Magnetic monopole as a spacetime defect

Suvikranth Gera[*] and Sandipan Sengupta[†]

Department of Physics, Indian Institute of Technology Kharagpur, Kharagpur-721302, India

We propose that the only possible realization of a magnetic pole is emergent, it being an artifact of a torsion defect in a curved spacetime. This special phase is built upon a class of degenerate metric spacetime solutions of first-order gravity in vacuum. The (apparent) magnetic charge is shown to have a topological origin, given by a lower-dimensional counterpart of the Nieh-Yan invariant. At the invertible metric phase at a distance, this topological charge gets reflected as the (magnetic) Reissner-Nordström charge to an asymptotic observer, even though the monopole defect itself remains hidden.

I. INTRODUCTION

Dirac's formulation of singular magnetic monopoles stands out as a remarkably elegant yet unconfirmed explanation of the quantization of electric charge [1,2]. As 't Hooft and Polyakov showed later, monopole fields without string singularities can emerge naturally in non-Abelian gauge theories as solitonic solutions of finite energy and size and a definite magnetic charge [3–6]. Their presence also appears to be fundamental in the formulation of grand unified theories. However, the lack of any experimental encouragement until now makes one wonder whether monopoles really do exist in nature, at least in the forms they have been theorized thus far.

Here we work up the slightly more radical idea that the magnetic charge in a curved spacetime could be a purely gravitational (emergent) effect. The apparent charge is sourced by a solution of first-order field equations of vacuum gravity theory, associated with a noninvertible tetrad [7,8]. To emphasize, this description does not involve any matter coupling such as non-Abelian gauge fields.

Based on the field equations of first-order vacuum gravity, we provide an emergent definition for the magnetic field and the associated magnetic charge. The noninvertible solution necessarily exhibits a nontrivial spatial contortion, whose form is constrained by eliminating the Dirac-string singularities in the spacetime geometry. The emergent three-geometry where the apparent charge lives is regular, as the evaluation of the characteristic curvature scalars explicitly shows.

The degenerate core of the apparent monopole cannot be observed directly. This is because it cannot be reached along any timelike radial trajectory (geodesic) by an observer outside, living in a standard Einsteinian geometry. Locally, this part of the spacetime is equivalent to the

(extremal) magnetic Reissner-Nordström solution almost everywhere. The full spacetime is spherically symmetric and smooth.

Since there is no genuine matter source, the apparent magnetic charge must be a reflection of a topological charge of the noninvertible metric solution in vacuum. We demonstrate that this is indeed the case, the relevant topological invariant being a lower dimensional counterpart of the Nieh-Yan number. In other words, what is manifested as a magnetic Reissner-Nordstrom charge in curved space-time away from the vacuum phase has a purely topological interpretation within this formulation. The implications of such a connection could be radical.

This construction of an emergent monopole as a space-time defect in gravity theory provides a potential theoretical justification as to why a free magnetic pole is not observed in nature. In this sense, this configuration supersedes the nonsingular 't Hooft–Polyakov monopole built upon a genuine matter source, which should in principle be detected experimentally.

II. MAGNETIC CHARGE IN FIRST-ORDER GRAVITY

In order to define a "static" solution in gravity theory, an appropriate setting is to consider a spacetime configuration where the metric has a degenerate eigenvalue along the timelike direction, which naturally precludes any temporal dynamics. In general, such three-geometries could emerge as solutions to the field equations of first-order gravity [7–13], where the invertibility of tetrads is not assumed or required *a priori*.[1]

Let us consider a spacetime geometry given by the following four-metric:

---

[*]suvikranthg@iitkgp.ac.in
[†]sandipan@phy.iitkgp.ac.in, sandipan422@gmail.com

[1]The tetrad compatibility condition would be assumed throughout.

$$ds^2 = 0 + F^2(u)du^2 + R^2(u)(d\theta^2 + \sin^2\theta d\phi^2) \quad \text{at } u < u_0$$

$$= -\left[1 - \frac{Q_m}{R(u)}\right]^2 dt^2 + \frac{R'^2(u)du^2}{[1 - \frac{Q_m}{R(u)}]^2}$$

$$+ R^2(u)[d\theta^2 + \sin^2\theta d\phi^2] \quad \text{at } u > u_0. \tag{1}$$

The smooth and monotonic functions $F(u)$, $R(u)$ and their nth derivatives satisfy the following boundary conditions:

$$F(u) \to 0, \qquad F^{(n)}(u) \to 0 \quad \text{as } u \to u_0,$$
$$R(u) \to Q_m, \qquad R^{(n)}(u) \to 0 \quad \text{as } u \to u_0,$$
$$R(u) \to \infty, \qquad R'(u) = \text{const} \quad \text{as } u \to \infty. \tag{2}$$

The function $R(u)$ may be interpreted as the common radius of the two-sphere almost everywhere.

We shall now show that the spacetime defined above reflects the presence of a (apparent) magnetic charge.

A. Magnetic phase in Einstein gravity

At $u > u_0$, the metric (1) is invertible (det $g_{\mu\nu} \neq 0$) and becomes flat at the asymptotic boundary $u \to \infty$. Under a local reparametrization of the coordinate $u \to R(u)$, this becomes equivalent to the extremal Reissner-Nordström metric at $u > u_0$. However, such an identification cannot hold everywhere, in particular, at the phase boundary $u = u_0$ where the metric smoothly degenerates.

The tetrad fields associated with this metric (the internal metric being $\eta_{IJ} = [-1, 1, 1, 1]$) completely determine the torsionless spin-connection fields $\omega_\mu^{IJ}(e)$, which in turn lead to the following field-strength components:

$$R^{01} = \frac{Q_m}{R^3}\left(2 - \frac{3Q_m}{R}\right)R'dt \wedge du,$$

$$R^{02} = -\frac{Q_m}{R^2}\left(1 - \frac{Q_m}{R}\right)^2 dt \wedge d\theta,$$

$$R^{03} = -\frac{Q_m}{R^2}\left(1 - \frac{Q_m}{R}\right)^2 \sin\theta dt \wedge d\phi,$$

$$R^{12} = -\frac{Q_m}{R^2}R'du \wedge d\theta,$$

$$R^{23} = \frac{Q_m}{R}\left(2 - \frac{Q_m}{R}\right)\sin\theta d\theta \wedge d\phi,$$

$$R^{31} = -\frac{Q_m}{R^2}R'\sin\theta d\phi \wedge du. \tag{3}$$

In the matter sector, the only nonvanishing component is the magnetic field:

$$F_{\theta\phi} = Q_m \sin\theta. \tag{4}$$

This is characteristic of a magnetic pole of charge "Q_m."

The complete specification of the field content at the invertible phase as given above still does not tell us what exactly seeds the charge or, namely, whether it is genuine matter or geometry. The real origin of this monopole field depends on its continuation at $u \leq u_0$. This is what concerns us in the subsequent analysis.

B. Spacetime solution in noninvertible phase

At $u \leq u_0$, the tetrad (metric) fields in Eq. (1) have no inverse. These, along with an appropriate set of connection fields, define a degenerate spacetime solution of the first-order Lagrangian density:

$$\mathcal{L} = \epsilon^{\mu\nu\alpha\beta}\epsilon_{IJKL}e_\mu^I e_\nu^J R_{\alpha\beta}{}^{KL}(\omega).$$

The associated field equations in vacuum read [7]

$$\epsilon^{\mu\nu\alpha\beta}\epsilon_{IJKL}e_\nu^K D_\alpha(\omega)e_\beta^L = 0,$$
$$\epsilon^{\mu\nu\alpha\beta}\epsilon_{IJKL}e_\nu^J R_{\alpha\beta}{}^{KL}(\omega) = 0, \tag{5}$$

where the covariant derivative $D_\mu(\omega)$ is defined with respect to the connection ω_μ^{IJ}, as implied by its argument.

In general, any degenerate metric solution to the field equations (5) could exhibit nontrivial contortion. More precisely, the general solution for the nonvanishing components of the connection fields is given by [8]

$$\omega_a{}^{ij} = \bar\omega_a{}^{ij}(e) + \epsilon^{ijk}e_a^l N_{kl}, \qquad \omega_a{}^{0i} = M^{il}e_{al}, \tag{6}$$

where the symmetric rank-three matrices N_{kl} and M_{kl} encode contortion. Here, however, it is sufficient to assume $M_{kl} = 0$, as these components play no role in the physics that concerns us here. The resulting expressions for the connection fields (with contortion) become

$$\omega^{12} = F\eta_2 du + \left(R\eta_1 - \frac{R'}{F}\right)d\theta + \gamma H \sin\theta d\phi,$$

$$\omega^{23} = F\alpha du + \eta_3 R d\theta + (\eta_2 R \sin\theta - \cos\theta)d\phi,$$

$$\omega^{31} = \eta_3 F du + \beta R d\theta + \left(R\eta_1 + \frac{R'}{F}\right)\sin\theta d\phi,$$

$$\omega^{0i} = 0, \tag{7}$$

where we use an explicit parametrization for the N_{kl} field as

$$N_{ij} = \begin{pmatrix} \alpha & \eta_3 & \eta_2 \\ \eta_3 & \beta & \eta_1 \\ \eta_2 & \eta_1 & \gamma \end{pmatrix}.$$

Let us now analyze the structure of the field strength based on these connection fields:

$$R^{0i} = 0,$$

$$R^{12} = \left[\frac{1}{FR}\left(\eta_1 R - \frac{R'}{F}\right)' + (\eta_3^2 - \alpha\beta)\right]e^1 \wedge e^2 + \left[\frac{(\gamma-\beta)}{R}\cot\theta + \left(\beta\eta_2 - \eta_3\left(\eta_1 R + \frac{R'}{FR}\right)\right)\right]e^2 \wedge e^3$$

$$+ \left[\frac{1}{FR}[-(\gamma R)' + \alpha R'] + (\alpha\eta_1 - \eta_2\eta_3) + \frac{\eta_3}{R}\cot\theta\right]e^3 \wedge e^1,$$

$$R^{23} = \left[\frac{(\eta_3 R)'}{FR} + \left(\eta_2\beta - \eta_1\eta_3 + \frac{\eta_3 R'}{FR}\right)\right]e^1 \wedge e^2 + \frac{1}{R^2}\left[\eta_2 R \cot\theta + \left(1 + (\eta_1^2 - \beta\gamma)R^2 - \left(\frac{R'}{F}\right)^2\right)\right]e^2 \wedge e^3$$

$$+ \left[-\frac{(\eta_2 R)'}{FR} + \left(\eta_3\gamma - \eta_2\left(\eta_1 + \frac{H'}{FR}\right)\right)\right]e^3 \wedge e^1,$$

$$R^{31} = \left[\frac{(\beta R)'}{FR} + \left(\alpha\left(\eta_1 - \frac{R'}{FR}\right) - \eta_2\eta_3\right)\right]e^1 \wedge e^2 + \frac{1}{R^2}\left[2\eta_1 R \cot\theta + \left((\eta_3\gamma - \eta_1\eta_2)R^2 + \frac{RR'}{F}\eta_2\right)\right]e^2 \wedge e^3$$

$$+ \left[-\frac{1}{FR}\left(\eta_1 R + \frac{R'}{F}\right)' + \eta_2^2 - \alpha\gamma - \frac{\eta_2}{R}\cot\theta\right]e^3 \wedge e^1. \tag{8}$$

An inspection of these reveals that the geometry exhibits a Dirac-string singularity along $\theta = 0$ or $\theta = \pi$, unless some of the contortion fields vanish:

$$\eta_1 = \eta_2 = \eta_3 = 0, \qquad \beta - \gamma = 0. \tag{9}$$

We shall impose this requirement from here on.

Given the fields above, all of the field equations are satisfied except the component $\mu = t$, $I = 0$ in the second set in Eq. (5). In terms of the metric functions $F(u)$, $R(u)$ introduced in Eq. (1), this reduces to

$$2R\left[\frac{R'}{F}\right]' + \left[\left(\frac{R'}{F}\right)^2 - 1\right]F + \beta[2\alpha + \beta]FR^2 = 0. \tag{10}$$

For any function $\Omega(u) \equiv \beta(2\alpha + \beta)(u)$, this equation has the general solution

$$F(u) = \frac{R'(u)}{\sqrt{1 - \frac{1}{R}\int du \Omega(u) R^2(u) R'(u) + \frac{k}{R(u)}}},$$

where $k \geq 0$ is an integration constant.

For simplicity, we shall consider the case where non-trivial contortion components α, β, γ are constant. With this, the solution for the four-metric at the noninvertible phase finally becomes

$$ds^2 = 0 + \frac{R'^2 du^2}{[1 - \frac{1}{3}\Omega R^2 + \frac{k}{R}]} + R^2[d\theta^2 + \sin^2\theta d\phi^2], \tag{11}$$

where the constants are given by $k = \frac{1}{3}[2\beta(\alpha-\beta)Q_m^2 - 3]Q_m$, $\Omega = [\beta(2\alpha + \beta)]$, and k is fixed by the continuity of fields across $u = u_0$. Since $k > 0$, the denominator in g_{uu} has (at least) one positive real root [corresponding to the surface

$R(u) = R_*(Q_m)$ for some $u = u_* > 0$], in general. This defines the inner boundary of the metric.

C. Smoothness of spacetime geometry

The tetrad components defined by the metric (1) are smooth by construction. Further, let us look at the torsion components at the zero determinant phase:

$$T^0 = 0,$$

$$T^1 = -\beta R^2 \sin\theta d\theta \wedge d\phi,$$

$$T^2 = -\frac{1}{2}(\alpha + \beta)FR \sin\theta d\phi \wedge du,$$

$$T^3 = -\frac{1}{2}(\alpha + \beta)FR \sin\theta du \wedge d\theta, \tag{12}$$

whereas the invertible phase has no torsion. Evidently, all components above smoothly go to zero as the phase boundary is approached, and are also finite at the inner boundary. Finally, the $SO(3,1)$ field-strength components in Eq. (8), upon using Eq. (9), read

$$R^{0i} = 0,$$

$$R^{12} = \left[-\left(\frac{R'}{F}\right)' - \alpha\beta FR\right]du \wedge d\theta$$
$$+ [\alpha R' - (\beta R)']\sin\theta d\phi \wedge du,$$

$$R^{23} = \left[1 - \beta^2 R^2 - \left(\frac{R'}{F}\right)^2\right]\sin\theta d\theta \wedge d\phi,$$

$$R^{31} = \left[-\left(\frac{R'}{F}\right)' - \alpha\beta FR\right]\sin\theta d\phi \wedge du$$
$$+ [(\beta R)' - \alpha R']du \wedge d\theta. \tag{13}$$

A comparison with Eq. (13) shows that these are smooth as well. Since these fields $(e_\mu^I, T_{\mu\nu}{}^I \equiv \frac{1}{2}D_{[\mu}e_{\nu]}^I, R_{\mu\nu}{}^{IJ})$ define the complete set of gauge-covariant fields in the first-order

formulation, the full spacetime is manifestly smooth everywhere.

III. EMERGENT MAGNETIC FIELD

The noninvertible phase given by Eq. (1) at $u \le u_0$ naturally defines a set of (invertible) triad fields:

$$\hat{e}_u^1 = F(u), \qquad \hat{e}_\theta^2 = R(u), \qquad \hat{e}_\phi^3 = R(u)\sin\theta.$$

These, along with the unique torsionless connection $\hat{\omega}_a^{ij}(\hat{e}) = \bar{\omega}_a^{ij} \equiv \frac{1}{2}[\hat{e}_i^b\partial_{[a}\hat{e}_{b]}^j - \hat{e}_j^b\partial_{[a}\hat{e}_{b]}^i - \hat{e}_l^a\hat{e}_i^b\hat{e}_j^c\partial_{[b}\hat{e}_{c]}^l]$, describe an effective Einstein theory in three-space. The contortion K_a^{ij}, however, is not a part of this geometry. Rather, it lives as a (emergent) matter field on it. The precise nature of such matter must emerge naturally from the basic equations within this formulation.

In view of the fact that all of the field equations (5) have already been solved, let us consider the identity involving torsion and the Hodge dual of the $SO(3,1)$ field strength:

$$\epsilon^{\alpha\beta\rho\sigma}D_\beta(\omega)T_{\rho\sigma}{}^I = \frac{1}{2}\epsilon^{\alpha\beta\rho\sigma}R_{\beta\rho}{}^{IJ}(\omega)e_{\sigma J}. \qquad (14)$$

The $\alpha = t$, $I = i$ component of the above, upon projection along a purely spatial unit vector n^i in the internal space, leads to

$$\partial_a\left[\frac{1}{2}\epsilon^{abc}T_{bc}{}^i n_i\right] = \frac{1}{4}\epsilon^{abc}R_{ab}{}^{ij}n_i e_{cj} + \frac{1}{2}\epsilon^{abc}T_{bc}{}^i D_a n_i. \qquad (15)$$

Given the even parity of torsion, this equation is precisely of the form $\partial_a(\hat{e}\hat{B}^a) = \hat{e}\hat{\rho}_m$ in curved space, where the (static) magnetic field is sourced by a nontrivial magnetic charge density, defined as

$$\hat{e}\hat{B}^a \equiv -\frac{1}{2}\epsilon^{abc}T_{bc}{}^i n_i \equiv \frac{1}{2}\epsilon^{abc}F_{bc},$$

$$\hat{e}\hat{\rho}_m \equiv \frac{1}{4}\epsilon^{abc}R_{ab}{}^{ij}n_i e_{cj} + \frac{1}{2}\epsilon^{abc}T_{bc}{}^i D_a n_i. \qquad (16)$$

To emphasize, the vector n^i defines the projection from the $SO(3)$ fields to the (emergent) $U(1)$ field strength in this context.

A radial field may now be obtained by choosing n^i to point radially: $n^i = (1,0,0)$. Inserting the solution at the degenerate phase constructed explicitly earlier, we find

$$\hat{F}_{\theta\phi} = \beta R^2 \sin\theta. \qquad (17)$$

Note that this is independent of α. Remarkably, it approaches a monopole field ($\hat{F}_{\theta\phi} \to Q\sin\theta$) at the outer boundary, and is nonsingular everywhere all the way up to

the inner boundary. The continuity of the field strength across $u = u_0$ fixes the free constant above as $\beta = \frac{1}{Q_m}$.

To see if the emergent definition (16) does reproduce the correct physics in this context, let us consider the integral of the charge density at the phase boundary,

$$\frac{1}{4\pi}\int d^3x\hat{e}\hat{\rho}_m = \frac{1}{4\pi}\int_{S^2} du d\theta d\phi \partial_u[\beta R^2 \sin\theta] = Q_m$$

$$= \frac{1}{4\pi}\int_{S^2} d\theta d\phi \hat{F}_{\theta\phi}, \qquad (18)$$

where in obtaining the first equality above we have used the following identities:

$$\frac{1}{4}\epsilon^{abc}R_{ab}{}^{ij}n_i\hat{e}_{cj} = (\beta - \alpha)RR'\sin\theta,$$

$$\frac{1}{2}\epsilon^{abc}T_{bc}{}^i D_a n_i = (\alpha + \beta)RR'\sin\theta.$$

Thus, the net flux through the (compact) phase boundary is indeed equal to the magnetic charge of the solution.

This completes the demonstration of how a static nonsingular monopole could emerge as an artifact of the noninvertible vacuum phase of gravity theory in general. Even though we have considered the simpler case of constant contortion, a generalization to the inhomogeneous case is straightforward. However, that does not add to the insights gained here, since the essential physics depends on their value at the phase boundary, which remains unaffected.

IV. A TOPOLOGICAL ORIGIN OF APPARENT MAGNETIC CHARGE

Now we explore if it is possible to construct a geometric invariant, which could encode the boundary (topological) effects induced by torsion at the surface separating the two phases. The boundary here has the topology of S^2. The only torsional invariant that may be defined on it is given by

$$N = -\frac{1}{8\pi}\int_{S_B^2} d^2x \epsilon^{ab}T_{ab}{}^i n^i, \qquad (19)$$

where $a \equiv (\theta, \phi)$. This is simply the two-integral of a unique gauge-invariant projection of the torsion two-form over a compact manifold. This, however, may be seen as a lower-dimensional counterpart of the Nieh-Yan invariant, which is a four-integral (over a compact manifold) of a gauge-invariant projection of the appropriate torsional four-form [14].

Next, let us evaluate the reduced Nieh-Yan charge (19) for the (monopole) geometry constructed earlier:

$$N = \frac{1}{4\pi}\int_{S_B^2} d\theta d\phi[K_\theta{}^{31}e_\phi^3 + K_\theta{}^{12}e_\theta^2] = Q_m, \qquad (20)$$

where we have used the expressions $K_\theta{}^{31} = \beta R$, $K_\theta{}^{12} = \gamma R\sin\theta = \beta R\sin\theta$ from Eq. (7). As remarkable as it seems, the topological Nieh-Yan number at the

noninvertible phase $u < u_0$ is precisely what gets reflected as the magnetic charge to an observer at the Einsteinian phase at $u > u_0$. The magnetic charge thus acquires a topological interpretation within a first-order formulation of gravity theory in the absence of matter.

The associated winding number corresponds to a map from the S^2 in the coordinate space to the S^2 in the internal space. This may be defined through a unit vector $m^i(\theta, \phi) \equiv (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$:

$$w = \frac{1}{8\pi} \int_{S_B^2} d^2x \epsilon_{ijk}\epsilon^{ab} m^i \partial_a m^j \partial_b m^k = 1.$$

Note that in terms of m^i, the emergent three-metric at the noninvertible phase could be given by the following set of vielbein fields:

$$E^i = R\partial_a m^i dx^a [a \equiv \theta, \phi], \qquad E^4 = \sqrt{\sigma}F du.$$

V. CONCLUSIONS

From a modern perspective, the unobservability of free magnetic poles sourced by matter fields could be turned into the assertion that such objects do not exist in nature, and their only manifestation could be emergent. Here we explicitly proposed such a realization in curved spacetime with defects[2] associated with a vanishing metric determinant. This description of an emergent magnetic pole is based on a special class of classical solutions to the first-order field equations of vacuum gravity, where the temporal direction exhibits a zero eigenvalue. Our construction has no analogue in standard Einsteinian gravity in vacuum based on invertible tetrad fields everywhere, since such a theory does not admit a classical solution with nontrivial torsion, as is necessarily the case here.

In addition, we provided a topological interpretation of the (apparent) magnetic charge in gravity theory, given by a lower-dimensional counterpart of the torsional Nieh-Yan topological number. The existence of a compact surface at the phase boundary makes such an identification possible.

The details of how the apparent magnetic charge gets manifested to the observer necessarily living in an Einstein (invertible) phase have been elucidated. Based on smoothness requirements, this geometry was found to be the magnetic Reissner-Nordström solution (extremal). Since

there are no analytic radial geodesics from this outer region to the noninvertible phase, the monopole defect remains hidden. However, the detection of the topological charge of the defect is still possible through the gravitational potential away from the phase boundary.

The emergent field and the three-geometry seeding the topological charge are both regular. This may be contrasted with the point curvature singularity characteristic of a magnetic Reissner-Nordström black hole. However, any statement regarding a possible resolution of the curvature singularity in a four dimensional sense needs to be formulated in terms of relevant fields superceding the four dimensional curvature invariants which cease to be well defined themselves due the noninvertibility of tetrad. It is an interesting question as to whether this regulating effect of the noninvertible phase could play any role in the renormalizability of quantum gravity in a first-order formulation where torsion is expected to introduce a scale.

A relevant criticism of our construction could be that it is based on spacetimes with noninvertible metric phases, which have no place in standard Einstein theory. However, it would not be logical to ignore them based on this ground only, since these could mediate topology change in classical theory, a possibility that need not be excluded in principle. As we have demonstrated here, some of these are special and could carry an apparent magnetic charge. It would certainly be important to understand the significance of these geometries beyond the classical realm. For instance, the inclusion of such "charged" saddle points in the functional integral could lead to fresh insights regarding the role of Nieh-Yan topological charge in quantum gravity.

Furthermore, when the timelike direction lies along the zero eigenvalue of the degenerate four-metric, one might as well be bothered by the apparent loss of causality at this phase. However, similar scenarios are typical in quantum field theory, which is known to admit virtual particles. Momenta of such particles need not lie on the relativistic mass shell, and these need not respect causality all along their respective trajectories. In other words, the deeper implications of the apparent acausality [16] associated with the noninvertible metric phase, once completely understood, could have their own virtue.

ACKNOWLEDGMENTS

---

[2]The reader may see Ref. [15] for one of the earliest discussions of torsion defects in a context different from this work. Such configurations, however, are not solutions to the vacuum field equations of gravity.

[1] P. A. M. Dirac, Proc. R. Soc. A **133**, 60 (1931).

[2] P. A. M. Dirac, Phys. Rev. **74**, 817 (1948).

[3] G. t Hooft, Nucl. Phys. **B79**, 276 (1974).

[4] A. M. Polyakov, Pis'ma Zh. Eksp. Teor. Fiz. **20**, 430 (1974) [JETP Lett. **20**, 194 (1974)], https://inspirehep.net/literature/90679.

[5] M. K. Prasad and C. M. Sommerfield, Phys. Rev. Lett. **35**, 760 (1975).

[6] E. B. Bogomolnyi, Sov. J. Nucl. Phys., **24**, 49 (1976), https://inspirehep.net/literature/101280.

[7] A. A. Tseytlin, J. Phys. A **15**, L105 (1982).

[8] R. K. Kaul and S. Sengupta, Phys. Rev. D **93**, 084026 (2016); **94**, 104047 (2016).

[9] I. Bengtsson, Classical Quantum Gravity **8**, 1847 (1991); Gen. Relativ. Gravit. **25**, 101 (1993).

[10] R. K. Kaul and S. Sengupta, Phys. Rev. D **96**, 104011 (2017).

[11] S. Sengupta, Phys. Rev. D **96**, 104031 (2017).

[12] S. Sengupta, Phys. Rev. D **97**, 124038 (2018).

[13] S. Gera and S. Sengupta, Phys. Rev. D **99**, 124038 (2019).

[14] H. T. Nieh and M. L. Yan, J. Math. Phys. (N.Y.) **23**, 373 (1982); H. T. Nieh, Int. J. Mod. Phys. A **22**, 5237 (2007).

[15] A. J. Hanson and T. Regge, Torsion and quantum gravity, in *Group Theoretical Methods in Physics*, edited by W. Beiglbck, A. Bhm, and E. Takasugi, Lecture Notes in Physics Vol. 94 (Springer, Berlin, Heidelberg, 1979).

[16] S. Sengupta, Phys. Rev. D **97**, 124038 (2018).