Monopole-antimonopole pair production in primordial magnetic fields

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We show that monopoles can be pair produced by cosmological magnetic fields in the early Universe. The pair production gives rise to relic monopoles and, at the same time, induces a self-screening of the magnetic fields. By studying these effects we derive limits on the monopole mass and also on the initial amplitude of primordial magnetic fields. Monopoles of GUT scale mass can even be produced if primordial magnetic fields exist at sufficiently high redshifts.

I. INTRODUCTION

The hypothesis that magnetic monopoles exist, albeit without any experimental evidence, has long been a subject of intense research. Monopoles would symmetrize Maxwell's equations and, moreover, their existence would be tied to the observed quantization of electric charge through the Dirac quantization condition $eg = 2\pi n$, $n \in \mathbb{Z}$ [1]. Besides the possibility that monopoles are elementary particles, they can be realized as topological solitons in spontaneously broken gauge theories as shown by 't Hooft and Polyakov [2,3]. The fact that such soliton solutions are contained in any grand unified gauge theory (GUT), in which the electromagnetic U(1) is embedded in a semisimple gauge group, makes monopoles an inevitable prediction of grand unification.

Despite the strong theoretical support for their existence, monopoles remain elusive in experimental searches. One reason is that their masses are expected to be superheavy. Solitonic monopoles have masses of the order of the symmetry breaking scale, which for GUT monopoles is typically 10^{16} GeV, and thus is far beyond the reach of terrestrial colliders. This does depend on the model and much lighter monopoles can also arise, for instance, in theories with several stages of symmetry breaking [4]. However, even with a small mass, producing solitonic monopoles at colliders has been argued to be strongly suppressed due to their high degree of compositeness [5,6]. It is also important to note that the computation of the production cross section of monopoles is a challenging task

in itself. This is because the Dirac quantization condition demands $|g| \gg 1$ for $|e| \ll 1$, rendering monopoles strongly coupled and perturbation theory invalid.

On the other hand, a symmetry breaking phase transition in the early Universe copiously produces solitonic monopoles with an abundance that would overdominate the present Universe, unless the symmetry breaking scale is very low. This was one of the motivations for inflationary cosmology, which dilutes away the monopoles by a period of a rapid cosmological expansion [7–10]. By solving the monopole problem, however, cosmic inflation also prohibits relic monopoles from a phase transition to be observed.

Another possible venue for monopole production is in strong magnetic fields, from which monopole-antimonopole pairs are nonperturbatively produced. This is the magnetic dual of the Schwinger process [11–13], and the rate of monopole pair production with arbitrary coupling was computed using an instanton method in [14,15]. Monopole production from magnetic fields in magnetars (highly magnetized neutron stars with fields up to $B \sim 10^{15}$ G $\sim 10^{-5}$ GeV^2) and heavy-ion collisions ($B \sim 10^{18}$ G $\sim 10^{-2}$ GeV^2 at CERN Super Proton Synchrotron) have been studied, by further taking into account finite-temperature effects into the calculations [16–19] (see [20] for a review). However, even in such environments the magnetic fields are not strong enough to produce monopoles with masses much larger than a GeV.

Magnetic fields also exist in the cosmic space on various length scales, with their origin still remaining a mystery. Spiral galaxies are known to host magnetic fields of $B \sim 10^{-5}$ G [21]. Recent gamma ray observations suggest the presence of magnetic fields even in intergalactic voids with strength $B \gtrsim 10^{-15}$ G coherent on Mpc scales or larger [22–24], giving strong indication that they are remnants of primordial magnetic fields produced in the early Universe. Importantly, if (some parts of) the cosmological magnetic fields are actually of primordial origin, then even if their

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present field strengths are weak, they could have been extremely strong in the early Universe.

In this paper we show that (even superheavy) monopole-antimonopole pairs can be produced by primordial magnetic fields and explore their cosmological implications. A strong enough primordial magnetic field dissipates energy by the monopole pair production, and also by accelerating the monopoles. By evaluating these effects, we obtain consistency conditions for primordial magnetic fields to survive until today to make up the observed magnetic fields, within physical theories that contain either elementary or solitonic monopoles. We also discuss the possibility of primordial magnetic fields producing an observable abundance of monopoles in the Universe, or even giving rise to a new kind of monopole problem. Based on these discussions, we derive lower bounds on the monopole mass, under the assumption that the observed cosmological magnetic fields have a primordial origin.

Our discussion regarding the dissipation of primordial magnetic fields is quite distinct from those of the so-called "Parker limit" on the monopole flux, obtained by requiring the survival of galactic magnetic fields [25,26]. (See also [27] which studied the Parker limit for primordial magnetic fields.) While these works assume a hypothetical abundance of preexisting monopoles, here the monopoles are produced by the primordial magnetic field itself, and thus the monopole abundance is uniquely determined.[1] This enables us to obtain a direct bound on the monopole mass, as a function of the primordial magnetic field strength.

This paper is organized as follows: In Sec. II we compute the number of monopoles produced in primordial magnetic fields. In Sec. III we evaluate the magnetic field dissipation by the monopoles, as well as the monopole relic abundance, and derive limits on the primordial magnetic field strength. In Sec. IV, the magnetic field limit is translated into limits on the monopole mass and the energy scale of magnetic field generation. We then conclude in Sec. V. In Appendix A we present a general formalism for analyzing the magnetic field dissipation by monopoles and also analyze effects that are not discussed in the main text. In Appendix B we give the relations between the Hubble rate, cosmic temperature, and redshift during the reheating and radiation-dominated epochs. In Appendix C we give a lower limit on the relic abundance of solitonic monopoles produced at a symmetry breaking phase transition.

Throughout this paper we use Heaviside-Lorentz units, with $c = \hbar = k_B = 1$. M_{Pl} refers to the reduced Planck mass $(8\pi G)^{-1/2}$. Unless explicitly noted, our discussions cover both elementary and solitonic monopoles. The magnetic charge of the monopole is typically large (e.g., $g \approx 21n$ for $e \approx 0.30$), however, most of the analyses apply even if g is tiny.

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[1]If there are additional monopole producing processes such as a thermal production [28], then our bound becomes tighter.

II. MONOPOLE PAIR PRODUCTION IN MAGNETIC FIELDS

A. Vacuum decay rate

Analyses of pair production in an external field often invokes a weak coupling, as was also assumed by Schwinger [13], however, this does not necessarily apply to monopoles due to the Dirac quantization condition. Using an instanton method, the authors of [14,15] derived an expression for the vacuum decay rate due to monopole-antimonopole pair production in a static magnetic field as

$$\Gamma = \frac{(gB)^2}{(2\pi)^3} \exp\left[-\frac{\pi m^2}{gB} + \frac{g^2}{4}\right], \qquad (2.1)$$

where B is the magnetic field strength, m is the monopole mass, and g is the amplitude of the magnetic coupling (thus g is non-negative hereafter). This result is valid for an arbitrary g, as long as the field is sufficiently weak such that

$$\frac{gB}{m^2} \lesssim 1, \qquad (2.2)$$

$$\frac{g^3 B}{m^2} \lesssim 4\pi. \qquad (2.3)$$

The second condition suggests that the expression (2.1) is valid while its exponent is negative, and this is stricter than the first condition if $g \gg 1$. It can be understood as the requirement that, in order for the semiclassical instanton techniques used to obtain (2.1) to be valid, the loop radius of the classical instanton solution [14,15],

$$R = \frac{m}{gB}, \qquad (2.4)$$

should be larger than the size of a monopole,[2]

$$r \sim \frac{g^2}{4\pi m}. \qquad (2.5)$$

The expression (2.1) can receive corrections also from finite-temperature effects, when the inverse of the temperature of the thermal bath is smaller than the instanton radius $1/T < R$; such thermal corrections to the monopole production rate have been computed in [16,18]. Gravitational effects on the monopole production are less studied, but one naively expects that the rate receives corrections when the curvature radius of the spacetime is smaller than R; in a

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[2]The classical radius of a vanilla 't Hooft-Polyakov monopole is of (2.5) [29]. It has been claimed that elementary monopoles should also have a similar spatial extension [30–32]; one simple argument is that the classical point-particle picture should break down at distances shorter than (2.5) since otherwise the sum of the rest energy and potential energy of a monopole-antimonopole pair can become negative and render the vacuum unstable.

Friedmann-Robertson-Walker (FRW) Universe, this condition is written as $1/H < R$, where H is the Hubble expansion rate.[3] Moreover, primordial magnetic fields redshift with the expansion of the Universe on a time scale of order the Hubble time; this time dependence can also modify the rate if $1/H < R$ (see e.g., [19] for discussions on pair production in spacetime-dependent fields). These corrections would enhance the pair production rate, even enabling the pair production to proceed via sphalerons, if the temperature and/or the Hubble scale are sufficiently high.

We should also note that in (2.1), m and g should be taken to be the renormalized quantities [15]. However, we ignore their runnings, which should be good enough for the approximate calculations in this paper.

B. Number density

In order to evaluate the number of monopoles produced from primordial magnetic fields, we identify the decay rate (2.1) with the rate of pair production per unit volume per unit time.[4] Then the number density n of monopole-antimonopole pairs follows

$$\dot{n} = -3Hn + \Gamma, \qquad (2.6)$$

where an overdot denotes a derivative with respect to physical time t, and the Hubble rate is $H = \dot{a}/a$ in terms of the scale factor a. Considering the magnetic field to be effectively homogeneous, this equation is integrated to yield

$$n(t) = \frac{1}{a(t)^3} \int_{t_i}^{t} dt' \, a(t')^3 \Gamma(t'), \qquad (2.7)$$

where t_i denotes the time when the magnetic field is switched on. Here, Γ depends on time through its dependence on the magnetic field, which redshifts with the expansion of the Universe. We parametrize the redshifting of the magnetic field strength as

$$B \propto a^{-p}, \qquad (2.8)$$

with p being a positive constant of order unity. Without any source, primordial magnetic fields redshift with $p = 2$,

however, different values of p can also be realized in the presence of matter or with stronger electric fields [36]. Here, let us also introduce a dimensionless quantity

$$\epsilon \equiv \frac{gB}{\pi m^2}, \qquad (2.9)$$

which obeys $\epsilon \ll 1$ when the first weak field condition (2.2) is well satisfied. Since the production rate Γ depends exponentially on B, it decays very quickly under weak fields on a time scale of $\Delta t_\Gamma = |\Gamma/\dot{\Gamma}| \simeq \epsilon/(pH) \ll 1/H$. Hence the integral in (2.7) is dominated by the lower limit, and we obtain an approximate expression for the pair number density valid for $t \gtrsim t_i + \Delta t_{\Gamma i}$ as[5]

$$n \sim \frac{(\Delta t_\Gamma a^3 \Gamma)_i}{a^3} = \frac{\epsilon_i \Gamma_i}{pH_i} \left(\frac{a_i}{a}\right)^3, \qquad (2.12)$$

where quantities measured at the initial time t_i is denoted by the subscript i.

The "initial time" t_i, which we defined as the moment when the magnetic field switches on and begins to redshift as (2.8), can be understood as the time when the magnetic field generation has concluded. To keep our discussion general we do not specify the concrete mechanism of primordial magnetic field generation, but the time when the generation process completes could, for instance, be at the end of inflation [37,38], after inflation when the Universe is dominated by an oscillating inflaton [39], or at cosmological phase transitions [40,41].

III. EFFECTS OF PRODUCED MONOPOLES

As one can read off from the expression (2.1) for Γ, the exponential suppression factor disappears and monopole production becomes significant as the magnetic field strength approaches the value,

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[3]Pair production of charged scalar particles by electric fields in de Sitter space was analyzed in [33]. It was found that the flat-space result is modified at $H^2 \gtrsim eE$, which is different from the above naive guess of $1/H < R$ (with the replacement $g, B \to e, E$). It would be interesting to explicitly compute the monopole production rate in a curved spacetime and check when gravitational effects become important.

[4]The two rates are not necessarily the same [34]. The pair production rate in an electric field is computed in [35], however, it is also found that this matches with the vacuum decay rate in the weak field limit. Hence we suppose that they also match for monopoles in weak fields.

[5]The integral in (2.7) can be directly performed when the background Universe has a constant equation of state w such that $H \propto a^{-3(1+w)/2}$, as

$$n = e^{\frac{g^2}{4}} \frac{1}{8\pi p} \frac{m^4}{H_i} \left(\frac{a_i}{a}\right)^3 \epsilon_i^{b+2} \left[\mathcal{G}\left(b, \frac{1}{\epsilon_i}\right) - \mathcal{G}\left(b, \frac{1}{\epsilon}\right)\right], \quad \text{where}$$

$$b = \frac{3(3+w)}{2p} - 2. \qquad (2.10)$$

Here we assumed $b > 0$, and $\mathcal{G}(b, z) = \int_z^\infty x^{b-1} e^{-x} dx$ is the incomplete gamma function. By using the asymptotic form $\mathcal{G}(b, 1/\epsilon) \sim \epsilon^{-b+1} e^{-1/\epsilon}$ in the weak field limit $\epsilon \to 0$, one obtains

$$n \sim \frac{\epsilon_i \Gamma_i}{pH_i} \left(\frac{a_i}{a}\right)^3 - \frac{\epsilon \Gamma}{pH}. \qquad (2.11)$$

The first term dominates at $t \gtrsim t_i + \Delta t_{\Gamma i}$, and then the expression reduces to (2.12).

$$B_\star = 4\pi \frac{m^2}{g^3}. \tag{3.1}$$

This is also the field strength which saturates the second weak field condition (2.3). In this section we show that primordial magnetic fields could not have been stronger than $B_\star$, by evaluating the backreaction of the monopoles on the magnetic field, and also the monopole relic abundance.

We will mostly consider times long before the electroweak phase transition, therefore, the primordial magnetic field and the monopoles are actually those of the hypercharge U(1) gauge field. When these are converted into the magnetic fields and monopoles of the electromagnetic U(1) at the electroweak phase transition, quantities such as the magnetic field strength and magnetic charge will change by a number of order unity that depends on the Weinberg angle, however, this will not be important for our discussions.

A. Magnetic field dissipation by monopole production

In terms of the energy density of the magnetic field,

$$\rho_B = \frac{B^2}{2}, \tag{3.2}$$

the magnetic field scaling (2.8) is rewritten as

$$(\dot\rho_B)_{\rm red} = -2pH\rho_B, \tag{3.3}$$

where we have added a subscript "red" to specify that this contribution to $\dot\rho_B$ represents the redshifting of the magnetic field due to the cosmic expansion. The time scale of redshifting, $\Delta t_{\rm red} = |\rho_B/(\dot\rho_B)_{\rm red}| = 1/2pH$, is of order the Hubble time.

Additionally, each time the field produces a monopole-antimonopole pair it looses energy corresponding to the rest energy of the pair, $\Delta\mathcal{E}_B = -2m$.[6] Thus the energy dissipation due to pair production per unit time and volume is

$$(\dot\rho_B)_{\rm prod} = -2m\Gamma. \tag{3.4}$$

This dissipation rate is smaller than the rate of redshifting, i.e., $2m\Gamma < 2pH\rho_B$, if

$$B < B_{\rm prod} = \frac{4\pi m^2}{g^3}\left[1 + \frac{4}{g^2}\ln\left(\frac{g^2}{4\pi^3 p}\frac{m}{H}\right)\right]^{-1}. \tag{3.5}$$

A magnetic field stronger than the right-hand side would quickly decay through monopole production on a time scale shorter than a Hubble time, until the field falls below $B_{\rm prod}$. Hence (3.5) gives an upper bound on the primordial magnetic field strength.

The expression for $B_{\rm prod}$ becomes negative if $(4/g^2)\ln[(g^2/4\pi^3 p)(m/H)] < -1$. This implies that in such a case the dissipation becomes significant only at very strong fields where the expression (2.1) for Γ breaks down. However, the logarithmic term is generically larger than -1 if $g \gg 1$: Even in the extreme case where the monopole mass saturates its lower bound $m \gtrsim 1$ GeV derived from heavy-ion collisions [17],[7] and the Hubble scale saturates the upper bound $H \lesssim 10^{14}$ GeV on the inflation scale [42], we get $(4/g^2)\ln(m/H) \gtrsim -0.5$ as long as $g \gtrsim 16$. Hence by supposing $g \gg 1$ and considering the logarithmic term to be either negligible or positive, we get $B_{\rm prod} \lesssim B_\star$, which guarantees that the weak field conditions (2.2) and (2.3) are satisfied at $B = B_{\rm prod}$.

One can also check that, under the weak field conditions, the ratio $B/B_{\rm prod}$ monotonically decreases in time, which indicates that the energy dissipation by the monopoles is more important at earlier times. Hence (3.5) also sets a lower limit on the time of magnetic field generation, as we will see explicitly in Section IV.

B. Magnetic field dissipation by monopole acceleration

After the monopoles are produced, they are accelerated by the magnetic fields and thus further deplete the magnetic field energy.[8] We first evaluate this effect by assuming that the (anti)monopoles move with relativistic velocities $v \simeq 1$ in the (reverse) direction of the magnetic field. Then each (anti)monopole gains kinetic energy of $\Delta\mathcal{E}_M = gB\Delta t$ and, in turn, the magnetic field loses energy per unit time and volume as

$$(\dot\rho_B)_{\rm R} = -2ngB. \tag{3.6}$$

Let us for the moment only consider pairs that are produced during an interval Δt_Γ around the time of consideration and substitute for the pair density [see discussions around (2.12)],

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[6]By equating the pair's rest energy with its potential energy due to the background magnetic field, one obtains the critical separation between the pair upon creation as $r_c = 2m/gB$, which is of the same order as the instanton radius (2.4). The depletion of the field energy $\Delta\mathcal{E}_B = -2m$ corresponds to the decrease in the net magnetic field strength due to a monopole-antimonopole pair separated by r_c. Under the weak field condition (2.3), the attractive force between a pair separated by r_c is weaker than the repulsive force imposed by the background field.

[7]A similar bound on m can also be derived by combining discussions on the thermal production of monopoles [28] with the lower limit on the reheating temperature from Big Bang Nucleosynthesis (BBN).

[8]This discussion of monopole acceleration follows that of [25,26]. There is, however, a key difference that here the monopole abundance is produced by the magnetic field itself and thus is uniquely determined.

$$n \to \frac{\epsilon \Gamma}{pH}. \qquad (3.7)$$

This amounts to ignoring energy dissipation by the accumulated abundance of monopoles produced in the past, and thus we will obtain a conservative bound on the field strength. Then one finds that the dissipation rate (3.6), due to accelerating relativistic monopoles, is smaller than the rate of redshifting (3.3), i.e., $2ngB < 2pH\rho_B$, when

$$B < B_{\mathrm{R}} = \frac{\pi m^2}{2g} \frac{1}{W(x_{\mathrm{R}})} \quad \text{with} \quad x_{\mathrm{R}} = e^{\frac{g^2}{8}} \frac{g}{4\pi p} \frac{m}{H}. \qquad (3.8)$$

Here, $W(x)$ is the Lambert W-function which is a solution of $We^W = x$; it is non-negative and increasing for $x \geq 0$. B_{R} also serves as an upper limit on the magnetic field strength, beyond which the field quickly decays by accelerating the monopoles the field itself has produced.

One can obtain an approximate expression for B_{R} if the first weak field condition (2.2) is well satisfied, i.e., $2gB_{\mathrm{R}}/\pi m^2 = 1/W(x_{\mathrm{R}}) \ll 1$. This is equivalent to $x_{\mathrm{R}} \gg 1$, for which we can use the rough approximation $W(x_{\mathrm{R}}) \sim \ln x_{\mathrm{R}}$ [43] to obtain

$$B_{\mathrm{R}} \sim \frac{4\pi m^2}{g^3} \left[1 + \frac{8}{g^2} \ln\left(\frac{g}{4\pi p} \frac{m}{H}\right)\right]^{-1}. \qquad (3.9)$$

This now takes a form similar to the upper bound (3.5) from pair production, except for the logarithmic factor.

In the above discussions we assumed the monopoles to be relativistic, however, one obtains similar results also for nonrelativistic monopoles. From the equation of motion of a nonrelativistic monopole/antimonopole, $m\ddot{z} = \pm gB$ where z is the direction of the magnetic field, the distance the monopole/antimonopole travels is $\Delta z = \pm (gB/2m)(\Delta t)^2$, supposing they are initially at rest. Here we ignored the time dependence of B, as well as the effect of the cosmological expansion on the monopole dynamics; this is because we are interested in cases where the magnetic field is dissipated on time scales comparable to, or shorter than, a Hubble time, and also because we use this computation only while the magnetic field strength changes by an order-unity factor.[9] Thus by accelerating n pairs from zero initial velocity, the magnetic field looses its energy density during Δt as

$$-\Delta \rho_B = 2ngB|\Delta z| = \frac{ng^2B^2}{m}(\Delta t)^2. \qquad (3.10)$$

Equating this with ρ_B, one obtains the characteristic time scale of magnetic energy dissipation,

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[9]When taking into account the backreaction of the monopoles, one obtains an oscillatory solution for the magnetic field; this magnetic field oscillation is expected to decay by a Landau damping [26].

$$\Delta t_{\mathrm{NR}} = \frac{1}{g}\sqrt{\frac{m}{2n}}. \qquad (3.11)$$

The condition for this to be longer than the time scale of redshifting $\Delta t_{\mathrm{red}} = 1/2pH$ [cf. (3.3)] is obtained by substituting (3.7) for n as

$$B < B_{\mathrm{NR}} = \frac{\pi m^2}{3g} \frac{1}{W(x_{\mathrm{NR}})} \quad \text{with} \quad x_{\mathrm{NR}} = e^{\frac{g^2}{12}} \frac{g^{2/3}}{6(2\pi)^{1/3} p} \frac{m}{H}. \qquad (3.12)$$

Under the first weak field condition (2.2), this upper limit is approximated by

$$B_{\mathrm{NR}} \sim \frac{4\pi m^2}{g^3} \left[1 + \frac{12}{g^2} \ln\left(\frac{g^{2/3}}{6(2\pi)^{1/3} p} \frac{m}{H}\right)\right]^{-1}. \qquad (3.13)$$

Since (3.9) and (3.13) differ only by the logarithmic factor, we conclude that the order of magnitude of the magnetic field limit does not depend on whether the monopoles are relativistic or not.

We have treated the primordial magnetic field as effectively homogeneous, by supposing the coherence length of the field to be larger than the Hubble radius at the time of consideration. In fact, the observationally hinted intergalactic magnetic field typically has a coherence length of Mpc scale or larger [22–24], which, if it is of primordial origin, reenters the horizon only at $a_0/a \lesssim 10^6$. However, if the primordial magnetic field had inhomogeneous components with subhorizon coherence lengths in the early Universe, then the monopoles would not always travel in the direction of the magnetic field, and thus the energy dissipation via monopole acceleration would be less effective (see [26] for similar discussions for monopoles in galactic magnetic fields).

We also remark that, in the above analyses, we only included monopoles produced "on the spot" and ignored monopoles that have already been produced in the past. Depending on the time evolution of the monopole velocity, the population of monopoles from the past may more effectively deplete the magnetic field energy, in which case, the bound on the magnetic field strength becomes even tighter. Discussions on this point, as well as a general formalism for analyzing the magnetic field dissipation by both the production and acceleration of all existing monopoles, are presented in Appendix A.

C. Monopole relic abundance and flux

Constraints on the relic density of the produced monopoles yield further limits on primordial magnetic fields. Supposing the monopoles today to be nonrelativistic, their relic density is obtained as

$$\rho_{M0} = 2mn_0 \sim \frac{2m\epsilon_i\Gamma_i}{pH_i}\left(\frac{a_i}{a_0}\right)^3, \qquad (3.14)$$

where we used (2.12), and the subscript "0" denotes values in the present Universe. Requiring the density of monopoles not to exceed that of dark matter, i.e., $\rho_{M0} < \rho_{\mathrm{dm}0} \approx 0.3\rho_{\mathrm{crit}0}$ [42], we obtain an upper limit on the initial magnetic field strength (the value when magnetic field generation concludes) as

$$B_i < B_{\mathrm{dm}} = \frac{\pi m^2}{3g}\frac{1}{W(x_{\mathrm{dm}})} \quad \text{with}$$

$$x_{\mathrm{dm}} = e^{\frac{g^2}{12}}\left(\frac{1}{108\pi p}\frac{m^5}{H_i\rho_{\mathrm{dm}0}}\right)^{1/3}\frac{a_i}{a_0}. \qquad (3.15)$$

In order to evaluate a_i/a_0, let us suppose that the generation of the primordial magnetic field concludes at the end of inflation or later, but before matter-radiation equality. We further assume a postinflation history starting with an epoch dominated by an oscillating inflaton, which eventually decays away and initiates the radiation-dominated epoch. We use the subscript "end" to denote quantities at the end of inflation, "dom" at the time when radiation domination takes over, and "eq" at matter-radiation equality.

The Hubble scale during the postinflation epochs as a function of the scale factor is given in Appendix B. Using (B2) and (B4) to rewrite a_i in terms of H_i, one obtains

$$x_{\mathrm{dm}} \sim e^{\frac{g^2}{12}}\frac{1}{p^{1/3}}\left(\frac{m}{10^9\,\mathrm{GeV}}\right)^{5/6}\left(\frac{m}{H_i}\right)^{5/6}\min\left\{1,\left(\frac{H_{\mathrm{dom}}}{H_i}\right)^{1/6}\right\}, \qquad (3.16)$$

where the last factor depends on whether the magnetic field generation completes during radiation domination ($H_i < H_{\mathrm{dom}}$), or in an earlier epoch ($H_i > H_{\mathrm{dom}}$). Further using the weak field condition, the magnetic field limit is thus approximately written as

$$B_{\mathrm{dm}} \sim \frac{4\pi m^2}{g^3}\left[1\right.$$
$$+ \frac{12}{g^2}\ln\left(\frac{1}{p^{1/3}}\left(\frac{m}{10^9\,\mathrm{GeV}}\right)^{5/6}\left(\frac{m}{H_i}\right)^{5/6}\right.$$
$$\left.\left.\times \min\left\{1,\left(\frac{H_{\mathrm{dom}}}{H_i}\right)^{1/6}\right\}\right)\right]^{-1}, \qquad (3.17)$$

which differs from the other bounds only by the logarithmic factor.

A few comments are in order. First, since the monopoles are continuously accelerated by the magnetic fields, they may withstand the Hubble damping and be moving with relativistic velocities in the current Universe (cf. Appendix A 2). In such a case, the relic density is larger than (3.14), and the relativistic monopoles serve as extra radiation which is constrained by cosmic microwave

background (CMB) observations and BBN, yielding a tighter bound on B_i. Secondly, we assumed that the annihilation of monopoles and antimonopoles does not significantly reduce their abundance. According to the analyses in [44,45], the annihilation only becomes relevant if the monopole number density is so large as to lead to an overabundance (unless the mass is very light). The discussion may be modified in the presence of cosmological magnetic fields, which pull the monopoles and antimonopoles apart. It will be interesting to study annihilation effects in a magnetic field background.

One can further compute the average flux of monopoles, $F = 2n_0v_0/4\pi$, with v_0 being the monopole velocity, and compare with various existing bounds [46,47] including the Parker limit [25–27]. Depending on the monopole mass, the flux bounds give stronger constraints than $\rho_{M0} < \rho_{\mathrm{dm}0}$. However, they do not drastically improve the limit on B_i, which depends on the bound on the monopole abundance only through the logarithmic factor.

D. Remarks on solitonic monopoles

For monopoles that are topological solitons of spontaneously broken gauge theories, the discussions above do not apply when the symmetry is unbroken, as then the monopole solution does not exist. Thus strong magnetic fields can exist without producing monopoles while the cosmic temperature and/or the Hubble rate is larger than the symmetry breaking scale, $\sigma < \max.\{T, H\}$. Even with a low temperature and Hubble rate, the magnetic field itself may restore the symmetry if it is stronger than $B_\star$ [48–50]. (The explosive production of monopole-antimonopole pairs at $B \sim B_\star$ may be related to this symmetry restoration.) However, if the symmetry is unbroken at some time during the postinflation era, then later at the symmetry breaking phase transition, monopoles are copiously produced and eventually overdominate the Universe, unless the symmetry breaking scale is very low. It should also be noted that this monopole problem is particularly severe if the phase transition happens prior to radiation domination (see Appendix C).

Hence, although the magnetic field limits can, in principle, be evaded by keeping the symmetry unbroken, one has to pay the price of endangering the Universe with the monopole problem. Therefore, it is safe to assume also for solitonic monopoles that primordial magnetic fields beyond the aforementioned upper limits could not have existed in the postinflation Universe.

We also remark that in a phase of broken symmetry, the magnetic field strength in a radiation-dominated Universe is bounded from above by the symmetry breaking scale, and thus is typically well below $B_\star$. On the other hand, stronger fields can exist in the preradiation-dominated era, during which solitonic monopoles can be abundantly produced. We will see this in detail in Sec. IV B.

E. Summary: A conservative bound

In the previous subsections we derived upper limits on the primordial magnetic field amplitude, beyond which the magnetic field self-screens by producing monopole-antimonopole pairs (3.5), or by accelerating the produced monopoles (3.9) and (3.13). A cosmological upper limit (3.17) was also derived by the requirement that the magnetic field does not overproduce monopoles in the Universe. These limits from self-screening and overproduction are all comparable to or smaller than $B_\star$, as one can check by following a discussion similar to that below (3.5).

Potential loopholes to the individual limits were discussed in each subsection, but let us give a few more remarks:

(i) *Corrections to pair production rate.* The expression (2.1) for Γ used in our analyses can break down at the values of B where self-screening or monopole overproduction takes place, if (a) the weak field conditions are violated, or (b) the cosmic temperature/Hubble scale are sufficiently high to induce finite-temperature/gravitational corrections. We discussed below (3.5) that (a) is unlikely for $g \gg 1$, however, there can still be some corrections to Γ since $B_\star$ only marginally satisfies the second weak field condition (2.3). Regarding (b), note that the radius (2.4) of the classical instanton solution at $B_\star$ is $R_\star = g^2/(4\pi m)$. For solitonic monopoles whose masses are related to the symmetry breaking scale typically via $m \sim g\sigma$, one finds during the symmetry broken phase, i.e., $\sigma > T, H$, that $(4\pi/g)R_\star \lesssim 1/T$, $1/H$. If $g \sim 10$, then we get $R_\star \lesssim 1/T, 1/H$, and hence we can safely use the zero-temperature and flat-space expression (2.1). The corrections, however, may become important if the actual limits such as B_{prod} are much smaller than $B_\star$, or in the late Universe when B is small. We also note that this discussion based on the symmetry breaking scale does not directly apply to elementary monopoles. When (2.1) breaks down, pair production tends to take place at a faster rate. Therefore, we expect that corrections to Γ, if any, can only make the upper limits on the primordial magnetic field more stringent.

(ii) *Interaction with thermal plasma.* We have ignored the interaction of monopoles with the thermal plasma, which can affect our discussions in the following ways: (a) The friction from the plasma may slow down the monopole (see e.g., [51]) and render the magnetic field dissipation via monopole acceleration less efficient. (b) Strong magnetic fields in the preradiation-dominated era (such as those generated in the magnetogenesis scenarios of [37–39]) can give away large energy to the monopoles, which in turn may raise the temperature of the plasma. If such a monopole-mediated reheating were to happen, then it would modify the perturbative reheating history we assumed for evaluating the monopole relic density.

(iii) *Thermal production.* Monopoles can also be thermally produced in the early Universe. In particular for solitonic monopoles, according to the analysis in [28], there can be a temperature window below the symmetry breaking scale where an observable monopole abundance is thermally produced. However, this analysis assumes entropy conservation after the monopole production, and thus is modified in the preradiation-dominated epoch. In any case, the existence of such an additional monopole population would further tighten the magnetic field limits we discussed.

(iv) *Effects on magnetic field generation.* Our field limits should be applied to primordial magnetic fields after their generation process has completed. This is because during the magnetic field generation, the field can grow faster than it is dissipated by the monopoles, and/or the U(1) gauge theory itself is modified such that the magnetic energy density does not take the form $\rho_B = B^2/2$ (as is typically the case for Weyl symmetry-breaking magnetogenesis scenarios). If the monopoles are solitonic, then modifications of the gauge theory can further affect the monopole solution itself. We also note that the calculation of the monopole relic density is modified if the magnetic field generation completes before the end of inflation. It would be interesting to study how monopoles affect various magnetic field generating mechanisms.

While most of the effects discussed here and in each subsection further tighten our magnetic field limits, some of them may weaken the limits. However, we also note that none of the effects seem capable of evading all limits in one go. Thus we conclude that if either elementary or solitonic monopoles are contained in the physical theory, then the amplitude of primordial magnetic fields in the postinflation Universe is always bounded from above as

$$B \lesssim B_\star = 4\pi \frac{m^2}{g^3}. \tag{3.18}$$

We stress that this is a conservative upper bound, and the magnetic self-screening and/or monopole overproduction can happen with weaker fields.

IV. LIMITS ON MONOPOLE MASS AND PRIMORDIAL MAGNETIC FIELDS

A. General limits

We now discuss the implications of the bound (3.18) for primordial magnetic field generation and monopoles. Below we suppose the magnetic field to redshift consistently as $B \propto a^{-2}$, i.e., (2.8) with $p = 2$, after being generated. Then $B_i < B_\star$ imposes a lower bound on the scale factor (or equivalently an upper bound on the redshift) when the magnetic field generation completes,

$$a_i > a_\star = a_0 \left(\frac{B_0}{B_\star}\right)^{1/2}, \qquad (4.1)$$

with the right-hand side expressed in terms of the present-day magnetic field strength B_0. This in turn sets an upper bound on the Hubble scale as $H_i < H(a_\star)$. (We remind the reader that H_i is the Hubble scale at the completion of the magnetic field generation; see discussions below (2.12). For instance, in inflationary magnetogenesis scenarios where the magnetic fields are excited during the inflation epoch, H_i is equal to the Hubble scale at the end of inflation, $H_{\rm end}$.)

We assume the magnetic field generation was completed either at the end of inflation, or during the subsequent reheating or radiation-dominated epochs (i.e., $t_{\rm end} \leq t_i < t_{\rm eq}$), and adopt the usual postinflation history based on perturbative reheating as described in Sec. III C or Appendix B. Then calculating $H(a_\star)$ using (B2) and (B4), we obtain an upper limit on H_i, whose form depends on whether $a_\star$ is smaller or larger than the scale factor upon radiation domination $a_{\rm dom}$,[10]

$$H_i \lesssim H_{\rm dom} \min\left\{ \left(\frac{a_{\rm dom}}{a_\star}\right)^2, \left(\frac{a_{\rm dom}}{a_\star}\right)^{3/2} \right\},$$

$$\frac{a_{\rm dom}}{a_\star} \sim \left(\frac{H_{\rm dom}}{10^{14}\ {\rm GeV}}\right)^{-1/2} \left(\frac{B_0}{10^{-15}\ {\rm G}}\right)^{-1/2} \left(\frac{mg^{-3/2}}{10^{11}\ {\rm GeV}}\right).$$

$$(4.2)$$

The combination $mg^{-3/2}$ derives from (3.18), which traces back to the ratio between the two terms in the exponent of Γ, cf. (2.1). This upper limit on H_i can also be written as a lower limit on the monopole mass,

$$\frac{m}{g^{3/2}} \gtrsim 10^{11}\ {\rm GeV} \left(\frac{B_0}{10^{-15}\ {\rm G}}\right)^{1/2} \left(\frac{H_i}{10^{14}\ {\rm GeV}}\right)^{1/2}$$

$$\times \max\left\{1, \left(\frac{H_i}{H_{\rm dom}}\right)^{1/6}\right\}. \qquad (4.3)$$

Thus we have obtained a consistency bound on monopoles (m, g) and primordial magnetic fields (B_0, H_i), for a given postinflation history characterized by $H_{\rm dom}$.

The temperature at the onset of radiation domination $T_{\rm dom}$, which is often referred to as the reheat temperature, is related to $H_{\rm dom}$ via (B4). It is required to lie within the range $10^{-3}\ {\rm GeV} \lesssim T_{\rm dom} \lesssim 10^{16}\ {\rm GeV}$, where the lower bound comes from BBN, and the upper bound is from the observational limit on the energy scale of inflation [42]. The reference value for the present-day magnetic field in

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[10]Since $a_\star/a_0 \sim 10^{-29}(B_0/10^{-15}\ {\rm G})^{1/2}(mg^{-3/2}/10^{11}\ {\rm GeV})^{-1}$, we can safely assume that $a_\star \ll a_{\rm eq} \approx a_0/3000$. On the other hand, depending on the inflation scale, $a_\star$ can even be smaller than $a_{\rm end}$. In such cases our $H(a_\star)$, obtained using (B2) which is valid for $t_{\rm end} < t < t_{\rm eq}$, gives a conservative upper limit on H_i.
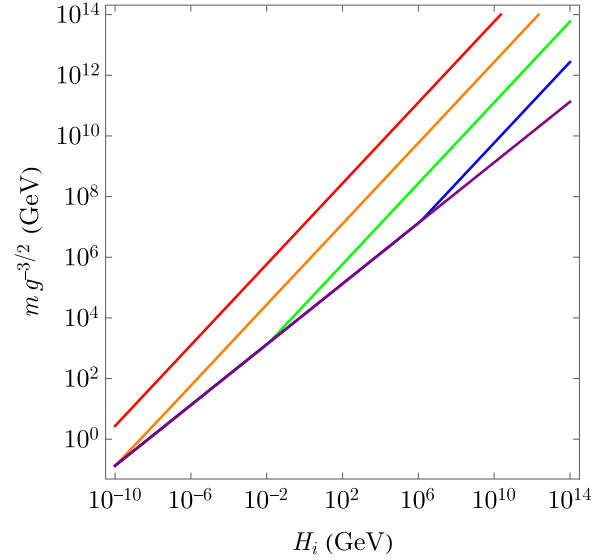

FIG. 1. Lower limit on $mg^{-3/2}$ (m: monopole mass, g: magnetic charge) as a function of the Hubble scale H_i when primordial magnetic fields are initially generated. The present-day magnetic field strength is taken as $B_0 = 10^{-15}$ G. The cosmic temperature when radiation domination takes over is varied as $T_{\rm dom} = 10^{16}$ GeV (purple), 10^{12} GeV (blue), 10^8 GeV (green), 10^4 GeV (orange), and 1 GeV (red).

the above expressions is taken from the claimed lower limit $B_0 \gtrsim 10^{-15}$ G on intergalactic magnetic fields from gamma ray observations [22–24]. (If the coherence length λ of the magnetic field is much smaller than a Mpc, then the lower limit improves as $\lambda^{-1/2}$.) We also remark that a primordial magnetic field, if homogeneous, is bounded from above as $B_0 \lesssim 10^{-9}$ G from CMB anisotropies [52], although it has also been claimed that this limit is relaxed in the presence of free-streaming particles like neutrinos [53].

In Fig. 1 we plot the lower limit (4.3) on $mg^{-3/2}$ as a function of H_i (or equivalently the upper limit (4.2) on H_i in terms of $mg^{-3/2}$). Here, the present-day magnetic field strength is fixed to the minimum value for intergalactic magnetic fields, $B_0 = 10^{-15}$ G, and the limits are shown for $T_{\rm dom} = 10^{16}$ GeV (purple), 10^{12} GeV (blue), 10^8 GeV (green), 10^4 GeV (orange), and 1 GeV (red). The colored lines in the plot overlap at $H_i \leq H_{\rm dom}$, where the limit becomes independent of $H_{\rm dom}$; in other words, the bend in the line is at $H_i = H_{\rm dom}$. As one goes towards larger H_i (magnetic field generation at earlier times), a stronger initial magnetic field is required to survive the more substantial redshifting, and therefore, the lower limit on $mg^{-3/2}$ becomes more stringent. The same is true for lower $T_{\rm dom}$ when $H_i > H_{\rm dom}$, which can be understood from the fact that the Universe expands more rapidly during an inflaton domination than radiation domination.

In the parameter regions slightly above the colored lines, an observable abundance of monopoles, but not so large as
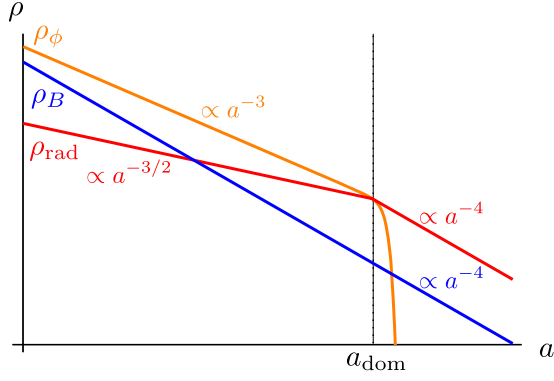
FIG. 2. Schematic of the evolution of energy densities in the reheating and radiation-dominated epochs as functions of the scale factor, in log-log scale. Shown are the energy densities of an oscillating inflaton (orange), radiation (red), and primordial magnetic fields (blue). Here the magnetic field energy density is extrapolated back to the left edge of the plot (which corresponds to some time during reheating), but there is actually a cutoff corresponding to the time when the magnetic fields are generated. See the text for details.

to overdominate the Universe, could be produced. One sees that even monopoles of GUT scale mass ($m = 10^{16}$ GeV with, say, $g = 10$ gives $mg^{-3/2} \sim 10^{14}$ GeV) are produced if magnetic field generation takes place at sufficiently high energy scales.

Our bound also sets an upper limit on the scale H_i of magnetic field generation, for a given value of $mg^{-3/2}$. Let us also comment on other bounds on H_i. Firstly, as we are assuming the magnetic field generation to conclude between the end of inflation and matter-radiation equality, the Hubble scale should lie within the range 10^{-37} GeV $\lesssim H_i \lesssim 10^{14}$ GeV, where the upper bound is the observational limit on the inflation scale. Secondly, we have assumed that the magnetic field only gives a subdominant contribution to the total energy density of the Universe. The time evolution of the energy densities in the postinflation era is illustrated in Fig. 2. Here the orange line denotes the energy density of an oscillating inflaton ρ_ϕ, and the red line denotes the radiation energy density $\rho_{\rm rad}$, which is created by the decay of the inflaton. Since the generation of primordial magnetic fields and reheating are, in general, different processes, we discuss the energy density of the magnetic field ρ_B separately from $\rho_{\rm rad}$ and denote it by the blue line in the figure. By extrapolating the magnetic energy density back in time as $\rho_B \propto a^{-4}$, it can overtake ρ_ϕ in the reheating epoch and dominate the Universe, which would signal that the cosmological expansion history was once significantly affected by the magnetic field. However, the scaling $\rho_B \propto a^{-4}$ is actually cut off at the time t_i, and we constrain this by requiring that the magnetic energy density never dominated the Universe. By using (B2) and (B4), the magnetic energy fraction at t_i is written as

$$\frac{\rho_{Bi}}{3M_{\rm Pl}^2 H_i^2} \sim 10^{-19} \left(\frac{B_0}{10^{-15}\,{\rm G}} \right)^2 \max\left\{ 1, \left(\frac{H_i}{H_{\rm dom}} \right)^{2/3} \right\}, \quad (4.4)$$

which is smaller than unity if $H_i \leq H_{\rm dom}$ and $B_0 \lesssim 10^{-6}$ G. However, in cases with $H_i > H_{\rm dom}$, then $\rho_{Bi} < 3M_{\rm Pl}^2 H_i^2$ requires[11]

$$H_i \lesssim 10^{10}\,{\rm GeV} \left(\frac{B_0}{10^{-15}\,{\rm G}} \right)^{-3} \left(\frac{T_{\rm dom}}{1\,{\rm GeV}} \right)^2, \quad (4.5)$$

where we have rewritten $H_{\rm dom}$ in terms of $T_{\rm dom}$. This condition is satisfied on the limits displayed in the plot; e.g., on the red line ($T_{\rm dom} = 1$ GeV), the condition is violated at $H_i \gtrsim 10^{10}$ GeV, which is around the upper edge and beyond. Both upper limits on H_i, (4.2) and (4.5), are tightened by a larger value of B_0; we will see this explicitly below.

B. Further limits for solitonic monopoles

For solitonic monopoles of spontaneously broken gauge theories, the mass limit (4.3) can be evaded by keeping the symmetry unbroken when the magnetic field is generated. However, in such a case the monopoles produced later at the symmetry breaking phase transition would induce a monopole problem, unless the symmetry breaking scale is very low[12] (see also discussions in Sec. III D and Appendix C). Thus we can combine the requirement to avoid a postinflation symmetry breaking with the mass limit and give further constraints for solitonic monopoles.

In the following, for concreteness, we study the vanilla 't Hooft-Polyakov monopole of an SO(3) gauge theory spontaneously broken to U(1) [2,3]. In this case the monopole mass is related to the vacuum expectation value of a triplet Higgs field σ, which we also refer to as the symmetry breaking scale, via $m \sim g\sigma$ (the exact value depends also on the Higgs self-coupling [29]). The magnetic charge is $g = 4\pi/e$ in terms of the gauge coupling e.

To avoid a symmetry breaking after inflation, the symmetry breaking scale should be high enough to satisfy $\sigma > \max\{T, H\}$ throughout the postinflation Universe. During radiation domination, this implies $\rho_B \sim B^2 \lesssim \rho_{\rm rad} \sim T^4 \lesssim \sigma^4$ (here we neglect numerical coefficients). Hence with $m \sim g\sigma$, we get $B \lesssim gB_\star$. Thus one sees that if, say, $g = \mathcal{O}(10)$, then unless the inequalities are close to being saturated, the magnetic field in the symmetry broken phase is well below the threshold value $B_\star$ for significant

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[11]If the coherence length of the primordial magnetic field happens to be close to the CMB scales, then the magnetic energy density is further restricted from discussions on curvature perturbations.

[12]A very low scale postinflation symmetry breaking might avoid cosmological issues while allowing for an initially very strong primordial magnetic field to survive until today, but we do not pursue this direction further herein.
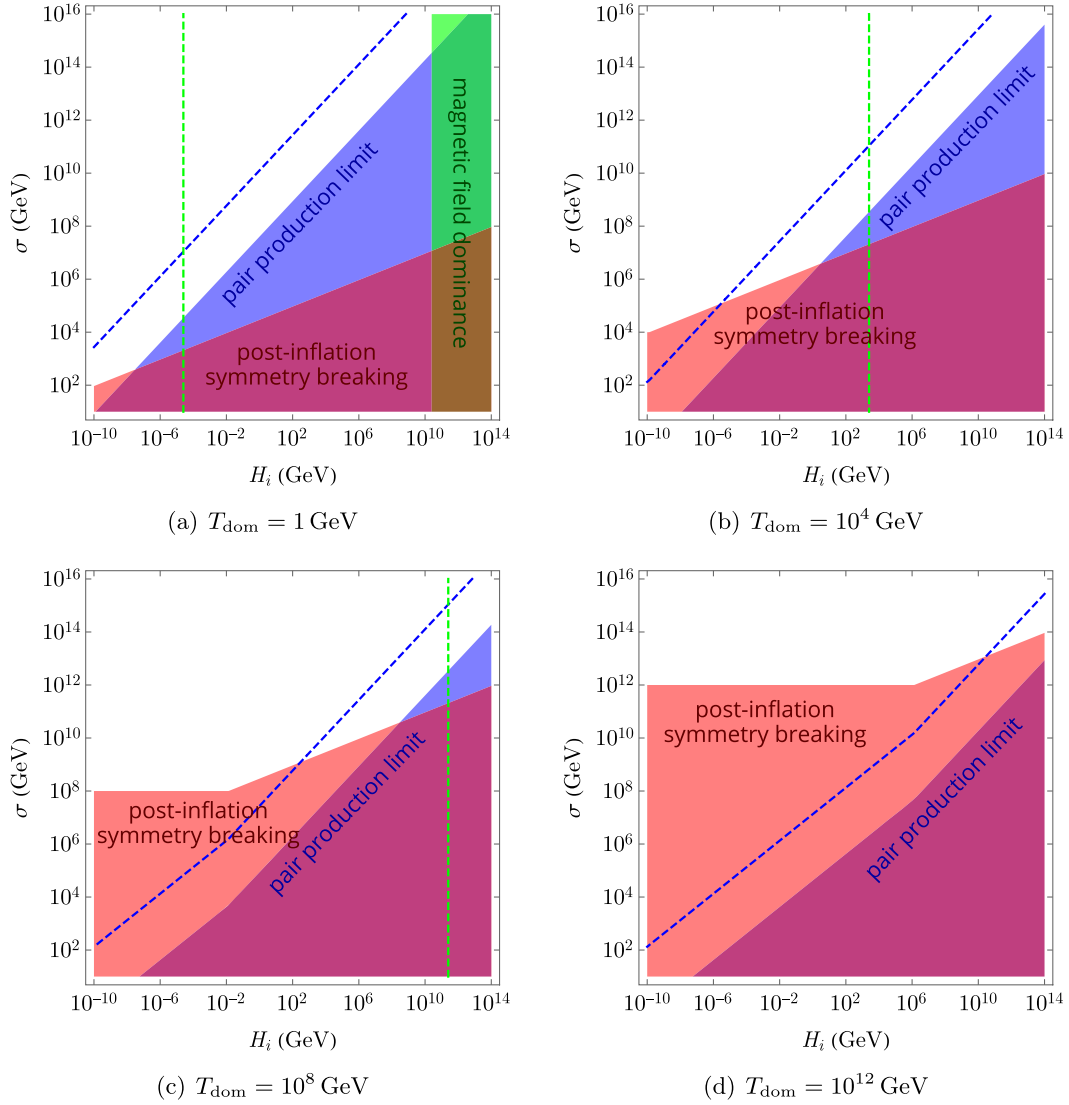
(a) $T_{\rm dom} = 1\,{\rm GeV}$

(b) $T_{\rm dom} = 10^4\,{\rm GeV}$

(c) $T_{\rm dom} = 10^8\,{\rm GeV}$

(d) $T_{\rm dom} = 10^{12}\,{\rm GeV}$

FIG. 3. Parameter space of 't Hooft-Polyakov monopoles in the plane of the Hubble scale H_i when primordial magnetic fields are initially generated, and the symmetry breaking scale σ. The magnetic charge is fixed to $g = 10$, and the present-day magnetic field strength to $B_0 = 10^{-15}$ G. The cosmic temperature $T_{\rm dom}$ when radiation domination begins is varied in the four plots. The blue region violates the monopole mass limit (4.3) obtained by the analyses of pair production in primordial magnetic fields. The green region is excluded by magnetic field dominance in the early Universe. In the red region the symmetry breaking phase transition happens after inflation. For a stronger magnetic field $B_0 = 10^{-10}$ G, the left edges of the blue and green regions shift to the positions depicted by the dashed lines.

monopole production (although there can still be non-negligible effects below $B_\star$ as discussed in the previous sections). However, this is no longer the case in the epoch prior to radiation domination, where ρ_B can be larger than $\rho_{\rm rad}$, while being smaller than the dominant inflaton energy density (cf. Fig. 2). This kind of situation can arise, for instance, in magnetic field generating mechanisms that invoke a violation of the Weyl invariance of the Yang-Mills action (see e.g., [37,38]); these take place only in a cold Universe, such as during inflation, since otherwise electrically charged particles in the thermal plasma freeze in the magnetic flux. While such mechanisms are in operation, the

energy density of the magnetic field is typically much larger than that of the radiation component.

In Fig. 3 we show the parameter space of 't Hooft-Polyakov monopoles in the $H_i - \sigma$ plane, where we took $m = g\sigma$, $g = 10$, and $B_0 = 10^{-15}$ G. $T_{\rm dom}$ is varied in the four plots as 1 GeV, 10^4 GeV, 10^8 GeV, and 10^{12} GeV. The blue region is excluded by the lower limit (4.3) on the monopole mass and corresponds to that shown in Fig. 1. The green region violates the magnetic field energy bound (4.5), which is seen only in the plot for $T_{\rm dom} = 1$ GeV, since for $T_{\rm dom} \gtrsim 10^2$ GeV the upper limit on H_i from this bound exceeds the highest possible inflation scale. The red

region shows where $\sigma < \max\{T_i, T_{\mathrm{dom}}\}$, indicating that the symmetry breaking takes place after inflation and thus possibly gives rise to a monopole problem. Here, T_i is given in terms of H_i through (B2) and (B3).[13] We do not show where $\sigma < \max\{H_i, H_{\mathrm{dom}}\}$ since it only gives constraints weaker than the other conditions in the displayed parameter regions.

As we have already discussed, the requirement of $\sigma > \max\{T_i, T_{\mathrm{dom}}\}$ gives a stronger constraint than the mass limit (4.3) during radiation domination ($H_i < H_{\mathrm{dom}}$), and serves as the dominant constraint in the entire displayed space in the plot for $T_{\mathrm{dom}} = 10^{12}$ GeV. For the other plots with lower T_{dom}, the mass limit dominates at $H_i \gg H_{\mathrm{dom}}$, i.e., if the primordial magnetic field is generated long before radiation domination. The combination of $\sigma > \max\{T_i, T_{\mathrm{dom}}\}$ and the mass limit (4.3) put severe constraints on symmetry breaking at intermediate and low scales. For instance, the necessary condition for $\sigma = 10^8$ GeV to evade the two constraints is that $T_{\mathrm{dom}} < 10^8$ GeV and $H_i \lesssim 10^3$ GeV are both satisfied.

The condition $\sigma > \max\{T_i, T_{\mathrm{dom}}\}$ is independent of B_0, while the limits (4.3) and (4.5) become stronger for a larger B_0. In the plots we also show (4.3) and (4.5) for $B_0 = 10^{-10}$ G, by the blue and green dashed lines, respectively. With this larger B_0, the monopole mass limit is tightened by about two orders of magnitude and overtakes the constraint from $\sigma > \max\{T_i, T_{\mathrm{dom}}\}$ in a wider parameter range. The magnetic energy bound is also tightened and is seen to constrain the high-H_i regions in the plots with T_{dom} up to 10^8 GeV.

Note that, to keep the discussion general, we have not specified the inflation scale. We focused on the time t_i at the end of magnetic field generation, which may coincide with the end of inflation, but can also be at some later time. Accordingly, we only imposed $\sigma > \max\{T_i, H_i, T_{\mathrm{dom}}, H_{\mathrm{dom}}\}$, i.e., the symmetry to be broken by the time when magnetic field generation completes or radiation domination begins, whichever happens earlier, instead of imposing $\sigma > \max\{T, H\}$ since the end of inflation. The actual lower bound on σ for evading a postinflation symmetry breaking would be tighter than shown in the figure, if radiation domination and magnetic field generation take place long after the end of inflation.

V. CONCLUSIONS

We showed that the process of pair production in primordial magnetic fields provides an excellent opportunity to confront magnetic monopoles with astrophysical observations. We analyzed two major consequences of the monopole pair production: (i) Primordial magnetic fields

dissipate energy by producing the monopole pairs and subsequently accelerating them. This fact that the field self-screens yields a consistency condition for primordial magnetic fields to survive until today and explain the observed magnetic fields. (ii) The pair produced monopoles can give rise to a new type of monopole problem, which gives a cosmological bound on monopoles and primordial magnetic fields. After evaluating the constraints from each effect, we used the most conservative bound on the primordial magnetic field amplitude (3.18) to derive a lower limit on the monopole mass (4.3):

$$
\begin{aligned}
m \gtrsim 10^{13} \text{ GeV} \left(\frac{g}{20}\right)^{3/2} \left(\frac{B_0}{10^{-15} \text{ G}}\right)^{1/2} \\
\times \left(\frac{H_i}{10^{14} \text{ GeV}}\right)^{1/2} \max\left\{1, \left(\frac{H_i}{H_{\mathrm{dom}}}\right)^{1/6}\right\}. \quad (5.1)
\end{aligned}
$$

Here g is the magnetic charge of the monopole, B_0 is the present-day magnetic field strength, H_i is the Hubble scale when the primordial magnetic field is initially generated, and H_{dom} is the Hubble scale when radiation domination begins. This limit also serves as an upper bound on the scale of magnetic field generation. A primordial magnetic field that seeds the observationally suggested intergalactic magnetic fields of $B_0 \gtrsim 10^{15}$ GeV imposes constraints on monopoles for a wide mass range (Fig. 1). This also sets a constraint on grand unified theories, which is particularly severe for models with intermediate and low scale symmetry breaking. Moreover, we showed that even super-heavy monopoles of $m \sim 10^{16}$ GeV can be abundantly produced if primordial magnetic fields exist at sufficiently high redshifts.

It is also important to know the exact abundance of monopoles produced in primordial magnetic fields, in order to make concrete predictions for monopole search experiments. Because the pair production rate depends exponentially on the magnetic field, the threshold field strengths for magnetic self-screening and monopole overabundance are typically of the same order. Moreover, this threshold value may only marginally satisfy the weak field condition invoked in the instanton calculation of the pair production rate. Therefore, a precise evaluation of the monopole abundance would require solving the full system, including the backreaction from the monopoles on the magnetic field, with possible corrections to the pair production rate for marginally weak fields. Additional effects that deserve careful studies are listed in Sec. III E. Taking into account all of them can be nontrivial, however, there may be fortunate circumstances where some effects decouple from the rest to simplify the analysis. Alternatively, if some effects can be argued to work only in a certain direction, such as to always enhance the pair production, then one can ignore those effects to derive conservative limits, which is the strategy adopted in this paper. Despite being conservative, our constraint should be useful since it

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[13]A perturbative reheating is assumed here. If instead the inflaton decays nonperturbatively (so-called preheating), then the evolution of the cosmic temperature at $T > T_{\mathrm{dom}}$ could be modified.

applies to monopoles with a wide mass range, including superheavy ones that are practically impossible to probe in colliders.

Magnetic field generation in the early Universe can be accompanied by a simultaneous generation of electric fields, which are considered to eventually short out during the reheating process. It would be interesting to study monopole production in primordial magnetic and electric fields before the latter vanish (see e.g., [54,55] for studies of Schwinger pair production in electric and magnetic fields). We also note that our analyses can be extended to the production of dyons [56] from primordial electromagnetic fields. Finally, we note that the pair production in primordial fields works equally effective for monopoles and magnetic fields of hidden U(1) gauge fields, therefore, it can provide a new production mechanism for hidden monopole dark matter [57].

Ultimately, one wishes to probe theories of monopoles and quantum vacuum instability via astrophysical measurements of cosmological magnetic fields and, in turn, to reveal the origin of cosmological magnetic fields by studying monopole pair production. This paper serves as a first step towards this goal.

APPENDIX A: GENERAL DISCUSSION OF MAGNETIC FIELD DISSIPATION BY MONOPOLES

We present a general discussion on the dissipation of cosmological magnetic fields by monopoles.

1. General formalism

The physical energy density of a spatially homogeneous magnetic field in a FRW background Universe obeys

$$d[\rho_B(t)a(t)^3] = -P_B(t)d[a(t)^3] - 2m\Gamma(t)a(t)^3 dt$$
$$- 2gB(t)dt \int_{-\infty}^{t} dt' \, a(t')^3 \Gamma(t') v(t',t), \quad \text{(A1)}$$

where P_B is the pressure of the magnetic fluid, and Γ is the rate of monopole-antimonopole pair production by the magnetic field. The second term in the right-hand side denotes the magnetic field energy being depleted by $2m$ for the production of each pair, assuming the pairs to be produced at rest [this term corresponds to (3.4)]. The third term represents the energy loss by accelerating the population of pairs produced from the infinite past to time t, where it should be noted that $a(t')^3\Gamma(t')dt'$ gives the

comoving number density of pairs produced between t' and $t' + dt'$. Moreover, we used $v(t',t)$ to denote the velocity of monopoles produced at t', measured at t ($\geq t'$), in the direction of the magnetic field [antimonopoles are taken to have charge $-g$ and velocity $-v(t',t)$]. We have ignored monopole-antimonopole annihilation.

Using $\rho_B = B^2/2$, and supposing a barotropic equation of state $P_B/\rho_B = (2p/3) - 1$ for the magnetic fluid, which amounts to supposing $B \propto a^{-p}$ in the absence of monopole production, then (A1) is rewritten as

$$\frac{\dot{\rho}_B}{\rho_B} = -\Pi_{\text{red}} - \Pi_{\text{prod}} - \Pi_{\text{acc}}, \quad \text{(A2)}$$

with the damping rates due to redshifting, monopole production, and monopole acceleration:

$$\Pi_{\text{red}}(t) = 2pH(t), \qquad \Pi_{\text{prod}}(t) = \frac{4m\Gamma(t)}{B(t)^2},$$

$$\Pi_{\text{acc}}(t) = \frac{4g}{a(t)^3 B(t)} \int_{-\infty}^{t} dt' \, a(t')^3 \Gamma(t') v(t',t). \quad \text{(A3)}$$

The depletion of the magnetic field energy can be studied by solving this equation, combined with the expression (A6) for $v(t',t)$ given below.

2. Monopole velocity

The motion of a monopole with magnetic charge g in a homogeneous magnetic field and a FRW background spacetime is described by the equation of motion,

$$\frac{m}{a}\frac{d}{dt}(a\gamma v) = gB, \qquad \gamma = \frac{1}{\sqrt{1-v^2}}, \quad \text{(A4)}$$

where v is the velocity in the direction of the magnetic field, and we neglected motion perpendicular to the magnetic field. This is integrated as

$$\gamma(t',t)v(t',t) = \frac{g}{ma(t)}\int_{t'}^{t} dt'' \, a(t'')B(t''), \quad \text{(A5)}$$

or equivalently,

$$v(t',t) = \frac{\frac{g}{ma(t)}\int_{t'}^{t} dt'' \, a(t'')B(t'')}{\sqrt{1 + [\frac{g}{ma(t)}\int_{t'}^{t} dt'' \, a(t'')B(t'')]^2}}. \quad \text{(A6)}$$

Here t is the time when the velocity is measured, and t' is when the monopole was initially produced at rest.

For example, if the magnetic field scales as $B \propto a^{-p}$, and the Hubble rate as $H \propto a^{-3(1+w)/2}$ with a constant equation of state w, then the integral can be directly performed as

$$\gamma v = \frac{1}{1-\nu}\frac{gB'}{mH'}\left[\left(\frac{a'}{a}\right)^{\nu} - \left(\frac{a'}{a}\right)\right], \qquad \nu = p - \frac{3(1+w)}{2},$$

(A7)

where $a = a(t)$, $a' = a(t')$, etc. The behavior of γv in the asymptotic future is as follows: If $\nu > 0$, then it decays in time as either $\gamma v \propto a^{-1}$ ($\nu > 1$), $\gamma v \propto a^{-1} \ln a$ ($\nu = 1$), or $\gamma v \propto a^{-\nu}$ ($0 < \nu < 1$). If $\nu < 0$, then it grows as $\gamma v \propto a^{|\nu|}$. If $\nu = 0$, i.e., the equation of state of the Universe equals that of the magnetic fluid, then it asymptotes to a constant value $\gamma v \to gB'/mH'$.

3. Case studies of dissipation by monopole acceleration

Let us evaluate the dissipation rate by monopole acceleration Π_{acc} under the simplifying assumption that the magnetic field is suddenly switched on at time t_i, then subsequently redshifts as $B \propto a^{-p}$ with a positive p of order unity (while the dissipation by monopoles is negligible). Then with Γ of the form (2.1), the integral for Π_{acc} is dominated by the contribution from $t_i \leq t' \lesssim t_i + \Delta t_{\Gamma i}$, where $\Delta t_{\Gamma i} = \epsilon_i/pH_i$ [$\ll 1/H_i$, see discussions below (2.9)]. Ignoring the variation of $a^3\Gamma$ during this period (which implicitly assumes $\Delta t_{\Gamma i} < \Pi_{\mathrm{prod}i}^{-1}, \Pi_{\mathrm{acc}i}^{-1}$), we get

$$\Pi_{\mathrm{acc}}(t) \simeq \frac{4g\Gamma_i}{B(t)}\left(\frac{a_i}{a(t)}\right)^3 \int_{t_i}^{t_i+\Delta t_{\Gamma i}} dt'\, v(t',t), \quad \text{(A8)}$$

for $t \geq t_i + \Delta t_{\Gamma i}$. For a further evaluation, we consider some limiting cases below.

a. Monopoles produced on the spot

We start by considering the times $\Delta t_{\Gamma i} \ll t - t_i \lesssim 1/H_i$, which are within a Hubble time after the magnetic field is switched on. During this period the expansion of the Universe can be ignored. Further supposing B to be nearly constant (which amounts to ignoring the backreaction from the monopoles), then (A8) is approximated as

$$\Pi_{\mathrm{acc}}(t) \simeq \frac{4g\Gamma_i}{B_i}\int_{t_i}^{t_i+\Delta t_{\Gamma i}} dt'\, v(t',t). \tag{A9}$$

Likewise, the monopole velocity (A6) in the integral is approximated as

$$v(t',t) \simeq \frac{\frac{gB_i}{m}(t-t')}{\sqrt{1 + [\frac{gB_i}{m}(t-t')]^2}}. \tag{A10}$$

If the initial magnetic field is sufficiently strong, such that the monopoles are relativistic at the time of consideration, i.e., $(gB_i/m)(t-t_i) \gg 1$, then $\int dt'\, v \simeq \Delta t_{\Gamma i}$ and we obtain

$$\Pi_{\mathrm{acc}} \simeq \frac{4g^2\Gamma_i}{\pi pm^2 H_i}. \tag{A11}$$

This matches with the dissipation rate $|(\dot\rho_B)_{\mathrm{R}}/\rho_B|$ of (3.6) with the substitution of (3.7) and $t = t_i$.

On the other hand for nonrelativistic monopoles, i.e., $(gB_i/m)(t-t_i) \ll 1$, then $\int dt'\, v \simeq (gB_i/m)\Delta t_{\Gamma i}(t-t_i)$ and we get

$$\Pi_{\mathrm{acc}} \simeq \frac{4g^3 B_i \Gamma_i}{\pi pm^3 H_i}(t-t_i). \tag{A12}$$

Using this to solve $\dot\rho_B/\rho_B = -\Pi_{\mathrm{acc}}$, one obtains $\rho_B = \rho_{Bi}\exp[-\{(t-t_i)/\Delta t_{\mathrm{acc}}\}^2]$ with the dissipation time scale given by

$$\Delta t_{\mathrm{acc}} = \left(\frac{\pi pm^3 H_i}{2g^3 B_i \Gamma_i}\right)^{1/2}. \tag{A13}$$

This matches with Δt_{NR} derived in (3.11) with the substitution of (3.7) and $t = t_i$.

b. Monopoles produced in the past

We now consider the times $t - t_i \gtrsim 1/H_i$. Here we assume for simplicity that $v(t',t) \simeq v(t_i,t)$ for $t_i \leq t' \leq t_i + \Delta t_{\Gamma i}$, i.e., most monopoles have the same velocity. Then $\int dt'\, v \simeq \Delta t_{\Gamma i} v(t_i,t)$, which gives

$$\Pi_{\mathrm{acc}} \simeq \frac{4g^2\Gamma_i}{\pi pm^2 H_i}\frac{B_i}{B(t)}\left(\frac{a_i}{a(t)}\right)^3 v(t_i,t). \tag{A14}$$

This denotes the rate of magnetic field dissipation by accelerating monopoles that have been produced at around the initial time t_i, which is well separated from the time t of consideration. We did not discuss this effect in the main text.

Let us focus on the time evolution of Π_{acc} with respect to the redshifting rate Π_{red},

$$\frac{\Pi_{\mathrm{acc}}}{\Pi_{\mathrm{red}}} \propto \frac{v(t_i,t)}{B(t)a(t)^3 H(t)}. \tag{A15}$$

For v, we can use (A7) when the Universe has an equation of state w and while the magnetic field scales as $B \propto a^{-p}$. For instance, with $p = 2$ and $w = 1/3$, the velocity v approaches a constant value, and thus the ratio grows asymptotically as $\Pi_{\mathrm{acc}}/\Pi_{\mathrm{red}} \propto a$. In this case, even if the dissipation by monopole acceleration is initially negligible, it can become important at later times. For $p = 2$ and $w = 0$, we get $\Pi_{\mathrm{acc}}/\Pi_{\mathrm{red}} \propto a^{1/2}$ while the monopoles are relativistic ($v \simeq 1$), and $\Pi_{\mathrm{acc}}/\Pi_{\mathrm{red}} = \text{const}$ when nonrelativistic ($v \propto a^{-1/2}$). It would be interesting to perform a systematic study of the dissipation effect by monopoles produced in the past.

APPENDIX B: HUBBLE SCALE DURING AND AFTER REHEATING

We give the expressions for the Hubble scale and cosmic temperature as functions of redshift during the reheating epoch and the subsequent radiation-dominated epoch. Here we assume that after inflation ends (at $t_{\rm end}$), the Universe is initially dominated by an oscillating inflaton field, which undergoes perturbative decay into radiation; the radiation component eventually comes to dominate the Universe ($t_{\rm dom}$), until it gives way to matter domination at matter-radiation equality ($t_{\rm eq}$). A case of an instantaneous reheating, i.e., a sudden decay of the inflaton at the end of inflation, is handled by setting $t_{\rm end} = t_{\rm dom}$ in the following discussions.

During radiation domination ($t_{\rm dom} \ll t \ll t_{\rm eq}$), we have $3M_{\rm Pl}^2 H^2 \simeq \rho_{\rm rad}$, where $\rho_{\rm rad} = (\pi^2/30)g_* T^4$ is the radiation energy density, and T is the radiation temperature. Combining this with the assumption that the entropy is conserved until today, namely, that the entropy density redshifts as $s = (2\pi^2/45)g_{*s}T^3 \propto a^{-3}$, one obtains

$$H \simeq \left(\frac{45}{128\pi^2}\right)^{1/6} \frac{g_*^{1/2} s_0^{2/3}}{g_{*s}^{2/3} M_{\rm Pl}}\left(\frac{a_0}{a}\right)^2, \quad T \simeq \left(\frac{45}{2\pi^2}\frac{s_0}{g_{*s}}\right)^{1/3}\frac{a_0}{a}.$$
(B1)

The subscript "0" denotes quantities in the present Universe.

When the Universe is dominated by an oscillating inflaton field ($t_{\rm end} \ll t \ll t_{\rm dom}$), it is effectively matter dominated and thus $H \propto a^{-3/2}$. The radiation density during this epoch is sourced by the perturbative decay of the inflaton, and thus redshifts as $\rho_{\rm rad} \propto a^{-3/2}$ when ignoring the time dependence of g_*; this can be checked by solving the continuity equation $\dot\rho_{\rm rad} + 4H\rho_{\rm rad} = \Gamma_\phi \rho_\phi$, with Γ_ϕ being the inflaton decay rate, and the energy density of the inflaton $\rho_\phi = \rho_{\phi{\rm end}}(a_{\rm end}/a)^3 e^{-\Gamma_\phi(t-t_{\rm end})}$ [58]. Hence the radiation temperature redshifts as $T \propto a^{-3/8}$.

Connecting the scaling behaviors in the two epochs at $t_{\rm dom}$, the Hubble rate and radiation temperature during the radiation-dominated epoch ($t_{\rm dom} < t < t_{\rm eq}$) and reheating epoch ($t_{\rm end} < t < t_{\rm dom}$) are collectively written as follows:

$$H \sim H_{\rm dom}\min\left\{\left(\frac{a_{\rm dom}}{a}\right)^2, \left(\frac{a_{\rm dom}}{a}\right)^{3/2}\right\}, \quad (B2)$$

$$T \sim T_{\rm dom}\min\left\{\left(\frac{a_{\rm dom}}{a}\right), \left(\frac{a_{\rm dom}}{a}\right)^{3/8}\right\}, \quad (B3)$$

where we have ignored the time variation of $g_{*(s)}$ in (B1). The relations between $H_{\rm dom}$, $T_{\rm dom}$, and $a_{\rm dom}$ can be obtained by extrapolating (B1) to the time $t_{\rm dom}$; after plugging in numbers for $M_{\rm Pl}$ and the cosmological parameters, one gets

$$\frac{a_0}{a_{\rm dom}} \sim 10^{29}\left(\frac{H_{\rm dom}}{10^{14}\ {\rm GeV}}\right)^{1/2},$$

$$T_{\rm dom} \sim 10^{16}\ {\rm GeV}\left(\frac{H_{\rm dom}}{10^{14}\ {\rm GeV}}\right)^{1/2}. \quad (B4)$$

We remark that these depend only weakly on $g_{*(s)}$, hence its detailed value does not affect the order-of-magnitude estimates.

APPENDIX C: MONOPOLE ABUNDANCE PRODUCED AT PHASE TRANSITIONS

In this appendix we consider solitonic monopoles produced at a symmetry breaking phase transition that happens after inflation. Hence the critical temperature $T_{\rm c}$ at the phase transition is assumed to be lower than the maximum temperature achieved during reheating, or the inflationary Hubble scale, i.e., $T_{\rm c} < \max\{T_{\rm max}, H_{\rm inf}\}$. We consider a postinflation history as discussed in Appendix B, and obtain lower limits on the monopole abundance for cases where the phase transition takes place during the reheating epoch and during the radiation-dominated epoch.

Considering that at least one monopole or antimonopole is created within a Hubble volume after the phase transition, the monopole number density n_M follows $n_{Mc} \geq H_{\rm c}^3$, where the subscript "c" denotes quantities at the phase transition. (Here we only compute a lower bound, but the actual density can be computed by evaluating the correlation length as discussed in [57,59,60].) Then supposing that monopole-antimonopole annihilation is negligible, and that the monopoles today are nonrelativistic, the lower bound on the relic density is

$$\rho_{M0} = mn_{M0} \geq mH_{\rm c}^3\left(\frac{a_{\rm c}}{a_0}\right)^3. \quad (C1)$$

If the phase transition happens during the radiation-dominated epoch, $t_{\rm dom} < t_{\rm c} < t_{\rm eq}$, by rewriting the Hubble rate and redshift in terms of the cosmic temperature using (B2), (B3), and (B4), one finds for the relic abundance,

$$\Omega_M h^2 \gtrsim 10^{-1}\left(\frac{m}{10^{13}\ {\rm GeV}}\right)\left(\frac{T_{\rm c}}{10^{11}\ {\rm GeV}}\right)^3. \quad (C2)$$

On the other hand if the phase transition happens prior to radiation domination, $t_{\rm end} < t_{\rm c} < t_{\rm dom}$,

$$\Omega_M h^2 \gtrsim 10^{-1}\left(\frac{m}{10^{13}\ {\rm GeV}}\right)\left(\frac{T_{\rm c}}{10^{11}\ {\rm GeV}}\right)^3\left(\frac{T_{\rm c}}{T_{\rm dom}}\right). \quad (C3)$$

Compared to (C2), this lower bound is enhanced by $T_{\rm c}/T_{\rm dom}$. This can be understood from the fact that for the same critical temperature $T_{\rm c}$, the Hubble scale at the phase transition $H_{\rm c}$ is larger during inflaton domination than during radiation domination, and thus the number of monopoles is enhanced.

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