Nucleon decay in the R-parity violating MSSM

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We present a reanalysis of nucleon decay in the context of the R-parity violating Minimal Supersymmetric Standard Model, updating bounds on R-parity violating parameters against recent experimental and lattice results. We pay particular attention to the derivation of these constraints and specifically to the hadronic matrix elements, which usually stand as the limiting factor in order to derive reliable bounds, except for these few channels that have been studied on the lattice.

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I. INTRODUCTION

The question of matter stability emerged sixty years ago from the realization that the observed baryon asymmetry of the Universe [1] required a violation of those symmetries forbidding proton decay [2]. While baryon number B is accidentally conserved in the Standard Model (SM) at the perturbative level (as well as lepton number L), it is an anomalous symmetry and thus broken by effects such as instanton or sphaleron processes [3–5]. On the other hand, B (or L) violation could reach more dramatic proportions in constructions of new physics, such as grand unified theories (GUTs) [6] or supersymmetric (SUSY) extensions of the SM [7,8], where the Lagrangian density is not even classically B (or L) invariant. In the former case, the typical pattern of proton decay is imprinted in its mediation by the gauge bosons of the extended gauge group [9–12]. In the second case, B or L conservation is conditioned to the explicit enforcement of these symmetries as a model-building ingredient.

The usual assumption in the Minimal Supersymmetric Standard Model (MSSM) consists in applying an R-parity (R_P) [13] on the Lagrangian density, making the lightest supersymmetric particle a stable dark-matter candidate by the same occasion. At the level of renormalizable terms, B and L would then again appear as accidental symmetries of the model. Nevertheless, if the MSSM is regarded as an effective field theory (EFT) at the electroweak (EW) and SUSY scales, nonrenormalizable operators acquire a legitimacy as markers of effects of higher energy (e.g., a GUT or string completion), so that R_P-conserving B/L-violating operators of dimension five could then develop and mediate proton decay [14–16]. Alternatively, R_P could be sacrificed altogether, leading to so-called R_P-violating (RpV) models [17,18], with (renormalizable) bilinear and/or trilinear either B- or L-violating terms in the superpotential. At this level, it is still possible to impose B or L invariance on the Lagrangian density, or accept proton decay as a phenomenological possibility.

These theoretical motivations, especially in the context of GUT models, have triggered extensive experimental interest in discovering B-violating decays of nucleons. Early experiments testing the law of B conservation, proposed by Weyl, Stueckelberg, and Wigner [19–21], actually predate the Sakharov paper from 1967 [2]: a first experiment was performed in 1954 by Goldhaber, and also by Reines, Cowan, and Goldhaber [22]. See Table I in Ref. [23] for a list of early experiments, 1954–1964, as well as the work by Gurr *et al.* in 1967. However, even the most recent results [24–34] have found no evidence for this phenomenon and place ever stronger bounds on individual proton or neutron decay channels. Recently, several new experiments [35–37] have been announced; they should be able to extend the current sensitivity considerably. See also the recent overview in the introduction of Ref. [12].

In R-parity conserving supersymmetry, dimension-five baryon-number violating operators have been considered [14]. However, they involve external superpartners, which at low energies must be converted to SM particles, thus reverting to dimension-six operators, with possibly more

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than one high-energy mass scale. A complete list of dimension-five lepton- or baryon-number violating operators is given in Ref. [16].

In this paper, we focus on nucleon decay from the RpV perspective, i.e., with low-energy renormalizable couplings and mediators relatively close to the EW scale. The superpotential of the R_p-conserving MSSM is extended by the following terms [14]:

$$W_{\not{R}_p} = \mu_i H_u \cdot L_i + \frac{1}{2}\lambda_{ijk} L_i \cdot L_j (E^c)_k + \lambda'_{ijk} L_i \cdot Q_j (D^c)_k + \frac{1}{2}\lambda''_{ijk}\varepsilon_{\alpha\beta\gamma}(U^c)_i^\alpha (D^c)_j^\beta (D^c)_k^\gamma, \quad (1)$$

where Q, U^c, D^c, L, and E^c denote the usual quark and lepton superfields, $\cdot$ is the $SU(2)_L$ invariant antisymmetric product, and $\varepsilon_{\alpha\beta\gamma}$ is the three-dimensional Levi-Civita symbol. The indices i, j, and k correspond to the three generations of flavor, while α, β, and γ refer to the color index. The parameters λ_{ijk} and λ''_{ijk} satisfy the following conditions without loss of generality: $\lambda_{ijk} = -\lambda_{jik}$, $\lambda''_{ijk} = -\lambda''_{ikj}$. The first three sets of terms of Eq. (1) violate L and the last one, B. The simultaneous existence of B- and L-violating couplings opens up decay channels of nucleons into mesons and (anti)leptons, where squarks appear as typical mediators at tree level. See Fig. 1, where we show the decay $p \to \pi^+\nu$ via an effective four-fermion interaction generated from the $U^cD^cD^c$ and LQD^c operators. Such nucleon decays have received attention for a long time in the RpV MSSM [38], see e.g., [18,39] for summaries. Original studies focused on (B–L)-conserving processes [40], then ($B + L$)-conserving ones [41,42]. Reference [43] observed that flavor flips associated to the charged weak interaction could be exploited to extend the limits to all flavor directions of the RpV couplings.[1] For related cosmological bounds see, for example, [5,46,47]. Beyond the "direct" nucleon decays mediated by a virtual squark exchange, slightly more complicated structures involving additional intermediate charginos and neutralinos were also considered [38,48–51]. In case such decays are kinematically allowed, these supersymmetric fermions [52], or more exotic new particles [53,54], could also replace the lepton in the final state. The case of a very light neutralino is still experimentally allowed [55–58] and can also be searched for in rare meson decays in various experiments [59–63].

Beyond nucleon decays, which violate the baryon number by one unit, processes violating B by two units, such as dinucleon decays or neutron-antineutron oscillations, provide relevant limits on single RpV couplings, in particular, of $U^cD^cD^c$ type. However, these phenomena also often require further sources of flavor violation in the squark sector. For this reason, we will not discuss them in detail below and refer the interested reader to the recent summary in Ref. [64].

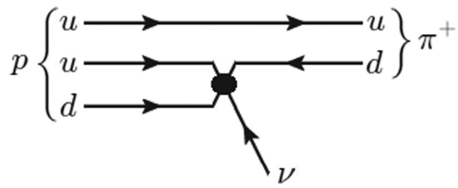

FIG. 1. Possible diagram for proton decay via an effective operator generated from the R-parity violating operators $U^cD^cD^c$ and LQD^c in the superpotential.

In the current paper, we attempt to update the status of the limits applying to the RpV couplings, providing a more detailed attention to the low-energy form factors, about which the RpV literature remains cursory, in general. We restrict ourselves to tree-level RpV contributions since a full one-loop matching would be much more involved. We also renounce a heuristic implementation of the quark-flavor changes, as proposed in, e.g., Ref. [43] since we believe that such limits depend on the renormalization scheme, i.e., on the formal definition of the tree-level couplings.[2] In the following section, we introduce the EFT encoding nucleon decays and derive the matching conditions. We also discuss the relevant low-energy hadronic matrix elements, referring to lattice evaluations, when available, then comparing these results to those of a static bag model, which we employ in other cases. Finally, in Sec. III, we apply up-to-date experimental bounds to specific decay channels and obtain limits on combinations of RpV couplings, before a short conclusion.

II. MATCHING THE RpV CONTRIBUTIONS ON THE $\Delta B = 1$ HAMILTONIAN

In this section, we review the general framework that we employ to compute the nucleon decay widths in the context of the RpV MSSM.

A. Low-energy EFT and QCD running

The classification of operators involving only SM fields and satisfying the SM gauge symmetries and violating B was performed in [65,66]. The operators of lowest dimension that do not conserve B are of dimension six and conserve $B - L$ [65]. Among them, we will be more particularly interested in

$$O^{(1)}_{mnpq} = \varepsilon_{\alpha\beta\gamma}[(\overline{d^c})^\alpha_m P_R (u)^\beta_n] \cdot [(\overline{u^c})^\gamma_p P_L (e)_q - (\overline{d^c})^\gamma_p P_L(\nu)_q],$$
$$O^{(5)}_{mnpq} = \varepsilon_{\alpha\beta\gamma}[(\overline{d^c})^\alpha_m P_R (u)^\beta_n][(\overline{u^c})^\gamma_p P_R (e)_q]. \quad (2)$$

[1] See also Refs. [44,45] for the effects of flavor flips on bounds on and also on signals of RpV, beyond proton decay.

[2] It is indeed possible to thus inflate the set of limits applying to individual (or pairs of) RpV couplings, but the actual bounds, in fact, constrain given directions in parameter space.

Note that u, d, e, and ν correspond to the usual four-component spinors representing quarks and leptons, with latin index relating to flavor and greek to color. Note that f^c ($f = u, d, e, \nu$) indicates charge conjugation: $f^c = C\bar{f}^T$, with C the charge-conjugation matrix. $P_{L,R}$ are chiral projectors. We note that the fields are defined in the gauge-eigenstate basis so that an additional Cabibbo-Kobayashi-Maskawa (CKM) rotation on, e.g., the down-type left-handed quarks and a Pontecorvo-Maki-Nakagawa-Sakata (PMNS) rotation on the neutrinos should be included, in case we wish to work in the mass eigenbasis.

Dimension-seven operators conserve $B + L$ [67]. Through a Higgs vacuum expectation value (VEV), they produce effective dimension-six operators, which violate the EW symmetry. We will encounter the following ones:

$$\begin{aligned}
Q^{(1)}_{mnpq} &= \varepsilon_{\alpha\beta\gamma}[(\overline{d^c})^\alpha_m P_R(u)^\beta_n][(\bar{\nu})_q P_R(d)^\gamma_p],\\
Q^{(2)}_{mnpq} &= \varepsilon_{\alpha\beta\gamma}[(\overline{d^c})^\alpha_m P_R(d)^\beta_n][(\bar{\nu})_q P_R(u)^\gamma_p],\\
Q^{(5)}_{mnpq} &= \varepsilon_{\alpha\beta\gamma}[(\overline{d^c})^\alpha_m P_R(d)^\beta_n][(\bar{e})_q P_L(d)^\gamma_p],\\
Q^{(6)}_{mnpq} &= \varepsilon_{\alpha\beta\gamma}[(\overline{d^c})^\alpha_m P_R(d)^\beta_n][(\bar{e})_q P_R(d)^\gamma_p]. \qquad (3)
\end{aligned}$$

Such terms are produced in the RpV MSSM via particle mixing, either in the squark or in the lepton-Higgsino-gaugino sectors. An EW-violating VEV is always needed to generate them. As long as the EW and SUSY scales are not resolutely in hierarchical ratio, i.e., $M_{\rm SUSY}/M_W$ is not too big, the associated suppression is not paramount. In fact, even the relic of a dimension-eight operator will show up in tree-level matching, though involving two orders of mixing:

$$R = \varepsilon_{\alpha\beta\gamma}[(\overline{d^c})^\alpha_m P_R(d)^\beta_n][(\overline{\nu^c})_q P_L(u)^\gamma_p]. \qquad (4)$$

In all the operators considered above, the lepton field can be replaced by an electroweakino field. In fact, due to the mixing appearing in the RpV context, the neutrinos and charged leptons could themselves be viewed as specific neutralino and chargino eigenstates. The resulting new operators could be genuine low-energy operators in the presence of, e.g., a light gaugino. If, on the contrary, the electroweakinos are very massive (as compared to the nucleon mass), then these operators with external electroweakinos are simply a step in the direction of typically higher-dimensional low-energy operators, as considered in, e.g., Ref. [51]. As long as no further quark (or gluon) lines are attached in this manner, the QCD aspects of the operators with external electroweakinos do not differ from those of operators with external leptons (up to momentum-dependent terms), so that the recipes discussed below continue to apply.

In the RpV MSSM, B-violating effects in nucleon decays are mediated by supersymmetric particles. At least the sfermions can be expected to be comparatively heavy with respect to the scale at which nucleon decay takes place. This means that, below the scale of the sfermions, we can summarize their impact in the B-violating processes by their contribution to the operators of Eqs. (2)–(3) (where, technically, the operators of Eqs. (3) should be restored to their full EW-conserving version). This defines the effective Hamiltonian as follows:

$$\mathcal{H}^{\rm eff} = \sum_{\Omega=O,Q} C_\Omega(\mu_R)\Omega(\mu_R), \qquad (5)$$

where μ_R denotes the renormalization scale. The Wilson coefficients C_Ω encode the short-distance effects and are obtained from integrating out the heavy fields. Large $\ln\frac{M_{\rm SUSY}}{M_N}$ corrections, where M_N denotes the nucleon mass, are expected to develop via radiative effects between the scale of the sfermions $\mu_R = M_{\rm SUSY}$, where the short-distance effects are defined, and the scale of the nucleon $\mu_R \approx M_N$, at which the operators mediate the hadronic process. The leading contributions to the anomalous dimension have been studied in Ref. [68]. Contrary to the case of GUTs, our high-energy boundary, the sfermion scale, is expected to be comparatively close to the EW scale, so that we can neglect the EW running and restrict ourselves to the sole QCD running. Following Ref. [68], all the operators then receive a simple scaling factor from the QCD corrections, which we can summarize as follows:

$$\begin{aligned}
C_\Omega(\mu_R) &= \eta_{\rm QCD} C_\Omega(M_{\rm SUSY}),\\
\eta_{\rm QCD} &= \left[\frac{\alpha_S(m_t)}{\alpha_S(M_{\rm SUSY})}\right]^{2/\beta_0[6]} \cdot \left[\frac{\alpha_S(m_b)}{\alpha_S(m_t)}\right]^{2/\beta_0[5]}\\
&\quad \cdot \left[\frac{\alpha_S(\mu_R)}{\alpha_S(m_b)}\right]^{2/\beta_0[4]},\\
\beta_0[N_F] &\equiv 11 - \frac{2}{3}N_F, \qquad (6)
\end{aligned}$$

with α_S the running QCD coupling. The low-energy scale μ_R cannot be set much below the charm mass m_c because of the perturbative description failing at low energy. Lattice calculations [69] employ $\mu_R = 2$ GeV. Then, the problem is factorized in two separate issues, the determination of the short-distance coefficients $C_\Omega(M_{\rm SUSY})$ (or matching to the high-energy model), which we perform in the next subsection, and the evaluation of the hadronic matrix element, which we discuss in the subsequent subsections.

B. Defining the Wilson coefficients

In order to define the Wilson coefficients, we consider partonic scattering amplitudes both in the full RpV MSSM and in the EFT, and identify them at the SUSY scale (matching). See Fig. 2 for example interactions.

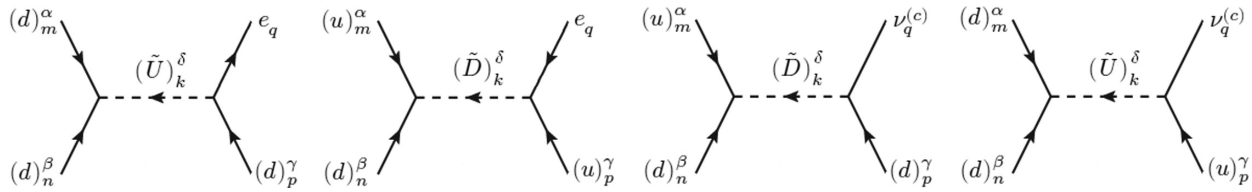

FIG. 2. Possible nucleon decays via the combination of couplings λ''_{kmn} and λ'_{qpk}. These can be seen as t- or s-channel processes.

1. Feynman amplitude in the RpV MSSM

These transition amplitudes can be easily written at tree level in terms of the couplings that are defined in the Appendix.

An internal sup (up-squark) line mediates a transition amplitude with three external down-type quarks (for simplicity, we write fields in the amplitudes below, while they should be replaced by four-component spinors in practice):

$$\begin{aligned}\mathcal{A}^{\mathrm{RpV}}[d_m^\alpha d_n^\beta d_p^\gamma e_q] = {}& \frac{i\varepsilon_{\gamma\alpha\beta}}{m^2_{\tilde{U}_k}}(g_R^{Udd})_{kmn} \\ & \times\{(g_L^{Ud\chi})_{kpq}[(\overline{d^c})_m^\alpha P_R(d)_n^\beta][(\bar{e})_q P_L(d)_p^\gamma] \\ & +(g_R^{Ud\chi})_{kpq}[(\overline{d^c})_m^\alpha P_R(d)_n^\beta][(\bar{e})_q P_R(d)_p^\gamma]\} \\ & +[(m,\alpha)\leftrightarrow(n,\beta)\leftrightarrow(p,\gamma)]. \end{aligned} \tag{7}$$

Here, $m_{\tilde{U}_k}$ denotes the sup mass of generation k, and the couplings $(g_R^{Udd})_{kmn}$, etc. are defined in Appendix B. The lepton spinor is one of the light states of the chargino-lepton system: $e_q = \chi^-_{q+2}$ (we omit the $+2$ above, as well as the $+4$ for the neutrinos among the neutralino states later on). We see that the result projects on the operators $Q^{(5)}_{mnpq}$ and $Q^{(6)}_{mnpq}$ of Eq. (3).

Similarly, with two entering up-type lines, one entering down-type line and a lepton, we have a diagram with an internal sdown line:

$$\begin{aligned}\mathcal{A}^{\mathrm{RpV}}[u_m^\alpha d_n^\beta u_p^\gamma e_q] = {}& -\frac{i\varepsilon_{\alpha\beta\gamma}}{m^2_{\tilde{D}_k}}(g_R^{uDd})_{mkn} \\ & \times\{(g_L^{Du\chi})_{kpq}[(\overline{u^c})_m^\alpha P_R(d)_n^\beta][(\overline{e^c})_q P_L(u)_p^\gamma] \\ & +(g_R^{Du\chi})_{kpq}[(\overline{u^c})_m^\alpha P_R(d)_n^\beta][(\overline{e^c})_q P_R(u)_p^\gamma]\} \\ & +[(m,\alpha)\leftrightarrow(p,\gamma)]. \end{aligned} \tag{8}$$

Here $m^2_{\tilde{D}_k}$ denotes the sdown mass and again the couplings $(g_R^{uDd})_{mkn}$ are given in Appendix B. For two entering down-type lines and one entering up-type line plus a neutrino, we first have a diagram with internal sdown line (the equation is somewhat abusive as we consider neutrino and antineutrino simultaneously):

$$\begin{aligned}\mathcal{A}^{\mathrm{RpV}}[u_m^\alpha d_n^\beta d_p^\gamma \nu_q] = {}& -\frac{i\varepsilon_{\alpha\beta\gamma}}{m^2_{\tilde{D}_k}}(g_R^{uDd})_{mkn} \\ & \times\{(g_L^{Dd\chi})_{kpq}[(\overline{u^c})_m^\alpha P_R(d)_n^\beta][(\overline{\nu^c})_q P_L(d)_p^\gamma] \\ & +(g_R^{Dd\chi})_{kpq}[(\overline{u^c})_m^\alpha P_R(d)_n^\beta][(\bar{\nu})_q P_R(d)_p^\gamma]\} \\ & +[(n,\beta)\leftrightarrow(p,\gamma)]. \end{aligned} \tag{9}$$

Then, there is a diagram with an internal sup line contributing to the same amplitude:

$$\begin{aligned}\mathcal{A}^{\mathrm{RpV}}[d_m^\alpha d_n^\beta u_p^\gamma \nu_q] = {}& \frac{i\varepsilon_{\gamma\alpha\beta}}{m^2_{\tilde{U}_k}}(g_R^{Udd})_{kmn} \\ & \times\{(g_L^{Uu\chi})_{kpq}[(\overline{d^c})_m^\alpha P_R(d)_n^\beta][(\overline{\nu^c})_q P_L(u)_p^\gamma] \\ & +(g_R^{Uu\chi})_{kpq}[(\overline{d^c})_m^\alpha P_R(d)_n^\beta][(\bar{\nu})_q P_R(u)_p^\gamma]\} \\ & +[(m,\alpha)\leftrightarrow(n,\beta)]. \end{aligned} \tag{10}$$

2. Amplitudes in the effective field theory

The corresponding amplitudes in the EFT read,

$$\begin{aligned}\mathcal{A}^{\mathrm{EFT}}[d_m^\alpha d_n^\beta d_p^\gamma e_q] = {}& i\varepsilon_{\alpha\beta\gamma}\{(C_{Q_5})_{mnpq} \\ & \times[(\overline{d^c})_m^\alpha P_R(d)_n^\beta][(\bar{e})_q P_L(d)_p^\gamma] \\ & +(C_{Q_6})_{mnpq}[(\overline{d^c})_m^\alpha P_R(d)_n^\beta][(\bar{e})_q P_R(d)_p^\gamma]\} \\ & +[(m,\alpha)\leftrightarrow(n,\beta)\leftrightarrow(p,\gamma)], \end{aligned} \tag{11}$$

$$\begin{aligned}\mathcal{A}^{\mathrm{EFT}}[u_m^\alpha d_n^\beta u_p^\gamma e_q] = {}& -i\varepsilon_{\alpha\beta\gamma}\{(C_{O_1})_{nmpq} \\ & \times[(\overline{u^c})_m^\alpha P_R(d)_n^\beta][(\overline{e^c})_q P_L(u)_p^\gamma] \\ & +(C_{O_5})_{nmpq}[(\overline{u^c})_m^\alpha P_R(d)_n^\beta][(\overline{e^c})_q P_R(u)_p^\gamma]\} \\ & +[(m,\alpha)\leftrightarrow(p,\gamma)], \end{aligned} \tag{12}$$

$$\begin{aligned}\mathcal{A}^{\rm EFT}[u_m^\alpha d_n^\beta d_p^\gamma \nu_q] = -i\varepsilon_{\alpha\beta\gamma}\{&(-C_{O_1})_{nmpq}\\ &\times[(\overline{u^c})_m^\alpha P_R (d)_n^\beta][(\overline{\nu^c})_q P_L (d)_p^\gamma]\\ &+(C_{Q_1})_{nmpq}[(\overline{u^c})_m^\alpha P_R (d)_n^\beta][(\bar\nu)_q P_R (d)_p^\gamma]\\ &-(C_R)_{mnpq}[(\overline{d^c})_m^\alpha P_R (d)_n^\beta][(\overline{\nu^c})_q P_L (u)_p^\gamma]\\ &-(C_{Q_2})_{mnpq}[(\overline{d^c})_m^\alpha P_R (d)_n^\beta][(\bar\nu)_q P_R (u)_p^\gamma]\}\\ &+[(n,\beta)\leftrightarrow(p,\gamma)]. \end{aligned}\quad (13)$$

3. Matching

Identifying Eqs. (11)–(13) with Eqs. (7)–(10), we obtain

$$\begin{aligned}(C_{O_1})_{nmpq} &= \frac{1}{m^2_{\tilde D_k}}(g_R^{uDd})_{mkn}(g_L^{Du\chi})_{kpq},\\ (C_{O_5})_{nmpq} &= \frac{1}{m^2_{\tilde D_k}}(g_R^{uDd})_{mkn}(g_R^{Du\chi})_{kpq},\\ (C_{Q_1})_{nmpq} &= \frac{1}{m^2_{\tilde D_k}}(g_R^{uDd})_{mkn}(g_R^{Dd\chi})_{kpq},\\ (C_{Q_2})_{mnpq} &= \frac{1}{m^2_{\tilde U_k}}(g_R^{Udd})_{kmn}(g_R^{Uu\chi})_{kpq},\\ (C_{Q_5})_{mnpq} &= \frac{1}{m^2_{\tilde U_k}}(g_R^{Udd})_{kmn}(g_L^{Ud\chi})_{kpq},\\ (C_{Q_6})_{mnpq} &= \frac{1}{m^2_{\tilde U_k}}(g_R^{Udd})_{kmn}(g_R^{Ud\chi})_{kpq},\\ (C_R)_{mnpq} &= \frac{1}{m^2_{\tilde U_k}}(g_R^{Udd})_{kmn}(g_L^{Uu\chi})_{kpq}.\end{aligned}\quad (14)$$

The contribution to O_1 is mediated directly by λ'' and λ' couplings. The contribution to O_5 is in fact of dimension eight: it involves a Higgs VEV from squark mixing and a second from chargino mixing. It is thus generated from the μ_i terms and receives additional mixing suppression. The contributions to Q_1 and Q_6 can be generated from a λ' coupling, in which case, the Higgs VEV is provided by squark mixing or from μ_i, in which case, the VEV comes from gaugino-Higgsino mixing. The contribution to Q_2 is essentially mediated by mixing of the lepton with the gauginos. The contributions to Q_5 and R are of the same order as that to O_5, i.e., they depend on secondary mixing of the leptons with the charginos/neutralinos. This counting is changed if we replace the external leptons by electroweakino states, as we see in Sec. III.

C. Low-energy operators

So far, we have kept generic flavor indices. However, assuming that valence quarks determine nucleon decays, we can restrict ourselves to the three light quark flavors (as well as the two lighter charged leptons), hence to a smaller set of operators:

$$\begin{aligned}\mathcal{O}_1^e &= \varepsilon_{\alpha\beta\gamma}[(\overline{d^c})^\alpha P_R u^\beta][(\overline{u^c})^\gamma P_L e],\\ \mathcal{O}_1^\nu &= \varepsilon_{\alpha\beta\gamma}[(\overline{d^c})^\alpha P_R u^\beta][(\overline{d^c})^\gamma P_L \nu],\\ \mathcal{O}_5^e &= \varepsilon_{\alpha\beta\gamma}[(\overline{d^c})^\alpha P_R u^\beta][(\overline{u^c})^\gamma P_R e],\\ \mathcal{Q}_1^\nu &= \varepsilon_{\alpha\beta\gamma}[(\overline{d^c})^\alpha P_R u^\beta][(\overline{d^c})^\gamma P_R \nu^c],\\ \hat{\mathcal{O}}_1^e &= \varepsilon_{\alpha\beta\gamma}[(\overline{s^c})^\alpha P_R u^\beta][(\overline{u^c})^\gamma P_L e],\\ \hat{\mathcal{O}}_1^\nu &= \varepsilon_{\alpha\beta\gamma}[(\overline{s^c})^\alpha P_R u^\beta][(\overline{d^c})^\gamma P_L \nu],\\ \hat{\mathcal{O}}_5^e &= \varepsilon_{\alpha\beta\gamma}[(\overline{s^c})^\alpha P_R u^\beta][(\overline{u^c})^\gamma P_R e],\\ \hat{\mathcal{O}}'^\nu_1 &= \varepsilon_{\alpha\beta\gamma}[(\overline{d^c})^\alpha P_R u^\beta][(\overline{s^c})^\gamma P_L \nu],\\ \hat{\mathcal{Q}}_1^\nu &= \varepsilon_{\alpha\beta\gamma}[(\overline{s^c})^\alpha P_R u^\beta][(\overline{d^c})^\gamma P_R \nu^c],\\ \hat{\mathcal{Q}}'^\nu_1 &= \varepsilon_{\alpha\beta\gamma}[(\overline{d^c})^\alpha P_R u^\beta][(\overline{s^c})^\gamma P_R \nu^c],\\ \hat{\mathcal{Q}}_2^\nu &= \varepsilon_{\alpha\beta\gamma}[(\overline{s^c})^\alpha P_R d^\beta][(\overline{u^c})^\gamma P_R \nu^c],\\ \hat{\mathcal{R}}^\nu &= \varepsilon_{\alpha\beta\gamma}[(\overline{s^c})^\alpha P_R d^\beta][(\overline{u^c})^\gamma P_L \nu],\\ \hat{\mathcal{Q}}_5^e &= \varepsilon_{\alpha\beta\gamma}[(\overline{s^c})^\alpha P_R d^\beta][(\overline{d^c})^\gamma P_L e^c],\\ \hat{\mathcal{Q}}_6^e &= \varepsilon_{\alpha\beta\gamma}[(\overline{s^c})^\alpha P_R d^\beta][(\overline{d^c})^\gamma P_R e^c].\end{aligned}\quad (15)$$

Operators with ^ (lower set) induce $\Delta S = 1$, while those without preserve strangeness. We neglect operators with $\Delta S \geq 2$. The lepton should be understood as generic, $e = e$, μ and $\nu = \nu_{e,\mu,\tau}$. We note that when twice the same quark is contracted in a scalar product, e.g., $\varepsilon_{\alpha\beta\gamma}[(\overline{d^c})^\alpha P_{L,R} d^\beta]$, then the operator is identically 0.

It is then straightforward to identify the Wilson coefficients of the operators of Eq. (15) with the low-energy coefficients of the original operators, Eqs. (2)–(3):

$$\begin{aligned}C[\mathcal{O}_1^e] &= (C_{O_1})_{111e},\\ C[\hat{\mathcal{O}}_1^e] &= (C_{O_1})_{211e},\\ C[\mathcal{O}_1^\nu] &= -V_{r1}^{\rm CKM}(C_{O_1})_{11r\nu},\\ C[\hat{\mathcal{O}}_1^\nu] &= -V_{r1}^{\rm CKM}(C_{O_1})_{21r\nu},\\ C[\hat{\mathcal{O}}'^\nu_1] &= -V_{r2}^{\rm CKM}(C_{O_1})_{11r\nu},\\ C[\mathcal{O}_5^e] &= (C_{O_5})_{111e},\\ C[\hat{\mathcal{O}}_5^e] &= (C_{O_5})_{211e},\\ C[\mathcal{Q}_1^\nu] &= (C_{Q_1})_{111\nu},\\ C[\hat{\mathcal{Q}}_1^\nu] &= (C_{Q_1})_{211\nu},\\ C[\hat{\mathcal{Q}}'^\nu_1] &= (C_{Q_1})_{112\nu},\\ C[\hat{\mathcal{Q}}_2^\nu] &= (C_{Q_2})_{211\nu} - (C_{Q_2})_{121\nu},\\ C[\hat{\mathcal{R}}^\nu] &= (C_R)_{211\nu} - (C_R)_{121\nu},\\ C[\hat{\mathcal{Q}}_5^e] &= V_{r1}^{\rm CKM}[(C_{Q_5})_{21re} - (C_{Q_5})_{12re}],\\ C[\hat{\mathcal{Q}}_6^e] &= (C_{Q_6})_{211e} - (C_{Q_6})_{121e}.\end{aligned}\quad (16)$$

Here, we have chosen to define the couplings such that the CKM rotation V^{CKM} is fully carried by the down-quark sector [70].

D. Hadronic matrix elements

In order to connect the parton level B-violating EFT to actual nucleon decays, it is necessary to evaluate the operators of Eq. (15) between the nucleon and its hadronic decay products, e.g., pions or kaons. Nonperturbative methods are needed to perform this step. Among the models employed in the 1980s, a first class [71–76] would consider nonrelativistic partons and exploit the $SU(6)$ flavor-spin symmetry. An alternative approach is that of the bag models [77–82], where partonic quarks are now relativistic. Major conceptual difficulties in these descriptions appear in association with, e.g., the treatment of a relativistic pion in the final state or the impact of the three-quark fusion process [80,83]. Partial conservation of axial vector currents was also employed for the estimate [83–86]. The formulation of a chiral model for baryon interaction [87–90] allowed to derive relations between the decay rates mediated by dimension-six operators and low-energy constants (LEC). How far the validity of the chiral model extends is not completely clear. Predictions are (very) roughly in agreement among these calculations.

1. Lattice evaluation

Lattice approaches to nucleon decays were also considered early on and have continued up to this day [69,91–96]. Corresponding calculations focus on dimension-six operators and nucleon (N) decays into one pseudoscalar meson (Π) and one (anti)lepton ($L^{(c)}$): $N\to\Pi+L^{(c)}$. The matrix elements can be represented by the following form factors:

$$\langle\Pi(p-\ell)|\Omega_{GH}|N(p)\rangle = P_H\left\{W^{N\to\Pi}_{0,[\Omega_{GH}]}(\ell^2) - \frac{i\not\ell}{M_N}W^{N\to\Pi}_{1,[\Omega_{GH}]}(\ell^2)\right\}u_N(p),$$
$$\Omega_{GH}\equiv\varepsilon_{\alpha\beta\gamma}[(\overline{q^c})^\alpha_i P_G(q)^\beta_j][P_H(q)^\gamma_k]. \quad (17)$$

Here, $q=u,\ d,\ s$; $G,H\in\{L,R\}$ are the indices of the chiral projectors; p and ℓ denote the four-momenta of the nucleon and (anti)lepton, respectively; $u_N(p)$ represents the spinor associated with the nucleon M_N, its mass, and $W^{N\to\Pi}_{(0,1),[\Omega_{GH}]}$ correspond to the form factors, which depend on the momentum transfer squared ℓ^2 between the nucleon and the meson. For commodity, we will write the left-hand side of Eq. (17) in the abbreviated form $\langle\Pi|(q_iq_j)_G(q_k)_H|N\rangle$ below. If $q_i=q_j$, then the operator is identically 0. Parity invariance of the strong interaction results in identities under $(L,R)\leftrightarrow(R,L)$ or $(L,L)\leftrightarrow(R,R)$ exchanges. The assumption of isospin invariance $u\leftrightarrow d$ produces further relations between different initial or final states: $p\leftrightarrow -n$, $\pi^+\leftrightarrow\pi^-$, $\pi^0\to-\pi^0$, $\eta\to\eta$, $K^+\leftrightarrow K^0$, $\rho^+\leftrightarrow\rho^-$, $\rho^0\to-\rho^0$, $\omega\to\omega$, $K^{*+}\leftrightarrow K^{*0}$.

The lattice results for all the relevant form factors $W^{N\to\Pi}_{0,[\Omega]}$ (including the renormalization scheme conversion) are presented in Table 4 of Ref. [69]. The $W^{N\to\Pi}_{1,[\Omega]}$ corrections are evidently suppressed for electrons and neutrinos in the final state but have been considered in the case of a muon (see Table 5 of the previous reference). We then obtain a complete list of matrix elements for the operators of Eq. (15) for the transitions of $N\to\Pi+L^{(c)}$ type:

$$\langle\Pi(p-\ell),L^{(c)}|\Omega_{G,H}|N(p)\rangle = \bar{w}_L(l)P_H\cdot\cdot\left\{W^{GH}_{0,[\Omega]}(\ell^2)-\frac{i\not\ell}{M_N}W^{GH}_{1,[\Omega]}(\ell^2)\right\}u_N(p); \quad (18)$$

$w_L=u_L,v^c_L$ denotes the lepton spinor. From there, it is straightforward to derive the decay amplitudes and decay widths:

$$\mathcal{A}[N\to\Pi+L^{(c)}] = i\sum_\Omega C[\Omega_{GH}]\langle\Pi(p-\ell),L^{(c)}|\Omega_{GH}|N(p)\rangle = i\bar{w}_L(l)P_H\sum_\Omega C[\Omega_{GH}]W^{N\to\Pi}_{[\Omega_{GH}]}u_N(p), \quad (19)$$

$$\Gamma[N\to\Pi+L^{(c)}] = \frac{1}{16\pi M_N}\sqrt{1-2\frac{M_\Pi^2+M_L^2}{M_N^2}+\left(\frac{M_\Pi^2-M_L^2}{M_N^2}\right)^2}\left[\frac{1}{2}\sum_{\mathrm{pol}}|\mathcal{A}[N\to\Pi+L^{(c)}]|^2\right]$$
$$\simeq\frac{M_N}{32\pi}\left(1-\frac{M_\Pi^2}{M_N^2}\right)^2\sum_{\Omega,\Omega'}(C[\Omega_{GH}]W^{N\to\Pi}_{[\Omega_{GH}]})^*(C[\Omega'_{G'H'}]W^{N\to\Pi}_{[\Omega'_{G'H'}]})\delta_{HH'}, \quad (20)$$

with Ω, Ω' scanning the list of Eq. (15), $\sum_{\mathrm{pol}}$ representing the sum over spinor polarizations, and M_X denoting the mass of particle X. We have neglected the lepton mass in the last line. W reduces to W_0 most of the time but includes the W_1 correction for muons in the final state.

TABLE I. Hadronic matrix elements for proton decays from lattice [(Lat.)] computations. Here the entries such as $(du)_G(u)_H$ denote the operator $\Omega_{G,H}$ of Eq. (18). We have not included the subscripts $\Omega_{G,H}$ on W_0 but just indicate the decay mode in brackets.

$(du)_G(u)_H$	$W_0[p\to\pi^0]$ $(\mathrm{GeV})^2$, lat.	$W_0[p\to\pi^0]$ $(\mathrm{GeV})^2$, bag	$W_0[p\to\eta]$ $(\mathrm{GeV})^2$, lat.	$W_0[p\to\eta]$ $(\mathrm{GeV})^2$, bag
LL, RR	0.134	0.015	0.113	0.074
LR, RL	−0.131	−0.015	0.006	0.005

$(du)_G(d)_H$	$W_0[p\to\pi^+]$ $(\mathrm{GeV})^2$, lat.	$W_0[p\to\pi^+]$ $(\mathrm{GeV})^2$, bag
LL, RR	0.189	0.022
LR, RL	−0.186	−0.018

$(su)_G(u)_H$	$W_0[p\to K^0]$ $(\mathrm{GeV})^2$, lat.	$W_0[p\to K^0]$ $(\mathrm{GeV})^2$, bag
LL, RR	0.057	0.010
LR, RL	0.103	0.012

$W_0[p\to K^+]$	$(su)_G(d)_H$ $(\mathrm{GeV})^2$, lat.	$(su)_G(d)_H$ $(\mathrm{GeV})^2$, bag	$(du)_G(s)_H$ $(\mathrm{GeV})^2$, lat.	$(du)_G(s)_H$ $(\mathrm{GeV})^2$, bag	$(sd)_G(u)_H$ $(\mathrm{GeV})^2$, lat.	$(sd)_G(u)_H$ $(\mathrm{GeV})^2$, bag
LL, RR	0.041	0.022	0.139	0.014	−0.098	−0.012
LR, RL	−0.049	−0.022	−0.134	−0.014	−0.054	−0.010

2. Nonlattice evaluation

If we want to consider other decay channels, e.g., involving vector mesons, we need to turn to other means of estimates of the hadronic matrix elements since corresponding lattice results are not available. However, this means that the uncertainties will be significantly larger for these additional channels. The lattice results are presented with comparatively small uncertainties $\sim O(10\%)$ [69]. This latter reference already mentions a factor 2 to 3 difference with the proton decays evaluated from LEC, showing that the precision drops considerably when resorting to nonlattice methods. In practice, we consider the static bag model described in the Appendix, from which we expect results of strictly qualitative value. On the other hand, considering the relatively large mass of the vector mesons, the static bag model should be performing almost in its regime of validity. We first compared the results of the bag model with those obtained in Ref. [78], which we essentially follow, and sensibly recovered the results of this paper for the operator considered in this reference. Then, we consider the bag predictions for the $N\to\Pi+L$ channels in order to compare with the lattice results and assess the reliability of this approach. The results are displayed in Table I for a proton radius of 5 GeV^{-1} and smaller meson radii (pion: 3.3 GeV^{-1}, kaon: 2.8 GeV^{-1}, and η: 4.7 GeV^{-1}). In the case of "narrow" mesons (pions and kaons), the results of the bag model are one order of magnitude below the central values originating from the lattice calculation and somewhat closer for "broad" mesons (η, a factor ~ 2). The difference in the case of the narrow mesons can be somewhat reduced if one employs the same bag radius of the proton for the mesons as well—essentially because the overlap between proton and meson wave functions is larger—and these values could be further tuned by varying the chosen proton radius (which we do not attempt). We thus observe that the bag model is not reliable on a quantitative basis. However, the corresponding results seem to always underestimate the hadronic matrix elements, so that the bounds that one obtains when using these values are conservative.

The hadronic matrix elements for the transitions into vector mesons are shown in Table II with meson radii 4.8 GeV^{-1} (rho, omega) and 3.0 GeV^{-1} (K^*). There, we define the form factors as follows:

$$\langle V(p-\ell)|\Omega_{GH}|N(p)\rangle = W^{N\to V}_{[\Omega_{GH}]}(\ell^2)P_H\gamma^\mu u_N(p)\varepsilon^{V*}_\mu(p-\ell), \tag{21}$$

where ε^{V*}_μ corresponds to the polarization vector of the vector meson. Including the lepton spinor $L^{(c)}$ in the matrix element, the decay amplitude and widths then read,

$$\begin{aligned}&\mathcal{A}[N\to V+L^{(c)}]\\ &= i\sum_\Omega C[\Omega_{GH}]\langle V(p-\ell),L^{(c)}|\Omega_{GH}|N(p)\rangle\\ &= i\sum_\Omega C[\Omega_{GH}]W^{N\to V}_{[\Omega_{GH}]}\bar{w}_L(l)P_H\gamma^\mu u_N(p)\varepsilon^{V*}_\mu(p-\ell),\end{aligned} \tag{22}$$

TABLE II. Hadronic matrix elements for proton to vector meson decays using the bag model. Otherwise, the notation is as in Table I.

$(du)_G(u)_H$	$W_0[p\to\rho^0]$ $(\mathrm{GeV})^2$, bag	$W_0[p\to\omega^0]$ $(\mathrm{GeV})^2$, bag	$(du)_G(d)_H$	$W_0[p\to\rho^+]$ $(\mathrm{GeV})^2$, bag
LL, RR	0.062	−0.041	LL, RR	0.088
LR, RL	−0.032	0.041	LR, RL	−0.048

$(su)_G(u)_H$	$W_0[p\to K^{*0}]$ $(\mathrm{GeV})^2$, bag	$W_0[p\to K^{*+}]$	$(su)_G(d)_H$ $(\mathrm{GeV})^2$, bag	$(du)_G(s)_H$ $(\mathrm{GeV})^2$, bag	$(sd)_G(u)_H$ $(\mathrm{GeV})^2$, bag
LL, RR	−0.005	LL, RR	0.013	0.022	−0.008
LR, RL	−0.006	LR, RL	0.009	−0.022	−0.002

$$\Gamma[N\to V+L^{(c)}]=\frac{1}{16\pi M_N}\sqrt{1-2\frac{M_V^2+M_L^2}{M_N^2}+\left(\frac{M_V^2-M_L^2}{M_N^2}\right)^2}\left[\frac{1}{2}\sum_{\mathrm{pol}}|\mathcal{A}[N\to V+L^{(c)}]|^2\right]$$
$$\simeq\frac{M_N^3}{64\pi M_V^2}\left(1-\frac{M_V^2}{M_N^2}\right)^2\left(1+2\frac{M_V^2}{M_N^2}\right)\sum_{\Omega,\Omega'}(C_\Omega^{GH}W^{N\to V}_{[\Omega_{GH}]})^*(C[\Omega'_{G'H}]W^{N\to V}_{[\Omega'_{G'H}]}). \tag{23}$$

We have again neglected the lepton mass in the last expression. Given that the matrix elements obtained with the bag model are suppressed, as compared to those derived on the lattice, the limits applying to nucleon to vector meson transitions will always prove subleading as compared to those of the nucleon to pseudoscalar meson channels. As the latter are more reliable, this situation is desirable so that derived bounds are conservative. On the other hand, we note that in the bag model, the branching ratios of the nucleon to vector meson transitions are larger than those of the nucleon decays into pseudoscalar mesons. Consequently, a more precise determination of the hadronic form factors for vector channels could eventually lead to competitive results, at least in certain directions of the B-violating operator basis.

The matrix elements for the neutron decays can be obtained from those of the proton channels by exploiting the isospin symmetry:

$$\begin{aligned}
\langle\pi^-|(du)_G(u)_H|n\rangle&=\langle\pi^+|(du)_G(d)_H|p\rangle,\\
\langle\pi^0|(du)_G(d)_H|n\rangle&=-\langle\pi^0|(du)_G(u)_H|p\rangle,\\
\langle\eta|(du)_G(d)_H|n\rangle&=\langle\eta|(du)_G(u)_H|p\rangle,\\
\langle K^+|(sd)_G(d)_H|n\rangle&=-\langle K^0|(su)_G(u)_H|p\rangle,\\
\langle K^0|(du)_G(s)_H|n\rangle&=\langle K^+|(du)_G(s)_H|p\rangle,\\
\langle K^0|(su)_G(d)_H|n\rangle&=-\langle K^+|(sd)_G(u)_H|p\rangle,\\
\langle K^0|(sd)_G(u)_H|n\rangle&=-\langle K^+|(su)_G(d)_H|p\rangle,\\
\langle\rho^-|(du)_G(u)_H|n\rangle&=\langle\rho^+|(du)_G(d)_H|p\rangle,\\
\langle\rho^0|(du)_G(d)_H|n\rangle&=-\langle\rho^0|(du)_G(u)_H|p\rangle,\\
\langle\omega|(du)_G(d)_H|n\rangle&=\langle\omega|(du)_G(u)_H|p\rangle,\\
\langle K^{*+}|(sd)_G(d)_H|n\rangle&=-\langle K^{*0}|(su)_G(u)_H|p\rangle,\\
\langle K^{*0}|(du)_G(s)_H|n\rangle&=\langle K^{*+}|(du)_G(s)_H|p\rangle,\\
\langle K^{*0}|(su)_G(d)_H|n\rangle&=-\langle K^{*+}|(sd)_G(u)_H|p\rangle,\\
\langle K^{*0}|(sd)_G(u)_H|n\rangle&=-\langle K^{*+}|(su)_G(d)_H|p\rangle.
\end{aligned}\tag{24}$$

E. Summary

For commodity, we introduce reduced decay widths $\tilde{\Gamma}$ for the nucleon (N) to pseudoscalar (Π) or vector (V) meson transitions, from which the actual decay widths (Γ) can be recovered as follows:

$$\Gamma[N\to\Pi+L^{(c)}]\equiv\frac{M_N\eta_{\mathrm{QCD}}^2}{32\pi}\left(1-\frac{M_\Pi^2}{M_N^2}\right)^2\times\tilde{\Gamma}[N\to\Pi+L^{(c)}], \tag{25}$$

$$\Gamma[N\to V+L^{(c)}]\equiv\frac{M_N^3\eta_{\mathrm{QCD}}^2}{64\pi M_V^2}\left(1-\frac{M_V^2}{M_N^2}\right)^2\times\left(1+2\frac{M_V^2}{M_N^2}\right)\tilde{\Gamma}[N\to V+L^{(c)}], \tag{26}$$

where η_{QCD} represents the QCD running factor of Eq. (6).

We may then summarize the calculation of the nucleon decay widths into a meson and a (an anti)lepton with the following equations:

$$\begin{aligned}
\tilde{\Gamma}[p \to (\pi^0, \eta, \rho^0, \omega^0) + e^+] &= |W^{N\to M}_{\mathcal{O}^e_1} C[\mathcal{O}^e_1]|^2 + |W^{N\to M}_{\mathcal{O}^e_5} C[\mathcal{O}^e_5]|^2 \\
&= \tilde{\Gamma}[n \to (\pi^-, \rho^-) + e^+], \\
\tilde{\Gamma}[p \to (\pi^+, \rho^+) + \nu^{(c)}] &= |W^{N\to M}_{\mathcal{O}^\nu_1} C[\mathcal{O}^\nu_1]|^2 + |W^{N\to M}_{\mathcal{Q}^\nu_1} C[\mathcal{Q}^\nu_1]|^2 \\
&= \tilde{\Gamma}[n \to (\pi^0, \eta, \rho^0, \omega^0) + \nu^{(c)}], \\
\tilde{\Gamma}[p \to K^{0(*)} + e^+] &= |W^{N\to M}_{\hat{\mathcal{O}}^e_1} C[\hat{\mathcal{O}}^e_1]|^2 + |W^{N\to M}_{\hat{\mathcal{O}}^e_5} C[\hat{\mathcal{O}}^e_5]|^2, \\
\tilde{\Gamma}[n \to K^{+(*)} + e^-] &= |W^{N\to M}_{\hat{\mathcal{Q}}^e_5} C[\hat{\mathcal{Q}}^e_5]|^2 + |W^{N\to M}_{\hat{\mathcal{Q}}^e_6} C[\hat{\mathcal{Q}}^e_6]|^2, \\
\tilde{\Gamma}[p \to K^{+(*)} + \nu^{(c)}] &= |W^{N\to M}_{\hat{\mathcal{O}}^\nu_1} C[\hat{\mathcal{O}}^\nu_1] + W^{N\to M}_{\hat{\mathcal{O}}'^\nu_1} C[\hat{\mathcal{O}}_1{}'^\nu] + W^{N\to M}_{\hat{\mathcal{R}}^\nu} C[\hat{\mathcal{R}}^\nu]|^2 \\
&\quad + |W^{N\to M}_{\hat{\mathcal{Q}}^\nu_1} C[\hat{\mathcal{Q}}^\nu_1] + W^{N\to M}_{\hat{\mathcal{Q}}'^\nu_1} C[\hat{\mathcal{Q}}'^\nu_1] + W^{N\to M}_{\hat{\mathcal{Q}}^\nu_2} C[\hat{\mathcal{Q}}^\nu_2]|^2 \\
&= \tilde{\Gamma}[n \to K^{0(*)} + \nu^{(c)}], \qquad (27)
\end{aligned}$$

where the $N \to M$ superscript corresponds to the nucleon (N) to meson (M) transition.

III. BOUNDS ON *R*-PARITY-VIOLATING PARAMETERS

A. Approximations

The ingredients described in the previous section can be included within a code and would allow to perform a test that takes the mixing effects into account to their full extent. Nevertheless, for simplicity, we perform several approximations below on the mixings in the SUSY sector and RpV violation, which allow to derive analytical bounds on the RpV couplings:

(i) We neglect secondary electroweakino-lepton mixing, only allowing for the leading Higgsino-lepton mixing generated by the bilinear RpV parameters.

(ii) We linearize the sfermion-electroweakino/lepton-quark couplings in terms of lepton-violating RpV parameters.

(iii) We neglect Yukawa couplings of the first generation.

(iv) We assume that the squark sector is aligned with the Yukawa structure of the quarks, so that the squark mass matrices do not involve new sources of flavor violation.

(v) We neglect left-right squark mixing, except for the third generation.

We stress that the assumptions on the squark sector eliminate most constraints originating in $\Delta B = 2$ processes. While flavor violation in the squark sector could be restored at the radiative level through charged electroweak(ino) loops, this feature would depend on the renormalization scheme adopted for the squark masses and would only be meaningful in a full one-loop analysis of B-violating processes. Under these conditions, $NN \to KK$ [97] appears as the main source of constraints from $\Delta B = 2$ phenomena. Depending on the SUSY spectrum, the latter place limits on the coupling λ''_{112} that can be read off in the left panel of Fig. 4 in Ref. [64].

Exploiting these hypotheses considerably simplifies the expression of the Wilson coefficients for the operators of Eq. (15):

$$\begin{aligned}
C[\mathcal{O}^{e_l}_1] &= -\lambda''_{1g1} V^{\mathrm{CKM}*}_{1f} \left[Y^f_d \delta_{fg} \frac{\mu^*_l}{\mu^*} - \lambda'^*_{lfg} \right] \frac{|X^{D_g}_{kR}|^2}{m^2_{\tilde{D}_k}} \\
C[\hat{\mathcal{O}}^{e_l}_1] &= -\lambda''_{1g2} V^{\mathrm{CKM}*}_{1f} \left[Y^f_d \delta_{fg} \frac{\mu^*_l}{\mu^*} - \lambda'^*_{lfg} \right] \frac{|X^{D_g}_{kR}|^2}{m^2_{\tilde{D}_k}} \\
C[\mathcal{O}^{\nu_l}_1] &= -\lambda''_{1g1} \lambda'^*_{l1g} \frac{|X^{D_g}_{kR}|^2}{m^2_{\tilde{D}_k}} \\
C[\hat{\mathcal{O}}^{\nu_l}_1] &= -\lambda''_{1g2} \lambda'^*_{l1g} \frac{|X^{D_g}_{kR}|^2}{m^2_{\tilde{D}_k}} \\
C[\hat{\mathcal{O}}'^{\nu_l}_1] &= \lambda''_{1g1} \left[Y^2_d \delta_{g2} \frac{\mu^*_l}{\mu^*} - \lambda'^*_{l2g} \right] \frac{|X^{D_g}_{kR}|^2}{m^2_{\tilde{D}_k}} \\
C[\mathcal{Q}^{\nu_l}_1] &= -\lambda''_{131} \lambda'^*_{l31} \frac{X^{D_3}_{kR} X^{D_3*}_{kL}}{m^2_{\tilde{D}_k}} \\
C[\hat{\mathcal{Q}}^{\nu_l}_1] &= -\lambda''_{132} \lambda'^*_{l31} \frac{X^{D_3}_{kR} X^{D_3*}_{kL}}{m^2_{\tilde{D}_k}} \\
C[\hat{\mathcal{Q}}'^{\nu_l}_1] &= -\lambda''_{131} \lambda'^*_{l32} \frac{X^{D_3}_{kR} X^{D_3*}_{kL}}{m^2_{\tilde{D}_k}} \\
C[\hat{\mathcal{Q}}^{e_l}_6] &= (\lambda''_{321} - \lambda''_{312}) \lambda'^*_{lf1} V^{\mathrm{CKM}}_{3f} \frac{X^{U_3}_{kR} X^{U_3*}_{kL}}{m^2_{\tilde{U}_k}} \\
C[\mathcal{O}^{e_l}_5] &= C[\hat{\mathcal{O}}^{e_l}_5] = C[\hat{\mathcal{Q}}^{\nu_l}_2] = C[\hat{\mathcal{Q}}^{e_l}_5] = C[\hat{\mathcal{R}}^{\nu_l}] = 0, \qquad (28)
\end{aligned}$$

where l is the lepton-flavor index while summation over repeated indices is implicit. The $X^{D_g}_{kR}$, etc. denote the sfermion mixing factors given in Appendix A. This list can be further reduced by neglecting sbottom mixing, CKM mixing, or the strange-quark mass. We note, however, that all the coefficients of Eq. (28) are linearly independent, i.e., that one can *a priori* find directions in parameter space where only one of these coefficients (and any of the nontrivial ones) is nonzero.

In Eq. (28), we have considered only the four-fermion operators mediating proton decay that explicitly include a lepton field. As argued before [49–51], it is meaningful to consider operators with an electroweakino field replacing the lepton, either because the electroweakino is light and, e.g., long-lived, or because a heavy electroweakino mediates higher-dimensional operators for proton decay. In the latter case, an electroweakino-fermion-sfermion coupling with further RpV sfermion-fermion-fermion interaction would indeed produce dimension-nine (or higher) operators.[3] Here, we specialize in a subsequent coupling of the LLE type since the hadronic matrix elements studied in the previous section would not apply if additional quarks were involved.[4] Factorizing out this step, we provide the Wilson coefficients for the operators of Eq. (15), where an electroweakino replaces the lepton. The subscript b, w, $h_{(\pm)}$, respectively, denote bino, wino, Higgsino states (with two states of mass $\pm\mu$ in the neutral case). Electroweakino mixing can be included in the picture by combining the various Wilson coefficients,

$$
\begin{aligned}
C[\mathcal{O}_1^{w^-}] &= -g_2 \lambda''_{131} V_{13}^{\mathrm{CKM}*} \frac{X_{kR}^{D_3} X_{kL}^{D_3*}}{m^2_{\tilde{D}_k}} \\
C[\hat{\mathcal{O}}_1^{w^-}] &= -g_2 \lambda''_{132} V_{13}^{\mathrm{CKM}*} \frac{X_{kR}^{D_3} X_{kL}^{D_3*}}{m^2_{\tilde{D}_k}} \\
C[\mathcal{O}_1^{h^-}] &= Y_d^g V_{1g}^{\mathrm{CKM}*} \lambda''_{1g1} \frac{|X_{kR}^{D_g}|^2}{m^2_{\tilde{D}_k}} \\
C[\hat{\mathcal{O}}_1^{h^-}] &= Y_d^g V_{1g}^{\mathrm{CKM}*} \lambda''_{1g2} \frac{|X_{kR}^{D_g}|^2}{m^2_{\tilde{D}_k}} \\
C[\hat{\mathcal{O}}'^{h_-}_1] &= -\frac{Y_d^2}{\sqrt{2}} \lambda''_{121} \frac{|X_{kR}^{D_2}|^2}{m^2_{\tilde{D}_k}} = -C[\hat{\mathcal{O}}'^{h_+}_1] \\
C[\mathcal{Q}_1^b] &= -\frac{\sqrt{2}}{3} g_1 \lambda''_{111} \frac{|X_{kR}^{D_1}|^2}{m^2_{\tilde{D}_k}} = 0 \\
C[\hat{\mathcal{Q}}_1^b] &= -\frac{\sqrt{2}}{3} g_1 \lambda''_{112} \frac{|X_{kR}^{D_1}|^2}{m^2_{\tilde{D}_k}} \\
C[\hat{\mathcal{Q}}'^b_1] &= -\frac{\sqrt{2}}{3} g_1 \lambda''_{121} \frac{|X_{kR}^{D_2}|^2}{m^2_{\tilde{D}_k}} \\
C[\hat{\mathcal{Q}}_2^b] &= \frac{\sqrt{2}}{3} g_1 (\lambda''_{121} - \lambda''_{112}) \frac{|X_{kR}^{U_1}|^2}{m^2_{\tilde{U}_k}} \\
C[\hat{\mathcal{Q}}_5^{h^+}] &= Y_u^g V_{g1}^{\mathrm{CKM}*} (\lambda''_{g21} - \lambda''_{g12}) \frac{|X_{kR}^{U_g}|^2}{m^2_{\tilde{U}_k}} \\
C[\mathcal{Q}_1^{w,h_\pm}] &= C[\hat{\mathcal{Q}}_1^{w,h_\pm}] = C[\hat{\mathcal{Q}}'^{w,h_\pm}_1] = 0 \\
C[\mathcal{O}_1^{b,w,h_\pm}] &= C[\hat{\mathcal{O}}_1^{b,w,h_\pm}] = C[\hat{\mathcal{O}}'^{b,w}_1] = 0 \\
C[\mathcal{O}_5^{w^-,h^-}] &= C[\hat{\mathcal{O}}_5^{w^-,h^-}] = 0 \\
C[\hat{\mathcal{Q}}_2^{w,h_\pm}] &= C[\hat{\mathcal{Q}}_5^{w^+}] = C[\hat{\mathcal{Q}}_6^{w^+,h^+}] = C[\hat{\mathcal{R}}^{b,w,h}] = 0. \quad (29)
\end{aligned}
$$

Here $g_1 = e/\sin\theta_W$ is the $U(1)_Y$ hypercharge gauge coupling, and g_2 is the $SU(2)_L$ gauge coupling. Unsurprisingly, the contributions are suppressed (i.e., require sfermion mixing or vanish) when the quantum numbers of the bino (SU(2)-singlet), wino (triplet) or Higgsinos (doublets) lead to an operator violating the SM gauge group. However, we note that the operators of type $Q_{1,2}$ with a bino are SM conserving, hence fully legitimate dimension-six objects. All these coefficients are only B violating, so that they do not involve λ' or μ_l couplings, in contrast to the coefficients of Eq. (28). Yet, unless the lightest neutralino is very light, the "decays" of the electroweakino line require additional RpV effects to contribute to nucleon decays.

B. Nucleon decays into a meson and a lepton

The experimental results for nucleon decay modes into a meson and a (an anti)lepton are collected in [34]. The channels with pseudoscalar mesons in the final state tend to be more constrained than those with vector mesons. Thus, in consideration of our conservative estimates of the hadronic matrix elements for the decays into vector meson, these latter processes hardly have a chance to compete in the current situation.[5] Therefore, the limits on the Wilson coefficients of Eq. (28) principally derive from the decays into pseudoscalar mesons:

[3] The loop diagrams of Ref. [51] would be inconsistent in the context of the tree-level matching that we perform here and obviously depend on the renormalization conditions defining the RpV couplings.

[4] On the other hand, hadronic matrix elements for B-violating operators of dimension nine would yield further limits on RpV parameters.

[5] However, we note that this would not be systematically the case if we also employed the hadronic matrix elements derived with the bag models for the decays into pseudoscalar mesons since the branching ratios associated with the vector mesons are then larger. A more precise determination of the hadronic matrix elements for the nucleon to vector meson transition could thus increase the relevance of these channels.

(i) From $p \to \pi^0 e^+$ [32], $\eta_{\rm QCD}|C[\mathcal{O}_1^e]| < 9.2 \times 10^{-32}$ GeV^{-2}.
(ii) From $p \to \pi^0 \mu^+$ [32], $\eta_{\rm QCD}|C[\mathcal{O}_1^\mu]| < 1.5 \times 10^{-31}$ GeV^{-2}.
(iii) From $n \to \pi^0 \nu$ [30], $\eta_{\rm QCD}[\sum_\nu |C[\mathcal{O}_1^\nu]|^2 + 1.05 \sum_\nu |C[\mathcal{Q}_1^\nu]|^2]^{1/2} < 3.5 \times 10^{-31}$ GeV^{-2}.
(iv) From $p \to K^0 e^+$ [28], $\eta_{\rm QCD}|C[\hat{\mathcal{O}}_1^e]| < 6.4 \times 10^{-31}$ GeV^{-2}.
(v) From $p \to K^0 \mu^+$ [29], $\eta_{\rm QCD}|C[\hat{\mathcal{O}}_1^\mu]| < 5.3 \times 10^{-31}$ GeV^{-2}.
(vi) From $n \to K^+ e^-$ [26], $\eta_{\rm QCD}|C[\hat{\mathcal{Q}}_6^e]| < 6.4 \times 10^{-30}$ GeV^{-2}.
(vii) From $n \to K^+ \mu^-$ [26], $\eta_{\rm QCD}|C[\hat{\mathcal{Q}}_6^\mu]| < 4.5 \times 10^{-30}$ GeV^{-2}.
(viii) From $p \to K^+ \nu$ [31], $\eta_{\rm QCD}[\sum_\nu |C[\hat{\mathcal{O}}'^\nu_1] + 0.366 C \times [\hat{\mathcal{O}}_1^\nu]|^2 + 1.05 \cdot \sum_\nu |C[\hat{\mathcal{Q}}'^\nu_1] + 0.295 C[\hat{\mathcal{Q}}_1^\nu]|^2]^{1/2} < 2.0 \times 10^{-31}$ GeV^{-2}.

Assuming a universal squark mass $m_{\tilde{Q}}$ and defining the mixing parameter $\Delta_{LR}^{\tilde{F}_g} \approx -[\mathcal{M}^2_{\tilde{F}_g}]_{LR}/m^2_{\tilde{Q}}$ (with $[\mathcal{M}^2_{\tilde{F}_g}]_{LR}$ the left-right squared-mass-matrix element, as defined in Appendix A) for $\tilde{F}_g = \tilde{U}_g, \tilde{D}_g$ ($g = 3$), we arrive at the following limits on combinations of RpV parameters (with implicit sum on repeated indices):

$$
\begin{aligned}
\left|\lambda''_{1g1} V^{\rm CKM*}_{1f}\left[Y_d^f \delta_{fg}\frac{\mu_1^*}{\mu^*} - \lambda'^*_{1fg}\right]\right| &< 2.9 \times 10^{-26}\left(\frac{m_{\tilde{Q}}}{1\ \text{TeV}}\right)^2 \frac{3.15}{\eta_{\rm QCD}} \\
\left|\lambda''_{1g1} V^{\rm CKM*}_{1f}\left[Y_d^f \delta_{fg}\frac{\mu_2^*}{\mu^*} - \lambda'^*_{2fg}\right]\right| &< 4.7 \times 10^{-26}\left(\frac{m_{\tilde{Q}}}{1\ \text{TeV}}\right)^2 \frac{3.15}{\eta_{\rm QCD}} \\
\left[|\lambda''_{1g1}\lambda'^*_{l1g}|^2 + 1.05|\lambda''_{131}\lambda'^*_{l31}\Delta_{LR}^{\tilde{D}_3}|^2\right]^{1/2} &< 1.1 \times 10^{-25}\left(\frac{m_{\tilde{Q}}}{1\ \text{TeV}}\right)^2 \frac{3.15}{\eta_{\rm QCD}} \\
\left|\lambda''_{1g2} V^{\rm CKM*}_{1f}\left[Y_d^f \delta_{fg}\frac{\mu_1^*}{\mu^*} - \lambda'^*_{1fg}\right]\right| &< 2.0 \times 10^{-25}\left(\frac{m_{\tilde{Q}}}{1\ \text{TeV}}\right)^2 \frac{3.15}{\eta_{\rm QCD}} \\
\left|\lambda''_{1g2} V^{\rm CKM*}_{1f}\left[Y_d^f \delta_{fg}\frac{\mu_2^*}{\mu^*} - \lambda'^*_{2fg}\right]\right| &< 1.7 \times 10^{-25}\left(\frac{m_{\tilde{Q}}}{1\ \text{TeV}}\right)^2 \frac{3.15}{\eta_{\rm QCD}} \\
|(\lambda''_{321} - \lambda''_{312}) V^{\rm CKM*}_{3f}\lambda'^*_{1f1}\Delta_{LR}^{\tilde{U}_3}| &< 2.0 \times 10^{-24}\left(\frac{m_{\tilde{Q}}}{1\ \text{TeV}}\right)^2 \frac{3.15}{\eta_{\rm QCD}} \\
|(\lambda''_{321} - \lambda''_{312}) V^{\rm CKM*}_{3f}\lambda'^*_{2f1}\Delta_{LR}^{\tilde{U}_3}| &< 1.4 \times 10^{-24}\left(\frac{m_{\tilde{Q}}}{1\ \text{TeV}}\right)^2 \frac{3.15}{\eta_{\rm QCD}} \\
\left[\left|\lambda''_{1g1}\left[Y_d^2\delta_{g2}\frac{\mu_2^*}{\mu^*} - \lambda'^*_{l2g}\right] - 0.366\lambda''_{1g2}\lambda'^*_{l1g}\right|^2 + 1.05|\Delta_{LR}^{\tilde{D}_3}|^2|\lambda''_{131}\lambda'^*_{l32} + 0.295\lambda''_{132}\lambda'^*_{l31}|^2\right]^{1/2} &< 6.4 \times 10^{-26}\left(\frac{m_{\tilde{Q}}}{1\ \text{TeV}}\right)^2 \frac{3.15}{\eta_{\rm QCD}}.
\end{aligned}
\tag{30}
$$

These constraints update earlier bounds (e.g., [18]) with somewhat stronger limits due, in particular, to the relatively recent results from the Super-Kamiokande experiment. In case the squark mediator is a strange squark ($g = 2$), one can further exploit the limit from dinucleon decays applying to λ''_{121} [64] to derive a bound on single couplings of the LQD^c type.

However, the theoretical uncertainties have not been taken into account in the derivation above. In addition to the uncertainties associated to the hadronic matrix elements (of order 10% according to [69]), one should add those uncertainties originating with the Wilson coefficients. While the leading QCD logarithms should be properly included in $\eta_{\rm QCD}$, finite and Next-to-leading order (NLO) QCD corrections are not considered and could easily amount to a ∼30% modification. Electroweak logarithms have also been neglected, as well as electroweak flavor-changing effects. We believe that the latter can only be consistently included by performing a matching of one-loop order and choosing a particular renormalization scheme (so that the RpV parameters are set to a specific definition). Finally, we should stress that the approximations on squark and electroweakino-lepton mixing oversimplify the system, so that the bounds of Eq. (30) cannot replace a full numerical test. In particular, the relevance of neglecting flavor-changing effects in the sfermion sector, while allowing flavor-violation in the RpV couplings, can be questioned. Therefore, these limits should be seen at a purely qualitative level.

C. Nucleon decays into a meson and a long-lived bino

In this section, we assume the existence of a light bino with mass comparable or below that of the muon and long-lived so that it would appear as an invisible particle in nucleon decays. Then, the dimension-six operators of Eq. (29) are constrained by the limits on $p \to K^+\nu$.[6] In principle, the decay width involving such a bino final state should be counted together with other decay channels involving an invisible final state, i.e., a neutrino or an antineutrino, which were presented in the previous subsection. However, for simplicity, we neglect the purely leptonic contribution to invisible decays below, e.g., because L-violating parameters vanish:

(i) From $p \to K^+\nu$ [31]

$$\eta_{\rm QCD}|C[\hat{\mathcal{Q}}'^b_1] + 0.295C[\hat{\mathcal{Q}}^b_1] - 0.705C[\hat{\mathcal{Q}}^b_2]| < 2.0\times 10^{-31}\ \mathrm{GeV}^{-2},$$

from which we deduce (using $\lambda''_{112} = -\lambda''_{121}$),

$$|\lambda''_{121}| < 3.9\times 10^{-25}\left(\frac{m_{\tilde{Q}}}{1\ \mathrm{TeV}}\right)^2 \frac{3.15}{\eta_{\rm QCD}}\frac{0.350}{g_1}. \qquad (31)$$

This limit is much stronger than that obtained from dinucleon decay in Ref. [64], of order $|\lambda''_{121}| \lesssim 10^{-6}(m_{\tilde{Q}}/1\ \mathrm{TeV})^2(M_3/1\ \mathrm{TeV})^{1/2}$. However, in the presence of a very light bino, one should not neglect the mediation of this light field (in place of the gluino) in the $\Delta B = 2$ process. The latter would consist of a low-energy diagram with two effective quartic bino-quark dimension-six vertices ($\hat{\mathcal{Q}}^{(\prime)b}_{1,2}$) connected by a light bino line, instead of the dimension-nine operators obtained by integrating out heavy new physics. In particular, this means that the momentum of the exchanged bino cannot be neglected but needs to be assessed as part of the (strong) low-energy dynamics. This would motivate a reanalysis of this constraint (which goes beyond the scope of the current work).

D. Nucleon decays into a meson and three leptons

The gauginos are now assumed to be heavy; they are thus intermediate and off shell in the decay and can themselves decay to three leptons. The Wilson coefficients of Eq. (29) can be combined with gaugino and RpV couplings of the LLE type in order to form the following dimension nine (or higher from the $SU(2)_L$-conserving perspective):

$$\begin{aligned}
\mathcal{S}_1 &= \varepsilon_{\alpha\beta\gamma}[(\overline{d^c})^\alpha P_R u^\beta][(\overline{u^c})^\gamma P_L e_l][\bar{e}_n P_L e_m],\\
\hat{\mathcal{S}}_1 &= \varepsilon_{\alpha\beta\gamma}[(\overline{s^c})^\alpha P_R u^\beta][(\overline{u^c})^\gamma P_L e_l][\bar{e}_n P_L e_m],\\
\hat{\mathcal{T}}_{11} &= \varepsilon_{\alpha\beta\gamma}[(\overline{s^c})^\alpha P_R u^\beta][(\overline{d^c})^\gamma P_R \nu^c_l][\bar{e}_m P_L e_n],\\
\hat{\mathcal{T}}'_{11} &= \varepsilon_{\alpha\beta\gamma}[(\overline{d^c})^\alpha P_R u^\beta][(\overline{s^c})^\gamma P_R \nu^c_l][\bar{e}_m P_L e_n],\\
\hat{\mathcal{T}}_{12} &= \varepsilon_{\alpha\beta\gamma}[(\overline{s^c})^\alpha P_R u^\beta][(\overline{d^c})^\gamma P_R e_l][\bar{e}_m P_L \nu_n],\\
\hat{\mathcal{T}}'_{12} &= \varepsilon_{\alpha\beta\gamma}[(\overline{d^c})^\alpha P_R u^\beta][(\overline{s^c})^\gamma P_R e_l][\bar{e}_m P_L \nu_n],\\
\hat{\mathcal{T}}_{13} &= \varepsilon_{\alpha\beta\gamma}[(\overline{s^c})^\alpha P_R u^\beta][(\overline{d^c})^\gamma P_R e_l][\bar{e}_m P_R \nu^c_n],\\
\hat{\mathcal{T}}'_{13} &= \varepsilon_{\alpha\beta\gamma}[(\overline{d^c})^\alpha P_R u^\beta][(\overline{s^c})^\gamma P_R e_l][\bar{e}_m P_R \nu^c_n],\\
\hat{\mathcal{T}}_{14} &= \varepsilon_{\alpha\beta\gamma}[(\overline{s^c})^\alpha P_R u^\beta][(\overline{d^c})^\gamma P_R e^c_l][\bar{\nu}_n P_R e_m],\\
\hat{\mathcal{T}}'_{14} &= \varepsilon_{\alpha\beta\gamma}[(\overline{d^c})^\alpha P_R u^\beta][(\overline{s^c})^\gamma P_R e^c_l][\bar{\nu}_n P_R e_m],\\
\hat{\mathcal{T}}_{15} &= \varepsilon_{\alpha\beta\gamma}[(\overline{s^c})^\alpha P_R u^\beta][(\overline{d^c})^\gamma P_R e^c_l][\overline{\nu^c}_n P_L e_m],\\
\hat{\mathcal{T}}'_{15} &= \varepsilon_{\alpha\beta\gamma}[(\overline{d^c})^\alpha P_R u^\beta][(\overline{s^c})^\gamma P_R e^c_l][\overline{\nu^c}_n P_L e_m],\\
\hat{\mathcal{T}}_{21} &= \varepsilon_{\alpha\beta\gamma}[(\overline{s^c})^\alpha P_R d^\beta][(\overline{u^c})^\gamma P_R \nu^c_l][\bar{e}_m P_L e_n],\\
\hat{\mathcal{T}}_{22} &= \varepsilon_{\alpha\beta\gamma}[(\overline{s^c})^\alpha P_R d^\beta][(\overline{u^c})^\gamma P_R e_l][\bar{e}_m P_L \nu_n],\\
\hat{\mathcal{T}}_{23} &= \varepsilon_{\alpha\beta\gamma}[(\overline{s^c})^\alpha P_R d^\beta][(\overline{u^c})^\gamma P_R e_l][\bar{e}_m P_R \nu^c_n],\\
\hat{\mathcal{T}}_{24} &= \varepsilon_{\alpha\beta\gamma}[(\overline{s^c})^\alpha P_R d^\beta][(\overline{u^c})^\gamma P_R e^c_l][\bar{\nu}_n P_R e_m],\\
\hat{\mathcal{T}}_{25} &= \varepsilon_{\alpha\beta\gamma}[(\overline{s^c})^\alpha P_R d^\beta][(\overline{u^c})^\gamma P_R e^c_l][\overline{\nu^c}_n P_L e_m]. \qquad (32)
\end{aligned}$$

The associated Wilson coefficients read (where we have neglected the Yukawa couplings of the light leptons and assumed universal slepton masses):

$$\begin{aligned}
C[\mathcal{S}_1] &= C[\mathcal{O}^{w^-}_1]\frac{g_2\lambda_{lmn}}{M_2 m^2_{\tilde{N}_l}}\\
C[\hat{\mathcal{S}}_1] &= C[\hat{\mathcal{O}}^{w^-}_1]\frac{g_2\lambda_{lmn}}{M_2 m^2_{\tilde{N}_l}}\\
C[\hat{\mathcal{T}}_{11}] &= -C[\hat{\mathcal{Q}}^b_1]\frac{g_1\lambda^*_{lmn}}{\sqrt{2}M_1 m^2_{\tilde{N}_l}}\\
C[\hat{\mathcal{T}}'_{11}] &= -C[\hat{\mathcal{Q}}'^b_1]\frac{g_1\lambda^*_{lmn}}{\sqrt{2}M_1 m^2_{\tilde{N}_l}}\\
C[\hat{\mathcal{T}}_{21}] &= -C[\hat{\mathcal{Q}}^b_2]\frac{g_1\lambda^*_{lmn}}{\sqrt{2}M_1 m^2_{\tilde{N}_l}}\\
C[\hat{\mathcal{T}}_{12}] &= C[\hat{\mathcal{Q}}^b_1]\frac{\sqrt{2}g_1\lambda_{lnm}\Delta^{\tilde{E}_l *}_{LR}}{M_1 m^2_{\tilde{E}_l}}\\
C[\hat{\mathcal{T}}'_{12}] &= C[\hat{\mathcal{Q}}'^b_1]\frac{\sqrt{2}g_1\lambda_{lnm}\Delta^{\tilde{E}_l *}_{LR}}{M_1 m^2_{\tilde{E}_l}}\\
C[\hat{\mathcal{T}}_{22}] &= C[\hat{\mathcal{Q}}^b_2]\frac{\sqrt{2}g_1\lambda_{lnm}\Delta^{\tilde{E}_l *}_{LR}}{M_1 m^2_{\tilde{E}_l}}
\end{aligned}$$

[6] In principle, the kinematics of the decay to a light, but massive, neutralino are different than to a neutrino. The detailed consideration is beyond the present work.

$$C[\hat{\mathcal{T}}_{13}] = C[\hat{\mathcal{Q}}_1^b]\frac{\sqrt{2}g_1\lambda^*_{nml}}{M_1 m^2_{\tilde{E}_l}}$$
$$C[\hat{\mathcal{T}}_{23}] = C[\hat{\mathcal{Q}}_2^b]\frac{\sqrt{2}g_1\lambda^*_{nml}}{M_1 m^2_{\tilde{E}_l}}$$
$$C[\hat{\mathcal{T}}'_{13}] = C[\hat{\mathcal{Q}}_1'^b]\frac{\sqrt{2}g_1\lambda^*_{nml}}{M_1 m^2_{\tilde{E}_l}}$$
$$C[\hat{\mathcal{T}}_{14}] = -C[\hat{\mathcal{Q}}_1^b]\frac{g_1\lambda^*_{lnm}}{\sqrt{2}M_1 m^2_{\tilde{E}_l}}$$
$$C[\hat{\mathcal{T}}'_{14}] = -C[\hat{\mathcal{Q}}_1'^b]\frac{g_1\lambda^*_{lnm}}{\sqrt{2}M_1 m^2_{\tilde{E}_l}}$$
$$C[\hat{\mathcal{T}}_{24}] = -C[\hat{\mathcal{Q}}_2^b]\frac{g_1\lambda^*_{lnm}}{\sqrt{2}M_1 m^2_{\tilde{E}_l}}$$
$$C[\hat{\mathcal{T}}_{15}] = -C[\hat{\mathcal{Q}}_1^b]\frac{g_1\lambda_{nml}\Delta^{\tilde{E}_l *}_{LR}}{\sqrt{2}M_1 m^2_{\tilde{E}_l}}$$
$$C[\hat{\mathcal{T}}'_{15}] = -C[\hat{\mathcal{Q}}_1'^b]\frac{g_1\lambda_{nml}\Delta^{\tilde{E}_l *}_{LR}}{\sqrt{2}M_1 m^2_{\tilde{E}_l}}$$
$$C[\hat{\mathcal{T}}_{25}] = -C[\hat{\mathcal{Q}}_2^b]\frac{g_1\lambda_{nml}\Delta^{\tilde{E}_l *}_{LR}}{\sqrt{2}M_1 m^2_{\tilde{E}_l}}. \quad (33)$$

Here the $m_{\tilde{N}_l}$ denote the sneutrino masses. Applying lifetime limits from inclusive nucleon decay channels with antileptons [98,99], we derive the following RpV limits (with $m^2_{\tilde{L}}$ the universal slepton mass):

(i) From $p \to \pi^0 e_l^+ e_m^- e_n^+$,

$$|\Delta^{\tilde{D}_3}_{LR}\lambda''_{131}|[|\lambda_{211}|^2 + |\lambda_{122}|^2]^{1/2} < 2.3\times 10^{-9}\left(\frac{m_{\tilde{Q}}}{1\ \text{TeV}}\right)^2\left(\frac{m_{\tilde{L}}}{1\ \text{TeV}}\right)^2 \times \frac{M_2}{1\ \text{TeV}}\frac{3.15}{\eta_{\text{QCD}}}\left(\frac{0.653}{g_2}\right)^2. \quad (34)$$

(ii) From $p \to K^0 e_l^+ e_m^- e_n^+$,

$$|\Delta^{\tilde{D}_3}_{LR}\lambda''_{132}|[|\lambda_{211}|^2 + |\lambda_{122}|^2]^{1/2} < 2.4\times 10^{-8}\left(\frac{m_{\tilde{Q}}}{1\ \text{TeV}}\right)^2\left(\frac{m_{\tilde{L}}}{1\ \text{TeV}}\right)^2 \times \frac{M_2}{1\ \text{TeV}}\frac{3.15}{\eta_{\text{QCD}}}\left(\frac{0.653}{g_2}\right)^2. \quad (35)$$

(iii) From $p \to K^+ \nu_l^{(c)} e_n^- e_m^+$,

$$|\lambda''_{112}\lambda^*_{lm2}| < 1.6\times 10^{-10}\left(\frac{m_{\tilde{Q}}}{1\ \text{TeV}}\right)^2 \times\left(\frac{m_{\tilde{L}}}{1\ \text{TeV}}\right)^2\frac{M_1}{1\ \text{TeV}}\frac{3.15}{\eta_{\text{QCD}}}\left(\frac{0.350}{g_1}\right)^2,$$
$$|\lambda''_{112}\lambda^*_{lm1}| < 7.0\times 10^{-10}\left(\frac{m_{\tilde{Q}}}{1\ \text{TeV}}\right)^2 \times\left(\frac{m_{\tilde{L}}}{1\ \text{TeV}}\right)^2\frac{M_1}{1\ \text{TeV}}\frac{3.15}{\eta_{\text{QCD}}}\left(\frac{0.350}{g_1}\right)^2. \quad (36)$$

Here, we employed the four-body final-state phase space derived in [100] and neglected the lepton masses.

E. Purely leptonic final states

Experimental constraints on purely leptonic decay channels are also available [27]. These could *a priori* be mediated by the strangeness-conserving dimension-nine operator of Eq. (32), $\mathcal{S}_1$. Additional contributions from the operators of Eq. (15) and an off shell photon seem difficult to assess in the context of nonperturbative QCD. We will not consider them. We then need to evaluate the nucleon decay constant $\langle 0|\mathcal{S}|N\rangle$. Note that $\langle 0|$ represents the QCD vacuum and $N = p, n$, the nucleon. For this, we exploit the LEC of the chiral model [87] and write

$$\langle 0|\varepsilon_{\alpha\beta\gamma}[(\overline{d^c})^\alpha P_R u^\beta][(\overline{u^c})^\gamma P_L]|p\rangle = \tilde{\alpha}[P_L u_p], \quad (37)$$

$$\langle 0|\varepsilon_{\alpha\beta\gamma}[(\overline{d^c})^\alpha P_R u^\beta][(\overline{d^c})^\gamma P_R]|n\rangle = \tilde{\beta}[P_R u_n], \quad (38)$$

with $\tilde{\alpha}$ and $\tilde{\beta}$ the LEC calculated in Ref. [69] (see Eq. (23) of this reference). These quantities are *a priori* valid in the limit of vanishing energy transfer so that their use in decay processes with energy comparable to the nucleon mass is highly unreliable. Reference [69] quotes a factor of 2–3 (on the conservative side) in the case of nucleon decay widths into a meson and a (an anti)lepton, as compared to the full lattice evaluation of the hadronic matrix elements. Thus, we again expect results of purely qualitative value.

The transitions mediated by the operators of type $\mathcal{S}_1$ lead to the following limits:

(i) From $p \to e^+\mu^+\mu^-$ [27],

$$|\Delta^{\tilde{D}_3}_{LR}\lambda''_{131}\lambda_{122}| < 5.1\times 10^{-11}\left(\frac{m_{\tilde{Q}}}{1\ \text{TeV}}\right)^2\left(\frac{m_{\tilde{L}}}{1\ \text{TeV}}\right)^2 \times \frac{M_2}{1\ \text{TeV}}\frac{3.15}{\eta_{\text{QCD}}}\left(\frac{0.653}{g_2}\right)^2. \quad (39)$$

(ii) From $p \to e^+\mu^+e^-$ [27],

$$|\Delta^{\tilde{D}_3}_{LR}\lambda''_{131}\lambda_{211}| < 4.2\times10^{-11}\left(\frac{m_{\tilde{Q}}}{1\ \text{TeV}}\right)^2\left(\frac{m_{\tilde{L}}}{1\ \text{TeV}}\right)^2 \times\frac{M_2}{1\ \text{TeV}}\frac{3.15}{\eta_{\text{QCD}}}\left(\frac{0.653}{g_2}\right)^2, \qquad (40)$$

which, for these specific directions, are stronger than the constraints obtained with the inclusive decay widths, including an antilepton.

IV. CONCLUSIONS

In this paper, we have revisited the constraints from nucleon decays on RpV parameters, updating the bounds with current lattice calculations and experimental limits. We have also paid more detailed attention to the derivation of these constraints than usually presented in the literature. Nucleon decays could take a very diverse pattern in the context of RpV, and the current sets of bounds are restricted by the limited knowledge of hadronic matrix elements. We have exhumed the bag model for an estimate of the transitions involving vector mesons, but the outcome suffers from the comparison with the precise lattice results available for decays into pseudoscalar mesons. For this reason—and the associated performance of experimental searches—limits from the nucleon transition to pseudoscalar meson and (anti)lepton (or invisible) place the most stringent limits on RpV parameters.

In the RpV MSSM, it is also possible to build L-conserving B-violating operators involving electroweakinos, opening further search modes. Once again, the full exploitation of these channels is limited by the absence of reliable evaluations of hadronic matrix elements for, e.g., purely leptonic nucleon decays or channels with multiple mesons in the final state. Constraints from $\Delta B = 2$ processes could provide complementary information, e.g., on such scenarios, in particular, in the presence of a very light bino liable to mediate such transitions close to resonance.

On the high-energy side, we have restricted ourselves to a pure tree-level matching of the Wilson coefficients, as a one-loop matching would be technically much more involved. As a consequence, the limits that we have derived in Sec. III should be seen as largely qualitative. In particular, we renounced flavor-violating loops enlarging the set of RpV couplings that can be constrained, as sometimes presented in the literature. More precise limits could naturally be derived in an analysis of one-loop order, but these should then also depend on the renormalization conditions chosen to fix the counterterms of the RpV parameters, a point that seems to have been overlooked in corresponding proposals.

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APPENDIX A: MSSM, RpV AND MIXING

1. Mixing in the squark sector

This is an R-parity conserving effect. The sfermion mixing matrices can be written in the $(\tilde{F}_L, \tilde{F}^{c*}_R)$ basis as follows:

$$\mathcal{M}^2_{\tilde{F}} = \begin{bmatrix} m^2_{F_L} + Y_f^2 v_f^2 + \frac{1}{2}\left(\frac{\mathcal{Y}^f_L}{2}g_1^2 - I^f_3 g_2^2\right)(v_u^2 - v_d^2) & Y_f v_f\left(A^*_f - \mu\frac{v_{f'}}{v_f}\right) \\ Y_f v_f\left(A_f - \mu^*\frac{v_{f'}}{v_f}\right) & m^2_{F_R} + Y_f^2 v_f^2 + \frac{\mathcal{Y}^f_R}{4}g_1^2(v_u^2 - v_d^2) \end{bmatrix}, \qquad (A1)$$

where f is the fermion corresponding to the sfermion $\tilde{F}$, while f' is its $SU(2)_L$ partner. Then, Y_f is the associated Yukawa coupling, $\mathcal{Y}^f_{L,R}$ the associated hypercharges, and I^f_3 the isospin. Finally, v_f denotes the VEV of the Higgs doublet to which the fermion f couples at tree level. In principle, each matrix element in Eq. (A1) should be understood as a 3×3 block in flavor space. With M_{SUSY} above the electroweak scale, left-right mixing in the squark sector is only relevant for $Y_f = Y_{t,b}$ (but A_f is still a matrix in flavor space, meaning that right-handed squarks of the third generation could still have a relevant mixing with left-handed squarks of any generation).

We define the (unitary) mixing matrix $X^{\tilde{F}}$, such that $\mathcal{M}^2_{\tilde{F}} = X^{\tilde{F}\dagger}\text{diag}[m^2_{\tilde{F}_i}]X^{\tilde{F}}$. Then, the gauge eigenstates are connected to the mass eigenstates through $\tilde{F}_i = X^{\tilde{F}*}_{iL}\tilde{F}_L + X^{\tilde{F}*}_{iR}\tilde{F}^{c*}_R$, and reciprocally, $\tilde{F}_L = X^{\tilde{F}}_{iL}\tilde{F}_i$, $\tilde{F}^c_R = X^{\tilde{F}*}_{iR}\tilde{F}^*_i$. The mass matrix of Eq. (A1) should be diagonalized in a fully unprejudiced fashion as to the magnitude of the matrix elements, in general. However, it is instructive to consider the expansion in terms of the electroweak VEVs. Then (neglecting intergeneration mixing), $m^2_{\tilde{F}_1} \approx m^2_{F_L}$, $m^2_{\tilde{F}_2} \approx m^2_{F_R}$, $X^{\tilde{F}}_{1L}, X^{\tilde{F}}_{2R} \approx 1$ and $X^{\tilde{F}}_{1R} \approx -X^{\tilde{F}*}_{2L} \approx -[\mathcal{M}^2_{\tilde{F}}]_{LR}/(M^2_{\tilde{F}_1} - M^2_{\tilde{F}_2})$. The left-right mixing is obviously associated with an electroweak VEV and is thus liable to generate contributions to dimension-seven operators.

2. Mixing in the chargino/lepton sector

This involves both R-parity conserving (wino-Higgsino mixing) and R-parity violating effects (Higgsino-lepton mixing) [16]. We work in the description where the sneutrino fields do not take a VEV. Then the mass terms read $-\mathcal{L} \ni (\tilde{w}^+, \tilde{h}^+_u, e^{cf}_R)\mathcal{M}_{\tilde{C}}(\tilde{w}^-, \tilde{h}^-_d, e^g_L)^T + \text{H.c.}$, with

$$\mathcal{M}_{\tilde{C}} = \begin{bmatrix} M_2 & g_2 v_d & 0 \\ g_2 v_u & \mu & \mu_g \\ 0 & 0 & Y^f_e \delta_{fg} v_d \end{bmatrix}. \tag{A2}$$

Note that f, g correspond to the flavor indices. Note also that μ_g is the RpV bilinear coupling. The mass matrix of Eq. (A2) is diagonalized with a pair of unitary matrices so that $\mathcal{M}_{\tilde{C}} = V^T \mathrm{diag}(m_{\chi^\pm_i})U$. The mass eigenstates are then defined as $\chi^+_i = V^*_{iw}\tilde{w}^+ + V^*_{ih}\tilde{h}^+_u + V^*_{ie_f} e^{cf}_R$, $\chi^-_i = U^*_{iw}\tilde{w}^- + U^*_{ih}\tilde{h}^-_d + U^*_{ie_f} e^f_L$, and the gauge eigenstates can be expressed in terms of the mass eigenstates through inversion.

In the hierarchical context $|M_2|, |\mu|, ||M_2| - |\mu|| \gg gv$, $|\mu_g|, Y^g_e v_d$, we can approximate these mixing elements by the following expressions:

$$\begin{aligned}
&U_{1w}, V_{1w}, U_{2h} \approx 1, \\
&V_{2h}, U_{3e_l}, V_{3e_l} \approx 1, \\
&U_{1h} \approx \frac{g_2(v_d M^*_2 + v_u \mu)}{|M_2|^2 - |\mu|^2}, \\
&V_{1h} \approx \frac{g_2(v_d \mu + v_u M^*_2)}{|M_2|^2 - |\mu|^2}, \\
&U_{2w} \approx -\frac{g_2(v_d M_2 + v_u \mu^*)}{|M_2|^2 - |\mu|^2}, \\
&U_{2e_l} \approx \frac{\mu_l}{\mu}; \\
&V_{2w} \approx -\frac{g_2(v_d \mu^* + v_u M_2)}{|M_2|^2 - |\mu|^2}, \\
&U_{3h} \approx -\frac{\mu^*_l}{\mu^*}, \\
&U_{1e_l}, V_{1e_l}, V_{2e_l} \approx 0; \\
&U_{3w}, V_{3w}, V_{3h} \approx 0,
\end{aligned} \tag{A3}$$

where the mass-indices 1, 2, and 3, respectively, refer to mostly wino, Higgsino, and lepton states.

3. Mixing in the neutralino/neutrino sector

This is largely comparable to that in the chargino/lepton sector, however, it leads to at least one massive neutrino [38,101,102]. The mass term is of Majorana type and, in the basis $(\tilde{b}^0, \tilde{w}^0, \tilde{h}^0_d, \tilde{h}^0_u, \nu^f_L)$, involves the 7×7 matrix:

$$\mathcal{M}_{\tilde{N}} = \begin{bmatrix} M_1 & 0 & -\frac{g_1}{\sqrt{2}}v_d & \frac{g_1}{\sqrt{2}}v_u & 0 \\ 0 & M_2 & \frac{g_2}{\sqrt{2}}v_d & -\frac{g_2}{\sqrt{2}}v_u & 0 \\ -\frac{g_1}{\sqrt{2}}v_d & \frac{g_2}{\sqrt{2}}v_d & 0 & -\mu & 0 \\ \frac{g_1}{\sqrt{2}}v_u & -\frac{g_2}{\sqrt{2}}v_u & -\mu & 0 & -\mu_g \\ 0 & 0 & 0 & -\mu_g & 0 \end{bmatrix}. \tag{A4}$$

This matrix is diagonalized via the 7×7 unitary matrix N according to $\mathcal{M}_{\tilde{N}} = N^T \mathrm{diag}(m_{\chi^0})N$, from which we deduce the mass eigenstates $\chi^0_i = N^*_{ib}\tilde{b} + N^*_{iw}\tilde{w}^0 + N^*_{ih_d}\tilde{h}^0_d + N^*_{ih_u}\tilde{h}^0_u + N^*_{i\nu_f}\nu^f_L$, $i = 1, \ldots, 7$. Again, in a hierarchical context, the mixing elements can be linearized to

$$\begin{aligned}
&N_{1b}, N_{2w}, N_{5\nu_l} \approx 1, \\
&N_{1w}, N_{1\nu_l}, N_{2b}, N_{2\nu_l} \approx 0, \\
&N_{5b}, N_{5w}, N_{5h_u} \approx 0 \\
&N_{1h_d} \approx -\frac{g_1}{\sqrt{2}}\frac{M^*_1 v_d + \mu v_u}{|M_1|^2 - |\mu|^2} \\
&N_{1h_u} \approx \frac{g_1}{\sqrt{2}}\frac{M^*_1 v_u + \mu v_d}{|M_1|^2 - |\mu|^2} \\
&N_{2h_d} \approx \frac{g_2}{\sqrt{2}}\frac{M^*_2 v_d + \mu v_u}{|M_2|^2 - |\mu|^2} \\
&N_{2h_u} \approx -\frac{g_2}{\sqrt{2}}\frac{M^*_2 v_u + \mu v_d}{|M_2|^2 - |\mu|^2}, \\
&N_{3b} \approx \frac{g_1}{2}\frac{(v_d - v_u)(M_1 - \mu^*)}{|M_1|^2 - |\mu|^2} \\
&N_{3w} \approx \frac{g_2}{2}\frac{(v_u - v_d)(M_2 - \mu^*)}{|M_2|^2 - |\mu|^2}
\end{aligned} \tag{A5}$$

$$\begin{aligned}
&N_{3h_d}, N_{3h_u}, N_{4h_u}, -N_{4h_d} \approx \frac{1}{\sqrt{2}}, \\
&N_{3\nu_l}, -N_{4\nu_l} \approx \frac{\mu_l}{\sqrt{2}\mu} \\
&N_{4b} \approx -\frac{g_1}{2}\frac{(v_d + v_u)(\mu^* + M_1)}{|M_1|^2 - |\mu|^2} \\
&N_{4w} \approx \frac{g_2}{2}\frac{(v_d + v_u)(\mu^* + M_2)}{|M_2|^2 - |\mu|^2} \\
&N_{5h_d} \approx -\frac{\mu^*_l}{\mu^*},
\end{aligned} \tag{A6}$$

where the indices 1, 2, 3, 4, and 5 correspond to mostly bino, wino, a pair of Higgsino, and neutrino states (in fact 5 covers three leptonic states).

APPENDIX B: FEYNMAN RULES

Below, we write the Weyl two-component spinors [103] with lower case letters and the four-component spinors with capital letters. The baryon-number-violating couplings involving sups read ($\lambda''_{mnp} \equiv -\lambda''_{mpn}$),

$$\mathcal{L} \ni \varepsilon_{\alpha\beta\gamma}\lambda''^{*}_{mnp}(\tilde{U}^c_R)^{\alpha *}_m(\bar{d}^c_R)^{\beta}_n(\bar{d}^c_R)^{\gamma}_p + \text{H.c.} \tag{B1}$$

$$\to \varepsilon_{\alpha\beta\gamma}\lambda''^{*}_{rnp}X^{U_r}_{mR}\tilde{U}^{\alpha}_m[(\overline{D^c})^{\beta}_n P_R (D)^{\gamma}_p] + \text{H.c.}, \tag{B2}$$

and for sdowns,

$$\mathcal{L} \ni +\varepsilon_{\alpha\beta\gamma}\lambda''^{*}_{mnp}(\bar{u}^c_R)^{\alpha}_m(\bar{d}^c_R)^{\beta}_n(\tilde{D}^c_R)^{\gamma}_p + \text{H.c.} \tag{B3}$$

$$\to \varepsilon_{\alpha\beta\gamma}\lambda''^{*}_{mpr}X^{D_r}_{nR}\tilde{D}^{\beta}_n[(\overline{U^c})^{\alpha}_m P_R (D)^{\gamma}_p] + \text{H.c.} \tag{B4}$$

Here $X^{U_r}_{mR}, X^{D_r}_{nR}$ denote the squark mixing coefficients, cf. Appendix A 1. We will use the notations $(g^{Udd}_R)_{mnp}$ and $(g^{uDd}_R)_{mnp}$ to denote the complete coefficients in the second lines of Eqs. (B1) and (B3), respectively. We have, furthermore, in the Lagrangians put the fields in parentheses.

The lepton-number-violating couplings involving sups, downs, and charginos read,

$$\begin{aligned}\mathcal{L} \ni\ & Y^f_d(\tilde{U}_L)^{\alpha *}_f(\bar{\tilde{h}}^+_d)(\bar{d}^c_R)^{\alpha}_f + \lambda'^{*}_{fgh}(\tilde{U}_L)^{\alpha *}_g(\bar{e}_L)_f(\bar{d}^c_R)^{\alpha}_h + Y^f_u(\tilde{U}^c_R)^{\alpha}_f(\tilde{h}^+_u)(d_L)^{\alpha}_f - g_2(\tilde{U}_L)^{\alpha *}_f(\tilde{w}^+)(d_L)^{\alpha}_f + \text{H.c.}\\ &\to \tilde{U}^{*\alpha}_m\{(\bar{D}^c)^{\alpha}_f[g^{Ud\chi}_L P_L + g^{Ud\chi}_R P_R]_{mfq}(\chi^+)_q\} + \text{H.c.}\end{aligned} \tag{B5}$$

$$g^{Ud\chi}_{L\,mfq} \equiv V^{\text{CKM}}_{gf}[Y^g_u X^{U_g\,*}_{mR} V_{qh} - g_2 X^{U_g\,*}_{mL} V_{qw}]; \tag{B6}$$

$$g^{Ud\chi}_{R\,mfq} \equiv V^{\text{CKM}}_{gr}[Y^f_d \delta_{fr} X^{U_g\,*}_{mL} U^*_{qh} + \lambda'^{*}_{lgf} X^{U_r\,*}_{mL} U^*_{qe_l}]. \tag{B7}$$

Note that $g_{1,2}$ are gauge couplings. The lepton-number-violating couplings involving sdowns, ups, and charginos read,

$$\mathcal{L} \ni Y^f_d(\tilde{D}^c_R)^{\alpha}_f(\tilde{h}^-_d)(u_L)^{\alpha}_f - g_2(\tilde{D}_L)^{\alpha *}(\tilde{w}^-)(u_L)^{\alpha}_f + \lambda'_{fgk}(\tilde{D}^c_R)^{\alpha}_k(e_L)_f(u_L)^{\alpha}_g + Y^f_u(\tilde{D}_L)^{\alpha *}(\bar{h}^-_u)(\bar{u}^c_R)^{\alpha}_f + \text{H.c.} \tag{B8}$$

$$\to \tilde{D}^{*\alpha}_m\{(\overline{U^c})^{\alpha}_f[g^{Du\chi}_L P_L + g^{Du\chi}_R P_R]_{mfq}(\chi^-)_q\} + \text{H.c.} \tag{B9}$$

$$g^{Du\chi}_{Lmfq} = V^{\text{CKM}*}_{fg}[Y^g_d X^{D_g\,*}_{mR} U_{qh} - g_2 X^{D_g\,*}_{mL} U_{qw} + \lambda'^{*}_{lgk} X^{D_k\,*}_{mR} U_{qe_l}] \tag{B10}$$

$$g^{Du\chi}_{Rmfq} = V^{\text{CKM}*}_{fg} Y^f_u X^{D_g\,*}_{mL} V^*_{qh}. \tag{B11}$$

The lepton-number-violating couplings involving sups, ups, and neutralinos read,

$$\mathcal{L} \ni -Y^f_u(\tilde{U}^c_R)^{\alpha}_f(\tilde{h}^0_u)(u_L)^{\alpha}_f - \frac{1}{\sqrt{2}}(\tilde{U}_L)^{\alpha *}_f\left[\frac{g_1}{3}(\tilde{b}) + g_2(\tilde{w}^0)\right](u_L)^{\alpha}_f - Y^f_u(\tilde{U}_L)^{\alpha *}_f(\bar{\tilde{h}}^0_u)(\bar{u}^c_R)^{\alpha}_f + \frac{2\sqrt{2}}{3}g_1(\tilde{U}^c_R)^{\alpha}_f(\bar{\tilde{b}})(\bar{u}^c_R)^{\alpha}_f + \text{H.c.} \tag{B12}$$

$$\to \tilde{U}^{*\alpha}_m\{(\overline{U^c})^{\alpha}_f[g^{Uu\chi}_L P_L + g^{Uu\chi}_R P_R]_{mfq}(\chi^0)_q\} + \text{H.c.} \tag{B13}$$

$$g^{Uu\chi}_{Lmfq} = -Y^f_u X^{U_f\,*}_{mR} N_{qh_u} - \frac{1}{\sqrt{2}} X^{U_f\,*}_{mL}\left[\frac{g_1}{3}N_{qb} + g_2 N_{qw}\right], \tag{B14}$$

$$g^{Uu\chi}_{Rmfq} = -Y^f_u X^{U_f\,*}_{mL} N^*_{qh_u} + \frac{2\sqrt{2}}{3} g_1 X^{U_f\,*}_{mR} N^*_{qb}. \tag{B15}$$

The lepton-number-violating couplings involving sdowns, downs, and neutralinos read,

$$\begin{aligned}
\mathcal{L} \ni & -Y^f_d (\tilde{D}^c_R)^\alpha_f (\tilde{h}^0_d)(d_L)^\alpha_f - \frac{1}{\sqrt{2}}(\tilde{D}_L)^{\alpha*}_f \left[\frac{g_1}{3}(\tilde{b}) - g_2(\tilde{w}^0)\right](d_L)^\alpha_f - \lambda'_{fgk}(\tilde{D}^c_R)^\alpha_k (\nu_L)_f (d_L)^\alpha_g - Y^f_d (\tilde{D}_L)^{\alpha*}_f (\bar{\tilde{h}}^0_d)(\bar{d}^c_R)^\alpha_f \\
& - \frac{\sqrt{2}}{3} g_1 (\tilde{D}^c_R)^\alpha_f (\bar{\tilde{b}})(\bar{d}^c_R)^\alpha_f - \lambda'^*_{fgk}(\tilde{D}_L)^{\alpha*}_g (\bar{\nu}_L)_f (\bar{d}^c_R)^\alpha_k + \text{H.c.} \\
& \to \tilde{D}^{*\alpha}_m \{(\bar{D}^c)^\alpha_f [g^{Dd\chi}_L P_L + g^{Dd\chi}_R P_R]_{mfq} (\chi^0)_q\} + \text{H.c.}
\end{aligned}$$

$$g^{Dd\chi}_{Lmfq} = -Y^f_d X^{D_f\,*}_{mR} N_{qh_d} - \frac{1}{\sqrt{2}} X^{D_f\,*}_{mL}\left[\frac{g_1}{3} N_{qb} - g_2 N_{qw}\right] - \lambda'_{gfk} X^{D_k\,*}_{mR} N_{q\nu_g},$$

$$g^{Dd\chi}_{Rmfq} = -Y^f_d X^{D_f\,*}_{mL} N^*_{qh_d} - \frac{\sqrt{2}}{3} g_1 X^{D_f\,*}_{mR} N^*_{qb} - \lambda'^*_{gkf} X^{D_k\,*}_{mL} N^*_{q\nu_g}. \tag{B16}$$

Omitting the slepton-Higgs mixing, the slepton-lepton/electroweakino couplings read,

$$\begin{aligned}
g^{\tilde{N}\chi^+\chi^-}_{Lmjk} &= Y^f_e X^{\tilde{N}_f}_{mL} V_{je_f} U_{kd} - g_2 X^{\tilde{N}_f\,*}_{mL} V_{jw} U_{ke_f} - \lambda_{fpq} X^{\tilde{N}_f}_{mL} V_{je_q} U_{ke_p} = (g^{\tilde{N}\chi^+\chi^-}_{Rmkj})^* \\
g^{\tilde{N}\chi^0\chi^0}_{Lmjk} &= \frac{g_1}{\sqrt{2}} X^{\tilde{N}_f\,*}_{mL}(N_{j\nu_f} N_{kb} + N_{jb} N_{k\nu_f}) - \frac{g_2}{\sqrt{2}} X^{\tilde{N}_f\,*}_{mL}(N_{j\nu_f} N_{kw} + N_{jw} N_{k\nu_f}) = (g^{\tilde{N}\chi^0\chi^0}_{Rmkj})^* \\
g^{\tilde{E}^*\chi^0\chi^-}_{Lmjk} &= Y^f_e X^{\tilde{E}_f\,*}_{mR}(N_{j\nu_f} U_{kd} - N_{jd} U_{ke_f}) + \frac{X^{\tilde{E}_f *}_{mL}}{\sqrt{2}}[(g_1 N_{jb} + g_2 N_{jw}) U_{ke_f} - g_2 N_{j\nu_f} U_{kw}], \\
-\lambda_{fpq} & X^{\tilde{E}_q\,*}_{mR} N_{j\nu_f} U_{ke_q} = (g^{\tilde{E}\chi^+\chi^0}_{Rmkj})^*, \\
g^{\tilde{E}^*\chi^0\chi^-}_{Rmjk} &= -(Y^f_e X^{\tilde{E}_f\,*}_{mL} N^*_{jd} + \sqrt{2} g_1 X^{\tilde{E}_f\,*}_{mR} N^*_{jb}) V^*_{ke_f} - \lambda^*_{fpq} X^{\tilde{E}_f\,*}_{mL} N^*_{j\nu_p} V^*_{ke_q} = (g^{\tilde{E}\chi^+\chi^0}_{Lmkj})^*.
\end{aligned} \tag{B17}$$

APPENDIX C: STATIC BAG APPROACH TO NUCLEON DECAYS

In the MIT bag description of hadrons [104,105], valence quarks are relativistic fermions trapped in a spherical potential well of radius R (we restrict ourselves to the flat infinite potential: $V(|\vec{x}| < R) = 0$ and $V(|\vec{x}| > R) = \infty$), the boundary of which is stabilized by a pressure term. The associated fields can then be decomposed in modes,

$$q(x) = \sum_{m,s} [a^q_{m,s} U_{m,s}(\vec{x}) e^{-i\omega_m t} + b^{q\dagger}_{m,s} V_{m,s}(\vec{x}) e^{i\omega_m t}],$$

with $s = \pm\frac{1}{2} = \uparrow\downarrow$ the spin and m indexing the solutions of the boundary conditions. Note that $a^q_{m,s}$ and $b^{q\dagger}_{m,s}$ are creation and destruction operators of (anti)quarks. Note also $\omega_m \equiv E_m/R$ with E_m denoting the energy of the mode. We will restrict ourselves to the mode of lowest energy,[7] which can be described by the four spinors in Dirac representation:

$$U_{0,s}(x) = i\sqrt{\frac{\omega_0^3}{8\pi R^3(\omega_0 - 1)\sin^2\omega_0}} \begin{pmatrix} j_0(\omega_0 \frac{|\vec{x}|}{R})\chi_s \\ i\frac{\vec{x}\cdot\vec{\sigma}}{|\vec{x}|} j_1(\omega_0 \frac{|\vec{x}|}{R})\chi_s \end{pmatrix} \tag{C1}$$

$$\begin{aligned}
V_{0,s}(x) &= C U^*_{0,s} \\
&= -i\sqrt{\frac{\omega_0^3}{8\pi R^3(\omega_0 - 1)\sin^2\omega_0}} \begin{pmatrix} -i\frac{\vec{x}\cdot\vec{\sigma}}{|\vec{x}|} j_1(\omega_0 \frac{|\vec{x}|}{R})\chi'_s \\ j_0(\omega_0 \frac{|\vec{x}|}{R})\chi'_s \end{pmatrix},
\end{aligned} \tag{C2}$$

[7]In the case of the strange quark, we include a quark mass—see Ref. [106]—of 0.1 GeV, which, however, has negligible impact as compared to the massless case.

with $j_0(x) = \frac{\sin x}{x}$ and $j_1(x) = \frac{\sin x}{x^2} - \frac{\cos x}{x}$ the first two spherical Bessel functions, and $\omega_0 \approx 2.04$ the first root of the equation $j_0(\omega) = j_1(\omega)$. $\chi_\uparrow = \binom{1}{0} = -\chi'_\downarrow$, $\chi_\downarrow = \binom{0}{1} = \chi'_\uparrow$.

The hadronic bag states can be constructed with creation operators of the valence quarks $a^{q\dagger}_{\alpha s}$ and antiquarks $b^{q\dagger}_{\alpha s}$ (α is the color index) acting on the vacuum and satisfying the usual anticommutation relations. For example,

proton,

$$|p_\uparrow\rangle = \frac{\varepsilon_{\alpha\beta\gamma}}{3\sqrt{2}} a^{u\dagger}_{\alpha\uparrow}(a^{u\dagger}_{\beta\uparrow}a^{d\dagger}_{\gamma\downarrow} - a^{u\dagger}_{\beta\downarrow}a^{d\dagger}_{\gamma\uparrow})|0\rangle;$$

neutron,

$$|n_\uparrow\rangle = -\frac{\varepsilon_{\alpha\beta\gamma}}{3\sqrt{2}} a^{d\dagger}_{\alpha\uparrow}(a^{d\dagger}_{\beta\uparrow}a^{u\dagger}_{\gamma\downarrow} - a^{d\dagger}_{\beta\downarrow}a^{u\dagger}_{\gamma\uparrow})|0\rangle;$$

neutral pion,

$$|\pi^0\rangle = \frac{1}{2\sqrt{3}}(b^{d\dagger}_{\alpha\uparrow}a^{d\dagger}_{\alpha\downarrow} - b^{u\dagger}_{\alpha\uparrow}a^{u\dagger}_{\alpha\downarrow} - b^{d\dagger}_{\alpha\downarrow}a^{d\dagger}_{\alpha\uparrow} + b^{u\dagger}_{\alpha\downarrow}a^{u\dagger}_{\alpha\uparrow})|0\rangle;$$

and neutral rho,

$$\begin{cases} |\rho^0_1\rangle = \frac{1}{\sqrt{6}}(b^{u\dagger}_{\alpha\uparrow}a^{u\dagger}_{\alpha\uparrow} - b^{d\dagger}_{\alpha\uparrow}a^{d\dagger}_{\alpha\uparrow})|0\rangle \\ |\rho^0_0\rangle = \frac{1}{2\sqrt{3}}(b^{u\dagger}_{\alpha\uparrow}a^{u\dagger}_{\alpha\downarrow} - b^{d\dagger}_{\alpha\uparrow}a^{d\dagger}_{\alpha\downarrow} + b^{u\dagger}_{\alpha\downarrow}a^{u\dagger}_{\alpha\uparrow} - b^{d\dagger}_{\alpha\downarrow}a^{d\dagger}_{\alpha\uparrow})|0\rangle \\ |\rho^0_{-1}\rangle = \frac{1}{\sqrt{6}}(b^{u\dagger}_{\alpha\downarrow}a^{u\dagger}_{\alpha\downarrow} - b^{d\dagger}_{\alpha\downarrow}a^{d\dagger}_{\alpha\downarrow})|0\rangle. \end{cases} \tag{C3}$$

Then, the matrix element of a partonic operator Ω between hadronic external states $< H_f| \int d\vec{x}\Omega(\vec{x})|H_i >$ at $t = 0$ can be evaluated from replacing the quark fields within Ω by their expression in the bag model, leading to the usual interplay of Wick contractions. Different bags are employed for the various hadrons, the typical radius being 5 GeV^{-1} for a nucleon and 3.3 GeV^{-1} for a pion. A Wick contraction between an external creation/annihilation operator and an internal quark field thus exports the bag wave function of the corresponding hadron under the $\int d\vec{x}$. Contractions between operators involving both external hadrons produce spectator quarks, leading to a separate integral representing the overlap between the two bag functions: for instance, $\langle 0|a^q_{\alpha\uparrow}[H_f]a^{q\dagger}_{\beta\uparrow}[H_i]|0\rangle = \int d\vec{y}U^{H_f\dagger}_\uparrow(\vec{y})U^{H_i}_\uparrow(\vec{y})\delta_{\alpha\beta}$. Below, we detail the case of the $p_\uparrow \to \rho^0_1 e^+$ transition mediated by an operator $\Omega_{\Gamma\Gamma'} = \varepsilon_{\alpha\beta\gamma}[(\overline{d^c})^\alpha\Gamma u^\beta][(\overline{u^c})^\gamma\Gamma' e]$, with Γ, Γ' representing generic spinor-algebra matrices,

$$\begin{aligned} \langle\rho^0_1|\Omega_{\Gamma\Gamma'}|p_\uparrow\rangle &= \langle 0|\frac{1}{\sqrt{6}}(b^u_{\alpha\uparrow}a^u_{\alpha\uparrow} - b^d_{\alpha\uparrow}a^d_{\alpha\uparrow})\int d\vec{x}\varepsilon_{mnl}[(\overline{d^c})^m\Gamma u^n][(\overline{u^c})^l\Gamma' e]\frac{\varepsilon_{\beta\gamma\delta}}{3\sqrt{2}}a^{u\dagger}_{\beta\uparrow}(a^{u\dagger}_{\gamma\uparrow}a^{d\dagger}_{\delta\downarrow} - a^{u\dagger}_{\gamma\downarrow}a^{d\dagger}_{\delta\uparrow})|0\rangle \\ &= -\frac{1}{\sqrt{3}}\int d\vec{y}U^{\rho\dagger}_\uparrow(\vec{y})U^p_\uparrow(\vec{y})\Big\{2\int d\vec{x}[\bar{V}^p_\downarrow(\vec{x})\Gamma U^p_\uparrow(\vec{x})][\bar{U}^\rho_\uparrow(\vec{x})\Gamma' e(\vec{x})] + 2\int d\vec{x}[\bar{V}^p_\downarrow(\vec{x})\Gamma V^\rho_\uparrow(\vec{x})][\bar{V}^p_\uparrow(\vec{x})\Gamma' e(\vec{x})] \\ &\quad - \int d\vec{x}[\bar{V}^p_\uparrow(\vec{x})\Gamma V^\rho_\uparrow(\vec{x})][\bar{V}^p_\downarrow(\vec{x})\Gamma' e(\vec{x})] - \int d\vec{x}[\bar{V}^p_\uparrow(\vec{x})\Gamma U^p_\downarrow(\vec{x})][\bar{U}^\rho_\uparrow(\vec{x})\Gamma' e(\vec{x})] \\ &\quad + \int d\vec{x}[\bar{U}^\rho_\uparrow(\vec{x})\Gamma U^p_\downarrow(\vec{x})][\bar{V}^p_\uparrow(\vec{x})\Gamma' e(\vec{x})] + \int d\vec{x}[\bar{U}^\rho_\uparrow(\vec{x})\Gamma U^p_\uparrow(\vec{x})][\bar{V}^p_\downarrow(\vec{x})\Gamma' e(\vec{x})]\Big\}. \end{aligned} \tag{C4}$$

The connection between this calculation in the bag model and the transition amplitude is not completely trivial and requires resorting to the wave packet formalism [78,107]. We apply the conversion factor in Eq. (12) of Ref. [78]. The outgoing lepton is regarded as free, so that its position dependence would be a simple $e^{-i\vec{k}\cdot\vec{x}}$, with $\vec{k}$ the associated momentum. However, in the static approximation, the frequency $|\vec{k}|$ leads to a very slow variation, hence this factor can be discarded, or kept [79,81], e.g., in an attempt to extend the prediction to the case of light pions. In this latter case, however, the static cavity description is not really suited, and tentative corrections should be seen as largely heuristic, such as the phenomenological suppression introduced in [78]. In our analysis, however, the decay channels into pions are already covered by the lattice description, so that the results of the bag model are only employed in the more suitable configuration with heavy mesons in the final state. To complete the calculation of the transition amplitude, we provide the free-lepton spinors in the Dirac representation:

$$u_s^\ell(\vec{k}) = \begin{pmatrix} \sqrt{E_{\vec{k}} + m_\ell}\chi_s \\ \frac{\vec{k}\cdot\vec{\sigma}}{\sqrt{E_{\vec{k}}+m_\ell}}\chi_s \end{pmatrix}, \tag{C5}$$

$$v_s^\ell(\vec{k}) = \begin{pmatrix} \frac{\vec{k}\cdot\vec{\sigma}}{\sqrt{E_{\vec{k}}+m_\ell}}\chi'_s \\ \sqrt{E_{\vec{k}} + m_\ell}\chi'_s \end{pmatrix}, \tag{C6}$$

$$E_{\vec{k}} \equiv \sqrt{\vec{k}^2 + m_\ell^2}; \tag{C7}$$

$$w_s^\nu(\vec{k}) = \sqrt{|\vec{k}|}\begin{pmatrix} \chi_s \\ -\chi_s \end{pmatrix}, \tag{C8}$$

with the standard normalization convention. Once all the matrix elements have been computed, it is possible to match them onto the form factors of the decay, e.g.,

$$\begin{aligned} \mathcal{A}^\Omega[p \to \rho^0 e^+] &\equiv \langle \rho^0, e^+ | C_\Omega \Omega | p\rangle \\ &\equiv W_{[\Omega]}^{p\to\rho^0}\, \bar{v}_e^c(\vec{k}) P_\Omega \gamma^\mu u_p(\vec{0}) \epsilon_\mu^{\rho *}(-\vec{k}). \end{aligned} \tag{C9}$$

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