermion BZ diagrams only, as done by Broggio *et al.*, we were able to reproduce their results for the type-II and type-X 2HDMs in the alignment limit. Fig. 1 updates their work and includes additional contributions. Although Broggio *et al.* did include electroweak precision fits, other constraints arise from B physics, Z physics and τ physics. These are discussed in the A2HDM, which contains the type-II and type-X models as special cases, in Ref. [29]. For the type-X model, the bounds in Fig. 1 of that work lead to an upper bound on $\tan\beta$ which rises from 30 to 50 as m_A rises from 0 to 20 GeV and then levels off at a value between 60 and 100, depending on the mass of the heavier scalars. This may cut off the upper part of the allowed parameter space in Fig. 1.[1]

III. 2HDM WITH VECTORLIKE LEPTONS

Let us now consider a 2HDM extension with vectorlike leptons $\chi_{L,R} = (NL^-)^T_{L,R} \sim (\mathbf{2}, -1/2)$ and $E_{L,R} \sim (\mathbf{1}, -1)$, with $N_{L,R}$ being neutral and $L^-_{L,R}, E_{L,R}$ charged. If taken within the context of the muon-specific 2HDM, the VLLs will have no quantum numbers under the underlying muon number conservation symmetry of that model. We choose to have no mixing between the VLLs and the usual leptons. This can be achieved, for instance, by imposing an extra Z_2 symmetry on the model, under which all VLL fields have charge -1, and all other fields have charge $+1$. As noted in the Introduction, the field content of the model is the same as in Refs. [19–22]. Refs. [20,21] do not have the extra Z_2 symmetry and thus allow mixing with the muon, while the other two references forbid such mixing.

The VLL Yukawa and mass terms are

$$-\mathcal{L}_{\rm VLL} = m_L \overline{\chi_L}\chi_R + m_E \overline{E_L} E_R + \lambda_L \overline{\chi_R}\Phi_1 E_L + \lambda_R \overline{\chi_L}\Phi_1 E_R + \text{H.c.}, \tag{6}$$

which, after electroweak symmetry breaking, lead to the following heavy charged-lepton mass matrix:

$$\mathcal{M} = \begin{pmatrix} m_L & \lambda_R v\cos\beta/\sqrt{2} \\ \lambda_L^* v\cos\beta/\sqrt{2} & m_E \end{pmatrix}, \tag{7}$$

in the $(L^- E)^T_{L,R}$ basis, while for the new neutral-lepton mass $M_N = |m_L|$. The above matrix can be diagonalized as $U_L^\dagger \mathcal{M} U_R = \mathrm{diag}(M_1, M_2)$, where $M_{1,2}$ are the masses of the $L^-_{1,2}$ heavy charged-lepton physical states, and $U_{L,R}$ are 2×2 unitary matrices. As for the new charged-current interactions, we have

$$\mathcal{L} = \frac{g}{\sqrt{2}}\bar{N}\gamma^\mu[(U_L)_{1a}P_L + (U_R)_{1a}P_R]L_a^- W_\mu^+ + \text{H.c.}, \tag{8}$$

where $g = e/\sin\theta_W$ is the $\mathrm{SU}(2)_L$ gauge coupling. The interactions with the neutral and charged physical scalars are

[1]After this paper was submitted for publication, we became aware of [48], where further updates to the 2HDM fit of a_μ are discussed.

$$-\mathcal{L} = \frac{\sqrt{2}}{v} H^+ \bar{N}[\xi_a M_a (U_R)_{2a} P_R + \xi_N M_N (U_L)_{2a} P_L] L_a^- + \frac{1}{v} \sum_{k,a,b} M_a y_{ab}^k \overline{L_a^-} P_R L_b^- S_k^0 + \text{H.c.}, \quad (9)$$

with $S_k^0 = \{h, H, A\}$, a, $b = 1$, 2 and

$$\xi_a = \xi_\ell \frac{\mathcal{M}_{12}}{M_a}, \qquad \xi_N = \xi_\ell \frac{\mathcal{M}_{21}^*}{M_N}, \qquad \xi_\ell = -\tan\beta,$$
$$y_{ab}^k = (\mathcal{R}_{k1} - \xi_\ell \mathcal{R}_{k2}) X_{ab}^+ + i \xi_\ell \mathcal{R}_{k3} X_{ab}^-. \quad (10)$$

For simplicity, we restrict ourselves to the case of real $\mathcal{M}$, for which $U_{L,R}$ are orthogonal and parametrized by two mixing angles $\theta_{L,R}$ with $(U_{L,R})_{11} = (U_{L,R})_{22} = c_{L,R}$ and $(U_{L,R})_{12} = -(U_{L,R})_{21} = s_{L,R}$ (the notation $\cos\theta_{L,R} \equiv c_{L,R}$, $\sin\theta_{L,R} \equiv s_{L,R}$ has been used). Under this assumption, and taking into account that the only relevant VLL couplings are the flavor-conserving ones ($a = b$), we have

$$X_{aa}^+ = c_R^2 s_L^2 + c_L^2 s_R^2 - \frac{1}{2} s_{2L} s_{2R} \left(\frac{M_1}{M_2}\right)^{(-1)^a},$$
$$X_{aa}^- = \frac{(-1)^{a+1}}{2} (c_{2L} - c_{2R}), \quad (11)$$

with $c_{2L,2R} \equiv \cos(2\theta_{L,R})$ and $s_{2L,2R} \equiv \sin(2\theta_{L,R})$. The rotation angles also allow us to express the couplings $\lambda_{L,R}$ in terms of the VLL mass eigenvalues,

$$\lambda_L = -\frac{\sqrt{2}}{v c_\beta} [c_R s_L M_1 - c_L s_R M_2],$$
$$\lambda_R = -\frac{\sqrt{2}}{v c_\beta} [c_L s_R M_1 - c_R s_L M_2]. \quad (12)$$

We see that high $\tan\beta$ will increase the magnitude of these couplings, as will higher values of M_1, M_2, unless some fine-tuning with the angles $\theta_{L,R}$ occurs.

In the absence of VLL mixing with the muon, there are no new one-loop contributions to Δa_μ besides those already present in the 2HDM. Still, BZ diagrams involving the new charged and neutral leptons must be taken into account (see Fig. 2). We have computed both contributions, and for Fig. 2(a), our result agrees with that of Ref. [15], doing the appropriate replacements according to the interactions shown in (9). Namely,[2]

$$\Delta a_\mu^{(a)} = \frac{\alpha m_\mu^2}{4\pi^3 v^2} \sum_{k,a} Q_a^2 \left[\mathrm{Re}(y_{aa}^k)\mathrm{Re}(y_\mu^k) \mathcal{F}^{(1)}\left(\frac{M_a^2}{M_k^2}\right) + \mathrm{Im}(y_{aa}^k)\mathrm{Im}(y_\mu^k) \mathcal{F}^{(2)}\left(\frac{M_a^2}{M_k^2}\right)\right], \quad (13)$$

[2]Our result for this diagram also agrees with that of Ref. [22].

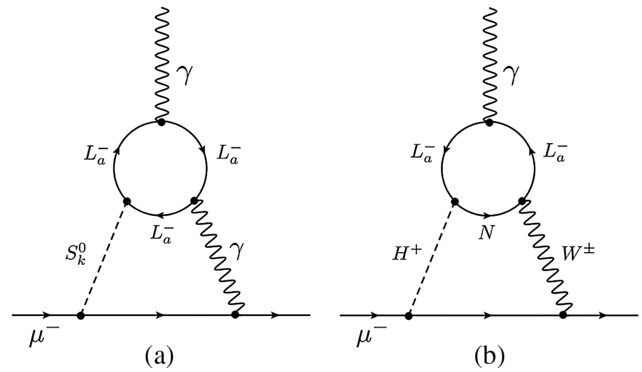

FIG. 2. BZ contributions to Δa_μ involving the new charged and neutral leptons L_a^- and N, respectively.

with

$$\mathcal{F}^{(1,2)}(r) = \frac{r}{2} \int_0^1 dx \frac{\mathcal{N}^{(1,2)}}{r - x(1-x)} \ln\left[\frac{r}{x(1-x)}\right], \quad (14)$$

and $\mathcal{N}^{(1)} = 2x(1-x) - 1$ and $\mathcal{N}^{(2)} = 1$. Regarding Fig. 2 (b), caution must be taken since, contrarily to what happens in the 2HDM, charged-current interactions are now left and right handed [see Eq. (8)]. We have computed the contribution from this diagram, the result being

$$\Delta a_\mu^{(b)} = \frac{\alpha m_\mu^2}{32\pi^3 s_W^2 v^2 (M_{H^+}^2 - M_W^2)} \sum_a \int_0^1 dx Q_a (1-x) \times \left[\mathcal{G}\left(\frac{M_N^2}{M_{H^+}^2}, \frac{M_a^2}{M_{H^+}^2}\right) - \mathcal{G}\left(\frac{M_N^2}{M_W^2}, \frac{M_a^2}{M_W^2}\right)\right] \times (\omega_L + \omega_R), \quad (15)$$

where the function $\mathcal{G}$ is

$$\mathcal{G}(r_1, r_2) = \frac{\ln\left[\frac{r_1 x + r_2(1-x)}{x(1-x)}\right]}{x(1-x) - r_1 x - r_2(1-x)}. \quad (16)$$

The term proportional to $\omega_{L(R)}$ takes the left (right)-handed component of the charged-current interactions with the new leptons, such that

$$\omega_L = \mathrm{Re}[(U_L)_{1a}^* \xi_a M_a (U_R)_{2a} \xi_\mu^*] M_a [(1-x) - (1-x)^2] - \mathrm{Re}[(U_L)_{1a}^* \xi_N M_N (U_L)_{2a} \xi_\mu^*] M_N x(1+x),$$
$$\omega_R = -\mathrm{Re}[(U_R)_{1a}^* \xi_N M_N (U_L)_{2a} \xi_\mu^*] M_a [(1-x) + (1-x)^2] + \mathrm{Re}[(U_R)_{1a}^* \xi_a M_a (U_R)_{2a} \xi_\mu^*] M_N x(1-x). \quad (17)$$

The first term is analogous to that for quarks presented in Ref. [15], while the second has the same structure as the general terms obtained in Ref. [49].

While fitting the experimental values of $(g-2)_\mu$ within the context of a 2HDM, we must make sure that all experimental and theoretical constraints of that model are satisfied. We have already mentioned B-physics constraints—particularly relevant for the type-II model, wherein $b \to s\gamma$ results force the charged Higgs mass to be very high—and bounded from below and unitarity bounds, which limit the values of the scalar potential's quartic couplings (see [8] for the explicit expressions). Equally important is to consider electroweak precision constraints, in the form of the Peskin-Takeuchi parameters S, T, and U [50]. Their explicit expression for the 2HDM can be found in numerous references {see, for instance, Eqs. (12) and (13) of [51]}, but if one considers vectorlike leptons present as well, their contributions to the T parameter must also be taken into account {see Eq. (4.5) of [22]};- the upshot of their inclusion in a 2HDM fit is that it usually is always possible for VLL masses M_1 and M_2 below ~500 GeV to find a vectorlike neutrino mass m_N to satisfy the current constraints on T. Indeed, these electroweak precision constraints, as is usual in the vanilla 2HDM, tend to reduce the splittings between the extra (scalar and VLL) masses in the theory.

With or without VLLs, LHC precision constraints on the properties of the 125 GeV h scalar are, with the exception of the diphoton branching ratio, trivially satisfied within the alignment limit, since h's tree-level couplings become identical to the SM Higgs when $\cos(\beta - \alpha) = 0$. The presence of charged scalars and two charged VLLs, however, contribute to the diphoton decay width. The decay amplitude $\mathcal{A}$ for $h \to \gamma\gamma$ includes therefore a contribution identical to the SM and another from the charged and VLL sector given by

$$\mathcal{A} = \mathcal{A}_{\rm SM} - \frac{\Lambda}{2m_{H^+}^2} A_0^H(\tau_+) + y_{11}^h A_{1/2}^H(\tau_1) + y_{22}^h A_{1/2}^H(\tau_2), \tag{18}$$

where $\tau_X = m_h^2/(4m_X^2)$, $\Lambda = 3m_h^2 - 2m_{H^+}^2 - 4m_{12}^2/s_{2\beta}$, and A_0^H, $A_{1/2}^H$ are the well-known scalar and fermionic form factors of the Higgs (see, for instance, [52]). Depending on the charged and VLL masses and couplings, their contributions to $h \to \gamma\gamma$ can actually cancel each other and will be less important for higher masses.

To illustrate the main features of the VLL models we are interested in, we focus on two benchmark cases (B_1 and B_2) within the type-II, type-X, and μSpec 2HDMs with heavy charged-lepton masses,

$$\begin{aligned} \mathrm{B}_1&: M_2 = 2.5M_1 = 250 \text{ GeV}, \\ \mathrm{B}_2&: M_2 = 1.5M_1 = 150 \text{ GeV}, \end{aligned} \tag{19}$$

taking $s_L(s_R) = 0.5(0.4)$. The neutral-lepton mass M_N turns out to be fixed since $M_N = |m_L| = |M_1 c_L c_R + M_2 s_L s_R|$, yielding $M_N \simeq 129(109)$ GeV for B_1 (B_2). The scalar masses m_{H^+} and m_H are the same as those considered in Fig. 1 (for μSpec, we adopt the same setting as for type X), and again, $\cos(\beta - \alpha) = 0$.[3] In Fig. 3, we show the $(m_A, \tan\beta)$ allowed regions in green and yellow (blue and red) for B_1 (B_2) at 1 and 2σ, respectively. Above the solid (dashed) horizontal line, $\lambda_{\rm max} \equiv \max\{|\lambda_L|, |\lambda_R|\} > 4\pi$ for B_1 (B_2).

The results show that in all cases the ranges of m_A can be substantially enlarged with respect to Fig. 1, while simultaneously shifting $\tan\beta$ to lower values. This effect is more pronounced for B_1 since M_2/M_1 is larger than for B_2 and so are the $L_2 L_2 S_k^0$ couplings [see Eqs. (10) and (11)]. The main drawback of this scenario is that the larger VLL coupling with the Higgs doublet Φ_1 is required to be close or above the perturbative limit, i.e., $\lambda_{\rm max} = 4\pi$. This can be easily understood noting that the coupling modifiers $\xi_{a,N}$ in Eq. (10) scale as $\lambda v \cos\beta/(\sqrt{2}M)$ (with respect to the pure 2HDM case), where λ and M stand for a generic VLL coupling and mass, respectively. Requiring that, at least, $\xi_{a,N} \simeq \tan\beta$ implies $\lambda v \cos\beta/(\sqrt{2}M) \simeq 1$. Thus, for $M \gtrsim 100$ GeV and large $\tan\beta$, λ must be large to avoid spoiling the $\tan\beta$ enhancement required to explain the Δa_μ discrepancy in 2HDMs. For the μSpec model, the effect of adding VLLs allows one to drastically lower $\tan\beta \gtrsim \mathcal{O}(1000)$ down to $\tan\beta \sim 40$–50 for $m_A \gtrsim 200$ GeV.

From an analysis of Fig. 3, we conclude that a strict requirement of perturbativity on the $\lambda_{L,R}$ couplings would provoke a drastic curtailment of the available parameter space; for the benchmarks presented, the μSpec model would have no available parameter space left, since the $1/2$-σ bands shown there always lie *above* the corresponding $\lambda_{\rm max} = 4\pi$ lines. For the type-II and type-X models, the only surviving region is the low pseudoscalar mass one (m_A smaller than roughly, respectively, 150 and 100 GeV for the second benchmark shown), albeit with larger masses than those obtained without VLLs. Nonetheless, the low masses for pseudoscalars obtained for the type-II model cannot be accommodated in that model, given that the $b \to s\gamma$ constraints imposed a lower bound on the charged mass of, generously, 600 GeV. A possible way to enlarge the allowed region would be to increase the number N of VLL generations, since *naïvely* one would expect that this limit would be improved by $\lambda_{L,R}/N$.

Since the benchmark analysis shows type X is favored for agreement with the muon anomaly, we have performed a general parameter space scan of the 2HDM for that model, considering extra scalars and VLLs (charged and neutral) to have masses ranging from 100 to 1000 GeV, as well as all possible values for the angles θ_L and θ_R. We only accepted combinations of parameters for which the model satisfies unitarity, boundedness from below, electroweak precision constraints, $b \to s\gamma$ constraints, and predicts a Higgs

[3]The impact on Δa_μ of considering $\cos(\beta - \alpha) \neq 0$ within the experimentally allowed limits is marginal. For instance, taking $\cos(\beta - \alpha) = 0.05$, the change in the results is almost imperceptible.

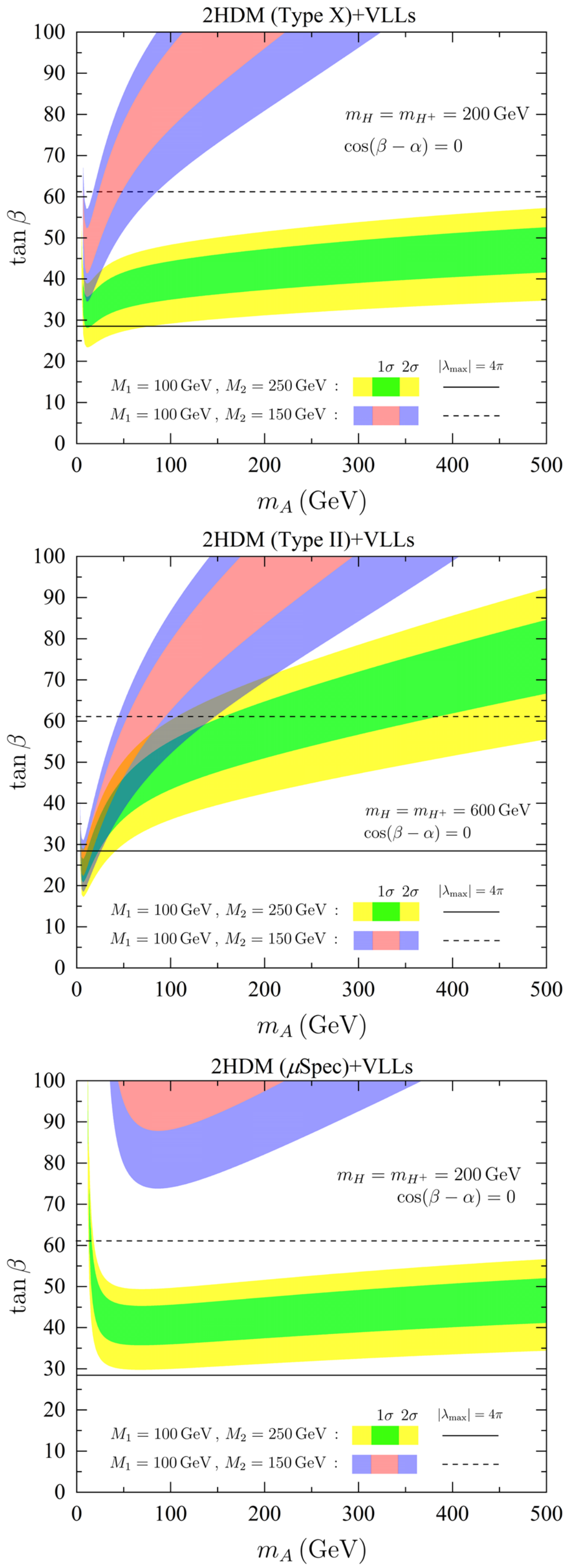

FIG. 3. $(m_A, \tan\beta)$ allowed regions for the type-X, type-II, and μSpec 2HDMs with VLLs (from top to bottom panel), taking the benchmarks B1 and B2 given in (19). The values $s_L = 0.5$, $s_R = 0.4$ are chosen. Above the solid (dashed) horizontal line, $\lambda_{\max} \equiv \max\{|\lambda_L|, |\lambda_R|\} > 4\pi$ for B_1 (B_2). See text for more details.

diphoton branching ratio within 10% of the SM values. Finally, since extra scalars are required to explain the $(g-2)_\mu$ anomaly, we need to verify whether current LHC searches would not have discovered those particles already. The B-decay constraints we mentioned take care of the charged Higgs; because we are working in the alignment limit, searches for the heavier CP-even scalar H in ZZ or WW are automatically satisfied; indeed, the channel which we need to worry the most would be searches for H and especially A decaying into tau pairs, having been produced via gluon fusion. Tau decays can be $\tan\beta$ enhanced and produce strong signal rates already excluded by LHC collaborations [53]. We verified that, after all other constraints had been taken into account, our scan included only a small fraction of points above current experimental limits on ditau searches for extra scalars and excluded those points from our $(g-2)_\mu$ calculations. The result of that scan shows that the type-X behavior shown in Fig. 3 can be reproduced by a wealth of different combinations of parameters; the values of allowed $\tan\beta$ are even more general, with, for instance, $\tan\beta \simeq 25$ permitted for $m_A = 400$ GeV. However, and also confirming the previous conclusions, although unitarity and perturbativity can be satisfied within the scalar sector, agreement with $(g-2)_\mu$ requires large VLL Yukawas; in particular, although λ_L can be found small, $|\lambda_R|$ is always found to be above ~20, and thus nonperturbative.

In [22], the authors work with a small mass difference between the charged-vectorlike masses and do not find any problems regarding perturbativity of the VLL Yukawas, which appears to contradict our results. Without taking into account all the above constraints, and working on that same region, we were able to reach that same result. However, an exhaustive scan as the one we have performed completely excludes such a region of the parameter space, and a small mass difference between M_1 and M_2 is not able to reproduce the $(g-2)_\mu$ anomaly. Finally, a word on limits on $\tan\beta$ stemming from tau and Z decays, as considered in [22]; although in that work mixing between VLLs and electrons is allowed and therefore a direct comparison is not practical, their Fig. 4 seems to show ample parameter space available for the case of a scalar spectrum without too large mass splittings, as is the case of our Fig. 3 or the result of eletroweak precision constraints in our general type-X scan. However, the point remains that the nonperturbativity found for the couplings $\lambda_{L,R}$ is a stronger limitation on the validity of this approach to fitting $g-2$ that the bounds stemming from tau and Z decays.

IV. CONCLUSIONS

We have studied the theory/experiment discrepancy of the muon's anomalous magnetic moment in the 2HDM. Without vectorlike leptons, the type-I and type-Y models cannot explain the discrepancy, and the type II requires light pseudoscalars that are in conflict with perturbativity, unitarity, and electroweak precision constraints. The type-X model can accommodate the discrepancy but require large values of $\tan\beta$ and also very light pseudoscalars. After revisiting some 2HDM results in light of the new result reported by the Muon g-2 Collaboration at Fermilab, we considered adding vectorlike leptons to the type-II, type-X, and muon-specific 2HDMs. We did not include any mixing between the vectorlike leptons and the muon. We have shown that the parameter space is substantially widened, and much larger values of the pseudoscalar mass are allowed. Here, $\tan\beta$ remains fairly large in all of these models. One problem of the VLL scenario analyzed in this work is that the VLL Yukawa couplings to the scalar doublet Φ_1 must be close to or above the perturbation theory limit. This can be alleviated by considering N families of vectorlike leptons instead of only one (very roughly, this would reduce the maximum values of these couplings by a factor of $1/N$). Alternatively, one can include VLL-muon mixing. This has been done in Refs. [17,21] in some detail, where it was also shown that the two-loop Barr-Zee diagrams are substantially less important than one-loop contributions and that smaller values of $\tan\beta$ can be accommodated. Finally, one can relax our assumption that a symmetry eliminates tree-level FCNC and consider the A2HDM mentioned in the Introduction; this would add additional parameters and might alleviate the problem of perturbativity.

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