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$N = 1$ supersymmetric $SU(12)_C \times SU(2)_L \times SU(2)_R$ models, $SU(4)_C \times SU(6)_L \times SU(2)_R$ models, and $SU(4)_C \times SU(2)_L \times SU(6)_R$ models from intersecting D6-branes

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For the first time we systematically discuss the $N = 1$ supersymmetric $SU(12)_C \times SU(2)_L \times SU(2)_R$ models, $SU(4)_C \times SU(6)_L \times SU(2)_R$ models, and $SU(4)_C \times SU(2)_L \times SU(6)_R$ models from the type IIA orientifolds on $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ with intersecting D6-branes. These gauge symmetries can be broken down to the Pati-Salam gauge symmetry $SU(4)_C \times SU(2)_L \times SU(2)_R$ via three $SU(12)_C/SU(6)_L/SU(6)_R$ adjoint representation Higgs fields, and further down to the Standard Model (SM) via the D-brane splitting and Higgs mechanism. We obtain three families of the SM fermions, and have the left-handed three-family SM fermion unification in the $SU(4)_C \times SU(6)_L \times SU(2)_R$ models, and the right-handed three-family SM fermion unification in the $SU(4)_C \times SU(2)_L \times SU(6)_R$ models. Utilizing mathematical analysis, we exclude the generalized $SU(12)_C \times SU(2)_L \times SU(2)_R$ models by requiring the conditions for constructing Minimal Supersymmetric Standard Model models. Moreover, the $SU(4)_C \times SU(6)_L \times SU(2)_R$ models and $SU(4)_C \times SU(2)_L \times SU(6)_R$ models are related by the left and right gauge symmetry exchanging, as well as a variation of type II T-duality. The hidden sector contains $USp(n)$ branes, which are parallel with the orientifold planes or their $\mathbb{Z}_2$ images and might break the supersymmetry via gaugino condensations.

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I. INTRODUCTION

Constructing the $N = 1$ supersymmetric Standard Models (SM) or SM from string theories has been the essential goal of string phenomenology. D-branes as boundaries of open strings plays an important role in phenomenologically interesting model building in type I, type IIA and type IIB string theories [1]. Conformal field theory provides the consistent constructions of four-dimensional supersymmetric $N = 1$ chiral models with non-Abelian gauge symmetry on type II orientifolds for the open string sectors. The chiral fermions on the world volume of the D-branes are located at orbifold singularities [2–8], and/or at the intersections of D-branes in the internal space [9] with a T-dual description in terms of magnetized D-branes as shown in [10,11]. Many nonsupersymmetric three-family SM-like models and generalized unified models have been constructed [12–25], within the intersecting D6-brane models on type IIA orientifolds [12–14]. These models typically suffer from the large Planck scale corrections at the loop level which results in the gauge hierarchy problem. A large number of the supersymmetric SM-like models and generalized unified models have been constructed [26–46], with the above problem solved. For a pedagogical introduction to phenomenologically interesting string models constructed with intersecting D-Branes, we refer to [47].

Along this direction, explicit models for the three-family $N = 1$ supersymmetric Pati-Salam models with type IIA orientifolds on $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ with intersecting D6-branes have been systematically constructed in [37]. The gauge symmetries all come from $U(n)$ branes, while the Pati-Salam gauge symmetries $SU(4)_C \times SU(2)_L \times SU(2)_R$ break down to $SU(3)_C \times SU(2)_L \times U(1)_{B-L} \times U(1)_{I_{3R}}$ via D6-brane splittings. It further breaks down to the SM via four-dimensional $N = 1$ supersymmetry via Higgs mechanism. This provides a way to realize the SM without any additional

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anomaly-free $U(1)$'s around the electroweak scale introduced. Note that there are also hidden sectors containing $USp(n)$ branes paralleling the orientifold planes or their $\mathbb{Z}_2$ images. These models normally are constructed with at least two confining gauge groups in the hidden sector, for which the gaugino condensation triggers supersymmetry breaking and (some) moduli stabilization. In particular, one of these type of models with a realistic phenomenology was found by Chen, Mayes, Nanopoulos and one of us (T. L.) in [42,44]. Its variations are also visited in [43]. Moreover, there are a few other potentially interesting constructions with possible massless vectorlike fields that might lead to the SM [37]. These vector fields do not arise from a $N = 2$ subsector, but can break the Pati-Salam gauge symmetry down to the SM or break the $U(1)_{B-L} \times U(1)_{I_{3R}}$ down to $U(1)_Y$. For such construction, large wrapping numbers are required because of the increased absolute values of the intersection numbers between $U(4)_C$ stack of D-branes and $U(2)_R$ stack(or its orientifold image). Therefore, more powerful scanning methods reaching large wrapping numbers are also requested in further investigations.

Employing our improved scanning methods, we systematically studied the three-family $N = 1$ supersymmetric Pati-Salam model building in type IIA orientifolds on $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ with intersecting D6-branes in which the $SU(4)_C \times SU(2)_L \times SU(2)_R$ gauge symmetries arise from $U(n)$ branes. In particular, we construct the new models with large wrapping numbers, and find that the approximate gauge coupling unification can be achieved at the string scale. The Pati-Salam gauge symmetries $SU(4)_C \times SU(2)_L \times SU(2)_R$ therein can be broken down to the SM via D-brane splitting as well as D- and F-flatness preserving the Higgs mechanism. The hidden sector contains $USp(n)$ branes with n equal to 4 or 2, which are parallel with the orientifold planes or their $\mathbb{Z}_2$ images. We find that the type II T duality in the previous study [37] is not an equivalent relation in Pati-Salam model building as most of the models are not invariant under $SU(2)_L$ and $SU(2)_R$ exchange. As follows, by swapping the b and c stacks of D6-branes, gauge couplings can be redefined and refine the gauge unification becomes possible. We systematically construct explicit new models with three families, which usually do not have gauge coupling unification at the string scale. We, for the first time, construct the Pati-Salam model with one large wrapping number reaching 5. In particular, we find that these models carry more refined gauge couplings, and with better approximate gauge coupling unification. Using the dimension reduction method "Latent Semantic Analysis", we show that the three-family $N = 1$ supersymmetric Pati-Salam models gather on islands, where more interesting models can be expected.

Distinct from the scanning methods we employed in Ref. [48], by explicit solving the conditions of the generalized version of Pati-Salam models, we for the first time systematically discuss the $N = 1$ supersymmetric $SU(12)_C \times SU(2)_L \times SU(2)_R$ models, $SU(4)_C \times SU(6)_L \times SU(2)_R$ models, and $SU(4)_C \times SU(2)_L \times SU(6)_R$ models from the type IIA orientifolds on $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ with intersecting D6-branes. These gauge symmetries can be broken down to the Pati-Salam gauge symmetry $SU(4)_C \times SU(2)_L \times SU(2)_R$ via three $SU(12)_C/SU(6)_L/SU(6)_R$ adjoint representation Higgs fields, and further down to the SM via the D-brane splitting and the Higgs mechanism. Also, we obtain three families of the SM fermions, and have the left-handed three-family SM fermion unification in the $SU(4)_C \times SU(6)_L \times SU(2)_R$ models, and the right-handed three-family SM fermion unification in the $SU(4)_C \times SU(2)_L \times SU(6)_R$ models. Moreover, the $SU(4)_C \times SU(6)_L \times SU(2)_R$ models and $SU(4)_C \times SU(2)_L \times SU(6)_R$ models are related by the left and right gauge symmetry exchanging, as well as a variation of type II T-duality n Ref. [48], but the $U(1)_Y$ gauge coupling are different. Furthermore, the hidden sector contains $USp(n)$ branes, which are parallel with the orientifold planes or their $\mathbb{Z}_2$ images and might break the supersymmetry via gaugino condensations.

This paper is organized as follows: We will first review the basic rules for supersymmetric intersecting D6-brane model building on type IIA $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifolds in Sec. II, as well as the tadpole cancellation conditions and the conditions for D6-brane configurations which preserve four-dimensional $N = 1$ supersymmetry in Sec. III. We present the generalized supersymmetric Pati-Salam model building in Sec. IV. We discuss the preliminary phenomenological consequences in Sec. V. The discussions and conclusion are in Sec. VI.

II. $\mathbf{T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)}$ ORIENTIFOLDS WITH INTERSECTING D6-BRANES

Before we construct the generalized verison of Pati-Salam models, let us briefly review the basic rules to construct the supersymmetric models on type IIA $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifolds with D6-branes intersecting at generic angles to obtain the massless open string state spectra as in [27,29]. In this construction, we consider the six-torus T^6 factorized as three two-tori $T^6 = T^2 \times T^2 \times T^2$ with complex coordinates for the ith two-torus to be z_i, $i = 1, 2, 3$ respectively. The θ and ω generators for the orbifold group $\mathbb{Z}_2 \times \mathbb{Z}_2$ are associated with the twist vectors $(1/2, -1/2, 0)$ and $(0, 1/2, -1/2)$ respectively. They act on the complex coordinates z_i in the form of

$$\begin{aligned} \theta &: (z_1, z_2, z_3) \to (-z_1, -z_2, z_3), \\ \omega &: (z_1, z_2, z_3) \to (z_1, -z_2, -z_3). \end{aligned} \tag{1}$$

Furthermore, we implement the orientifold projection by gauging the ΩR symmetry. In which, Ω is world-sheet parity, and R acts on the complex coordinates as

$$R\colon\ (z_1, z_2, z_3) \to (\bar{z}_1, \bar{z}_2, \bar{z}_3). \tag{2}$$

In total, there are four kinds of orientifold 6-planes (O6-planes) for the actions of ΩR, $\Omega R\theta$, $\Omega R\omega$, and $\Omega R\theta\omega$ respectively. In addition, three stacks of N_a D6-branes wrapping on the factorized three-cycles are introduced to cancel the Ramond-Ramond (RR) charges of these O6-planes. As discussed in [13,27,29,44], these two-tori have two kinds of complex structures: rectangular or tilted, which are consistent with the orientifold projection. The homology classes of the three cycles which are wrapped by the D6-brane stack take the form $n_a^i[a_i] + m_a^i[b_i]$ and $n_a^i[a_i'] + m_a^i[b_i]$ for the rectangular and tilted tori respectively, with $[a_i'] = [a_i] + \frac{1}{2}[b_i]$. Therefore, a generic one cycle are labeled by (n_a^i, l_a^i) in terms of the wrapping numbers, $l_a^i \equiv m_a^i$ and $l_a^i \equiv 2\tilde{m}_a^i = 2m_a^i + n_a^i$ for a rectangular and tilted two-torus, respectively. Moreover, $l_a^i - n_a^i$ is even for tilted two-tori.

We note the wrapping number for stack a of D6-branes along the cycle to be (n_a^i, l_a^i), and their ΩR images a' stack of N_a D6-branes have wrapping numbers $(n_a^i, -l_a^i)$. The homology three cycles for stack a of D6-branes and its orientifold image a' takes the form of

$$[\Pi_a] = \prod_{i=1}^{3}(n_a^i[a_i] + 2^{-\beta_i} l_a^i[b_i]),$$
$$[\Pi_{a'}] = \prod_{i=1}^{3}(n_a^i[a_i] - 2^{-\beta_i} l_a^i[b_i]), \tag{3}$$

where $\beta_i = 0$ for the rectangular and $\beta_i = 1$ for the tilted ith two-torus. The homology three-cycles wrapped by the four O6-planes are in terms of

$$\Omega R\colon\ [\Pi_{\Omega R}] = 2^3[a_1] \times [a_2] \times [a_3], \tag{4}$$

$$\Omega R\omega\colon\ [\Pi_{\Omega R\omega}] = -2^{3-\beta_2-\beta_3}[a_1] \times [b_2] \times [b_3], \tag{5}$$

$$\Omega R\theta\omega\colon\ [\Pi_{\Omega R\theta\omega}] = -2^{3-\beta_1-\beta_3}[b_1] \times [a_2] \times [b_3], \tag{6}$$

$$\Omega R\theta\colon\ [\Pi_{\Omega R}] = -2^{3-\beta_1-\beta_2}[b_1] \times [b_2] \times [a_3]. \tag{7}$$

The intersection numbers are related with the wrapping numbers in the term of

$$I_{ab} = [\Pi_a][\Pi_b] = 2^{-k}\prod_{i=1}^{3}(n_a^i l_b^i - n_b^i l_a^i), \tag{8}$$

$$I_{ab'} = [\Pi_a][\Pi_{b'}] = -2^{-k}\prod_{i=1}^{3}(n_a^i l_b^i + n_b^i l_a^i), \tag{9}$$

$$I_{aa'} = [\Pi_a][\Pi_{a'}] = -2^{3-k}\prod_{i=1}^{3}(n_a^i l_a^i), \tag{10}$$

$$\begin{aligned} I_{aO6} &= [\Pi_a][\Pi_{O6}] \\ &= 2^{3-k}(-l_a^1 l_a^2 l_a^3 + l_a^1 n_a^2 n_a^3 + n_a^1 l_a^2 n_a^3 + n_a^1 n_a^2 l_a^3), \end{aligned} \tag{11}$$

TABLE I. General massless particle spectrum for intersecting D6-branes at generic angles.

Sector	Representation
aa	$U(N_a/2)$ vector multiplet 3 adjoint chiral multiplets
$ab + ba$	I_{ab} $(\Box_a, \bar{\Box}_b)$ fermions
$ab' + b'a$	$I_{ab'}$ $(\Box_a, \Box_b)$ fermions
$aa' + a'a$	$\frac{1}{2}(I_{aa'} - \frac{1}{2}I_{a,O6})$ $\Box\!\Box$fermions
	$\frac{1}{2}(I_{aa'} + \frac{1}{2}I_{a,O6})$ $\begin{smallmatrix}\Box\\\Box\end{smallmatrix}$ fermions

where $k = \beta_1 + \beta_2 + \beta_3$ is the total number of the tilted two-tori, while $[\Pi_{O6}] = [\Pi_{\Omega R}] + [\Pi_{\Omega R\omega}] + [\Pi_{\Omega R\theta\omega}] + [\Pi_{\Omega R\theta}]$ is the sum of four O6-plane homology three-cycles.

On the model building side, the massless particle spectrum for intersecting D6-branes at general angles can be expressed in terms of the intersection numbers shown in Table I. The representations refer to $U(N_a/2)$, the gauge symmetry results from $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold projection [27]. For Pati-Salam models with type IIA orientifolds on $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$, when the intersecting D6-branes are of numbers $N_a = 8$, $N_b = 4$, $N_c = 4$, this gives $U(N_x/2)$ gauge groups for $x = a$, b, c. The chiral supermultiplets contain both scalars and fermions in the supersymmetric constructions, while the positive intersection numbers refer to the left-handed chiral supermultiplets. The two main constraints on the four-dimensional $N = 1$ supersymmetric model built from type IIA orientifolds with intersecting D6-branes are: RR tadpole cancellation conditions and $N = 1$ supersymmetry preservation in four dimensions, which we will discuss in the following section with our generalized construction.

III. GENERALIZED D6-BRANE CONSTRUCTIONS

In [48], we observe that new models with three generations of particles can also be constructed when n_x^i and l_x^i have the common factor 3, while x refers to a, b, c stacks of branes and i refers to 1,2,3 for different wrapping directions. For example, when n_a^1 and l_a^1 have common factor 3, the intersection numbers consequently will also have cofactor 3 for Eqs. (8) and (9), and cofactor 9 for Eq. (10). Dividing by this co-factor 3 for the stack a of D-brane, the generalized gauge symmetry resulting from the D6-branes becomes $SU(12)_C \times SU(2)_L \times SU(2)_R$. The a-stack brane's gauge $U(12)$ can be broken down to $U(4)$ with proper orientations, i.e., by taking vacuum expectation values of an adjoint Higgs field with respect to the Cartan generators of U(12). When the wrapping number n_b^1, l_b^1 for the b stack of the brane have common factor 3, dividing out the cofactor 3 results in the gauge symmetry becoming $SU(4)_C \times SU(6)_L \times SU(2)_R$. To break it down to the Pati-Salam symmetry $SU(4)_C \times SU(2)_L \times SU(2)_R$, similar orientations needed to be performed to break the

$U(6)_L$ gauge to $U(2)_L$; details will be given in Sec. IV. Also, we obtain three families of the SM fermions, and have the left-handed three-family SM fermion unification in the $SU(4)_C \times SU(6)_L \times SU(2)_R$ models, and the right-handed three-family SM fermion unification in the $SU(4)_C \times SU(2)_L \times SU(6)_R$ models. As follows, the Pati-Salam gauge symmetries $SU(4)_C \times SU(2)_L \times SU(2)_R$ can be broken down to SM via D-brane splitting as well as D- and F-flatness preserving the Higgs mechanism.

We will present the two main constraints on the four-dimensional $N = 1$ supersymmetric model building from type IIA orientifolds with intersecting D6-branes, namely the RR tadpole cancellation conditions, and $N = 1$ supersymmetry preservation in four dimensions with our generalized gauge modifications.

A. The RR tadpole cancellation conditions

In the standard Pati-Salam models, the tadpole cancellation conditions lead to the $SU(N_a)^3$ cubic non-Abelian anomaly cancellation as shown in [15,16,27], while the cancellation of $U(1)$ mixed gauge and gravitational anomaly (or $[SU(N_a)]^2 U(1)$ gauge anomaly) can be achieved by the Green-Schwarz mechanism mediated by the untwisted RR fields as shown in [15,16,27]. The D6-branes and the orientifold O6-planes are the sources of RR fields and restricted by the Gauss law in a compact space. The sum of the RR charges from D6-branes must cancel with those from the O6-planes due to the conservation of the RR field flux lines. The conditions for RR tadpole cancellations take the form of

$$\sum_a N_a[\Pi_a] + \sum_a N_a[\Pi_{a'}] - 4[\Pi_{O6}] = 0, \qquad (12)$$

where the last term arises from the O6-planes have −4 RR charges in the D6-brane charge unit. To simplify the discussion of the following tadpole cancellation, we define the following products of wrapping numbers as

$$\begin{aligned} A_a &\equiv -n_a^1 n_a^2 n_a^3, & B_a &\equiv n_a^1 l_a^2 l_a^3, & C_a &\equiv l_a^1 n_a^2 l_a^3, & D_a &\equiv l_a^1 l_a^2 n_a^3, \\ \tilde{A}_a &\equiv -l_a^1 l_a^2 l_a^3, & \tilde{B}_a &\equiv l_a^1 n_a^2 n_a^3, & \tilde{C}_a &\equiv n_a^1 l_a^2 n_a^3, & \tilde{D}_a &\equiv n_a^1 n_a^2 l_a^3. \end{aligned} \qquad (13)$$

In order to cancel the RR tadpoles, D6-branes wrapping cycles along the orientifold planes are introduced as the so-called "filler branes". This contributes to the RR tadpole cancellation conditions, and trivially satisfy the four-dimensional $N = 1$ supersymmetry conditions. They are chosen such that the tadpole conditions are satisfied in the manner of

$$\begin{aligned} &-2^k N^{(1)} + \sum_a N_a A_a = -2^k N^{(2)} + \sum_a N_a B_a \\ &= -2^k N^{(3)} + \sum_a N_a C_a = -2^k N^{(4)} + \sum_a N_a D_a = -16, \end{aligned} \qquad (14)$$

where $2N^{(i)}$ is the number of filler branes wrapping along the ith O6-plane. The filler branes representing the USp group, carry the wrapping numbers as one of the O6-planes shown in Table II. The filler branes with nonzero A, B, C or D refer to the A-, B-, C- or D-type USp group, respectively.

TABLE II. The wrapping numbers for four O6-planes.

Orientifold Action	O6-Plane	$(n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$
ΩR	1	$(2^{\beta_1}, 0) \times (2^{\beta_2}, 0) \times (2^{\beta_3}, 0)$
$\Omega R\omega$	2	$(2^{\beta_1}, 0) \times (0, -2^{\beta_2}) \times (0, 2^{\beta_3})$
$\Omega R\theta\omega$	3	$(0, -2^{\beta_1}) \times (2^{\beta_2}, 0) \times (0, 2^{\beta_3})$
$\Omega R\theta$	4	$(0, -2^{\beta_1}) \times (0, 2^{\beta_2}) \times (2^{\beta_3}, 0)$

Note that for our generalized version of Pati-Salam models, the wrapping numbers (n_x^i, l_x^i) wrap three times for one stack of D6-branes in one direction compared to the standard Pati-Salam model. Therefore, the relevant terms in the tadpole cancellation condition as shown in Eqs. (12) and (14) will also get rescaled. To be precise, when the wrapping numbers (n_x^i, l_x^i) wrap three times, the terms in Eq. (13) will be rescaled accordingly. And since the number of brane stacks N_a, N_b, N_c appear together with the wrapping numbers, this corresponds to absorbing a factor of 3-form the wrapping numbers into the number of D6-brane stacks. Although N-stacks of brane wrapping $3l$ times and $3N$-stacks of brane wrapping l times appear same in the brane picture, it represents gauge construction from $SU(4)$ to $SU(12)$ or $SU(2)$ to $SU(6)$. It is obvious that when the first two terms in Eq. (12) got rescaled with 3 factor, to satisfy the tadpole cancellation conditions, less number of orientifold planes are expected. This will be confirmed with our precise analysis to obtain the generalized Pati-Salam models in the next section. We will discuss their phenomenology aspects after the models are presented.

B. Conditions for four-dimensional $N = 1$ supersymmetric D6-brane

For the four-dimensional $N = 1$ supersymmetric models, the 1/4 supercharges are required to be preserved from the ten-dimensional type I T-dual. Namely, under the orientation projection of the intersecting D6-branes and the

$\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold projection on the background manifold these $1/4$ supercharges have the same value. It was shown in [9] that the four-dimensional $N = 1$ supersymmetry can be preserved after the orientation projection if and only if the rotation angle of any D6-brane with respect to the orientifold plane is an element of $SU(3)$. Namely, $\theta_1 + \theta_2 + \theta_3 = 0 \bmod 2\pi$, where θ_i is the angle between the $D6$-brane and the orientifold-plane in the ith two-torus. The four-dimensional N = 1 supersymmetry will automatically survive the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold projection [29], and the SUSY conditions can therefore be written as

$$x_A\tilde{A}_a + x_B\tilde{B}_a + x_C\tilde{C}_a + x_D\tilde{D}_a = 0,$$
$$A_a/x_A + B_a/x_B + C_a/x_C + D_a/x_D < 0, \qquad (15)$$

where $x_A = \lambda$, $x_B = \lambda 2^{\beta_2+\beta_3}/\chi_2\chi_3$, $x_C = \lambda 2^{\beta_1+\beta_3}/\chi_1\chi_3$, $x_D = \lambda 2^{\beta_1+\beta_2}/\chi_1\chi_2$, in which $\chi_i = R_i^2/R_i^1$ represent the complex structure moduli for the ith two-torus. Moreover, the positive parameter λ is introduced to put all the variables A, B, C, D on an equal footing. All the possible D6-brane configurations preserving four-dimensional $N = 1$ supersymmetry can be classified into three types:

1. Filler brane with the same wrapping numbers as one of the O6-planes from Table II. The gauge symmetry reveals to be USp group. When there is only one of the wrapping number products A, B, C or D has nonzero and negative value and we refer to the USp group as A-, B-, C- or D-type USp group accordingly.
2. When there is one zero wrapping number, two negative and two zero values in A, B, C and D we refer to it as the Z-type D6-brane.
3. When there are three negative value and one positive value in A, B, C and D we refer to this case as NZ-type D6-brane. Depending on which one is positive, we note the NZ-type branes as A-, B-, C- and D-type NZ branes. Each type has two forms of wrapping numbers which are noted as follows:

$$A1\colon (-,-)\times(+,+)\times(+,+), \qquad A2\colon (-,+)\times(-,+)\times(-,+), \qquad (16)$$

$$B1\colon (+,-)\times(+,+)\times(+,+), \qquad B2\colon (+,+)\times(-,+)\times(-,+), \qquad (17)$$

$$C1\colon (+,+)\times(+,-)\times(+,+), \qquad C2\colon (-,+)\times(+,+)\times(-,+), \qquad (18)$$

$$D1\colon (+,+)\times(+,+)\times(+,-), \qquad D2\colon (-,+)\times(-,+)\times(+,+). \qquad (19)$$

For our generalized construction, the wrapping number (n_x^i, l_x^i) will rescale the relevant terms in Eq. (15), and thus the supersymmetry condition will need to be checked in the scaled manner. Moreover, although T-duality is not the focus of our work, we would like to note that if the three two-tori of the two models and their corresponding wrapping numbers for all the D6-branes are correlated by an element of the permutation group S_3 acting on three two-tori, we consider these two models to be equivalent. This applies to our generalized Pati-Salam models in the same way. For more details about T-duality, D6-brane sign equivalent principle and the equivalence of dual models, we refer to [37,48].

IV. GENERALIZED SUPERSYMMETRIC PATI-SALAM MODEL BUILDING

A. Construction of generalized supersymmetric Pati-Salam models

In the standard Pati-Salam models, to construct the SM or SM-like models from the intersecting D6-brane scenarios (besides the $U(3)_C$ and $U(2)_L$ gauge symmetries from stacks of D6-branes) we construct two extra $U(1)$ gauge groups for both supersymmetric and nonsupersymmetric models to have the correct quantum number for right-handed charged leptons as shown in [16,27–29]. One $U(1)_L$ represents the lepton number symmetry, while the other $U(1)_{I_{3R}}$ behaves as the third component of right-handed weak isospin. The hypercharge is then given by

$$Q_Y = Q_{I_{3R}} + \frac{Q_B - Q_L}{2}, \qquad (20)$$

where $U(1)_B$ arises from the overall $U(1)$ in $U(3)_C$. The $U(1)$ gauge symmetry (coming from a non-Abelian symmetry) is anomaly free and its gauge field is massless. Thus, to stop the gauge field of $U(1)_{I_{3R}}$ from obtaining a mass via $B \wedge F$ couplings, $U(1)_{I_{3R}}$ can only arise from the non-Abelian part of $U(2)_R$ or USp gauge symmetry. Similarly, to obtain an anomaly-free $U(1)_{B-L}$ gauge symmetry, the $U(1)_L$ gauge symmetry should come from a non-Abelian group as well. Note that the $U(1)_L$ stack should be parallel to the $U(3)_C$ stack on at least one two-torus (we can obtain it by splitting one $U(4)$ stack of branes into $U(1)_L$ and $U(3)_C$ stacks). The $U(3)_C$ gauge symmetry is also generated in the mean time. When $U(1)_{I_{3R}}$ gauge arises from the stack of D6-branes on top of orientifold, there exist at least 8 pairs of SM Higgs doublets, and two extra anomaly-free $U(1)$ gauge symmetries from the USp group [27,28]. These $U(1)$ gauge symmetries could be spontaneously broken by the Higgs mechanism via the scalar components of the chiral superfields with the quantum numbers of the right-handed neutrinos. However, they also break the D-flatness conditions and thus break supersymmetry. Therefore, the symmetry breaking scale is around the electroweak scale. We typically do not have any other candidates, which can preserve the D-flatness and F-flatness conditions, and break these gauge symmetries at an intermediate scale.

Distinguished from the model building in Ref. [37,48], we for the first time construct a generalized version of Pati-Salam models with three times of wrapping for one stack of D6-branes construction, then realize standard Pati-Salam models via the Higgs mechanism. As follows, we can concentrate on the deriving Pati-Salam models in which $U(1)_{I_{3R}}$ arises from the $U(2)_R$ symmetry as usual. As it is difficult to construct phenomenology interesting models with $SU(2)_L$ from the D6-branes on the top of O6-plane [37], we instead discuss the gauge symmetries $SU(12)_C \times SU(2)_L \times SU(2)_R$, $SU(4)_C \times SU(6)_L \times SU(2)_R$, $SU(4)_C \times SU(2)_L \times SU(6)_R$ from three time of wrapping respectively on a, b or c stacks of D6-branes, which are not on the top of orientifold planes in a generalized construction. Namely, we introduce three stacks of D6-branes, a, b, c with D6-brane numbers 24, 4, 4; 8, 12, 4; and 8, 4, 12 which respectively give us the gauge symmetries as above. Then it can break down to three stacks of D6-branes, a, b, c with D6-brane numbers 8,4,4 with gauge symmetry $SU(4)_C \times SU(2)_L \times SU(2)_R$, and break the resulting Pati-Salam gauge symmetry down to $SU(3)_C \times SU(2)_L \times U(1)_{B-L} \times U(1)_{I_{3R}}$ from D6-brane splitting. The SM gauge symmetry can be realized via the Higgs mechanism with Higgs particles from a $N = 2$ subsector as for standard Pati-Salam models [37].

Moreover, the gauge anomalies from three $U(1)$s are cancelled by the generalized Green-Schwarz mechanism, and these $U(1)$ gauge fields obtain masses via the linear $B \wedge F$ couplings. Furthermore, to obtain three families of the SM fermions, we require the intersection numbers to satisfy

$$\begin{aligned} I_{ab} + I_{ab'} &= 3, \\ I_{ac} = -3, \qquad I_{ac'} &= 0, \end{aligned} \tag{21}$$

where the conditions $I_{ab} + I_{ab'} = 3$ and $I_{ac} = -3$ give us three generations of the SM fermions, whose quantum numbers under $SU(4)_C \times SU(2)_L \times SU(2)_R$ (for example with gauge symmetries) are $(\mathbf{4}, \mathbf{2}, \mathbf{1})$ and $(\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2})$ in our generalized construction. Moreover, in our generalized construction, to have three families of the SM fermions for whose quantum numbers under $SU(4)_C \times SU(2)_L \times SU(6)_R$ are $(\mathbf{4}, \mathbf{2}, \mathbf{1})$ and $(\bar{\mathbf{4}}, \mathbf{1}, \mathbf{6})$, we require the intersection numbers to satisfy

$$\begin{aligned} I_{ab} + I_{ab'} &= 3, \\ I_{ac} = -1, \qquad I_{ac'} &= 0, \end{aligned} \tag{22}$$

where the conditions $I_{ab} + I_{ab'} = 3$ and $I_{ac} = -1$ give us three generations of the SM fermions. Similarly, to have three families of the SM fermions for whose quantum numbers under $SU(4)_C \times SU(6)_L \times SU(2)_R$ are $(\mathbf{4}, \mathbf{6}, \mathbf{1})$ and $(\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2})$, we require the intersection numbers to satisfy

$$\begin{aligned} I_{ab} + I_{ab'} &= 1, \\ I_{ac} = -3, \qquad I_{ac'} &= 0, \end{aligned} \tag{23}$$

where the conditions $I_{ab} + I_{ab'} = 1$ and $I_{ac} = -3$ give us three generations of the SM fermions. To have three families of the SM fermions for whose quantum numbers under $SU(12)_C \times SU(2)_L \times SU(2)_R$ are $(\mathbf{12}, \mathbf{2}, \mathbf{1})$ and $(\overline{\mathbf{12}}, \mathbf{1}, \mathbf{2})$, we require the intersection numbers to satisfy

$$\begin{aligned} I_{ab} + I_{ab'} &= 1, \\ I_{ac} = -1, \qquad I_{ac'} &= 0, \end{aligned} \tag{24}$$

where the conditions $I_{ab} + I_{ab'} = 1$ and $I_{ac} = -1$ give us three generations of the SM fermions.

Since the intersection numbers are in terms of the wrapping numbers as shown in Eqs. (8), (9), (10), and our generalized Pati-Salam models are involved with common factor 3 in (n_x^i, l_x^i), it is expected that there exists much less generalized Pati-Salam models with three families of SM fermions than the standard Pati-Salam models. However, in our model building, the common factor is 3, rather than another number, it is natural to be understood because of the three family conditions have factor 3 also. This discussion for model building with a three family of SM fermions not only applies to the generalized gauge symmetries $SU(12)_C \times SU(2)_L \times SU(2)_R$, but also applies to the generalized gauge symmetries $SU(4)_C \times SU(6)_L \times SU(2)_R$ and $SU(4)_C \times SU(2)_L \times SU(6)_R$. Furthermore, a more concrete influence of the generalized factor 3 to the three families of SM fermions condition can be found in the next section with concrete models presented.

Similarly, as for the standard D-brane construction, to satisfy the $I_{ac'} = 0$ condition, the stack a D6-branes are constructed to be parallel to the orientifold (ΩR) image c' of the c stack of D6-branes along at least one two-torus. We choose this to be the third two-torus in our convention and the open strings stretch between the a and c' stacks of D6-branes. When the minimal distance square $Z^2_{(ac')}$ (in $1/M_s$ units) between these two stacks on the third two-torus is small, the minimal length squared of the stretched string is small. The light scalars with squared-masses $Z^2_{(ab')}/(4\pi^2\alpha')$ arise from the Neveu-Schwarz sector, while the light fermions with the same masses arise from R sector as discussed in [15,16,36]. These scalars and fermions form four-dimensional $N = 2$ hypermultiplets. One obtains $I^{(2)}_{ac'}$

number of vectorlike pairs for the chiral superfields with quantum numbers $(\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2})$ and $(\mathbf{4}, \mathbf{1}, \mathbf{2})$. These vectorlike particles contribute as the Higgs fields breaking the Pati-Salam gauge symmetry down to the SM gauge symmetry, with four-dimensional $N = 1$ supersymmetry preserved. In particular, these fields are massless under the condition $Z^2_{(ac')} = 0$. Note that the model with intersection numbers $I_{ac} = 0$ and $I_{ac'} = -3$ are equivalent to the models with $I_{ac} = -3$ and $I_{ac'} = 0$ under the symmetry transformation $c \leftrightarrow c'$.

From the phenomenology aspect, we now briefly review the procedures to break our generalized Pati-Salam gauge symmetry to the SM via realizing the Pati-Salam models. Firstly, we take models with gauge symmetry $U(4) \times U(6)_L \times U(2)_R \times USp(2)$ and $U(4) \times U(2)_L \times U(6)_R \times USp(2)$ as an example to discuss using the Higgs mechanism to break the $U(6)$ gauge to $U(2)$ and thus to discuss the resulting Pati-Salam models' phenomenology. Consider a $U(6)$ gauge theory with a scalar field in the adjoint representation. By taking proper orientations which commute with the $U(6)$ generators, we can break $U(6)$ spontaneously to $U(2) \times U(2) \times U(2)$, and then to $U(2) \times U(2)$ and in the end to $U(2)$. In the following, we will show the orientation matrix and the masses of the massive bosons. Firstly, we take the orientation for the $U(6)$ scalar field to act on the vacuum expectation value Φ_0 as

$$\mathbf{\Phi}_0 = \phi_{\mathbf{U(6)}} \cdot \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 0 & 0 & -3 \end{pmatrix}, \qquad (25)$$

we found that the $U(6)$ gauge symmetry spontaneously breaks to $U(2) \times U(2) \times U(2)$ by checking the commutator of the orientation and the $U(6)$ generators. As follows, to simplify the following discussion, we construct the $U(2) \times U(2) \times U(2)$ generators with the Pauli matrix for each $U(2)$ and apply the orientation matrix in the manner of

$$\mathbf{\Phi}_0 = \phi_{\mathbf{U(2) \times U(2) \times U(2)}} \cdot \begin{pmatrix} 0 & 0 & V_{12} & 0 & 0 & 0 \\ 0 & 0 & 0 & V_{12} & 0 & 0 \\ V_{12} & 0 & 0 & 0 & 0 & 0 \\ 0 & V_{12} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \qquad (26)$$

we found that the above expectation value breaks $U(2) \times U(2) \times U(2)$ to $U(2) \times U(2)$ and leaves the gauge bosons corresponding to the $U(2) \times U(2)$ generators massless. By taking a third orientation to the $U(2) \times U(2)$ gauge field

$$\mathbf{\Phi}_0 = \phi_{\mathbf{U(2) \times U(2)}} \cdot \begin{pmatrix} 0 & 0 & 0 & 0 & V_{13} & 0 \\ 0 & 0 & 0 & 0 & 0 & V_{13} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ V_{13} & 0 & 0 & 0 & 0 & 0 \\ 0 & V_{13} & 0 & 0 & 0 & 0 \end{pmatrix}, \qquad (27)$$

we break the $U(2) \times U(2)$ gauge symmetry to the $U(2)$ and acquire the masses

$$m^2 = (4g|\phi|)^2 \left(V_{12}^2 + V_{13}^2 \pm \sqrt{V_{12}^4 - V_{12}^2 V_{13}^2 + V_{13}^4} \right). \qquad (28)$$

For the models with gauge symmetry $SU(12)_C \times SU(2)_L \times SU(2)_R$, the Higgs mechanism for breaking $U(12) \rightarrow U(4) \times U(4) \times U(4) \rightarrow U(4) \times U(4) \rightarrow U(4)$ will follow the same procedure also. By splitting the a stack of D6-branes into a_1 and a_2 stacks with 6 and 2 D6-branes, the $U(4)_C$ gauge symmetry breaks in to $U(3)_C \times U(1)$ as shown in [44]. The gauge fields and three chiral multiplets in the adjoint representation of $SU(4)_C$ will be broken down to the adjoint representations of $SU(3)_C$ as well as the gauge field and three singlets of $U(1)_{B-L}$. We note the number of symmetric representations for $SU(4)_C$ as $n^a_{\Box\Box}$, while the antisymmetric representations noted as $n^a_{\begin{smallmatrix}\Box\\ \Box\end{smallmatrix}}$. Similar convention applies to $SU(3)_C$, $SU(2)_L$, and $SU(2)_R$. These chiral multiplets for $SU(4)_C$ are broken down to the $n^a_{\Box\Box}$ and $n^a_{\begin{smallmatrix}\Box\\ \Box\end{smallmatrix}}$ chiral multiplets in symmetric and antisymmetric representations for $SU(3)_C$, and $n^a_{\Box\Box}$ chiral multiplets with $U(1)_{B-L}$ charge ± 2. Moreover, we have $I_{a_1 a_2'}$ new fields with quantum number $(\mathbf{3}, \mathbf{-1})$ under $SU(3)_C \times U(1)_{B-L}$ from the open strings at the intersections of a_1 and a_2' stacks of D6-branes, while the other particles spectrum stay the same. The anomaly free gauge symmetries $SU(3)_C \times U(1)_{B-L}$ arise from a_1 and a_2 stacks of D6-branes as the $SU(4)_C$ subgroup. To break the $U(2)_R$ gauge symmetry, we split the c stack of D6-branes into c_1 and c_2 stacks, and each with two D6-branes. The gauge fields and three chiral multiplets in the adjoint representation of $SU(2)_R$ break down to the gauge field and three singlets of $U(1)_{I_{3R}}$. The $n^c_{\Box\Box}$ chiral multiplets in the symmetric representation of $SU(2)_R$ break down to the $n^c_{\square\square}$ chiral multiplets with $U(1)_{I_{3R}}$ charge, while the $n^c_{\substack{\square\\ \square}}$ chiral multiplets in antisymmetric representation $SU(2)_R$ vanish. Arising from the open strings at the intersections of c_1 and c_2' stacks of D6-brane, there are $I_{c_1 c_2'}$ new fields that are neutral under $U(1)_{I_{3R}}$. As follows, the anomaly free gauge symmetry from c_1 and c_2 stacks of D6-branes is $U(1)_{I_{3R}}$ (the $SU(2)_R$ Cartan subgroup). One obtains the $SU(3)_C \times SU(2)_L \times U(1)_{B-L} \times U(1)_{I_{3R}}$ gauge symmetry as in the above D6-brane splittings. To break the gauge symmetry further to SM gauge symmetry, we consider the minimal distance square $Z^2_{(a_2 c_1')}$ to be small, and obtain $I^{(2)}_{a_2 c_1'}$ pairs of chiral multiplets with quantum numbers $(\mathbf{1}, \mathbf{1}, \mathbf{-1}, \mathbf{1/2})$ and $(\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{-1/2})$ under $SU(3)_C \times SU(2)_L \times U(1)_{B-L} \times U(1)_{I_{3R}}$. Light open string stretches between the a_2 and c_1' stacks of D6-branes, and produces vectorlike particles that can then break the $SU(3)_C \times SU(2)_L \times U(1)_{B-L} \times U(1)_{I_{3R}}$ gauge symmetry down to the SM. In the meantime, D- and F-flatness is kept, as their quantum numbers are the same as those of the right-handed neutrino and its complex conjugate. In summary, the complete chains for symmetry breaking of our generalized Pati-Salam models read

$$\left.\begin{array}{r} SU(12)\times SU(2)_L\times SU(2)_R \\ SU(4)\times SU(6)_L\times SU(2)_R \\ SU(4)\times SU(2)_L\times SU(6)_R \end{array}\right\} \begin{array}{ll} \xrightarrow{\qquad\qquad\qquad}_{\text{Higgs Mechanism}} & SU(4)\times SU(2)_L\times SU(2)_R \\ \xrightarrow{\qquad\qquad}_{a\to a_1+a_2} & SU(3)_C\times SU(2)_L\times SU(2)_R\times U(1)_{B-L} \\ \xrightarrow{\qquad\qquad}_{c\to c_1+c_2} & SU(3)_C\times SU(2)_L\times U(1)_{I_{3R}}\times U(1)_{B-L} \\ \xrightarrow{\qquad\qquad\qquad}_{\text{Higgs Mechanism}} & SU(3)_C\times SU(2)_L\times U(1)_Y. \end{array} \tag{29}$$

For more details of the dynamical supersymmetry breaking of type IIA orientifolds with intersecting D6-branes, we refer to [33]. In the D6-brane models, the filler branes carrying USp gauge symmetries which are confining, and thus could allow for supersymmetry breaking via gaugino condensation. However, in our construction, we generalize this to supersymmetry breaking via other mechanisms and not restricting it to confining filler branes. The gauge kinetic function for a generic stack x of D6-branes takes the form of [33]

$$f_x = \frac{\mathbf{1}}{\mathbf{4}}\left[n_x^1 n_x^2 n_x^3 S - \left(\sum_{i=1}^{3} 2^{-\beta_j-\beta_k} n_x^i l_x^j l_x^k U^i\right)\right], \tag{30}$$

in which the real parts of dilaton S and moduli U^i are

$$\mathrm{Re}(S) = \frac{M_s^3 R_1^1 R_1^2 R_1^3}{2\pi g_s}, \tag{31}$$

$$\mathrm{Re}(U^i) = \mathrm{Re}(S)\chi_j \chi_k, \tag{32}$$

where $i \neq j \neq k$, and g_s to be the string coupling. We note the gauge coupling constant associated with a stack x is

$$g^{-2}_{D6_x} = |\mathrm{Re}(f_x)|. \tag{33}$$

In our generalized construction, the holomorphic gauge kinetic functions for $SU(12)_C$, $SU(2)_L$ and $SU(2)_R$ are associated with stacks a, b, and c, respectively. Recall that the holomorphic gauge kinetic function before our generalization is shown in [13,44,48,49], we have our holomorphic gauge kinetic function for $U(1)_Y$ as linear combination of these for $SU(4)$ and $SU(2)_R$ in the form

$$f_Y = \frac{2}{3} f_{SU(4)_a} + f_c. \tag{34}$$

Due to the pair of (n_x^i, l_x^i) in the gauge kinetic function Eq. (30), the kinetic function f_a will have an overall factor of 3, and rescale the value of $U(1)_Y$ gauge kinetic functions in Eq. (34) for gauge couplings of Minimal Supersymmetric Standard Model (MSSM). By taking care of the common factor of 3 between the kinetic function of $SU(12)$ and $SU(4)$, $SU(6)_L$ and $SU(2)_L$, $SU(6)_R$ and $SU(2)_R$, the tree-level MSSM gauge couplings take the form of

$$\begin{gathered} g^2_{SU(4)_a} = \alpha g^2_{SU(2)_b} = \beta \frac{5}{3} g^2_Y = \gamma[\pi e^{\phi_4}], \\ g^2_{SU(12)_a} = 3 g^2_{SU(4)_a}, \qquad g^2_{SU(6)_{Lb}} = 3 g^2_{SU(2)_{Lb}}, \qquad g^2_{SU(6)_{Rc}} = 3 g^2_{SU(2)_{Rc}} \end{gathered} \tag{35}$$

where $g^2_{SU(4)_a}$, $g^2_{SU(2)_b}$, and $\frac{5}{3}g^2_Y$ are the strong, weak and hypercharge gauge couplings respectively, and α, β, γ are the ratios between them.[1] Moreover, the Kähler potential reads

$$K = -\ln(S+\bar{S}) - \sum_{I=1}^{3}\ln(U^I + \bar{U}^I). \qquad (36)$$

Three stacks of D6-branes (carrying $U(4)_C \times U(2)_L \times U(2)_R$ gauge symmetry) determine the complex structure moduli χ_1, χ_2, and χ_3 due to the four-dimensional $N = 1$ supersymmetry conditions, with only one independent modulus field. To stabilize the moduli, one usually constructs the models with at least two USp groups with negative β functions which can be confined and then allow for gaugino condensations as was discussed in [50–52]. However, we will later present models not only with two and more USp groups, but also with only one USp group having a negative β function and still realize three family of particles. In general, the one-loop beta function for the $2N^{(i)}$ filler branes which are constructed on top of ith O6-plane and carry the $USp(N^{(i)})$ group are represented by [37]

$$\begin{aligned}\beta_i^g &= -3\left(\frac{N^{(i)}}{2}+1\right) + 2|I_{ai}| + |I_{bi}| + |I_{ci}| + 3\left(\frac{N^{(i)}}{2}-1\right) \\ &= -6 + 2|I_{ai}| + |I_{bi}| + |I_{ci}|. \qquad (37)\end{aligned}$$

When supersymmetry is broken via gaugino condensations on the condition of at least two confining gauge groups in the hidden sector, we may need to consider gauge mediation since gravity mediation is much smaller. Thus, the supersymmetry CP problem may be solved as well. In the models with only one confining USp gauge groups, the supersymmetry need to be broken with alternative mechanisms rather than gaugino condensations.

As we found in [48] that for a three-family supersymmetric Pati-Salam model, the corresponding new three-family supersymmetric Pati-Salam models by exchanging b and c stacks of D6-branes is not an equivalent model but leads to new gauge coupling behaviors. We will show that in the same way, one can improve the gauge couplings in our new model buildings and present the new models in the next subsection.

B. Generalized supersymmetric Pati-Salam models

Based on the generalized construction as we presented above, we now show the new generalized Pati-Salam models with their exact wrapping numbers. Distinct from the standard constructed Pati-Salam models such as in Ref. [37,48], we introduce three stacks of D6-branes, a, b, and c with three times the wrapping numbers of D6-brane for one of the stacks. Thus the corresponding gauge symmetries will be $U(12)_C \times U(2)_L \times U(2)_R$, $U(4)_C \times U(6)_L \times U(2)_R$, or $U(4)_C \times U(2)_L \times U(6)_R$.

To obtain the particle spectra with odd generations of the SM fermions, and satisfying the RR tadpole cancellation conditions, we again focus on the construction with only one tilted torus as was discussed in Ref. [37,48]. In our convention, we choose the third two-torus to be tilted and study the generalized Pati-Salam models in the following.

1. Mathematical search for generalized Pati-Salam models

Now we present the mathematical analysis behind the search for generalized supersymmetric Pati-Salam models. We take the construction with gauge group $U(12)_C \times U(2)_L \times U(2)_R$ as example for detailed discussion. The aim is to search for the solution of the physical conditions, i.e., common solutions of the RR tadpole cancellation conditions, supersymmetry conditions, and three generation conditions. The same ideas can be used to reduce the search space to a finite set when searching for models with the factor 3 at the b stack or the c stack, namely for gauge group $U(4)_C \times U(6)_L \times U(2)_R$, or $U(4)_C \times U(2)_L \times U(6)_R$. Such systematic search leads to the physical solution of models IV, V, VI, VII.

When the factor 3 appears at the a stack with gauge group $U(12)$, the tadpole conditions become

$$\begin{cases} 4 + 6A_a + A_b + A_c \geq 0, \\ 4 + 6B_a + B_b + B_c \geq 0, \\ 4 + 6C_a + C_b + C_c \geq 0, \\ 4 + 6D_a + D_b + D_c \geq 0. \end{cases} \qquad (38)$$

As we discussed in Sec. III B, because of the SUSY condition, there are only three possibilities for the choice of signs of A_a, B_a, C_a, D_a:

1. three numbers of A_a, B_a, C_a, D_a are zero and the other one is negative.
2. there are 2 negative values and 2 zero values among A_a, B_a, C_a, D_a.
3. there are 3 negative values and 1 positive values among A_a, B_a, C_a, D_a.

The first option is impossible since in this case (see Table II), $\tilde{A}_a = \tilde{B}_a = \tilde{C}_a = \tilde{D}_a = 0$, which contradicts with the three generation condition,

$$I_{ab} + I_{ab'} = -(\tilde{A}_a A_b + \tilde{B}_a B_b + \tilde{C}_a C_b + \tilde{D}_a D_b). \qquad (39)$$

The third possibility can be quickly ruled out via applying the tadpole condition Eq. (38). Without loss of generality, we may assume that A_a, B_a, C_a have negative values which implies that

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[1] Note that in the following discussion on the specified models, we use g_a^2, g_b^2, g_c^2 to denote the strong, weak and hypercharge gauge couplings respectively.

$$\begin{cases} A_b + A_c \geq 2, \\ B_b + B_c \geq 2, \\ C_b + C_c \geq 2. \end{cases}$$

However this is impossible because there is at most one positive value among A_b, B_b, C_b and at most one among A_c, B_c, C_c.

For the second possibility, we assume without loss of generality that

$$A_a < 0, \qquad B_a < 0 \quad \text{and} \quad C_a = D_a = 0, \tag{40}$$

which corresponds to $l_a^1 = 0$, and hence $\tilde{A}_a = \tilde{B}_a = 0$. Then the tadpole condition Eq. (38) implies

$$\begin{cases} A_b + A_c \geq 2, \\ B_b + B_c \geq 2, \end{cases} \tag{41}$$

which requires at least one positive value among A_b and A_c and at least another one among B_b and B_c. Hence, there is exactly one positive value among A_b and B_b and exactly one positive value among A_c and B_c, implying

$$C_b < 0, \qquad D_b < 0, \qquad C_c < 0, \qquad D_c < 0. \tag{42}$$

Now recall the SUSY equality condition Eq. (15), (x_A, x_B, x_C, x_D) is solution to the linear system

$$\begin{cases} x_A\tilde{A}_a + x_B\tilde{B}_a + x_C\tilde{C}_a + x_D\tilde{D}_a = 0, \\ x_A\tilde{A}_b + x_B\tilde{B}_b + x_C\tilde{C}_b + x_D\tilde{D}_b = 0, \\ x_A\tilde{A}_c + x_B\tilde{B}_c + x_C\tilde{C}_c + x_D\tilde{D}_c = 0. \end{cases} \tag{43}$$

Crámer's rule tells us, provided that the linear system has rank 3, (x_A, x_B, x_C, x_D) is proportional to (y_A, y_B, y_C, y_D) where

$$y_A = \begin{vmatrix} \tilde{B}_a & \tilde{C}_a & \tilde{D}_a \\ \tilde{B}_b & \tilde{C}_b & \tilde{D}_b \\ \tilde{B}_c & \tilde{C}_c & \tilde{D}_c \end{vmatrix}, \qquad y_B = -\begin{vmatrix} \tilde{A}_a & \tilde{C}_a & \tilde{D}_a \\ \tilde{A}_b & \tilde{C}_b & \tilde{D}_b \\ \tilde{A}_c & \tilde{C}_c & \tilde{D}_c \end{vmatrix}, \qquad y_C = \begin{vmatrix} \tilde{A}_a & \tilde{B}_a & \tilde{D}_a \\ \tilde{A}_b & \tilde{B}_b & \tilde{D}_b \\ \tilde{A}_c & \tilde{B}_c & \tilde{D}_c \end{vmatrix}, \qquad y_D = -\begin{vmatrix} \tilde{A}_a & \tilde{B}_a & \tilde{C}_a \\ \tilde{A}_b & \tilde{B}_b & \tilde{C}_b \\ \tilde{A}_c & \tilde{B}_c & \tilde{C}_c \end{vmatrix}.$$

More precisely, the solution of the SUSY equality condition in terms of the linear system Eq. (43) can be solved by

$$\begin{cases} x_A = \lambda, \\ x_B = \lambda y_B/y_A, \\ x_C = \lambda y_C/y_A, \\ x_D = \lambda y_D/y_A. \end{cases} \tag{44}$$

Recall that $l_a^1 = 0$, one can check that

$$\begin{aligned} y_A y_C = {} & (A_aB_aC_bD_bC_c + \tilde{D}_a^2C_bD_bD_c + A_aB_aC_bC_cD_c + \tilde{D}_a^2D_bC_cD_c)(A_b + A_c) \\ & + (A_aB_a\tilde{B}_b^2C_c + \tilde{D}_a^2\tilde{B}_b^2D_c + A_aB_a\tilde{B}_c^2C_b + \tilde{D}_a^2\tilde{B}_c^2D_b)(B_b + B_c). \end{aligned}$$

Combined with Eqs. (40), (41), and (42), we find $y_Ay_C < 0$. Hence the linear system Eq. (43) has indeed rank 3 and x_A and x_C have opposite signs. This contradicts the supersymmetry condition Eq. (15), where $x_A = \lambda, x_B = \lambda 2^{\beta_2+\beta_3}/\chi_2\chi_3$, $x_C = \lambda 2^{\beta_1+\beta_3}/\chi_1\chi_3$, $x_D = \lambda 2^{\beta_1+\beta_2}/\chi_1\chi_2$, in which $\chi_i = R_i^2/R_i^1$ represent the complex structure moduli for the ith two-torus, the values of x_A, x_B, x_C, x_D are required to be all positive. Therefore, the construction with gauge group $U(12)_C \times U(2)_L \times U(2)_R$ collapses with all the physical conditions satisfied.

When the factor 3 appears at the b stack with gauge group $U(6)_L$, the tadpole conditions become

$$\begin{cases} 4 + 2A_a + 3A_b + A_c \geq 0, \\ 4 + 2B_a + 3B_b + B_c \geq 0, \\ 4 + 2C_a + 3C_b + C_c \geq 0, \\ 4 + 2D_a + 3D_b + D_c \geq 0. \end{cases} \tag{45}$$

In this case, like the previous one, we start by discussing the signs of the wrapping number products A_i, B_i, C_i, D_i; $i = a, b, c$. Recall that each A_i, B_i, C_i, D_i falls into one of the three categories presented in Sec. IV B 1. Taking into consideration the tadpole conditions Eq. (45), we can easily list all the seven possibilities for the signs of the 12 wrapping number products, up to permutations of the A, B, C, D. We treat each possibilities one by one. For example, one of the possibilities is

$$\begin{aligned} & A_a, A_c, B_c, C_a, C_b, D_b, D_c < 0, \\ & \quad \text{and} \quad B_a = D_a = A_b = B_b = 0, \qquad C_c > 0, \end{aligned} \tag{46}$$

This implies $l_a^2 = n_b^1 = 0$ and $\tilde{A}_a = \tilde{C}_a = \tilde{C}_b = \tilde{D}_b = 0$.

This combined with Eq. (45) implies $A_a = -1$, $A_c \in \{-1, -2\}$, $B_c \in \{-1, -2, -3, -4\}$, $D_b = -1$ and $D_c = -1$. Hence the factor wrapping numbers of A_a, A_c,

TABLE III. D6-brane configurations and intersection numbers in model III, and its MSSM gauge coupling relation is $g_a^2 = \frac{671}{30} g_b^2 = \frac{631}{50} g_c^2 = \frac{3155}{1412} (\frac{5}{3} g_Y^2) = \frac{8\times 2^{1/4}\times 703^{3/4}}{75} \pi e^{\phi^4}$.

Model III	$U(12) \times U(2)_L \times U(2)_R \times USp(2)$								
Stack	N	$(n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$	$n_{\square\square}$	$n_{\begin{smallmatrix}\square\\\square\end{smallmatrix}}$	b	b'	c	c'	1
a	24	$(1, 1) \times (1, 0) \times (1, -1)$	0	0	0	1	0	−1	0
b	4	$(3, -2) \times (2, -1) \times (-1, 1)$	13	35	…	…	…	−8	−2
c	4	$(-1, 2) \times (-3, -1) \times (1, -1)$	−2	−22	…	…	0	…	−2
1	2	$(1, 0) \times (1, 0) \times (2, 0)$			$x_A = \frac{37}{4} x_B = \frac{37}{38} x_C = \frac{37}{4} x_D$ $\beta_2^g = -2$ $\chi_1 = \sqrt{\frac{37}{38}}, \chi_2 = \frac{\sqrt{1406}}{4}, \chi_3 = \sqrt{\frac{74}{19}}$				

B_c, D_b, D_c can be restricted to a finite set. To have three families of the SM fermions with quantum numbers under $SU(4)_C \times SU(6)_L \times SU(2)_R$, the intersection numbers need to satisfy the three generation conditions Eqs. (23). This equation will then determine the value of l_a^1 and l_a^3. At this stage, all the wrapping numbers except n_b^2 and l_b^3 are determined. To bound the last two wrapping numbers, notice that we know already the value of C_a and C_c and therefore $-4 - 2C_a - C_c \leq 3D_b < 0$ restricts D_b and hence n_b^2 and l_b^3 to a finite set. To summarize, we have reduced the search space to a finite set. A computer program will quickly find all the solutions under the condition Eq. (46). The other possibilities can either be treated similarly or ruled out using the argument in the discussion about $U(12)_C \times U(2)_L \times U(2)_R$.

In this way, we can significantly reduce the search space to a finite set while searching for models with gauge group $U(4)_C \times U(6)_L \times U(2)_R$, or $U(4)_C \times U(2)_L \times U(6)_R$ and we obtain the solutions as shown in models IV, V, VI, VII. This mathematical analysis method could also be utilized for other generalized Pati-Salam odmels or MSSM models constructed from other grand gauge groups.

2. Examples of generalized Pati-Salam models

In the following, we discuss the construction for gauge group $U(12) \times U(2)_L \times U(2)_R$ in Table III. We show that although this model satisfies all the constructing conditions such as the RR tadpole cancellation conditions, supersymmetry conditions, and three generation conditions, the solution for x_A, x_B, x_C, x_D appear to be negative values which are in conflict with the requirement in Eq. (15). While when we require the mathematical solutions to be satisfied with x_A, x_B, x_C, x_D positive, this will violate the RR tadpole cancellation conditions as we mathematically ruled out. For the models with gauge symmetries $U(4)_C \times U(6)_L \times U(2)_R$, and $U(4)_C \times U(2)_L \times U(6)_R$, we present the complete solutions of our generalized version of Pati-Salam models in Tables IV–VII.[2]

In the first column for each table, we denote the gauge constructions of D6-branes as a, b, and c stacks, and 1, 2, 3, and 4 stacks for the filler branes along ΩR, $\Omega R\omega$, $\Omega R\theta\omega$, $\Omega R\theta$ orientifold planes representing the $USp(N)$ gauge symmetries. In the second column, N represents the number of D6-branes for each stack. When 24, 12, 12 appear here, it means there are three times of wrapping than the standard Pati-Salam models. Moreover, in the third column we present the wrapping numbers of all the D6-branes and specify the third set of wrapping numbers as for the tilted two-torus. In the remaining right columns, we show the intersection numbers between different stacks, with b' and c' denote the ΩR images of b and c, respectively. In the last columns (denoted with number 1,2,3,4), we list the intersection numbers for the a, b, c stacks of branes intersecting with the 1,2,3,4 stacks of filler branes. In addition, we also present the relation among the moduli parameters imposed by the four-dimensional $N = 1$ supersymmetry conditions, and the one-loop β functions (β_i^g) for the hidden sector gauge symmetries in the table. In particular, we also give the MSSM gauge couplings in the caption of each model for checking the gauge coupling unification. Note that here the MSSM gauge coupling refers to the gauge coupling after generalized gauge construction breaking, i.e., $U(12) \to U(4)$, $U(6)_L \to U(2)_L$, $U(6)_R \to U(2)_R$.

The Higgs particles in models V, VII arise from $N = 2$ subsectors at the intersections of b and c' stacks of D6-branes, while the Higgs particles in models IV, VI arise from $N = 2$ subsectors at the intersections of b and c stacks of D6-branes. For example, there exist 5 exotic Higgs-like particles in model IV from $N = 2$ subsectors at the intersections of b and c stacks of D6-branes. We show that while there are 24 D6-branes constructed in stack a, the

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[2] To be precise, we consider the models dual to these models via T-dualities and with the same value of gauge couplings as equivalent model, and thus will not present them anymore.

TABLE IV. D6-brane configurations and intersection numbers in model IV, and its MSSM gauge coupling relation is $g_a^2 = 10g_b^2 = 2g_c^2 = \frac{10}{7}(\frac{5}{3}g_Y^2) = \frac{24\sqrt{3}}{5}\pi e^{\phi^4}$.

Model IV					$U(4)\times U(2)_L\times U(6)_R\times USp(8)$				
Stack	N	$(n^1,l^1)\times(n^2,l^2)\times(n^3,l^3)$	$n_{\square\square}$	$n_{\substack{\square\\ \square}}$	b	b'	c	c'	3
a	8	$(-1,-1)\times(-1,0)\times(1,-1)$	0	0	0	3	0	−1	0
b	4	$(-1,-1)\times(2,-1)\times(-1,-4)$	3	29	…	…	0	6	−1
c	12	$(0,-1)\times(2,-1)\times(-1,1)$	−1	1	…	…	…	…	0
3	8	$(0,-1)\times(1,0)\times(0,2)$			$x_A = 2x_B = \frac{1}{9}x_C = 2x_D$				
					$\beta_3^g = -5$				
					$\chi_1 = \frac{1}{3}, \chi_2 = 6, \chi_3 = \frac{2}{3}$				

TABLE V. D6-brane configurations and intersection numbers in model V, and its MSSM gauge coupling relation is $g_a^2 = 2g_b^2 = 10g_c^2 = \frac{50}{23}(\frac{5}{3}g_Y^2) = \frac{24\sqrt{3}}{5}\pi e^{\phi^4}$.

Model V					$U(4)\times U(6)_L\times U(2)_R\times USp(8)$				
Stack	N	$(n^1,l^1)\times(n^2,l^2)\times(n^3,l^3)$	$n_{\square\square}$	$n_{\substack{\square\\ \square}}$	b	b'	c	c'	4
a	8	$(1,-1)\times(1,1)\times(1,0)$	0	0	0	1	−3	0	0
b	12	$(0,1)\times(-1,-1)\times(2,1)$	1	−1	…	…	−6	0	0
c	4	$(1,1)\times(-1,-4)\times(-2,1)$	3	29	…	…	…	…	−1
4	8	$(0,-1)\times(0,1)\times(2,0)$			$x_A = 2x_B = 2x_C = \frac{1}{9}x_D$				
					$\beta_4^g = -5$				
					$\chi_1 = \frac{1}{3}, \chi_2 = \frac{1}{3}, \chi_3 = 12$				

TABLE VI. D6-brane configurations and intersection numbers in model VI, and its MSSM gauge coupling relation is $g_a^2 = 5g_b^2 = g_c^2 = \frac{5}{3}g_Y^2 = \frac{12\sqrt{6}}{5}\pi e^{\phi^4}$.

Model VI					$U(4)\times U(2)_L\times U(6)_R\times USp(2)\times USp(6)$					
Stack	N	$(n^1,l^1)\times(n^2,l^2)\times(n^3,l^3)$	$n_{\square\square}$	$n_{\square\square}$	b	b'	c	c'	1	3
a	8	$(-1,-1)\times(-1,0)\times(1,-1)$	0	0	0	3	0	−1	0	0
b	4	$(-1,-1)\times(1,-1)\times(-1,-4)$	0	16	…	…	0	3	4	−1
c	12	$(0,-1)\times(1,-1)\times(-1,1)$	0	0	…	…	…	…	−1	0
1	2	$(1,0)\times(1,0)\times(2,0)$			$x_A = x_B = \frac{1}{9}x_C = x_D$					
3	6	$(0,-1)\times(1,0)\times(0,2)$			$\beta_1^g = -1, \beta_3^g = -5$					
					$\chi_1 = \frac{1}{3}, \chi_2 = 3, \chi_3 = \frac{2}{3}$					

TABLE VII. D6-brane configurations and intersection numbers in model VII, and its MSSM gauge coupling relation is $g_a^2 = g_b^2 = 5g_c^2 = \frac{25}{13}(\frac{5}{3}g_Y^2) = \frac{12\sqrt{6}}{5}\pi e^{\phi^4}$.

Model VII					$U(4)\times U(6)_L\times U(2)_R\times USp(2)\times USp(6)$					
Stack	N	$(n^1,l^1)\times(n^2,l^2)\times(n^3,l^3)$	$n_{\square\square}$	$n_{\substack{\square\\ \square}}$	b	b'	c	c'	1	4
a	8	$(1,-1)\times(1,1)\times(1,0)$	0	0	0	1	−3	0	0	0
b	12	$(0,1)\times(-1,-1)\times(1,1)$	0	0	…	…	−3	0	1	0
c	4	$(1,1)\times(-1,-4)\times(-1,1)$	0	16	…	…	…	…	4	−1
1	2	$(1,0)\times(1,0)\times(2,0)$			$x_A = x_B = x_C = \frac{1}{9}x_D$					
4	6	$(0,-1)\times(0,1)\times(2,0)$			$\beta_1^g = -1, \beta_4^g = -5$					
					$\chi_1 = \frac{1}{3}, \chi_2 = \frac{1}{3}, \chi_3 = 6$					

gauge group yields to $U(12) \times U(2)_L \times U(2)_R$. With gauge breaking, $U(12) \to U(4)$, we obtain the standard Pati-Salam model gauge $U(4) \times U(2)_L \times U(2)_R$ and its MSSM gauge couplings after generalized gauge construction breaking. Models IV and V, and models VI and VII have their b and c stacks of D6-branes swapped, and as expected, the gauge coupling gets rescaled and refined. In particular, model VI has $g_a^2 = g_c^2$ and model VII has $g_a^2 = g_b^2$. By exchanging the b and c stacks of D6-branes, the unification of strong and hypercharged gauge couplings are swapped to the unification of the strong and weak couplings. Similarly, if the models have their gauge coupling from the b and c stacks of D6-branes equal, i.e., $g_b^2 = g_c^2$, the b and c stacks swapping would not change the MSSM gauge couplings.[3]

Now we continue with the examples of generalized gauge construction for $U(4) \times U(2)_L \times U(6)_R \times USp(2)$ and $U(4) \times U(6)_L \times U(2)_R \times USp(2)$ with gauge breaking $U(6)_R \to U(2)_R$ and $U(6)_L \to U(2)_L$ as shown in models IV and V. We observe that the generalized gauge construction got shifted from $U(6)_R$ to $U(6)_L$ and the $U(1)_Y$ gauge coupling $\frac{5}{3} g_Y^2$ got rescaled from $\frac{7}{10} g_a^2$ to $\frac{23}{50} g_a^2$ while g_a^2 remain the same. For models VI and VII, the generalized gauge constructions are $U(4) \times U(2)_L \times U(6)_R \times USp(2) \times USp(6)$ and $U(4) \times U(6)_L \times U(2)_R \times USp(2) \times USp(6)$, which got shifted from $U(6)_R$ to $U(6)_L$ and the $U(1)_Y$ gauge coupling $\frac{5}{3} g_Y^2$ got rescaled from g_a^2 to $\frac{13}{25} g_a^2$, while g_a^2 remains the same. Note that this construction is not simply swapping the b and c stacks of D6-branes, but nontrivial T-dualities are also performed to obtain models with three families of particles and tadpole cancellation conditions fulfilled.

For models with more than one orientifold plane in the generalized Pati-Salam models, e.g., with two confining gauge groups, such as models VI and VII, there may exist stable extrema with moduli stabilization and supersymmetry breaking via gaugino condensations, which are very interesting from the phenomenological points of view.

V. PRELIMINARY PHENOMENOLOGICAL STUDIES

In this section, we discuss the phenomenological features of our generalized Pati-Salam models. Among these models, models IV and V have one confining $USp(8)$ group, while models VI and VII have two confining groups $USp(2) \times USp(6)$. The β functions of these USp groups are all negative. For these models with two confining USp groups, one can break supersymmety via gaugino condensation, and decouple the exotic particles.

Comparing with the standard Pati-Salam models, we now discuss the generalized spectrum as we studied in [48] with gauge symmetry $SU(4) \times SU(2)_L \times SU(R)$. We start with models IV and V, which are constructed with gauge symmetry $U(6) \times U(2)_L \times U(6)_R \times USp(8)$ and $U(6) \times U(6)_L \times U(2)_R \times USp(8)$. Their explicit spectrum are shown in Table VIII and Table IX respectively. For these models, it is obvious that for model IV the Higgs multiplets therein are from the intersection of b and c stacks of D6-branes, while for model V the Higgs multiplets are from the intersection of b and c' stack of branes. Taking the pair of models VI and VII as example, we show the spectrum with two confining gauge groups $USp(2) \times USp(6)$. The models VI and VII are constructed with gauge symmetries $U(4) \times U(6)_L \times U(2)_R \times USp(2) \times USp(6)$ and $U(4) \times U(2)_L \times U(6)_R \times USp(2) \times USp(6)$ respectively. We present the explicit spectrum for model VI in Table X and the spectrum for model VII in Table XI. For model VI, it is obvious that the Higgs multiplets therein are from the intersection of b and c stacks of D6-branes, while for model VII the Higgs multiplets therein are from the intersection of b and c' stacks of D6-branes. For both of these models, there are two confining $USp(N)$ gauge groups. A general analysis of the nonperturbative superpotential with tree-level gauge couplings can be performed. It was shown that there can exist extrema with the stabilizations of dilaton and complex structure moduli [33]. However, these extrema of such model might be saddle points and thus do not break supersymmetry. For further investigation, if the models have three or four confining $USp(N)$ gauge groups, the nonperturbative superpotential allows for the moduli stabilization and supersymmetry breaking at the stable extremum in general [33].

As follows, we show in Table XII and XIII the new composite states formed due to the strong forces from the hidden sector for models VI and VII. They have one confining gauge group $USp(2)$ with two charged intersections and $USp(6)$ gauge. Therefore, besides self-confinement, the mixed-confinement between different intersections is also possible, which yields the chiral supermultiplets (1,2,6,1,1) or (1,6,2,1,1).

All the models we presented contain exotic particles that are charged under the hidden gauge groups. The strong coupling dynamics in the hidden sector at a certain intermediate scale might provide a mechanism for all these particles to form bound states or composite particles. These are compatible with anomaly cancellation conditions, such that we do not have extra anomalies introduced. Moreover, similar to the quark condensation in QCD, these particles will only be charged under the SM gauge symmetry [30].

In general, these USp groups have two kinds of neutral bound states. The first one is the pseudo inner product of

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[3] Note that from Eq. (34), it is easy to check that when one model has $g_Y^2 = g_a^2$, by applying b and c stacks of brane swapping, the new model has $g_a^2 = g_b^2$, and vice versa. This we have shown with detailed examples in [48]. In principle, by replacing the holomorphic gauge function f_c with f_b in Eq. (34), one can compute and predict all the gauge couplings behaviors after b and c stacks of D6-branes swapping without reconstructing the dual model.

TABLE VIII. The chiral spectrum in the open string sector for model IV.

Model IV	$SU(4)\times SU(2)_L\times SU(6)_R\times USp(8)$	Q_4	Q_{2L}	Q_{6R}	Q_{em}	$B-L$	Field
ab'	$3\times(4,2,1,1)$	1	1	0	$-\frac{1}{3},\frac{2}{3},-1,0$	$\frac{1}{3},-1$	Q_L, L_L
ac'	$1\times(\bar{4},1,\bar{6},1)$	−1	0	−1	$\frac{1}{3},-\frac{2}{3},1,0$	$-\frac{1}{3},1$	Q_R, L_R
bc'	$6\times(1,2,6,1)$	0	1	1	−1, 0, 0, 1	0	H'
$b3$	$1\times(1,\bar{2},1,8)$	0	−1	0	$\pm\frac{1}{2}$	0	
$b_{\Box\Box}$	$3\times(1,3,1,1)$	0	2	0	$0,\pm1$	0	
$b_{\begin{smallmatrix}\Box\\\Box\end{smallmatrix}}$	$29\times(1,1,1,1)$	0	2	0	0	0	
$c_{\overline{\Box\Box}}$	$1\times(1,1,\overline{21},1)$	0	0	−2	$0,\pm1$	0	
$c_{\begin{smallmatrix}\Box\\\Box\end{smallmatrix}}$	$1\times(1,1,15,1)$	0	0	2	0	0	
bc	$5\times(1,2,\bar{6},1)$	0	1	−1			
	$5\times(1,\bar{2},6,1)$	0	−1	1	−1, 0, 0, 1	0	H_u^i, H_d^i

TABLE IX. The chiral spectrum in the open string sector for model V.

Model V	$SU(4)\times SU(6)_L\times SU(2)_R\times USp(8)$	Q_4	Q_{6L}	Q_{2R}	Q_{em}	$B-L$	Field
ab'	$1\times(4,6,1,1)$	1	1	0	$-\frac{1}{3},\frac{2}{3},-1,0$	$\frac{1}{3},-1$	Q_L, L_L
ac	$3\times(\bar{4},1,2,1)$	−1	0	1	$\frac{1}{3},-\frac{2}{3},1,0$	$-\frac{1}{3},1$	Q_R, L_R
bc	$6\times(1,\bar{6},2,1)$	0	−1	1	−1, 0, 0, 1	0	H'
$c4$	$1\times(1,1,\bar{2},8)$	0	0	−1	$\pm\frac{1}{2}$	0	
$b_{\Box\Box}$	$1\times(1,3,1,1)$	0	2	0	$0,\pm1$	0	
$b_{\overline{\begin{smallmatrix}\Box\\\Box\end{smallmatrix}}}$	$1\times(1,1,1,1)$	0	−2	0	0	0	
$c_{\Box\Box}$	$3\times(1,1,21,1)$	0	0	2	$0,\pm1$	0	
$c_{\begin{smallmatrix}\Box\\\Box\end{smallmatrix}}$	$29\times(1,1,15,1)$	0	0	2	0	0	
bc'	$5\times(1,6,2,1)$	0	1	1			
	$5\times(1,\bar{6},\bar{2},1)$	0	−1	−1	−1, 0, 0, 1	0	H_u^i, H_d^i

TABLE X. The chiral spectrum in the open string sector for model VI.

Model VI	$SU(4)\times SU(2)_L\times SU(6)_R\times USp(2)\times USp(6)$	Q_4	Q_{2L}	Q_{6R}	Q_{em}	$B-L$	Field
ab'	$3\times(4,2,1,1,1)$	1	1	0	$-\frac{1}{3},\frac{2}{3},-1,0$	$\frac{1}{3},-1$	Q_L, L_L
ac'	$1\times(\bar{4},1,\bar{6},1,1)$	−1	0	−1	$\frac{1}{3},-\frac{2}{3},1,0$	$-\frac{1}{3},1$	Q_R, L_R
bc'	$3\times(1,2,6,1,1)$	0	1	1	−1, 0, 0, 1	0	H'
$b1$	$4\times(1,2,1,2,1)$	0	1	0	$\pm\frac{1}{2}$	0	
$b3$	$1\times(1,\bar{2},1,1,6)$	0	−1	0	$\pm\frac{1}{2}$	0	
$c1$	$1\times(1,1,\bar{6},2,1)$	0	0	−1	$\pm\frac{1}{2}$	0	
$b_{\begin{smallmatrix}\Box\\\Box\end{smallmatrix}}$	$16\times(1,1,1,1,1)$	0	2	0	$0,\pm1$	0	
bc	$5\times(1,2,\bar{6},1,1)$	0	1	−1			
	$5\times(1,\bar{2},6,1,1)$	0	−1	1	−1, 0, 0, 1	0	H_u^i, H_d^i

two fundamental representations generated by decomposing the rank two antisymmetric representation. This can be considered as the reminiscent of a meson formed by the inner product of one pair of fundamental and antifundamental representations of $SU(3)_C$ in QCD. This applies to our modelc IV and V. The second kind has rank $2N$ antisymmetric representation of $USp(2N)$ group for $N\geq 2$, which is an $USp(2N)$ singlet and somewhat similar to a baryon, as a rank three antisymmetric representation of $SU(3)_C$ in QCD. This case appears in models IV and V via the form of $USp(8)$ and in models VI and VII via the form of $USp(6)$.

TABLE XI. The chiral spectrum in the open string sector for model VII.

Model VII	$SU(4) \times SU(6)_L \times SU(2)_R \times USp(2) \times USp(6)$	Q_4	Q_{2L}	Q_{6R}	Q_{em}	$B-L$	Field
ab'	$1 \times (4, 6, 1, 1, 1)$	1	1	0	$-\frac{1}{3}, \frac{2}{3}, -1, 0$	$\frac{1}{3}, -1$	Q_L, L_L
ac	$3 \times (\bar{4}, 1, 2, 1, 1)$	−1	0	1	$\frac{1}{3}, -\frac{2}{3}, 1, 0$	$-\frac{1}{3}, 1$	Q_R, L_R
bc	$3 \times (1, \bar{6}, 2, 1, 1)$	0	−1	1	−1, 0, 0, 1	0	H'
$b1$	$1 \times (1, 6, 1, 2, 1)$	0	1	0	$\pm\frac{1}{2}$	0	
$c1$	$4 \times (1, 1, 2, 2, 1)$	0	0	1	$\pm\frac{1}{2}$	0	
$c4$	$1 \times (1, 1, \bar{2}, 1, 6)$	0	0	−1	$\pm\frac{1}{2}$	0	
$c_{\square\square}$	$16 \times (1, 1, 1, 1, 1)$	0	0	2	$0, \pm 1$	0	
bc'	$5 \times (1, 6, 2, 1, 1)$	0	1	1			
	$5 \times (1, \bar{6}, \bar{2}, 1, 1)$	0	−1	−1	−1, 0, 0, 1	0	H_u^i, H_d^i

TABLE XII. Composite particle spectrum for model VI.

Model VI		$SU(4) \times SU(2)_L \times SU(6)_R \times USp(2) \times USp(6))$	
Confining Force	Intersection	Exotic Particle Spectrum	Confined Particle Spectrum
$USp(2)_1$	$b1$	$4 \times (1, 2, 1, \bar{2}, 6)$	$4 \times (1, 1, 1, 1, 1)$, $4 \times (1, 3, 1, 1, 1)$, $4 \times (1, 2, 6, 1, 1)$
	$c1$	$1 \times (1, 1, 6, 2, 1)$	$4 \times (1, 1, 15, 1, 1)$, $4 \times (1, 1, 21, 1, 1)$
$USp(6)_3$	$c1$	$1 \times (1, 1, 6, 2, 1)$	$1 \times (1, 1, 15, 1, 1)$, $1 \times (1, 1, 21, 1, 1)$

TABLE XIII. Composite particle spectrum for model VII.

Model VII		$SU(4) \times SU(6)_L \times SU(2)_R \times USp(2) \times USp(6)$	
Confining Force	Intersection	Exotic Particle Spectrum	Confined Particle Spectrum
$USp(2)_1$	$b1$	$1 \times (1, 6, 1, \bar{2}, 1)$	$4 \times (1, 1, 1, 1, 1)$, $4 \times (1, 1, 3, 1, 1)$, $4 \times (1, 6, 2, 1, 1)$
	$c1$	$4 \times (1, 1, 2, \bar{2}, 6)$	$4 \times (1, 15, 1, 1, 1)$, $4 \times (1, 21, 1, 1, 1)$
$USp(6)_4$	$c1$	$1 \times (1, 1, 2, 2, 1)$	$1 \times (1, 15, 1, 1, 1)$, $1 \times (1, 21, 1, 1, 1)$

VI. DISCUSSIONS AND CONCLUSION

We generalized the construction of three-family $N = 1$ supersymmetric Pati-Salam models from type IIA orientifolds on $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ with intersecting D6-branes, where the $SU(12)_C \times SU(2)_L \times SU(2)_R$, $SU(4)_C \times SU(6)_L \times SU(2)_R$ or $SU(4)_C \times SU(2)_L \times SU(6)_R$ gauge symmetries arise from the stacks of D6-branes with $U(n)$ gauge symmetries. Firstly, via Higgs mechanism we can break the generalized Pati-Salam gauge symmetry $U(12) \to U(4) \times U(4) \times U(4) \to U(4) \times U(4) \to U(4)$ and $U(6) \to U(2) \times U(2) \times U(2) \to U(2) \times U(2) \to U(2)$, with new massive bosons obtained in this procedure, and resulting in the standard Pati-Salam gauge symmetry. Taking the gauge group $U(6)$ and breaking it to $U(2)$ as an example, we studied the gauge symmetry breaking in detail from the generalized models to the standard Pati-Salam models, and computed the masses of the gauge bosons in this Higgs mechanism. The Pati-Salam gauge symmetry can then be broken down to the $SU(3)_C \times SU(2)_L \times U(1)_{B-L} \times U(1)_{I_{3R}}$ via D6-brane splittings, and further down to the SM gauge symmetry via the D- and F-flatness preserving Higgs mechanism in which Higgs fields are the massless open string states from a specific $N = 2$ subsector. Moreover, models IV and V, and models VI and VII are constructed with their b and c stacks of D6-branes swapped respectively. The $U(1)_Y$ and $SU(4)_C$ gauge couplings can be constructed closer to unification at the string scale through the swapping. Furthermore, for models VI and VII this swapping leads to strong and hypercharge gauge unification shift to weak and hypercharge gauge unification. In the generalized Pati-Salam model building, we reduced the search parameter space to a finite set by utilizing mathematical analysis, and obtained the models from common solutions of the RR tadpole cancellation conditions, supersymmetry conditions, and three generation conditions. It would be interesting to employ our mathematical analysis methods to search for the Pati-Salam models, trinification models, $SU(5)$ models, and flipped $SU(5) \times U(1)_X$ models. We also would like to address that another interesting scenario worthwhile investigating is constructing the $SU(2)_L$ and/or $SU(2)_R$ gauge symmetries from filler branes, namely, $SU(2)_{L,R} = USp(2)_{L,R}$. As follows, the number of the SM Higgs doublet pairs might be decreased.

In this case, one usually shall not construct the $SU(2)_{L,R}$ gauge symmetries from the splittings of higher rank $USp(N)$ ($N \geq 4$) branes, as it generally leads to an even number of families, and the absolute value for one wrapping number of $U(4)$ branes larger than 2 cannot be avoided. This makes it difficult due to the tadpole cancellation conditions for model building and calls for more powerful scanning methods for model buildings.

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