Distribution of supersymmetry μ parameter and Peccei-Quinn scale f_a from the landscape

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A scan of soft supersymmetry (SUSY) breaking parameters within the string theory landscape with the minimal supersymmetric standard model assumed as the low energy effective field theory—using a power-law draw to large soft terms coupled with an anthropic selection of a derived weak scale to be within a factor of 4 of our measured value—predicts a peak probability of $m_h \simeq 125$ GeV with sparticle masses typically beyond the reach of LHC Run 2. Such multiverse simulations usually assume a fixed value of the SUSY conserving superpotential μ parameter to be within the assumed anthropic range, $\mu \lesssim 350$ GeV. However, depending on the assumed solution to the SUSY μ problem, the expected μ term distribution can actually be derived. In this paper, we examine two solutions to the SUSY μ problem. The first is the gravity-safe-Peccei-Quinn model based on an assumed $\mathbb{Z}_{24}^R$ discrete R-symmetry which allows a gravity-safe accidental, approximate Peccei-Quinn global symmetry to emerge which also solves the strong CP problem. The second case is the Giudice-Masiero solution wherein the μ term effectively acts as a soft term and has a linear draw to large values. For the first case, we also present the expected landscape distribution for the Peccei-Quinn scale f_a; in this case, weak scale anthropics limits its range to the cosmological sweet zone of around $f_a \sim 10^{11}$ GeV.

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I. INTRODUCTION

One of the curiosities of nature pertains to the origin of mass scales. Naively, one might expect all mass scales to be of order the fundamental Planck mass scale $m_{\mathrm{Pl}} = 1.2 \times 10^{19}$ GeV as occurs in quantum mechanics and in its relativistic setting: string theory. For instance, one expects the cosmological constant $\Lambda_{cc} \sim m_{\mathrm{Pl}}^2$, whereas its measured value is over 120 orders of magnitude smaller. The only plausible explanation so far is by Weinberg [1] in the context of the eternally inflating multiverse wherein each pocket universe has a different value of Λ_{cc} ranging from $-m_{\mathrm{Pl}}^2$ to $+m_{\mathrm{Pl}}^2$: if Λ_{cc} were too much larger than its measured value, then the early universe would have expanded so quickly that structure in the form of galaxies, and hence observers, would not occur. This anthropic explanation finds a natural setting in the string theory landscape of vacuum solutions [2] where of order 10^{500} [3] (or many, many more [4]) solutions may be expected from string flux compactifications [5].

A further mystery is the origin of the weak scale: why is $m_{\mathrm{weak}} \sim m_{W,Z,h} \sim 100$ GeV instead of 10^{19} GeV? A similar environmental solution has been advocated by Agrawal, Barr, Donoghue, and Seckel (ABDS) [6,7]: if m_{weak} was a factor 2–5 greater than its measured value, then quark mass differences would be affected such that complex nuclei, and hence atoms as we know them, could not form (atomic principle).

This latter solution has been successfully applied in the context of weak scale supersymmetry (WSS) [8] within the string theory landscape. The assumption here is to adopt a fertile patch of landscape vacua where the minimal supersymmetric standard model (MSSM) forms the correct weak scale effective field theory (EFT), but wherein the soft supersymmetry (SUSY) breaking terms would scan in the landscape. For perturbative SUSY breaking where no nonzero F-term or D-term is favored over any other in the landscape, then soft terms are expected to scan as a power law [9–11]:

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$$f_{\rm SUSY} \sim m_{\rm soft}^n, \tag{1}$$

where $n = 2n_F + n_D - 1$ with n_F the number of SUSY breaking hidden sector F-terms and n_D is the number of SUSY breaking hidden sector D-terms. The factor of 2 comes from the fact that F-terms are distributed as complex values while the D-breaking fields are distributed as real numbers. Even for the textbook value $n_F = 1$ and $n_D = 0$, already one expects a statistical draw from the landscape to large soft SUSY breaking terms and one might expect soft terms at the highest possible scale, perhaps at the Planck scale.

However, such huge soft terms would generically result in a Higgs potential with either charge-or-color breaking minima (CCB) or no electroweak symmetry breaking (EWSB) at all. For vacua with appropriate EWSB, then one typically expects the pocket universe value of the weak scale $m_{\rm weak}^{\rm PU} \gg m_{\rm weak}^{\rm OU}$ in violation of the atomic principle (where $m_{\rm weak}^{\rm OU}$ corresponds to the measured value of the weak scale in our universe). Here, for specificity, we will evaluate the expected weak scale value in terms of $m_Z^{\rm PU}$ as calculated for each pocket universe via the SUSY EWSB minimization conditions, which read

$$\frac{(m_Z^{\rm PU})^2}{2} = \frac{m_{H_d}^2 + \Sigma_d^d - (m_{H_u}^2 + \Sigma_u^u)\tan^2\beta}{\tan^2\beta - 1} - \mu^2 \tag{2}$$

$$\simeq - m_{H_u}^2 - \Sigma_u^u(\tilde{t}_{1,2}) - \mu^2. \tag{3}$$

Here, $m_{H_u}^2$ and $m_{H_d}^2$ are squared soft SUSY breaking Lagrangian terms, μ is the superpotential Higgsino mass parameter, $\tan\beta = v_u/v_d$ is the ratio of Higgs field vacuum expectation values (VEVs), and the Σ_u^u and Σ_d^d contain an assortment of radiative corrections, the largest of which typically arise from the top squarks. Expressions for the Σ_u^u and Σ_d^d are given in the Appendix of Ref. [12].

To remain in accord with the atomic principle according to Refs. [6,7], we will require, for a derived value of μ (so that μ is not available for the usual fine-tuning in Eq. (3) needed to gain the measured value of $m_Z^{\rm OU}$), that $m_Z^{\rm PU} < 4m_Z^{\rm OU}$ where $m_Z^{\rm OU} = 91.2$ GeV. This constraint is then the same as requiring the electroweak naturalness parameter [12,13] $\Delta_{\rm EW} \lesssim 30$. Thus, the anthropic condition is that—for various soft term values selected statistically according to Eq. (1)—there must be appropriate EWSB (no CCB or non-EWSB vacua) and that $m_Z^{\rm PU} < 4m_Z^{\rm OU}$. These selection requirements have met with success within the framework of gravity-mediation (NUHM2) models [14] and mirage mediation (MM) [15] in that the probability distribution for the Higgs mass m_h ends up with a peak around $m_h \sim 125$ GeV with sparticle masses typically well beyond LHC limits. Such results are obtained for $n = 1$, 2, 3, and 4 and even for a $\log(m_{\rm soft})$ distribution [16,17].

These encouraging results were typically obtained by fixing the SUSY conserving μ parameter at some natural value $\mu \lesssim 4m_Z^{\rm OU} \sim 350$ GeV so that the atomic principle is not immediately violated. But what sort of distribution of SUSY μ parameter is expected from the landscape? The answer depends on what sort of solution to the SUSY μ problem is assumed in the underlying model (a recent review of 20 solutions to the SUSY μ problem is given in Ref. [18]). Recall that since μ is SUSY conserving and not SUSY breaking, then one might expects its value to be far higher than $m_{\rm weak}$, perhaps as high as the reduced Planck mass m_P. But phenomenologically, its value ought to be at or around the weak scale in order to accommodate appropriate EWSB [19].

In this paper, our goal is to calculate the expected μ parameter probability distribution expected from the string landscape from two compelling solutions to the SUSY μ problem. We will first examine the so-called gravity-safe Peccei-Quinn (GSPQ) model[1] which is based upon a discrete R-symmetry $\mathbb{Z}_{24}^R$ from which the global PQ emerges as an accidental, approximate symmetry; it then solves the SUSY μ problem and the strong CP problem in a gravity-safe manner [20].[2] The second solution is perhaps most popular: the Giudice-Masiero (GM) mechanism [24] wherein the μ parameter arises from nonrenormalizable terms in the Kähler potential.

II. DISTRIBUTION OF μ PARAMETER AND PQ SCALE FOR THE GSPQ MODEL

The first μ term solution we will examine is the so-called gravity-safe PQ (GSPQ) model which was specified in Ref. [20]. The GSPQ model is based upon a discrete $\mathbb{Z}_{24}^R$ R-symmetry to at first forbid the μ parameter. The set of discrete R symmetries that allows for all anomaly cancellations in the MSSM (up to Green-Schwarz terms) and is consistent with $SO(10)$ or $SU(5)$ grand unified theory (GUT) matter assignments were catalogued by Lee *et al.* in Ref. [25] and found to consist of $\mathbb{Z}_4^R$, $\mathbb{Z}_6^R$, $\mathbb{Z}_8^R$, $\mathbb{Z}_{12}^R$, and $\mathbb{Z}_{24}^R$. These discrete R-symmetries as follows: 1. forbid the SUSY μ term; 2. forbid all R-parity-violating operators; 3. suppress dimension-five proton decay operators while 4. allowing for the usual superpotential Yukawa and neutrino mass terms.

The superpotential for the GSPQ model introduces two additional PQ sector fields X and Y and is given by

$$\begin{aligned} W_{\rm GSPQ} = {} & f_u Q H_u U^c + f_d Q H_d D^c + f_\ell L H_d E^c \\ & + f_\nu L H_u N^c + M_N N^c N^c/2 \\ & + \lambda_\mu X^2 H_u H_d/m_P + f X^3 Y/m_P, \end{aligned} \tag{4}$$

[1] The GSPQ model [20] is a hybrid between the CCK [21] and BGW [22] models.

[2] See also Harigaya *et al.* [23].

where $f_{u,d,\ell,\nu}$ are the usual MSSM + right-hand-neutrino (RHN) Yukawa couplings and M_N is a Majorana neutrino mass term which is essential for the SUSY neutrino seesaw mechanism. Since the μ term arises from the PQ sector of the superpotential [second line of Eq. (4)], this is an example of the Kim-Nilles solution to the SUSY μ problem [26]. The GSPQ model is a hybrid between the Choi-Chun-Kim (CCK) [21] radiative PQ breaking model and the Babu-Gogoladze-Wang model (BGW) [22] based on discrete gauge symmetries. For the case of $\mathbb{Z}_{24}^R$ symmetry applied to the GSPQ model, then it was also found that all further nonrenormalizable contributions to $W_{\rm GSPQ}$ are suppressed by powers up to $1/m_P^7$: terms such as X^8Y^2/m_P^7 and X^4Y^6/m_P^7 being allowed. These terms contribute to the scalar potential with terms suppressed by powers of $1/m_P^8$. The wonderful result is that the Peccei-Quinn symmetry needed to resolve the strong CP problem emerges as an accidental, approximate symmetry much as baryon and lepton numbers emerge in the SM as a result of the SM gauge symmetries. The $\mathbb{Z}_{24}^R$ symmetry is strong enough to sufficiently suppress PQ breaking terms in $W_{\rm GSPQ}$ such that a very sharp PQ symmetry emerges: enough to guarantee that PQ-violating contributions to the strong CP violating $\bar{\theta}$ parameter keep its value below $\bar{\theta} \lesssim 10^{-10}$ in accord with neutron electric dipole moment measurements. Thus, the GSPQ model based on $\mathbb{Z}_{24}^R$ discrete R-symmetry yields a *gravity-safe* global PQ symmetry.

The PQ symmetry ends up being violated when SUSY breaking also breaks the $\mathbb{Z}_{24}^R$ discrete R-symmetry, leading to the emergence of the μ parameter with value $\mu \sim \lambda_\mu v_X^2/m_P$. In the GSPQ model, the F-term part of the scalar potential

$$V_F = |3f\phi_X^2\phi_Y/m_P|^2 + |f\phi_X^3/m_P|^2 \tag{5}$$

is augmented by SUSY breaking soft term contributions

$$V_{\rm soft} \ni m_X^2|\phi_X|^2 + m_Y^2|\phi_Y|^2 + (fA_f\phi_X^3\phi_Y/m_P + {\rm H.c.}). \tag{6}$$

SUSY breaking with a large value of trilinear soft term $-A_f$ leads to $\mathbb{Z}_{24}^R$ breaking (allowing a μ term to develop) and consequent breaking of the approximate, accidental PQ symmetry, leading to the pseudo-Goldstone boson axion a (a combination of the X and Y fields).

The GSPQ scalar potential minimization conditions are [27] (neglecting the Higgs field contributions which lead to VEVs at far lower mass scales)

$$0 = \frac{9|f|^2}{m_P^2}|v_X^2|^2 v_Y + \frac{f^* A_f^*}{m_P} v_X^{*3} + m_Y^2 v_Y, \tag{7}$$

$$0 = \frac{3|f|^2}{m_P^2}|v_X^2|^2 v_X + \frac{18|f|^2}{m_P^2}|v_X|^2|v_Y|^2 v_X + \frac{3f^* A_f^*}{m_P} v_X^{*2} v_Y^* + m_X^2 v_X. \tag{8}$$

To simplify, we will take A_f and f to be real so that the VEVs v_X and v_Y are real as well. Then, the first of these may be solved for v_Y and substituted into the second equation to yield a cubic polynomial in v_X^4 which can be solved for either analytically or numerically. Viable solutions can be found for $|A_f| \geq \sqrt{12}m_0 \simeq 3.46m_0$ (where for simplicity, we assume a common scalar mass $m_X = m_Y = m_{3/2} \equiv m_0$). Then, for typical soft terms of order $m_{\rm soft} \sim 10$ TeV and $f = 1$, we develop VEVs $v_X \sim v_Y \sim 10^{11}$ GeV. For instance, for $m_X = m_Y = 10$ TeV, $f = 1$, and $A_f = -35.5$ TeV, then $v_X = 10^{11}$ GeV, $v_Y = 5.8 \times 10^{10}$ GeV, $v_{\rm PQ} \equiv \sqrt{v_X^2 + v_Y^2} = 1.15 \times 10^{11}$ GeV, and the PQ scale $f_a = \sqrt{v_X^2 + 9v_Y^2} = 2 \times 10^{11}$ GeV. The μ parameter for $\lambda_\mu = 0.1$ is given as $\mu = \lambda_\mu v_X^2/m_P \simeq 417$ GeV.

In Fig. 1, we plot contours of the derived value of μ in the $m_{3/2}$ vs $-A_f$ parameter space for $\lambda_\mu = 0.1$. The gray-shaded region does not yield admissible vacuum solutions while the right-hand region obeys the above bound $|-A_f| \gtrsim \sqrt{12}m_{3/2}$. From the plot we see that, for any fixed value of gravitino mass $m_{3/2}$, low values of μ occur for the lower allowed range of $|A_f|$. There is even a tiny region with $\mu < 100$ GeV in the lower left which may be ruled out by negative search results for pair production of Higgsino-like charginos at LEP2. As $|A_f|$ increases, then the derived value of μ increases beyond the anthropic limit of $\mu \lesssim 350$ GeV and would likely lead to too large a value of the weak scale unless an unnatural fine-tuning is invoked in $m_Z^{\rm PU}$.

A. GSPQ model in the multiverse

To begin our calculation of the expected distribution of the μ parameter from the landscape, we adopt the two-extra-parameter nonuniversal Higgs SUSY model NUHM2 [28–33] where matter scalar soft masses are unified to m_0

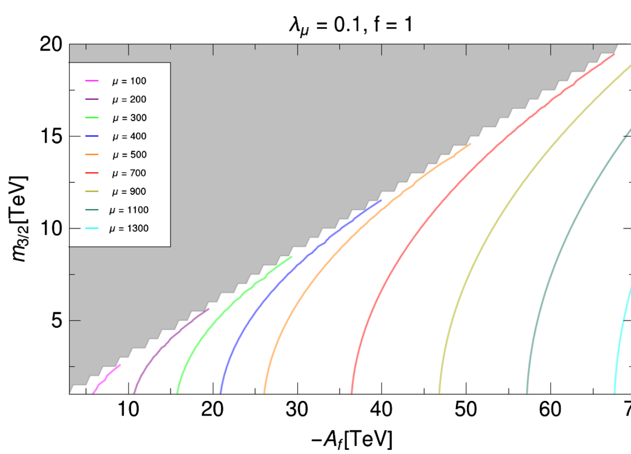

FIG. 1. Calculated value of SUSY μ parameter from the GSPQ model in the $m_{3/2}$ vs $-A_f$ plane for $f = 1$ and $\lambda_\mu = 0.1$.

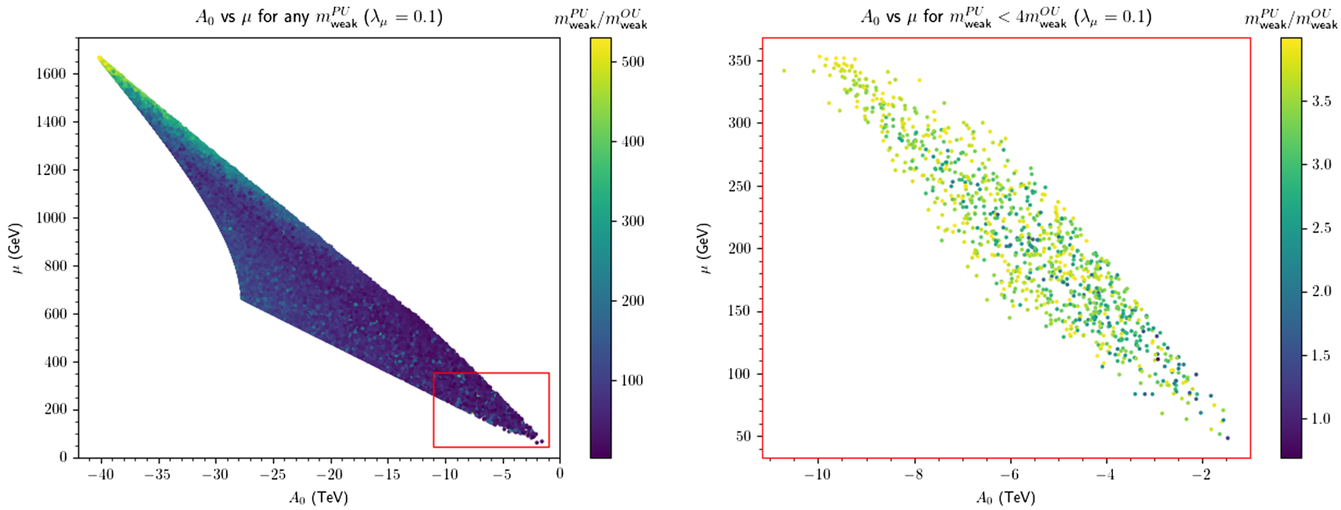

FIG. 2. Locus of $n = 1$ landscape scan points in the GSPQ + MSSM model in the A_0 vs μ plane for (a) all values of $m^{\rm PU}_{\rm weak}$ and (b) points with $m^{\rm PU}_{\rm weak} < 4m^{\rm OU}_{\rm weak}$. We take $f = 1$ and $\lambda_\mu = 0.1$.

while Higgs soft masses m_{H_u} and m_{H_d} are independent.[3] The latter soft Higgs masses are usually traded for weak scale parameters μ and m_A so the parameter space is given by

$$m_0,\quad m_{1/2},\quad A_0,\quad \tan\beta,\quad \mu,\quad m_A \quad \text{(NUHM2)}. \tag{9}$$

We will scan soft SUSY breaking terms with the $n = 1$ landscape power-law draw, with an independent draw for each category of soft term [34]. The scan must be made with parameter space limits beyond those which are anthropically imposed. Our p-space limits are given by

$$m_0\colon\ 0.1\text{–}20\ \text{TeV}, \tag{10}$$

$$m_{1/2}\colon\ 0.5\text{–}5\ \text{TeV}, \tag{11}$$

$$-A_0\colon\ 0\text{–}50\ \text{TeV}, \tag{12}$$

$$m_A\colon\ 0.3\text{–}10\ \text{TeV}, \tag{13}$$

$$\tan\beta\colon\ 3\text{–}60 \quad \text{(uniform scan)}. \tag{14}$$

A crucial assumption is that the matter scalar masses in the PQ sector are universal with the matter scalar masses in the visible sector: hence, we adopt that $m_0 = m_X = m_Y \equiv m_{3/2}$. We also assume correlated trilinear soft terms: $A_f = 2.5A_0$. This latter requirement is forced upon us by requiring $|A_f| \geq \sqrt{12}m_0$ to gain a solution in the PQ scalar potential while in the MSSM sector if $|A_0|$ is too large, then top squark soft-squared masses are driven tachyonic leading to CCB vacua. We also adopt $f = 1$ throughout.

For our anthropic requirement, we will adopt the atomic principle from Agrawal *et al.* [7] where $m^{\rm PU}_{\rm weak} \lesssim (2\text{–}5)m^{\rm OU}_{\rm weak}$. To be specific, we will require $m^{\rm PU}_Z < 4m^{\rm OU}_Z$ (which corresponds to the fine-tuning measure $\Delta_{\rm EW} < 30$ [12,13]). The fine-tuned solutions are possible but occur rarely compared to non-fine-tuned solutions in the landscape [35]. The anthropic requirement results in upper bounds on soft terms such as to maintain a pocket-universe weak scale value not-too-displaced from its measured value in our universe. We also must require no CCB minima and also an appropriate breakdown in electroweak symmetry (i.e., that $m^2_{H_u}$ is actually driven negative such that EW symmetry is indeed broken). Given this procedure, then the value of μ can be calculated from the GSPQ model scalar potential minimization conditions and then the entire SUSY spectrum can be calculated using the Isajet/Isasugra package [36]. The resulting spectra can then be accepted or rejected according to the above anthropic requirements.

1. Results for GSPQ model with $\lambda_\mu = 0.1$

In this subsection, we restrict our results to parameter scans with $\lambda_\mu = 0.1$. In Fig. 2, we show the distribution of scan points in the A_0 vs μ plane (a) for all derived weak scale values $m^{\rm PU}_{\rm weak}$ and (b) for only points with $m^{\rm PU}_{\rm weak} < 4m^{\rm OU}_{\rm weak}$. From frame (a), we see that only the colored portion of parameter space yields appropriate EWSB, albeit mostly with a huge value of $m^{\rm PU}_{\rm weak}$ well beyond the ABDS anthropic window. The points with too low a value of $-A_0$ do not yield viable GSPQ vacua (unless compensated for with an appropriately small value of m_0) while points with too large

[3] It is more realistic to allow independent generations $m_0(1)$, $m_0(2)$ and $m_0(3)$ but these will hardly affect our results here. They do play a big role in a landscape solution to the SUSY flavor and CP problems where $m_0(1)$ and $m_0(2)$ are drawn to common upper bounds in the 20–50 TeV range leading to a mixed decoupling/quasi-degeneracy solution to the aforementioned problems.

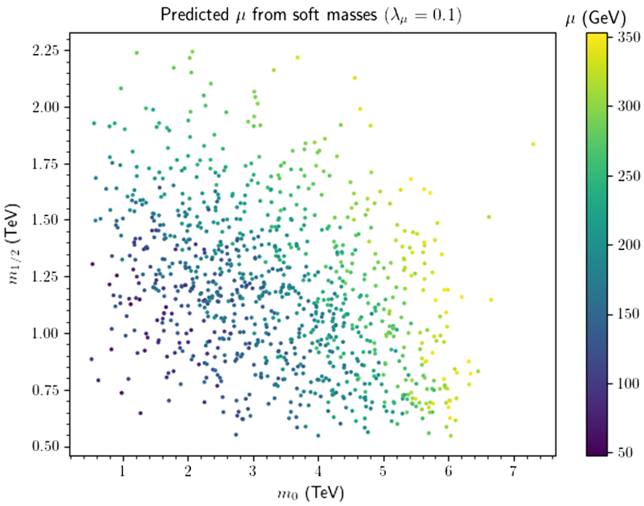

FIG. 3. Locus of $n = 1$ landscape scan points in the GSPQ + MSSM model in the m_0 vs $m_{1/2}$ plane for points with $m_{\rm weak}^{\rm PU} < 4m_{\rm weak}^{\rm OU}$. The color coding follows the magnitude of the μ parameter. We take $f = 1$ and $\lambda_\mu = 0.1$.

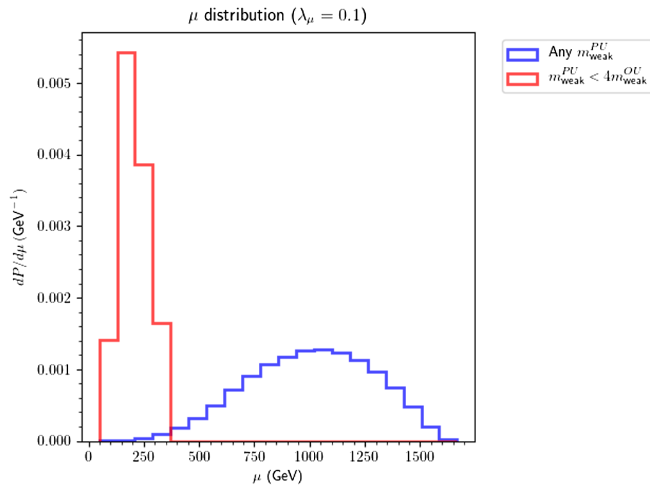

FIG. 4. Probability distribution for SUSY μ parameter in the GSPQ + MSSM model from an $n = 1$ landscape draw to large soft terms with $f = 1$ and $\lambda_\mu = 0.1$.

a value of $-A_0$ typically yield CCB minima in the MSSM scalar potential. The surviving points are color coded according to the value of $m_{\rm weak}^{\rm PU}$ with the dark blue points yielding the lowest values of $m_{\rm weak}^{\rm PU}$, which occur in the lower-right corner. In frame (b)—which is a blowup of the red-bounded region from frame (a)—we add the anthropic condition $m_{\rm weak}^{\rm PU} < 4m_{\rm weak}^{\rm OU}$. In this case, the range of $-A_0$ and μ values becomes greatly restricted since the large μ points require large values of m_0 and $m_{1/2}$, leading to too large values of $\Sigma_u^u(\tilde{t}_{1,2})$. This can be seen from Fig. 3, where we plot the color-coded μ values in the m_0 vs $m_{1/2}$ plane for $\lambda_\mu = 0.1$. From the right-hand scale, the dark purple dots have $\mu \lesssim 100$ GeV (and so would be excluded by LEP2 chargino pair searches which require $\mu \gtrsim 100$ GeV). The green and yellow points all have large values of $\mu \sim 300$–350 GeV, but these occur at the largest values of m_0 and $m_{1/2}$. For even larger m_0 and $m_{1/2}$ values, the derived μ value exceeds 365 GeV; and absent fine-tuning, such points would lead to $m_{\rm weak}^{\rm PU}$ lying beyond the ABDS window, in violation of the atomic principle.

In Fig. 4, we plot the distribution of derived values of μ for the GSPQ + NUHM2 model for all values of $m_{\rm weak}^{\rm PU}$ (blue histogram) and for the anthropically limited points with $m_{\rm weak}^{\rm PU} < 4m_{\rm weak}^{\rm OU}$ (red histogram). We see the blue histogram prefers huge values of μ, and only turns over at high values due to the artificial upper limits we have placed on our soft term scan values. However, once the anthropic constraint is applied, then we obtain the red distribution which varies between $\mu \sim 50$ and 365 GeV with a peak at $\mu^{PU} \sim 200$ GeV followed by a falloff to larger values.

In Fig. 5, we plot the derived value of the PQ scale f_a from all models with appropriate EWSB (blue) and those models with $m_{\rm weak}^{\rm PU} < 4m_{\rm weak}^{\rm OU}$ (red). In this case, the PQ scale comes out in the cosmological sweet spot where there are comparable relic abundances of SUSY Dine-Fischler-Srednicki-Zhitnitsky (DFSZ) axions and Higgsino-like weakly interacting massive particle (WIMP) dark matter [37]. The unrestricted histogram ranges up to values of $f_a \sim (2$–$4) \times 10^{11}$ GeV. This differs from an earlier work which sought to derive the PQ scale from the landscape by imposing anthropic conditions using constraints on an overabundance of mixed axion-neutralino dark matter [37]. In the present case, the GSPQ soft terms are correlated with the visible sector soft terms and the latter are restricted by requiring the derived weak scale to lie within the ABDS window. The fact that the present results lie within the cosmological sweet zone then resolves a string theory quandary as to why the PQ scale is not up around the GUT/

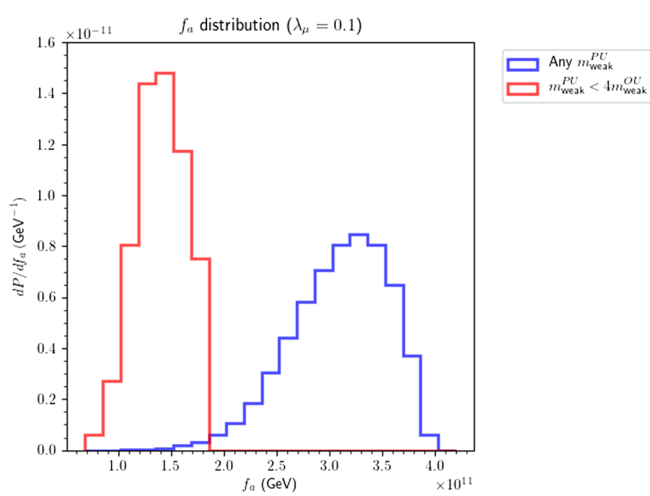

FIG. 5. Probability distribution for PQ scale f_a in the GSPQ + MSSM model from an $n = 1$ landscape draw to large soft terms with $f = 1$ and $\lambda_\mu = 0.1$.

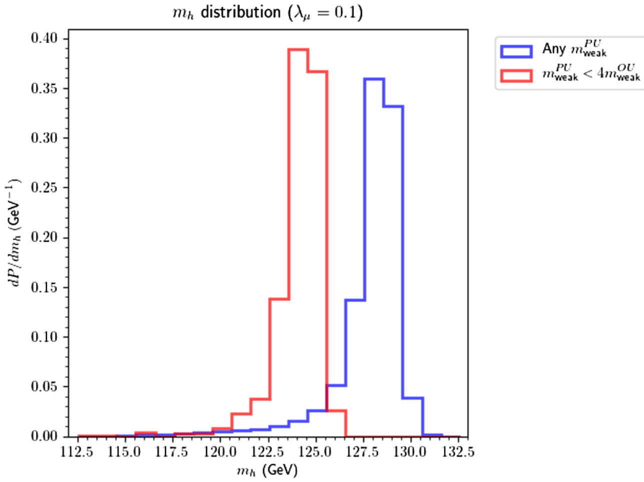

FIG. 6. Probability distribution for m_h in the GSPQ + MSSM model from an $n = 1$ landscape draw to large soft terms with $f = 1$ and $\lambda_\mu = 0.1$.

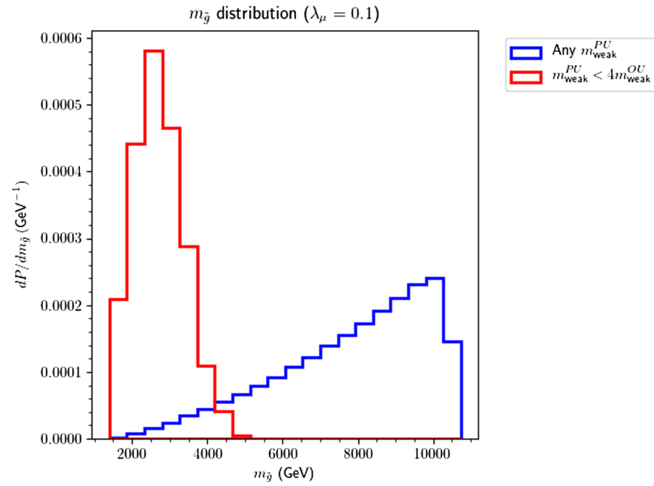

FIG. 7. Probability distribution for $m_{\tilde g}$ in the GSPQ + MSSM model from an $n = 1$ landscape draw to large soft terms with $f = 1$ and $\lambda_\mu = 0.1$.

Planck scale [38]. By including the weak scale ABDS anthropic requirement, the red histogram becomes rather tightly restricted to lie in the range $f_a : (1\text{–}2) \times 10^{11}$ GeV.

In Fig. 6, we show the expected distribution in light Higgs mass m_h without (blue) and with (red) the anthropic constraint. For the blue histogram, the upper bound on soft terms is set by a combination of our scan limits but also the requirement of getting an appropriate breakdown of PQ symmetry (as in lying outside the gray-shaded region of Fig. 1). In this case, the distribution peaks around $m_h \sim 128$ GeV with only a small probability down to $m_h \sim 125$ GeV. When the anthropic constraint $m_{\rm weak}^{\rm PU} < 4m_{\rm weak}^{\rm OU}$ is imposed, then we gain instead the red histogram which features a prominent peak around $m_h \sim 125$ GeV, which is supported by the ATLAS/CMS measured value of m_h [39].

In Fig. 7, we show the expected distribution in gluino mass $m_{\tilde g}$. For the blue curve, without the anthropic constraint, we have a strong statistical draw from the landscape for large gluino masses which is only cut off by our artificial upper scan limits along with the requirement of appropriate PQ breaking. Once the anthropic condition is imposed, then the $m_{\tilde g}$ distribution peaks around $m_{\tilde g} \sim 3$ TeV with a tail extending up to about 5 TeV. The ATLAS/CMS requirement that $m_{\tilde g} \gtrsim 2.2$ TeV only restricts the lowest portion of the derived $m_{\tilde g}$ probability distribution.

2. Results for other values of λ_μ

We have repeated our calculations to include other choices of $\lambda_\mu = 0.02$, 0.05, 0.1, and 0.2. By lowering the value of λ_μ, then correspondingly larger GSPQ soft term values (and hence NUHM2 soft term values) may lead to acceptable vacua. In Fig. 8, we show the derived μ parameter distribution for three choices of λ_μ after the anthropic weak scale condition is applied. A fourth histogram for $\lambda_\mu = 0.02$ actually peaks below ~ 100 GeV, and so the bulk of this distribution would be ruled out by LEP2 limits which require $\mu \gtrsim 100$ GeV due to negative searches for chargino pair production. As λ_μ increases, then the μ distribution becomes correspondingly harder: for $\lambda_\mu = 0.2$, then the distribution actually peaks around $\mu \sim 250\text{–}300$ GeV. This could offer an explanation as to why ATLAS and CMS have not yet seen the soft dilepton plus jets plus $\not{E}_T$ signature which arises from Higgsino pair production [40–44] at LHC [45,46]. Current limits on this process from ATLAS extend out to $\mu \sim 200$ GeV for $m_{\tilde\chi_2^0} - m_{\tilde\chi_1^0}$ mass gaps of ~ 10 GeV [45,46].

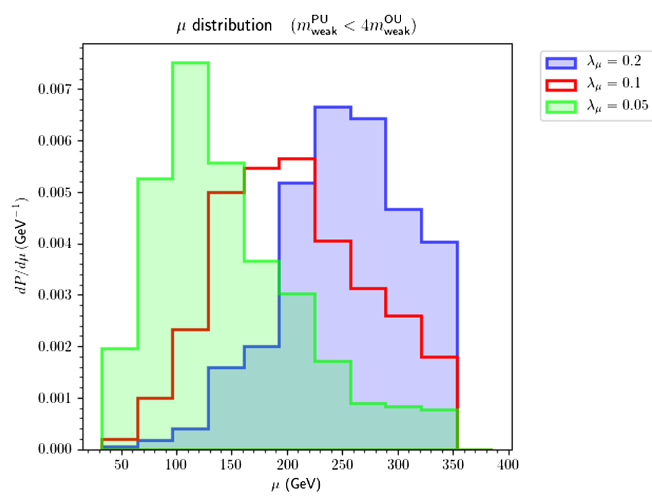

FIG. 8. Probability distribution for SUSY μ parameter in the GSPQ + MSSM model from an $n = 1$ landscape draw to large soft terms with $f = 1$ for $\lambda_\mu = 0.05$, 0.1, and 0.2.

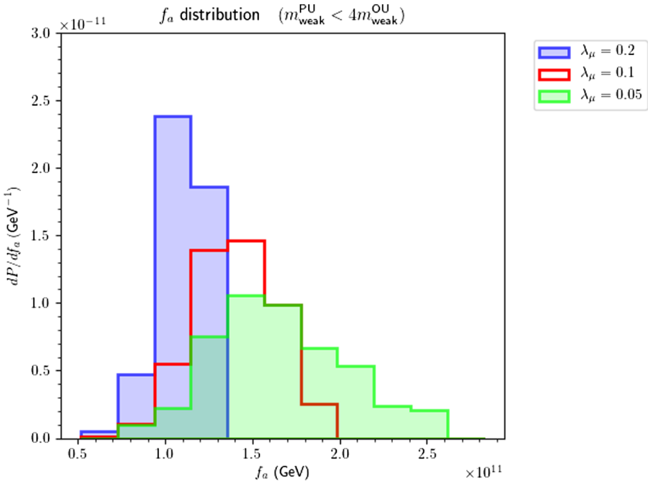

FIG. 9. Probability distribution for PQ scale f_a in the GSPQ + MSSM model from an $n = 1$ landscape draw to large soft terms with $f = 1$ and $\lambda_\mu = 0.05$, 0.1, and 0.2.

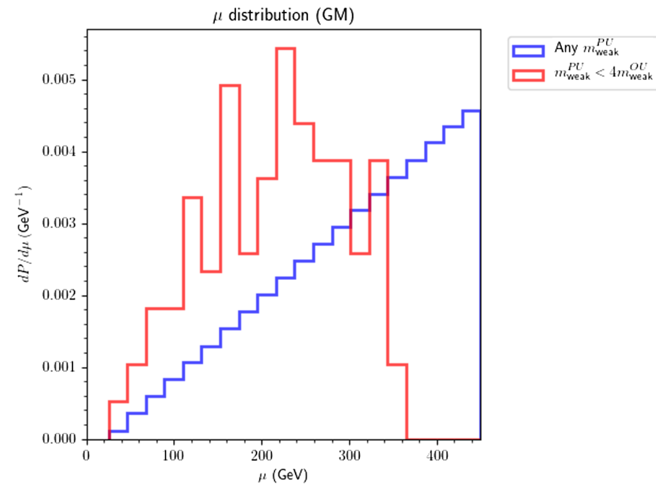

FIG. 10. Distribution of SUSY μ parameter in the GM model with $\lambda_{\rm GM} = 1$ with and without the anthropic constraint that $m^{\rm PU}_{\rm weak} < 4m^{\rm OU}_{\rm weak}$.

In Fig. 9, we show the distribution in f_a for the three different values of λ_μ. Here the model is rather predictive with the PQ scale lying at $f_a \sim (0.5\text{–}2.5) \times 10^{-11}$ GeV, corresponding to an axion mass of $m_a \sim 144$–720 μeV. Unfortunately, in the PQMSSM, the axion coupling $g_{a\gamma\gamma}$ is highly suppressed compared to the non-SUSY DFSZ model due to canceling contributions from Higgsino states circulating in the $a\gamma\gamma$ axion coupling triangle diagram [47]. Thus, axion detection at experiments such as axion dark matter search experiment may require new advances in sensitivity in order to eke out a signal.

III. DISTRIBUTION OF μ PARAMETER IN GIUDICE-MASIERO MODEL

For GM, one assumes first that the μ parameter is forbidden by some symmetry (R-symmetry or PQ symmetry?). Then one assumes that in the SUSY Kähler potential K, there is a Planck suppressed coupling of the Higgs bilinear to some hidden sector field h_m which gains a SUSY-breaking VEV:

$$K_{\rm GM} \ni \lambda_{\rm GM} h_m^\dagger H_u H_d / m_P + \text{c.c.}, \tag{15}$$

where $\lambda_{\rm GM}$ is some Yukawa couping of order ~ 1. When h_m develops a SUSY breaking VEV $F_h \sim m^2_{\rm hidden}$ with the hidden sector mass scale $m_{\rm hidden} \sim 10^{11}$ GeV, then a weak scale value of

$$\mu_{\rm GM} \simeq \lambda_{\rm GM} m^2_{\rm hidden}/m_P \tag{16}$$

would ensue, where m_P is the reduced Planck mass $m_P = m_{\rm Pl}/\sqrt{8\pi} \simeq 2.4 \times 10^{18}$ GeV. In the GM model, since $\mu \propto F_h$ (a single F-term), then one would expect also that $\mu_{\rm GM}$ would scale as $m^1_{\rm soft}$ in the landscape. Nowadays, models invoking the μ-forbidding PQ global symmetry are expected to lie within the swampland of string-inconsistent theories since quantum gravity admits no global symmetries [48–50]. Discrete or continuous R-symmetries or gauge symmetries may still be acceptable; the former are expected to emerge from compactification of manifolds with higher dimensional spacetime symmetries.

In Fig. 10, we show the expected distribution of the $\mu_{\rm GM}$ parameter (μ in the GM model) without (blue) and with (red) the anthropic constraint that $m^{\rm PU}_{\rm weak} < 4m^{\rm OU}_{\rm weak}$. The blue histogram is just a linear expectation of the μ parameter up to the upper scan limit. Thus, for the GM model in the landscape, one expects a huge μ parameter. Varying the coupling $\lambda_{\rm GM}$ just rescales the $\mu_{\rm GM}$ distribution. And since the $\mu_{\rm GM}$ sector effectively decouples from the visible sector (unlike for the GSPQ model), we do not find that varying $\lambda_{\rm GM}$ has any effect on the expected $\mu_{\rm GM}$ distribution from the landscape.

Next, the $\mu_{\rm GM}$ distribution must be tempered by the anthropic constraint which then places an upper limit of $\mu \lesssim 365$ GeV, but also excludes some parameter space with too large Σ^u_u values. Here, for $\lambda_{\rm GM} = 1$, we see the expected μ parameter distribution peaks around ~ 250 followed by a dropoff to ~ 360 GeV.

IV. SUMMARY AND CONCLUSIONS

In this paper we have explored the origin of several mass scale mysteries within the MSSM as expected from the string landscape. Soft SUSY breaking terms are expected to be distributed as a power-law or log distribution (although in dynamical SUSY breaking they are expected to scale as $1/m_{\rm soft}$ [51]). But other mass scales arise in supersymmetric models: the SUSY conserving μ parameter, the PQ scale f_a (if a solution to the strong CP problem is to be included), and the Majorana neutrino scale M_N. Here, we have examined the expected distribution of the SUSY μ parameter from the well-motivated GSPQ model which invokes a discrete $\mathbb{Z}_{24}^{R}$ symmetry to forbid the μ term (along with R-parity violating terms and while suppressing dangerous p-decay operators). It also generates an accidental, approximate global PQ symmetry which is strong enough to allow for the theta parameter $\bar{\theta} \lesssim 10^{-10}$ (hence it is gravity-safe [52–55]). The breaking of SUSY in the PQ sector then generates a weak scale value for the μ parameter and generates a gravity-safe PQ solution to the strong CP problem. For the GSPQ model, we expect the PQ sector soft terms to be correlated with visible sector soft terms which scan on the landscape and are susceptible to the anthropic condition that $m_{\rm weak}^{\rm PU} < 4m_{\rm weak}^{\rm OU}$ in accord with the ABDS window. Thus, a landscape distribution for both the μ parameter and the PQ scale f_a are generated. For small values of Yukawa coupling λ_μ, then the μ distribution is stilted toward low values $\mu \sim 100$ GeV which now seems ruled out by recent ATLAS/CMS searches for the soft-dilepton plus jets plus $\not{E}_T$ signature which arises from light Higgsino pair production at LHC. For larger values of $\lambda_\mu \sim 0.1$–0.2, then the μ distribution is stilted toward large values $\mu \sim 200$–300 GeV in accord with LHC constraints. The PQ scale f_a also ends up lying in the cosmological sweet zone so that dark matter would be composed of an axion/Higgsino-like WIMP admixture [47,56–58].

We also examined the μ distribution expected from the Giudice-Masiero solution. In this case, the μ parameter is expected to scan as $m_{\rm soft}^{1}$ with a distribution peaking around $\mu \sim 200$–300 GeV.

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