Nonlocal non-Abelian gauge theory: Conformal invariance and *β*-function

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This paper focuses on extending our previous discussion of an Abelian U(1) gauge theory involving infinite derivatives to a non-Abelian SU(N) case. The renormalization group equation (RGEs) of the SU(N) gauge coupling is calculated and shown to reproduce the local theory *β*-function in the limit of the nonlocal scale M → ∞. Interestingly, the gauge coupling stops its running beyond the scale *M*, approaching an asymptotically conformal theory.

I. INTRODUCTION

With no clinching evidence of new particles in physics beyond the Standard Model (BSM) by any of the current searches at experiments world-wide, such as the Large Hadron Collider (LHC), an alternative philosophy for BSM could be the modification of the standard canonical kinetic terms through the introduction of infinite derivatives, instead of introducing new particles (new states). Motivated by string field theory [1–10], infinite derivative formulation is expressed in the form of an entire function [11] and the higher order derivatives are accompanied by a suppression by a scale *M*, which we call the "nonlocal scale." The choice of this derivative function does not appear to be unique, as long as it acts to suppress terms in the high energy regime.

We have previously considered a nonlocal Abelian U(1) gauge theory within this framework and have shown that the evolution of the gauge coupling becomes fixed or "UV-insensitive" in the energy regime well above the nonlocal scale *M* [12]. It was also shown that Higgs vacuum instability problem [13] is cured, leading to a stable Abelian Higgs theory. The theory is ghost-free [14] and predicts a unique scattering phenomenology leading to transmutation of energy scales which has its own cosmological and astrophysical implications [15]. The phenomenology of dark matter in this theory is also investigated in Ref. [16] and shown DM experiments can be a novel probe for the scale on nonlocality. Strongly coupled regime of the

theory was considered in Refs. [17,18] in Higgs and Yang-Mills versions and it was found that the mass gap generated gets diluted due to nonlocal effects. On aspects of gravity, Ref. [19] showed that the most general quadratic curvature gravitational action (parity-invariant and torsion-free) with infinite covariant derivatives makes the gravitational sector free from the Weyl *ghost* and is devoid of any classical singularities, such as black hole [19–28][1] and cosmological singularities [29–34].[2]

In this paper, we extend the same idea involving infinite series of higher-order derivatives to a non-Abelian SU(N) gauge theory, investigate the gauge invariance, and compute the running of the SU(N) gauge coupling. Here too we expect that within this framework, the standard renormalization group equations (RGEs) should be reproduced in the local limit $(M \to \infty)$.

II. NON-ABELIAN EXTENSION

For local non-Abelian SU(N) gauge theory, the Lagrangian includes the gauge boson kinetic term,

$$\mathcal{L}_g = -\frac{1}{2}\text{tr}[F^{a\mu\nu}F_{a\mu\nu}] = -\frac{1}{4}F^{a\mu\nu}F_{a\mu\nu}. \quad (1)$$

The trace is over the SU(N) group indices and the field-strength tensor is given by

$$F^a_{\mu\nu} = \partial_{[\mu}A^a_{\nu]} - gf^{abc}A^b_\mu A^c_\nu, \quad (2)$$

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[1]For previous arguments related to nonsingular solutions,see Refs. [8,10].

[2]For supersymmetric versions on nonlocality in the matter section, see for instance, Refs. [35,36].

where the f^{abc} represents the group structure constant. For implementation of the nonlocal modification, we follow our approach in Ref. [12]. The gauge boson kinetic term is then described as

$$\mathcal{L}_g = -\frac{1}{2}\text{tr}[F^{a\mu\nu}e^{-\frac{D^2}{M^2}}F_{a\mu\nu}] + \text{H.c.,} \qquad (3)$$

where the covariant derivative is given by $D_\mu = \partial_\mu - igT^aA_\mu^a$. The fermionic part of the Lagrangian is given by the standard form as in Refs. [11,12]:

$$\mathcal{L} = \bar{\psi}e^{\frac{D^2}{M^2}}i\gamma^\mu D_\mu\psi \qquad (4)$$

where $D^2 = \eta_{\mu\nu}D^\mu D^\nu$ ($\mu,\nu = 0, 1, 2, 3$), assuming all gauge and fermionic particles being massless.

The exponential term is introduced by the nonlocal modification and the Lagrangian includes an infinite series of higher dimensional operators that are all suppressed by the nonlocal scale M. As a result, their contribution can be largely ignored at energies lower than M. In other words, the conventional Lagrangian is reproduced in the limit of $M \to \infty$. We take the metric with $\eta = \text{diag}(+1, -1, -1, -1)$ to implement our procedure for UV completion upon the Wick rotation.

A. Non-Abelian gauge field propagator

To find the massless non-Abelian gauge boson propagator, we follow the same gauge-fixing prescription as in Ref. [11], and the non-Abelian gauge and ghost Lagrangians are given by

$$\mathcal{L}_{\text{ghost}} = -\bar{c}^af(\Box)(\partial^\mu D_\mu^{ab})c^b, \qquad (5)$$

and

$$\mathcal{L}_g = \frac{1}{2}A_\mu^a e^{-\frac{\Box}{M^2}}(\Box\eta^{\mu\nu} - \partial^\mu\partial^\nu)A_\nu^a$$
$$+ \frac{1}{2\xi}A_\mu^a(f(\Box))^2\partial^\mu\partial^\nu A_\nu^a, \qquad (6)$$

where ξ is the gauge fixing parameter and $D_\mu^{ab} = \partial_\mu\delta^{ab} - igA_\mu^c(T^c)^{ab}$ is the covariant derivative in the adjoint representation. In order to have consistency with the standard gauge fixing procedure, we choose the entire function $f(\Box) = e^{-\frac{\Box}{2M^2}}$. The nonlocal Faddeev-Popov procedure is given in the Appendix. In the Euclidean space, the gauge boson and ghost propagators have the following forms:

$$\Pi_g(p^2) = \frac{i\eta_{\mu\nu}\delta^{ab}e^{-\frac{p^2}{M^2}}}{p^2 + i\epsilon},$$

$$\Pi_{\text{ghost}}(p^2) = \frac{i\delta^{ab}e^{-\frac{p^2}{2M^2}}}{p^2 + i\epsilon}, \qquad (7)$$

in the Feynman-'t Hooft gauge $\xi = 1$. And the massless fermion propagator is given by [11]:

$$\Pi_\psi(p_E) = -\frac{i\not{p}_E e^{\frac{p_E^2}{M^2}}}{p_E^2 + i\epsilon}. \qquad (8)$$

III. GAUGE COUPLING RUNNING

In Ref. [12], we have obtained the RGE for the gauge coupling in the Abelian U(1) gauge theory with one Dirac fermion having a unit U(1) charge:

$$\mu\frac{dg}{d\mu} = \frac{1}{16\pi^2}\left(\frac{4}{3}\right)g^3 e^{-2\frac{\mu^2}{M^2}}. \qquad (9)$$

The standard result for the beta function is obtained in the local limit of $M \to \infty$. Interestingly, for the nonlocal U(1) theory is "UV-complete," as the beta function is vanishing beyond the nonlocal scale. This behavior is opposed to the local U(1) theory where the running is asymptotically nonfree. In the case of a non-Abelian theory, we expect a similar behavior, namely, the gauge coupling stops running beyond the nonlocal scale. The beta function for the non-Abelian gauge coupling incorporates the gauge and ghost field contributions; we break up the derivation of each contribution below for clarity.

A. Gauge wave function renormalization

First we consider the wave function renormalization for the gauge fields. We use the standard group theory parameterizations: $f^{acd}f^{bcd} = T(A)\delta^{ab}$ and $\text{Tr}[T^aT^b] = T(R)\delta^{ab}$ with the Dynkin indices for the adjoint ($T(A) = N$) and the fundamental ($T(R) = 1/2$) representations, respectively. The quadratic Casimir for the adjoint representatio is $C_2(A) = N$. All the following calculations are performed with the Feynman gauge ($\xi = 1$).

Our methodology for calculating the wave function renormalization follows from the same procedure in Ref. [12]. In our calculation, we employ the cutoff regularization scheme. Although one should use a gauge invariant regularization scheme such as the dimensional regularization, the cutoff regularization scheme can be practically used, since there is a one-to-one correspondence between the two schemes (at the one-loop level), $\log\Lambda^2 = \frac{1}{\epsilon}$, where Λ is the cutoff parameter and ϵ is the dimensional regularization parameter. For the gauge-gauge-gauge contribution:

$$\Gamma_2 = \frac{-g^2 T(A)\delta^{ab}}{2}\int\frac{d^4p}{(2\pi)^4}\frac{N^{\mu\nu}}{p^2(p+q)^2}e^{-\beta(p)^2}e^{-\beta(p+q)^2},$$

where $\beta \equiv 1/M^2$, and $N^{\mu\nu}$ is given by

$$N^{\mu\nu} = [g^{\mu\rho}(q-p)^\sigma + g^{\rho\sigma}(2p+q) + g^{\sigma\mu}(-p-2q)^\rho]$$
$$\times [\delta^\nu_\rho(p-q)_\sigma + g_{\rho\sigma}(-2p-q)^\nu + \delta^\nu_\sigma(p+2q)_\rho]. \quad (10)$$

Using the Schwinger parameters, α_1 and α_2, the integral is recast as

$$\Gamma_2 = \frac{-g^2 T(A)\delta^{ab}}{32\pi^2} \int p^2 dp^2 \int d\alpha_1 d\alpha_2 N^{\mu\nu}$$
$$\times e^{-(\beta(p^2+(p+q)^2)+\alpha_1(p+q)^2+\alpha_2 p^2)}. \quad (11)$$

It is convenient to introduce new parameters, $s:\{0,\infty\}$ and $\alpha:\{0,1\}$, which are defined as $\alpha_1 + \alpha_2 = s$ and $\alpha = \alpha_1 s$ [which gives $\alpha_2 = s(1-\alpha)$]. In addition, we take the momentum shift $p \to p - \frac{\alpha s + \beta}{s + 2\beta} q$ and then

$$\Gamma_2 = \frac{-g^2 T(A)\delta^{ab}}{32\pi^2} \int p^2 dp^2 \int_0^1 d\alpha \int_0^\infty s ds \bar{N}^{\mu\nu}$$
$$\times e^{-((s+2\beta)p^2 + \frac{\alpha(1-\alpha)s^2 + \beta s + \beta^2}{s+2\beta} q^2)}, \quad (12)$$

where $\bar{N}^{\mu\nu}$ is given by

$$\bar{N}^{\mu\nu} = [g^{\mu\rho}(Aq-p)^\sigma + g^{\rho\sigma}(2p+Bq) + g^{\sigma\mu}(-p-Cq)^\rho]$$
$$\times [\delta^\nu_\rho(p-Aq)_\sigma + g_{\rho\sigma}(-2p-Bq)^\nu + \delta^\nu_\sigma(p+Cq)_\rho], \quad (13)$$

and $A = 1 + \frac{\alpha s + \beta}{s+2\beta}$, $B = 1 - \frac{2\alpha s + 2\beta}{s+2\beta}$, and $C = 2 - \frac{\alpha s + \beta}{s+2\beta}$. To find the corrections ΔZ_{Gauge}, we focus on the coefficient of terms only proportional to $q^\mu q^\nu$, which gives

$$\Gamma_2 = \frac{-g^2 T(A)\delta^{ab} q^\mu q^\nu}{16\pi^2} \int p^2 dp^2 \int_0^1 d\alpha \int_0^\infty s ds (A(C-B) + BC - 2B^2) \times e^{-(s+2\beta)p^2}[1 + O(q^2)]. \quad (14)$$

Here, $O(q^2)$ contains higher order momentum terms which are subleading and ignored:

$$\Gamma_2^{\text{gauge}} \approx \frac{-g^2 T(A)\delta^{ab} q^\mu q^\nu}{16\pi^2} \int p^2 dp^2 \int_0^1 d\alpha \int_0^\infty s ds$$
$$\times (1 + 5\alpha - 5\alpha^2) e^{-(s+2\beta)p^2}$$
$$\approx \frac{-g^2 T(A)\delta^{ab} q^\mu q^\nu}{16\pi^2} \left(\frac{11}{6}\right) \int_0^{\Lambda^2} d^2 p \frac{e^{-2\beta p^2}}{p^2} \quad (15)$$

Employing the same procedure for the ghost contribution, we find

$$\Gamma_2^{\text{ghost}} = \frac{-g^2 T(A)\delta^{ab}}{16\pi^2} \int p^2 dp^2 \int_0^1 d\alpha \int_0^\infty s ds$$
$$\times \left(p + \left(1 - \frac{\alpha s + \beta'}{s + 2\beta'}\right) q\right)^\mu \left(p - \frac{\alpha s + \beta'}{s + 2\beta'} q\right)^\nu$$
$$\times e^{-(s+2\beta')p^2}[1 + O(q^2)], \quad (16)$$

where $\beta' = \beta/2$ due to the ghost propagator's exponential factor in (B4). Again, picking up the $q^\mu q^\nu$ terms and ignoring the further subleading terms,

$$\Gamma_2^{\text{ghost}} \approx \frac{g^2 T(A)\delta^{ab} q^\mu q^\nu}{16\pi^2} \int p^2 dp^2 \int_0^1 d\alpha \int_0^\infty s ds$$
$$\times \frac{((1-\alpha)s + \beta')(\alpha s + \beta')}{(2\beta' + s)^2} e^{-(s+2\beta')p^2}$$
$$\approx \frac{g^2 T(A)\delta^{ab} q^\mu q^\nu}{16\pi^2} \left(\frac{1}{6}\right) \int_0^{\Lambda^2} dp^2 \frac{e^{-\beta p^2}}{p^2}. \quad (17)$$

Contributions from the fermion loops are the same as in Ref. [12] with the addition of the SU(N) group factor $T(R)$, for N_F fermions

$$\Gamma_2^{\text{fermion}} \approx \frac{g^2 N_F T(R)\delta^{ab} q^\mu q^\nu}{16\pi^2} \left(\frac{4}{3}\right) \int_0^{\Lambda^2} dp^2 \frac{e^{-2\beta p^2}}{p^2}. \quad (18)$$

Extracting all the relevant terms from (15) and (17),

$$\Delta Z_{\text{gauge}} = \frac{g^2}{16\pi^2} \int_0^{\Lambda^2} \frac{dp^2}{p^2} \left\{ \left[-\frac{11}{6} T(A) + \frac{4}{3} N_F T(R) \right] \times e^{-2\beta p^2} + \frac{1}{6} e^{-\beta p^2} T(A) \right\}. \quad (19)$$

B. Fermion wave function renormalization

Proceeding in a similar manner, lastly, the non-Abelian gauge contribution to the fermion wave function renormalization is

$$\Delta Z_{\text{fermion}} = \frac{g^2 C_2(R)}{16\pi^2} \int_0^{\Lambda^2} \frac{dp^2}{p^2} e^{-2\beta p^2}. \quad (20)$$

C. Non-Abelian vertex correction

Next we consider the corrections to the non-Abelian fermion-fermion-gauge vertex. They are evaluated in a similar fashion, with the main difference being external momentum is set to zero. We work in the Feynman-'t Hooft gauge, and find

$$\Delta g = \frac{g^3}{16\pi^2} \int_0^{\Lambda^2} \frac{dp^2}{p^2} \left\{ \left(C_2(R) - \frac{1}{2} T(A) \right) + \frac{3}{2} T(A) \right\} e^{-3\beta p^2} \quad (21)$$

Having derived wave function renormalization and vertex correction we proceed to investigate the β-function studies.[3]

D. Nonlocal non-Abelian beta function

Bringing these contributions together in the standard way we culminate with the beta function[4] for the gauge couplings in the infinite derivative framework.

According to the standard QFT procedure, we extract the β-function from Γ_2 by taking $\partial/\partial \log \Lambda$ and replacing the cutoff Λ with the renormalization scale μ.[5]

$$\mu \frac{dg}{d\mu} = \frac{-g^3}{16\pi^2} \left\{ -\frac{1}{6} C_2(A) e^{-\beta\mu^2} \right.$$
$$- \left(-\frac{11}{6} C_2(A) + \frac{4}{3} N_F T(R) + 2C_2(R) \right) e^{-2\beta\mu^2}$$
$$\left. + 2(C_2(R) + T(A)) e^{-3\beta\mu^2} \right\}. \quad (22)$$

In the limit $M \to \infty$ ($\beta \to 0$) we recover the standard SU(N) RGE, which is what we expect in the infrared (IR) limit of the nonlocal theory:

$$\mu \frac{dg}{d\mu} = \frac{-g^3}{16\pi^2} \left\{ \frac{11}{3} N - \frac{2}{3} N_F \right\}. \quad (23)$$

In Fig. 1 we show the gauge coupling running of the nonlocal (local) non-Abelian SU(N) gauge theory represented by the solid (dashed) line, where we have set $N = 3$ and $N_F = 6$ & $N_F = 0$. Beyond the nonlocal scale that we set $M = 10^5$ GeV, the running becomes "conformally complete," in the sense that the running becomes frozen. On the other hand, the standard gauge coupling running exhibits the usual asymptotic free behavior.

IV. CONCLUSIONS AND DISCUSSIONS

The central attractive feature of our nonlocal extension of QFT is that the theory becomes scale free (scale invariant) at energies beyond the nonlocal scale M. In other words, the theory becomes conformal, and M signifies the UV-fixed point.[6] The UV behavior of the non-Abelian Higgs model is expected to be very similar to what we have discussed in the Abelian case [12]. The RG evolution of the
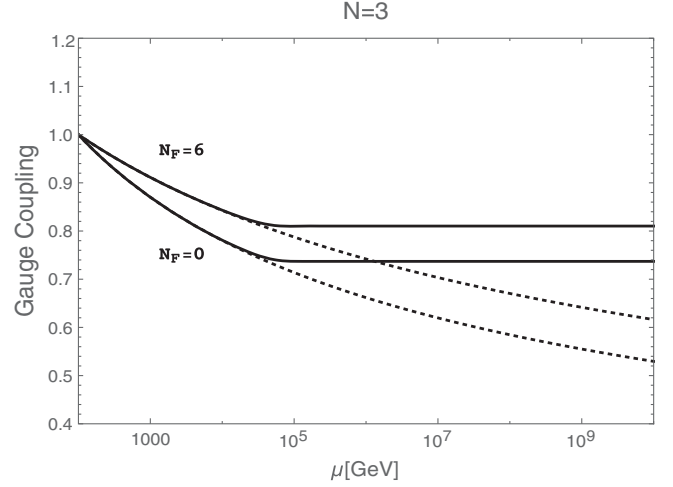

FIG. 1. The SU(3) gauge coupling running with $N_F = 6$ and $N_F = 0$, shown in solid (dashed) black lines for the nonlocal (local) theories. Here, we have set $M = 10^5$ GeV.

Higgs self-coupling freezes beyond M and the Higgs potential never develops any instability.

Classically scale-invariant models are of immense interest in QFT. Usually the conformal symmetries are anomalous in QFT in four-dimensions except for a specific system such as N = 4 supersymmetric Yang-Mills theory. We may classify scale-invariant theories as:

(i) Exact scale invariance is always maintained;

(ii) Scale Invariance breaks only at the quantum level.

In infinite derivative non-Abelian gauge theory, we find a unique scenario where the theory does not possess the scale invariance in the IR, but theory becomes scale invarinat in the UV whose scale is set by the nonlocal scale M. Thus, the symmetry breaking (to generate a scale) maybe considered as an artefact of only the low energy behavior of the theory leading to the concept of scale-dependence, or in other words, the running of the coupling constants. This is very similar to the classical scale invariant theory with "soft" symmetry-breaking, which would not suffer from the naturalness issue arising from the UV sensitivity of scalar mass squared corrections. However, in nonlocal theory, the beta function is exponentially suppressed for $\mu > M$, so that energy M practically works as the conformal fixed point. This leads to scale invariance in the UV *in spite of* "quantum" interactions and the presence of "scale" in the IR, thereby denoting a "scale-insensitivity" of the *tree-level* action.[7]

In the infinite derivative theory, the Higgs mass squared corrections are exponentially suppressed at the scale beyond M, and the nonlocal scale works as an effective

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[3]In deriving the RGE, one may consider 1-loop corrections to the gauge boson self-interactions or 1-loop corrections to the gauge coupling of the ghost. Thanks to the gauge invariance, the RGE devised in this way is coincide with the one presented in this paper.

[4]The RGE is described as $\delta g = \Delta g - g(\Delta Z_{\text{fermion}} + \frac{1}{2}\Delta Z_{\text{gauge}})$.

[5]Also see Ref. [12] for details of the procedure.

[6]See Refs. [37,38] for UV-fixed points in supersymmetric context.

[7]The idea of scale invariance in context to Higgs naturalness issue was proposed long time ago [39,40], it has recently received great attention with respect to UV-complete framework to address the hierarchy problem [41–57].

cutoff for the corrections, $\Delta m_H^2 \sim M^2$ [11]. This means that the Higgs mass fine-tuning is reduced to $\frac{M_{\text{Ew}}^2}{M^2}$ from the usual $\frac{M_{\text{Ew}}^2}{M_{\text{Planck}}^2}$, with M_{Planck} being the Planck mass. Beyond the scale of nonlocality to infinite energy, all the beta-functions are vanishing and all the couplings are approaching a fixed point, determined by the nonlocal scale (see Fig. 1). This means that *no Landau-pole* exists in the theory.

We end our discussion with the speculation that the nonlocal extension of gauge field theories may allow us to provide a unified framework of *Conformal Invariance* without having to encounter with Landau poles, and thus paves a theoretical pathway for theories being perturbatively stable and valid to infinite energy. However, the details are beyond the scope of the current investigation, and we will take this issue up explicitly in a future publication.

APPENDIX A: NL NON-ABELIAN FADDEEV-POPOV PROCEDURE

Focusing solely on the kinetic term of the non-Abelian gauge field, the partition function in the Euclidean space is given by

$$\mathcal{Z}(J) = \int DA^a \int D\alpha^a \delta(G^a(x)) det\left(\frac{\delta G^a(x)}{\delta \alpha^a}\right)$$
$$\times e^{-\int d^4 x (-\frac{1}{4} F^{a\mu\nu} e^{-\frac{\Box}{M^2}} F_{a\mu\nu})}, \tag{A1}$$

where $G^a(x) = f(\Box)\partial^\mu A_\mu^a(x) - w^a(x)$ and $G^a(x)$ transforms as $G^a(x) \to f(\Box)\partial^\mu(A_\mu^a(x) + D_\mu^{ab}\alpha^b(x)) - w^a(x)$ under a SU(N) gauge transformation. Because the function $\mathcal{Z}(J)$ is independent of $w^a(x)$ we introduce the arbitrary functional dependent on $w^a(x)$ and employ the usual gauge fixing procedure to arrive at

$$\mathcal{Z}(J) \propto \int Dw \int DA$$
$$\times \int D\alpha\delta(G(x))det(f(\Box)\partial^\mu D_\mu^{ab}\delta^4(x-y))$$
$$\times e^{-\int d^4 x (-\frac{1}{4} F^{a\mu\nu} e^{-\frac{\Box}{M^2}} F_{a\mu\nu} - \frac{1}{2\xi}w^2)}. \tag{A2}$$

Using the familiar relation between a functional determinant and a path integral over complex Grassmann variables, we obtain the Lagrangians given in Eqs. (5) and (6). In order to provide consistency with the gauge fixing procedure in the local limit, we have chosen $f(\Box) = e^{-\Box/2M^2}$.

APPENDIX B: RENORMALIZABILITY AND BRST INVARIANCE

1. Power counting

In conventional quantum field theories, the kinetic terms of gauge fields contain up to two derivatives. In momentum space this means that the propagators behave as k^{-2}. In four dimensions each momentum loop provides a k^4 factor in any quantum loop integral. The superficial degree of divergence of a Feynman diagram in the local theory is therefore given by

$$D = \text{no. of factors of internal momentum in the numerator}$$
$$- \text{no. of factors in the denominator} = 4L - 2I + 2V,$$

where L is the number of loops, V is the number of vertices, and I is the number of internal propagators.

When one has exponential factors in the loops, vertices and propagators, an exponential suppression will always be dominant over a polynomial growth. Therefore, we see that as long as these exponential factors come with a negative power, the integrals should be manifestly convergent. In the counting of the superficial degree of divergence in the infinite derivative theory, we note that each propagator comes with an exponential suppression, (see Eq. (7) and Ref. [12]) and each vertex also comes with an exponential suppression. Thus, the power of exponential suppression factor should be:

$$E = -V - I. \tag{B1}$$

By using the topological relation:

$$V = I + 1 - L, \tag{B2}$$

we have

$$E = 1 - 2V - L. \tag{B3}$$

Since V is an integer, $E < 0$, for all loops and the corresponding scattering amplitudes are superficially convergent.

2. BRST invariance

Next we discuss the convergence in the complete BRST-invariant infinite-derivative gauge theory action. The conclusion is exactly the same. The quantized action is of the form:

$$S_{\text{total,quantized}} = S_{\text{g}} + S_{\text{Gauge-Fixing}} + S_{\text{ghost}}$$

$$= \int d^4x \left(-\frac{1}{4}(F^a_{\mu\nu} e^{\frac{-D^2}{M^2}}(F^a)^{\mu\nu}) \right.$$

$$\left. + \frac{\xi}{2}(B^a)^2 + B^a \partial^\mu A^a_\mu + \bar{c}^a(-\partial^\mu e^{\frac{-\Box}{2M^2}} D^{ac}_\mu)c^c \right),$$

(B4)

where ξ is the gauge fixing parameter, B is the auxiliary field, and c and $\bar{c}$ are the ghost and anti-ghost fields, respectively. The BRST transformations for non-Abelian gauge theories express a residual symmetry of the effective action which remains after the original gauge invariance has been broken by the addition of the gauge-fixing and ghost action terms. In our theory, we introduce the following BRST transformations:

$$\delta_{\text{BRST}}(A^\mu)^a = (D^\mu_c)^a c^c \delta\lambda, \quad (B5)$$

$$\delta_{\text{BRST}} c^a = -\frac{1}{2} g f^{abc} c^a c^b \delta\lambda, \quad (B6)$$

$$\delta_{\text{BRST}} \bar{c}^a = (\delta\lambda) e^{\frac{\Box}{M^2}} B^a, \quad (B7)$$

$$\delta_{\text{BRST}} B^a = 0. \quad (B8)$$

where $\delta\lambda$ is an infinitesimal anti-commuting constant parameter. We show the BRST-invariance of $S_{\text{total,quantized}}$ (following Refs. [58,59]): the BRST transformation of the gauge field is just a gauge transformation of A_μ generated by $c_a\delta\lambda$. Therefore, any gauge-invariant functionals of $F_{\mu\nu}$, like the first term in Eq. (B4) gives $\delta_{\text{BRST}}(-\frac{1}{4}(F^a_{\mu\nu} e^{\frac{-D^2}{M^2}}(F^{\mu\nu})^a)) = 0$. The second term in Eq. (B4) gives $\delta_{\text{BRST}}(\frac{\xi}{2}(B^a)^2) = 0$ from Eq. (B8). For the third term in Eq. (B4), the transformation of A^a_μ cancels the transformation of $\bar{c}$ in the last term, due to Eqs. (B5), (B6) and (B7) leaving us with

$$\delta_{\text{BRST}}(D^{ac}_\mu c^c) = D^{ac}_\mu \delta c^c + g f^{abc} \delta A^b_\mu c^c, \quad (B9)$$

which is equal to 0, using the *Jacobi identity* (see Ref. [60]). The transformation of c^σ is nilpotent,

$$\delta_{\text{BRST}}(\partial_\mu c^a c^b) = 0, \quad (B10)$$

while the transformation of A^μ is also nilpotent,

$$\delta_{\text{BRST}}((D^\mu_b)^a c^b) = 0. \quad (B11)$$

Hence, the action in Eq. (B4) is BRST-invariant. Noting the fact that the only part of the ghost action which varies under the BRST transformations is that of the antighost $(\bar{c}_a)$, the central idea behind our proof of BRST-invariance is that we have chosen the BRST variation of the antighost

$(\bar{c}_a)$ [see Eqs. (B7)] to cancel the variation of the gauge-fixing term.

Next we proceed in the same manner as in Ref. [59] and introduce BRST-invariant couplings of the ghosts and gauge bosons to external fields K_μ (anticommuting) and L_σ (commuting), so that the effective action $\tilde{S}$ is

$$\tilde{S} = S_{\text{total,quantized}} + K_\mu D^\mu_a c^a + L_\sigma f^{abc} c^a c^b, \quad (B12)$$

which is also BRST-invariant.

Let us now compute the superficial degree of divergence for the BRST-invariant action. To proceed, we introduce the following notations:

(i) n_A is the number of gauge boson vertices,
(ii) n_G is the number of antighost-gauge boson-ghost vertices,
(iii) n_K is the number of K-gauge boson-ghost vertices,
(iv) n_L is the number of L-ghost-ghost vertices,
(v) I_A is the number of internal gauge boson propagators,
(vi) I_G is the number of internal ghost propagators,
(vii) E_c is the number of external ghosts,
(viii) $E_{\bar{c}}$ is the number of external antighosts.

By counting the exponential contributions of the propagators and the vertex factors, as discussed earlier, we can now obtain the superficial degree of divergence, which is given by

$$E = -n_A - I_A. \quad (B13)$$

By using the following topological relation,

$$L = 1 + I_A + I_G - n_A - n_G - n_K - n_L, \quad (B14)$$

we obtain

$$E = 1 - L + I_G - n_G - n_K - n_L - 2n_A. \quad (B15)$$

Employing the momentum conservation law for ghost and antighost lines,

$$2I_G - 2n_G = 2n_L + n_K - E_c - E_{\bar{c}}, \quad (B16)$$

we obtain

$$E = 1 - L - \frac{1}{2}(n_K + E_c + E_{\bar{c}}) - 2n_A. \quad (B17)$$

It is clear that the degree of divergence reduces as n_K, E_c and $E_{\bar{c}}$ increase. Thus, one may conclude the most divergent diagrams are those for which $n_K = E_c = E_{\bar{c}} = 0$, i.e., the diagrams whose external lines are all gauge bosons. In this case, the degree of divergence is given by $E = 1 - 2n_A - L$. Since n_A is an integer, $E < 0$, namely, the corresponding loop amplitudes $(L \geq 1)$ are superficially convergent.

Here, we have utilized *power-counting-in-exponentials* to understand the degree of divergence. In the local QFT limit of $M \to \infty$, all the exponential suppression factors disappear, and we will obtain divergences according to *power-counting-in-polynomials* as usual in the standard QFT. We justify our procedure in the way that by taking $M \to \infty$, our results should reproduce the local theory results.

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Correction: Equation numbers in the Appendixes were set incorrectly during the production cycle and have been fixed to adhere to proper formatting.