Black holes and uptunneling suppress Boltzmann brains

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Eternally inflating universes lead to an infinite number of Boltzmann brains but also an infinite number of ordinary observers. If we use the scale factor measure to regularize these infinities, the ordinary observers dominate the Boltzmann brains if the vacuum decay rate of each vacuum is larger than its Boltzmann brain nucleation rate. Here, we point out that nucleation of small black holes should be counted in the vacuum decay rate, and this rate is always larger than the Boltzmann brain rate, if the minimum Boltzmann brain mass is more than the Planck mass. We also discuss nucleation of small, rapidly inflating regions, which may also have a higher rate than Boltzmann brains. This process also affects the distribution of the different vacua in eternal inflation.

I. INTRODUCTION

If the observed dark energy is in fact a cosmological constant, our Universe will expand forever and will soon approach de Sitter space. There will be an infinite volume in which many types of objects may nucleate. In particular, there will be an infinite number of Boltzmann brains [1],[1] human brains (or perhaps computers that simulate brains) complete with our exact memories and thoughts, that appear randomly as quantum fluctuations. Human beings (and their artifacts) can arise in the ordinary way for only a certain period of time after the big bang, when there are still stars and other necessities of life, but Boltzmann brains can arise at any time in the future. So one might conclude that the Boltzmann brains infinitely outnumber ordinary humans, and thus, that we are Boltzmann brains, a non-sensical conclusion [3] because our observations on which we base this conclusion would have no connection to the actual Universe in which we live.

However, in any scenario such as the above, it is also possible for new inflating regions to nucleate, leading to eternal inflation. In that case, there will be an infinite number of ordinary observers in addition to the infinite number of Boltzmann brains, so we may hope to avoid the conclusion that we are Boltzmann brains.[2] In order to know what to expect in cases with infinite numbers of observers, we need a measure: a procedure to regulate the infinities

and produce a sensible probability distribution. Any measure faces a number of difficulties [5–7], and we do not have any principle to tell us which measure is correct. An obvious selection criterion is that the measure should not make predictions that are in conflict with observation. This removes most of the measures that have been suggested so far. The proper time measure suffers from the "youngness paradox," predicting that the CMB temperature should be much higher than observed [8]; the causal patch measure predicts that the cosmological constant should be negative with an overwhelming probability [9]; the pocket based measure suffers from a "Q catastrophe," predicting either extremely small or large values of the density fluctuation amplitude Q [10,11]. A measure that fares reasonably well is the scale factor cutoff measure [12–14]. Other measures that have not been ruled out by observations (such as the light cone time cutoff, apparent horizon cutoff and four-volume cutoff measures) make predictions very similar to the scale factor cutoff. (For more details and references, see, e.g., [15].)

In the present paper, we shall adopt the scale factor cutoff measure, which we will discuss in more detail below. In this measure, the ratio of Boltzmann brains to ordinary observers in a given vacuum is roughly given by the ratio of the Boltzmann brain nucleation rate Γ_i^{BB} to the total decay rate of that vacuum Γ_i. Here, Γ_i^{BB} is the rate at which Boltzmann brains form per unit (physical) volume of vacuum i, and Γ_i is proportional to the total rate at which volume flows out of vacuum i (a precise definition of Γ_i will be given below).

We point out here that there are two processes that are not always considered that influence the vacuum decay rate. The first is the nucleation of small black holes. This process removes volume from the vacuum and so contributes to Γ_i. The rate is largest for the smallest black holes.

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[1]Similar ideas appeared earlier in [2].
[2]Don Page [4] argued for a more drastic solution: vacuum decay on a scale similar to or shorter than the present Hubble time avoids the infinite volume in which Boltzmann brains may form.

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As we will discuss below, it is always larger than the Boltzmann brain nucleation rate, if the minimum Boltzmann brain mass is larger than the Planck mass,[3] so the Boltzmann brain problem is solved in that case.

The other process is the nucleation of small regions of higher-energy inflating false vacuum. In the usual Lee-Weinberg [17] process, a region larger than the Hubble distance in the old vacuum tunnels to the new vacuum. But here we are considering a localized fluctuation that yields a region of the new vacuum large enough to inflate but much smaller than the old Hubble distance [18–20]. The higher the energy of the new vacuum, the smaller the region of it that is necessary for inflation. Thus, this process (unlike Lee-Weinberg tunneling) is least suppressed when the daughter vacuum energy is the highest. The most likely process is to produce the highest energy inflating vacuum. If this is at the Planck scale, suppression is similar to that of Planck-scale black hole production. Otherwise, it is more suppressed than that.

Nucleation of small high-energy regions is not discussed in most treatments of multiverse physics. We shall comment on the reason for that below and explain why we believe it should be included. This process upends the conventional wisdom that low-energy vacua are most likely to tunnel to other low-energy vacua. Uptunneling is still suppressed when the parent vacuum energy is small, but now the most likely daughters are the ones with the highest energy. To compute the probabilities in the scale factor measure, we construct a transition matrix between vacua and find its eigenvector whose eigenvalue is least negative. This is usually made up almost entirely by a single "dominant vacuum" [21,22], whose total decay rate is the least. The measures of other vacua depend on tunneling processes leading to them from the dominant vacuum. When we take into account production of small high-energy inflating regions, we can still find the dominant vacuum, but the details do not matter. The likeliest transition out of the dominant vacuum is to jump directly to the highest energy possible. At very high energies, transitions are little suppressed, so all vacua are quickly populated. The chance of any specific low-energy (and in particular anthropically allowed) vacuum depends now on how it may be reached by a sequence of transitions from high energies, with little effect from the details of the dominant vacuum.

The rest of this paper is organized as follows. In the next section, we review the scale factor measure and the resulting distribution of the different vacua and discuss the effects of nucleating black holes and small high-energy

regions. In Sec. III, we discuss the nucleation rates of black holes, Boltzmann brains, and regions of different vacuum. We discuss the effects of these processes on the Boltzmann brain problem in Sec. IV and on the distribution of the different vacua in Sec. V. We conclude in Sec. VI.

II. THE SCALE FACTOR CUTOFF

The scale factor measure was introduced by Linde and collaborators (e.g., [12]) and was worked out in detail in [13,14,23]. It is based on constructing a scale factor time that represents (the logarithm of) the total expansion that each point in spacetime has experienced. To make it well defined, we must start with some initial spacelike hypersurface Σ and follow a congruence of geodesics orthogonal to Σ. The scale factor time is then given by

$$\eta = \int_0^t \frac{\theta}{3} dt', \tag{1}$$

where t is proper time, and the expansion $\theta = u^\mu{}_{;\mu}$, with $u^\mu = dx^\mu/dt$ the tangent vector to the geodesics. In a homogeneous region of the Universe, the scale factor a is just $\exp \eta$.

To use this as a measure, we consider all events that take place before some cutoff time η_c. There are a finite number of these, so assigning probabilities is straightforward. Then, we take the limit of the probabilities as η_c grows without bound.

Unfortunately, when structures form, the local universe contracts instead of expanding, so η is not monotonic; we must make some provision for this case [14]. One plan would be to use a modified scale factor time $\tilde{\eta}$, where $\tilde{\eta}(x)$ is given by maximizing η over all points in the causal past of x. Thus, $\tilde{\eta}$ cannot decrease, and we avoid the possibility that an event allowed by the cutoff is in the future of a point excluded by the cutoff.[4] A number of other possibilities have been suggested [14,23].

It will not be important to our analysis here exactly how this issue is resolved, but for definiteness, we will use the above "maximum η" prescription. At any given scale factor, almost all the volume is in regions that are expanding. In such a region, the distance between two geodesics of the congruence is just their distance on the initial surface times the expansion of the scale factor. If we select an evenly spaced, very large but finite set of representative geodesics on the initial surface, all of these geodesics will represent equal volumes on the cutoff surface. Thus, the fraction of volume in each type of region is just the fraction of the initial geodesics that are there.

We will be interested in the number of Boltzmann brains and ordinary observers that appear in different vacua. Let us

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[3]A similar argument was made in [16] in the context of the "watcher measure." This measure makes the assumption that the big crunch singularities in anti–de Sitter (AdS) bubbles lead to bounces, where contraction is followed by expansion, so that geodesics can be continued through the crunch regions. We do not adopt this assumption in the present paper.

[4]Such a situation would lead to an inverse Guth-Vanchurin [6] paradox, where an observer may wake up without ever having gone to sleep.

start by defining $f_i(\eta)$ to be the fraction of comoving volume in vacuum i at time η. In expanding regions, the expansion factor a is the same everywhere on the constant-η surface, so f_i gives the fraction of physical volume. In contracting regions, we must make some adjustment, as described above. But such regions will not matter, as we discuss later.

The f_i obey the rate equation [21],

$$\frac{df_j}{d\eta} = \sum_i (-\kappa_{ij} f_j + \kappa_{ji} f_i), \qquad (2)$$

where κ_{ij} is the fraction of volume currently in vacuum j that transitions into vacuum i per unit scale factor time or equivalently, the chance per unit scale factor time for an observer in vacuum j to transition to vacuum i.

We can express κ_{ij} in terms of Γ_{ij}, the rate of tunneling events that produce vacuum i per unit physical spacetime volume of vacuum j. In general,

$$\kappa_{ij} = \frac{V_{ij}}{H_j} \Gamma_{ij}, \qquad (3)$$

where V_{ij} is the volume of space at a given time where a given tunneling event would lead to a given observer transitioning to the new vacuum. The expansion rate H_j of vacuum j in the denominator is the conversion between scale factor time and physical time.

In the Coleman-De Luccia process, a small region of lower-energy vacuum appears by tunneling and then expands to the horizon size. In the Lee-Weinberg process, a superhorizon region of a higher-energy vacuum appears by tunneling and then contracts in comoving size so the final comoving volume is just the comoving horizon at the time of nucleation. In either case, $V_{ij} = (4\pi/3) H_j^{-3}$.

Here, we will discuss two more processes. The first is the nucleation of black holes. A certain set of geodesics will fall into the black hole, hit the singularity, and be removed from the congruence.[5] They will reach a maximum η before they start to converge near the black hole; for larger η they will not be counted in the scale factor measure. They thus represent a flow of volume fraction out of the vacuum in which the black holes nucleate. In that sense, the process is similar to the creation of anti–de Sitter vacua that then collapse. We will describe black hole nucleation by a

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[5]Quantum gravity may resolve the singularity. It is not clear whether geodesics are still meaningful in such a scenario, but if they are we must ask where the geodesics go. A similar question affects the fate of geodesics after the crunch in anti–de Sitter regions. This was analyzed in [24,25] with the conclusion that the geodesics may travel through into new inflating vacua. Such a result would not affect our conclusions here, but if the geodesics instead return to the parent universe and resume expanding, black hole nucleation would not remove volume fraction.

transition rate κ_{0j} for each vacuum j and include it in Eq. (2) by including $i = 0$ in the sum.

If a black hole of mass M lives for a time long compared to the Hubble time, it will capture all geodesics within radius [16],

$$r_c \sim \left(\frac{GM}{H^2}\right)^{1/3}, \qquad (4)$$

which is the radius at which the attraction of the black hole gravity is balanced by the repulsive force due to the cosmological constant. The volume of geodesics absorbed is thus

$$V_c \sim \frac{GM}{H^2}. \qquad (5)$$

If the black hole is short-lived compared to the Hubble time, we can neglect the cosmological constant. A particle starting from rest at radius r will fall into the black hole on a time scale,

$$t \sim r^{3/2} (GM)^{-1/2}. \qquad (6)$$

We want $t < t_e$, where the evaporation time is[6] $t_e \sim G^2 M^3$. From this, we find that the capture radius and volume are

$$r_c \sim G^{5/3} M^{7/3} \sim GM \left(\frac{M}{M_{\text{Pl}}}\right)^{4/3} \qquad (7)$$

$$V_c \sim G^5 M^7 \sim (GM)^3 \left(\frac{M}{M_{\text{Pl}}}\right)^4. \qquad (8)$$

We will also consider the formation of regions of higher cosmological constant Λ_i that are smaller than the horizon of the parent vacuum j but larger than their own horizon. Such a region will inflate inside, but the outside will collapse into a black hole. As we mentioned in the Introduction, this nucleation process is often omitted in studies of multiverse dynamics. The main reason is that it does not fit into the standard Coleman-De Luccia formalism, where tunneling transitions are described by instantons. There are no known instantons corresponding to nucleation of small high-energy inflating regions. However, quantum transitions allowed by the conservation laws should occur with some nonzero probability. The state of a quantum field in de Sitter space is similar to a thermal state, and one expects that fluctuations of the scalar field ϕ and/or its velocity $\dot{\phi}$ will occur in localized regions of

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[6]We note that magnetically charged black holes may be much more stable. They can lose their magnetic charge only by emission of magnetic monopoles, which typically have large masses, so their emission may be strongly suppressed. The black hole may even be absolutely stable if monopole solutions of corresponding magnetic charge do not exist.

space. If the fluctuation is large enough, the field may acquire enough energy to fly over a potential barrier into a high-energy vacuum. And if the fluctuation extends over a superhorizon region in the new vacuum, it will produce an inflating baby universe [19,20].

Geometrically, it is clear that a rapidly inflating daughter region will be connected by a wormhole to the slowly inflating parent universe. The wormhole will close up in about one light crossing time, and both of its mouths will be seen as black holes.[7] After the black hole evaporates, the new inflating region is disconnected from the original universe, but there is no problem in applying the scale factor measure to the resulting set of disconnected universes.

If we could ignore gravitational effects, it would be easy to compute the energy necessary to create such a region. Let U_i be the energy density of the daughter vacuum. The expansion rate is thus $H_i \sim \sqrt{GU_i} = \sqrt{U_i}/M_{\rm Pl}$. The minimal volume to inflate would be a sphere of radius H_i^{-1}, which thus contains a volume,

$$V_i^{\rm nuc} \sim M_{\rm Pl}^3/U_i^{3/2}, \qquad (9)$$

and a mass,

$$M_i^{\rm nuc} \sim M_{\rm Pl}^3/U_i^{1/2}. \qquad (10)$$

This is modified by gravitation, but we will assume here that the effect is only to change the numerical factors that we did not compute, and so Eqs. (9) and (10) give the correct order of magnitude.

The new inflating volume must be surrounded by a bubble wall that interpolates between the two vacua. This is the same wall as in Lee-Weinberg and the inside-out version of the Coleman-De Luccia bubble wall. Suppose it is possible, as one normally expects, for a small bubble of vacuum j to form inside a Hubble volume of vacuum i. That means that the energy of this wall around a sphere of radius smaller than $1/H_i$ is less than the energy of the displaced volume of vacuum i. In the present case, we have a larger sphere, of radius $1/H_i$, which increases the ratio of volume to surface energy. So the wall energy will be much less than $M_i^{\rm nuc}$, which is the energy of the sphere of vacuum i of radius $1/H_i$, and there is no important correction to $M_i^{\rm nuc}$, from the wall.

Upon nucleation, we expect that the geodesics inside volume $V_i^{\rm nuc}$ will travel into the new inflating region.[8]

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[7]This process is similar to that described in [26]. It is also related to the process of [27], but in that paper, the authors propose deliberately constructing a region of high-energy vacuum that is not large enough to inflate and hoping that it tunnels to the inflating state, while here we propose creating the region as a fluctuation in de Sitter space.

[8]A geodesic congruence is not well defined when the spacetime undergoes a discontinuous change, as in quantum tunneling. But it should be possible to estimate, by an order of magnitude, what fraction of the initial comoving volume goes into each vacuum. This is typically all one needs in any anthropic analysis.

Shortly after that, the nucleated region will collapse into a black hole, and more geodesics will later fall into the black hole and end at the singularity, according to Eq. (7). So this process gives

$$\kappa_{ij} \sim \frac{V_i^{\rm nuc}}{H_j} \Gamma_{ij} \sim \Gamma_{ij} H_j^{-1} H_i^{-3}, \qquad (11)$$

and in addition, contributes

$$\sim \Gamma_{ij} H_j^{-1} M_{\rm Pl}^3 H_i^{-6}, \qquad (12)$$

to κ_{0j}.

Calculation of the relative abundance of Boltzmann brains and ordinary observers involves comparisons of extremely small numbers, such as tunneling transition rates κ_{ij} and Boltzmann brain nucleation rates. The tunneling actions are typically large, so these rates are double exponentially suppressed. The preexponential factors have therefore little effect, even though they can be very small or large. For this reason, the factors multiplying Γ_{ij} in Eqs. (3), (11), and (12) can be ignored, and we will omit them from now on.

III. NUCLEATION RATES

In this section, we review the usual nucleation rates for the Coleman-De Luccia and Lee-Weinberg cases and discuss nucleation of black holes, small inflating regions, and Boltzmann brains.[9]

A. Coleman-De Luccia and Lee-Weinberg nucleation

A metastable vacuum j may decay to a lower energy vacuum i through bubble nucleation. If we ignore the effects of gravity, we have the situation discussed by Coleman [32]. It proceeds by forming a bubble whose total energy is zero because the decreased energy of the vacuum inside compensates for the energy in the bubble wall. Including gravitation [33] leads to corrections, but these are small if the bubble size is small compared to the Hubble distance in both parent and daughter vacua. After formation, the bubble will expand rapidly because the force on the wall due to the difference in vacuum energies is larger than the effect of surface tension. Disregarding the preexponential factor, the bubble nucleation rate is given by

$$\Gamma_{ij} \sim e^{-I - S_j}, \qquad (13)$$

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[9]There is some controversy about the nucleation of objects in de Sitter space. References [28,29] claimed that there is no such nucleation, while [30] critiqued the claims of [29]. Reference [31] discussed the possibility that Boltzmann brains might form even in Minkowski space. Here, we will adopt the conventional view that objects nucleate in de Sitter space with the usual Boltzmann suppression and do not appear in Minkowski space.

where $I < 0$ is the instanton action and $S_j = \pi/H_j^2$ is the Gibbons-Hawking entropy of the parent vacuum j.

Lee and Weinberg [17] have argued that the same instanton should describe the inverse transition from i to j, where the daughter vacuum has a higher energy than the parent vacuum. The corresponding transition rate is

$$\Gamma_{ji} \sim e^{-I - S_i}. \qquad (14)$$

It follows that the upward and downward transition rates are related by

$$\Gamma_{ji}/\Gamma_{ij} \sim e^{S_j - S_i}. \qquad (15)$$

If the two vacuum energies are significantly different, the upward transition rate is very strongly suppressed. Equation (15) can be interpreted as an expression of detailed balance between vacuum transitions in the multiverse. It fits well with the widely accepted picture of quantum de Sitter space as a thermal state [2].

Analytic continuation of the instanton to the Lorentzian regime indicates that in the case of upward tunneling, the initial size of the bubble is larger than the parent vacuum horizon H_i^{-1}. The high-energy bubble is pushed inward because the vacuum energy density outside is smaller than the density inside, and thus, the inside pressure is more negative. But since the bubble is outside the Hubble distance, it is carried outward by the Hubble expansion, even though locally it accelerates inward.

B. Black hole nucleation

In general, we expect an arbitrary object of mass M that is much smaller than the Hubble distance to appear in de Sitter space at a rate proportional to

$$\exp(-2\pi M/H) = \exp(-M/T). \qquad (16)$$

The latter expression gives the likelihood of finding such an object in a thermal bath in the Gibbons-Hawking temperature $T = H/(2\pi)$.[10] The former expression has been found by instanton calculations; for example, see [34] for the nucleation of monopoles and [35] for the nucleation of black holes. The calculation of black hole nucleation rate in [35] is somewhat controversial, since it is based on an instanton with a conical singularity. Exclusion of such instantons leads to the conclusion that only maximal black holes of horizon radius equal to the cosmological horizon can nucleate in de Sitter space [36]. However, regular instantons do exist for nucleation of electrically or magnetically charged black holes of submaximal mass [37].

In the limit of a small mass, the corresponding nucleation rate is given by Eq. (16). We note also that Eq. (16) would give the rate to nucleate a distribution of dust that would collapse into a black hole.

C. Small inflating regions

As discussed above, it is possible to nucleate a much smaller bubble of higher energy vacuum i. As seen from the outside, the force on the bubble wall will cause it to shrink, leading the bubble to collapse into a black hole. However, if the bubble volume is larger than V_i^{nuc}, it will inflate on the inside, leading to a new inflating region of vacuum i.

What is the rate at which such regions are produced? The simple conjecture is that it is proportional to

$$e^{-M_i^{\mathrm{nuc}}/T} \sim e^{-M_{\mathrm{Pl}}^3/(T\sqrt{U_i})} \sim e^{-M_{\mathrm{Pl}}^2/(H_i H_j)}, \qquad (17)$$

as we would expect for any object of mass M_i^{nuc}. However, there are some caveats. New small inflating regions cannot be produced by any classical process, because their production violates the null energy condition [38]. Thus, a classical thermal state would not produce regions such as these, perhaps casting some doubt on the use of a thermal expression above. This is a fundamentally quantum process, so perhaps it can be described by an instanton, but such an instanton is not known.[11] A similar situation was discussed by Farhi *et al.* [27], who considered tunneling from a small initial false vacuum seed in asymptotically flat space to an inflating baby universe inside of a black hole. They constructed an instanton for this process but found that its metric is degenerate. The instanton action could still be calculated, but it is not clear that such pathological instantons are legitimate. Fischler *et al.* [40] considered the same problem using the Hamiltonian formalism and found no inconsistencies. The nucleation rate they found agrees with the result of [27] based on the degenerate instanton. But this issue remains controversial.

Nucleation of high-energy inflating regions can also be pictured as a two-step process. First a bubble of high-energy vacuum i having radius $R < H_i^{-1}$ spontaneously nucleates in the parent vacuum j, and then this bubble tunnels to an inflating baby universe contained inside of a black hole by the process discussed in [27,40]. One expects that the rate for the first step is $\Gamma \sim \exp(-2\pi M/H_j)$, where M is the mass of the bubble, and the tunneling action is estimated as [27,40] $S \sim (M_{\mathrm{Pl}}/H_i)^2$. Farhi *et al.* [27] find that the minimal bubble mass required for the tunneling is $M_{\mathrm{Pl}}^2/2H_i$; then the nucleation rate is $\Gamma \sim \exp(-\pi M_{\mathrm{Pl}}^2/H_i H_j)$. For $H_i \gg H_j$, this is the dominant factor determining the nucleation rate of baby universes. This is in agreement with the estimate in Eq. (17).

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[10]More precisely, the nucleation rate is proportional to $\exp(-F/T) = \exp(-M/T + S)$, where F is the free energy and S is the entropy of the nucleating object. This takes account of the possibility of nucleating the object in various microstates. The correction, however, is small in cases of interest to us here.

[11]Euclidean solutions with small false vacuum bubbles have been discussed in [39], but their interpretation as instantons is somewhat questionable.

Another possible objection to nucleation of inflating baby universes is that it is in conflict with the detailed balance condition (15). This condition however does not follow from any fundamental principle. It is violated in particular by transitions between de Sitter and anti–de Sitter vacua, which are necessarily present in any multiverse theory.

As we argued above, inflating baby universes should nucleate at some nonzero rate, even in the absence of instantons, because this process is allowed by all conservation laws. A calculation of their nucleation rate was attempted in [20].[12] This calculation appears to be reliable when the energies of the two vacua and the height of the barrier separating them are all sub-Planckian and are comparable to one another. But in the opposite limit, when $H_i \gg H_j$, the initial fluctuation is strongly influenced by gravitational effects and the calculation of its probability requires a quantum theory of gravity. Here, we shall assume that the nucleation rate in this case is given by Eq. (17), which seems to be a plausible guess.

D. Boltzmann brains

Finally, we expect Boltzmann brains to appear at the rate given by Eq. (16) with brain mass M_{BB}. This process is dominated by the lightest brains that need to be considered for anthropic reasoning. These may not actually be brains, *per se*, but tiny computers that stimulate human thought sufficiently well to be considered in anthropics.[13] We will assume here that $M_{BB} > M_{Pl} \approx 2 \times 10^{-5}$ g. This is correct for a human brain and for any computer that we have built so far. It is not correct if the only restriction is the fundamental-physics limit on the number of bits that the computer can store [23], i.e., if we are not concerned with what this computer might be made of and how it can operate. The minimum mass of a working computer is uncertain. See [23] for further discussion.

IV. THE BOLTZMANN BRAIN PROBLEM

To avoid domination by Boltzmann brains requires that the rate of Boltzmann brain production Γ_i^{BB} is less than the vacuum decay rate $\Gamma_i = \sum_j \Gamma_{ij}$ in every vacuum i [14,23]. Let us review the basic argument. Consider some vacuum i in which there are ordinary observers. First, we rewrite Eq. (2),

$$\frac{df_i}{d\eta} = M_{ij} f_j, \tag{18}$$

where $M_{ij} = \kappa_{ij} - \delta_{ij} \kappa_i$. In the limit where the cutoff grows without bound, this situation can be analyzed by finding the least negative eigenvalue $-q$ of the matrix M and the

corresponding eigenvector s so that $\sum_j \kappa_{ij} s_j - \kappa_i s_i = -q s_i$. The fraction of volume near the cutoff surface in each vacuum i is then given by s_i. The number of Boltzmann brains in that vacuum is proportional to $s_i \Gamma_i^{BB}$ because most of the volume is near the cutoff surface. Meanwhile, the number of ordinary observers is proportional to the rate at which new vacuum of type i is created, which is $\sum_j \kappa_{ij} s_j = (\kappa_i - q) s_i$. Ordinary observers are generally found in collapsed regions, which require some adjustment to the scale factor measure. However, this adjustment is insignificant compared to the double-exponential nature of Γ_i and Γ_i^{BB}, so it will not be important here.

Now q is generally smaller than the total decay rate of the dominant vacuum, which is less than that of vacuum i. (Since there is only one dominant vacuum, it is very unlikely that it is able to support Boltzmann brains. If it is, Boltzmann brains would certainly dominate [14,23].) Both κ_i and q are tiny numbers, and generally, they are quite far apart. So q can be ignored, and we find $\kappa_{ij} s_j \approx \kappa_i s_i$; i.e., the rate of creation and the rate of decay are nearly equal. We are not concerned with differences in prefactors, so κ_i and Γ_i are interchangeable, and the condition to avoid Boltzmann brain domination in vacuum i is that $\Gamma_i^{BB} < \Gamma_i$. References [14,23] show that the condition to avoid Boltzmann brain domination overall is that $\Gamma_i^{BB} < \Gamma_i$ in every vacuum.

Included in κ_i is κ_{0i}, the rate of formation of black holes. The rate for black holes of mass M is proportional to $\exp(-M/T)$, so it is dominated by the smallest black hole possible. Let us say this has a mass M_{min}^{BH}, which is around the Planck mass. Thus, κ_i is at least of order $\exp(-M_{Pl}/T)$. Meanwhile, Γ_i^{BB} is of order $\exp(-M_{BB}/T)$, where M_{BB} is the minimum Boltzmann brain mass. With our assumption that $M_{BB} > M_{Pl}$, it follows that $\Gamma_i^{BB} \ll \Gamma_i$, and there is no problem with Boltzmann brains.

The question of Boltzmann brain dominance involves comparing the number of Boltzmann brains and ordinary observers before the cutoff, so it may be counterintuitive that it is affected by the production of black holes, which are neither of these. Here is a way to understand how this happens. Consider a multiverse up until a scale factor cutoff. Most of the volume is near the cutoff, so we only need to look there. Most ordinary observers are in regions that were created not long before the cutoff and thus still have conditions where observers can live. But Boltzmann brains are in regions that were formed long ago, so we need to know how much volume these regions have.

Let us put a large but finite number of evenly spaced fiducial particles on the initial surface, traveling along the geodesics that we used to define the scale factor measure. In expanding regions, equal scale factor time means an equal amount of expansion, so each particle represents the same amount of spatial volume. Then the ratio of different volumes is just the relative number of particles that they contain.

The effect of the black holes is to swallow up some of these particles so that they do not reach the cutoff surface.

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[12]Nucleation of small inflating regions was discussed earlier by Carroll and Chen [18] and by Brown and Dahlen [19], but their estimates of the nucleation rate were much smaller than Eq. (17).

[13]See [41] for some discussion of the difficulty in determining what systems should be included in anthropic reasoning.

The result is that the volume of a given vacuum on the cutoff surface is smaller than it would be without black hole formation. Thus, s_i, the fraction of the cutoff surface in volume i, is inversely proportional to the decay rate Γ_i. Black hole nucleation increases Γ_i and so decreases s_i. The number of Boltzmann brains in this vacuum is then proportional to $\Gamma^{\mathrm{BB}}/\Gamma_i$. This leads to the criterion used above.

The process of removing particles by black hole formation is extremely slow. A Planck-scale black hole removes only a fraction of order H^3/M_{Pl}^3 of the Hubble volume where it forms. More importantly, such a black hole only occurs once in every $\exp(M_{\min}^{\mathrm{BH}}/T)$ Hubble volumes. Thus, we must wait a time of order $\exp(M_{\min}^{\mathrm{BH}}/T)$ Hubble times before this effect is important. During this time, the universe expands by a factor $\exp(\exp(M_{\min}^{\mathrm{BH}}/T))$. In our present Universe, this is about $\exp(\exp(10^{60}))$, a remarkably large number. Nevertheless, the scale factor measure instructs us to consider the limit where the scale factor goes to infinity, so the required scale factor to reach a steady-state situation does not matter.

Also included in κ_i is the nucleation of small inflating regions. This removes particles from the parent vacuum at a rate $\sim \exp(-M_{\mathrm{Pl}}^2/(H_m T))$, where H_m is the highest inflation rate in the landscape. It is often assumed that $H_m \sim M_{\mathrm{Pl}}$, in which case this effect is comparable to that of black hole nucleation.

V. VACUUM DYNAMICS

The possibility of less-suppressed tunneling to higher energy vacua changes the distribution of different possible states and thus the results to be expected under anthropic reasoning. The fraction of the volume in some vacuum i according to the scale factor measure depends on the tunneling rates to get from the dominant vacuum to vacuum i [21]. To reach any anthropically allowed vacuum from the dominant vacuum, we generally need an upward jump, or many such jumps, followed by many downward jumps.[14] Which vacua are easily reached depends on which process we consider.

If we consider only the Lee-Weinberg process, there is a large suppression factor given by Eq. (15). This suppression is less important when the two vacua are close in energy. Thus, the favored vacua are those which can be reached from the dominant vacuum by small upward jumps followed by downward jumps. Depending on the structure of the landscape, these vacua may be sparse enough that the anthropic

explanation of the cosmological constant does not work [21,22].

However, when we consider the formation of small regions of high-energy vacuum j, the mass of the region is $M_j^{\mathrm{nuc}} \sim M_{\mathrm{Pl}}^2/H_j$, and the suppression goes as $\exp(-M_{\mathrm{Pl}}^2/(H_i H_j))$. Thus, the least suppressed transitions are those to the largest H_i. Furthermore, there is little dependence on which is the dominant vacuum, because wherever one starts, the same high-energy vacua are preferred. From those vacua, one must then drop, generally in a number of steps, to the anthropic region. This pattern of transitions generally leads to a much smoother distribution of probabilities for the different vacua [43].

VI. CONCLUSION

In an eternally inflating universe, there is the possibility of Boltzmann brain domination, meaning that anthropic reasoning would lead to the nonsensical conclusion that we are Boltzmann brains. In the scale factor measure, this disaster is avoided when the rate of Boltzmann brain nucleation is smaller than the vacuum decay rate in each vacuum (and the dominant vacuum does not support Boltzmann brains). If one considers decay only by the Coleman-De Luccia and Lee-Weinberg processes, this may not be the case (but see [44] for a claim that vacuum decay rates in string theory are always larger than Γ^{BB}). However, we showed above that black hole nucleation should be included in the vacuum decay rate, and this process is much less suppressed than Boltzmann brain production, under a rather mild assumption that the mass of a Boltzmann brain should be greater than the Planck mass. Thus, we should not expect to be Boltzmann brains.

We also discussed the nucleation of small regions of inflating high-energy vacuum. If vacua of high enough energies exist, this process also would prevent Boltzmann brain domination. In any case, it modifies the probability distribution of the various vacua, likely giving a more uniform distribution for different anthropic possibilities and guaranteeing that anthropic explanations of the smallness of the cosmological constant are not affected by highly nonuniform probability distributions across anthropic vacua.

We finally mention the swampland conjectures which have been intensively discussed in recent years (see [45] for an up-to-date review and references). According to these conjectures, metastable de Sitter vacua do not exist, and many models of eternal inflation are also ruled out. However, it was shown in [46] that eternal inflation driven by inflating domain walls may still be possible. It would be interesting to apply the considerations of the present paper to this kind of multiverse models.

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[14]The dominant vacuum is likely to have a very low supersymmetry breaking scale η_*. Its energy density $U_* \lesssim \eta_*^4$ is then likely to be extremely small. It is also reasonable to expect that this nearly supersymmetric vacuum can support neither ordinary observers nor Boltzmann brains. For a discussion of the expected properties of the dominant vacuum in string theory, see [42] and references therein.

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