Decay properties of Roper resonance in the holographic QCD

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We investigate the one pion decay of the Roper resonance $N^*(1440) \to N\pi$ in the Sakai-Sugimoto model of the holographic QCD. The nucleon and Roper resonance emerge as ground and first excited states of the collective radial motion of the instanton in the four dimensional space with one extra dimension. It is found that the ratio of the πNN^* and πNN couplings, and hence the ratio of $g_A^{NN^*}$ and g_A^{NN}, is well reproduced in comparison with the experimental data. The mechanism of this result is due to the collective nature of excitations, which is very different from that of the single particle nature of the constituent quark model. Our results are obtained in the large-N_c and large λ ('t Hooft coupling) limit which are useful to test how baryon resonances share what are expected in these limits.

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I. INTRODUCTION

The Roper resonance $N^*(1440)$ is the first excited state of the nucleon with the spin and parity $J^P = 1/2^+$ [1]. Its mass smaller than the negative parity nucleon $N(1535)$ has attracted great amount of interests because the naive quark model predicts the mass of the Roper resonance much higher than that of the negative parity state. To resolve this problem, and also to reproduce the electromagnetic transitions, the importance of the meson cloud has been emphasized [2,3]. Turning to strong decays, an almost vanishing partial decay width of one pion emission when computed by the leading order terms of nonrelativistic expansion of the pion-quark interaction disagree with the large value of the experimental data. While it has been pointed out recently that higher order corrections can improve this significantly [4], this problem should be further investigated.

The relatively low mass has lead to the idea of collective vibrational mode along the radial direction [5]. Extensive discussions were made in the Skyrme model in the 1980s, where the soliton's radial vibrations were investigated in various context [6–9].

Later the solitonic picture of baryons has been further strengthened by the holographic QCD. The Sakai-Sugimoto model is one of successful descriptions of hadrons in the holographic QCD based on the D4-D8 brane construction [10,11]. They have derived an effective action of the flavor gauge field in the five dimensional space (four space-time and one extra dimension), implementing the spontaneous breaking of chiral symmetry leading to the successful low-energy effective action of hadrons. Moreover the extra dimension of the model naturally accommodates various excited states of hadrons.

In the holographic model, baryons emerge as instantons of the five-dimensional space [12], which is very much the same as the Skyrme mode, baryons as chiral solitons [13,14]. Such a baryon structure looks very different from the one of the quark model. Baryon dynamics is dominated by the collective motions of instantons/solitons, while that of the quark model by single-particle motions of quarks. Interestingly, it was found that the resulting Roper and the negative parity resonance [12] are degenerate and appear very close to the observed masses. This is one of good features of the holographic QCD for baryons.

The holographic baryons have been further studied by Hata *et al.* [15] and by Hashimoto *et al.* [16] for various static properties of the nucleon including electromagnetic and weak coupling constants. Inspired by these works, we would like to further study the properties of the Roper resonance in the holographic model. In this paper, we investigate the one pion emission decay. It is the axial transition between the Roper resonance and the nucleon, and is dictated by the transition matrix element of the axial current. Following Ref. [16], we define chiral currents by introducing the external gauge field that couples to the currents. By calculating the matrix elements of the obtained axial current, the axial coupling and hence decay width are calculated. The results are compared with the experimental data. The model and computation procedures are realized in

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the large-N_c and large 't Hooft coupling λ limits. Hence our study provides the measure to what extend hadron properties share the features of these limits.

This paper is organized as follows. In Sec. II, we present the actions used in this paper and derive the solutions of the equations of motion. Then we define the chiral currents and obtain their concrete expressions by the solutions. In Sec. III, we compute matrix elements of the axial currents for the nucleon and that of the Roper to the nucleon transitions. The resulting decay width is compared with the experimental data. We discuss the ratio of $g_A^{NN^*}$ and g_A^{NN}, and compare with the data carefully. Finally, Sec. IV is for some discussions and summary of the present work.

II. AXIAL CURRENT

A. Classical solutions and collective quantization

Let us start by briefly summarizing how the baryon states are obtained in the Sakai-Sugimoto model by collectively quantizing the instanton solution. The action of hadron effective theory is composed of the Yang-Mills term $S_{\rm YM}$ and the Chern-Simons term $S_{\rm CS}$,

$$S = S_{\rm YM} + S_{\rm CS} \tag{1}$$

where

$$\begin{aligned} S_{\rm YM} &= -\kappa \int d^4x dz {\rm tr}\left[\frac{1}{2}h(z)\mathcal{F}_{\mu\nu}^2 + k(z)\mathcal{F}_{\mu z}^2\right], \\ S_{\rm CS} &= \frac{N_c}{24\pi^2}\int_{M^4\times\mathbb{R}} \omega_5(\mathcal{A}), \\ \kappa &= \frac{\lambda N_c}{216\pi^3} = a\lambda N_c. \end{aligned} \tag{2}$$

In these equations N_c is the number of colors, λ the 't Hooft coupling, and the indices μ, $\nu = 0$, 1, 2, 3 are for the 4-dimensional space-time. The curvatures along the extra dimension z are defined by

$$h(z) = (1+z^2)^{-1/3}, \qquad k(z) = 1+z^2. \tag{3}$$

The 1-form $\mathcal{A} = \mathcal{A}_\alpha dx^\alpha$ expresses $\mathcal{A} = A_\alpha dx^\alpha + \hat{A}_\alpha dx^\alpha$ which consists of the flavor SU(2) part A_α and the U(1) part $\hat{A}_\alpha$ with $\alpha = 0, 1, 2, 3, z$, and the field strength of the $\mathcal{A}_\alpha$ is $\mathcal{F}_{\alpha\beta} = \partial_\alpha\mathcal{A}_\beta - \partial_\beta\mathcal{A}_\alpha + i[\mathcal{A}_\alpha, \mathcal{A}_\beta]$. The Chern-Simons 5-form is given by

$$\omega_5(\mathcal{A}) = {\rm tr}\left(\mathcal{A}\mathcal{F}^2 - \frac{i}{2}\mathcal{A}^3\mathcal{F} - \frac{1}{10}\mathcal{A}^5\right). \tag{4}$$

In general, it is difficult to analytically solve the equations of motion in the presence of the curvatures $h(z)$ and $k(z)$. However, it is known that the instanton size is proportional to $\lambda^{-1/2}$ and is small for large λ. Then in the vicinity of $z \sim 0$, the metric can be approximated as $h(z) = k(z) \sim 1$, where the Belavin, Polyakov, Schwartz, and Tyupkin (BPST) [17] instanton solution is available. Therefore, for $M = 1, 2, 3, z$,

$$\begin{aligned} A_M^{cl}(\mathbf{x}, z) &= -if(\xi)g\partial_M g^{-1}, \\ A_0^{cl} &= 0, \end{aligned} \tag{5}$$

$$\begin{aligned} \hat{A}_M^{cl} &= 0, \\ \hat{A}_0^{cl} &= \frac{1}{8\pi^2 a}\frac{1}{\xi^2}\left[1 - \frac{\rho^4}{(\xi^2+\rho^2)^2}\right], \end{aligned} \tag{6}$$

where

$$g(\mathbf{x}, z) = \frac{(z-Z) - i(\mathbf{x}-\mathbf{X})\cdot\boldsymbol{\tau}}{\xi},$$

with $(\boldsymbol{X}, Z)$ and ρ the location and size of the instanton, respectively. The profile function $f(\xi)$ is given by

$$\begin{aligned} f(\xi) &= \xi^2/(\xi^2+\rho^2), \\ \xi &= \sqrt{(\mathbf{x}-\mathbf{X})^2 + (z-Z)^2}. \end{aligned}$$

The classical instanton solution needs to be quantized for the physical nucleon and Roper resonances. This can be done by the collective coordinate method, where the relevant time dependent dynamical variables are, $\boldsymbol{X}$, Z, ρ and the SU(2) orientation $V(t, x^M; a(t))$ with $V(z\to\pm\infty)\to \boldsymbol{a}(t)$ related to the rotational variable $\boldsymbol{a}(t) = a_4(t) + ia_a(t)\tau^a$ in the isospin and spin space. As shown in Ref. [12] the time dependent gauge field is given by

$$A_M(t, x^N) = VA_M^{cl}(x^N; X^N(t), \rho(t))V^{-1} - iV\partial_M V^{-1}, \tag{7}$$

where V is defined by

$$\begin{aligned} -iV^{-1}\dot{V} &= -\dot{X}^M(t)A_M^{cl} + \chi^a f(\xi) g\frac{\tau^a}{2}g^{-1}, \\ \chi^a &= -i{\rm tr}(\tau^a \boldsymbol{a}^{-1}\dot{\boldsymbol{a}}). \end{aligned}$$

By substituting this gauge field for the action (2), integrating over the space of (x^μ, z), and quantizing the system of the above collective coordinates, we find the collective Hamiltonian as

$$\begin{aligned} H &= -\frac{1}{2M_0}(\partial_{\vec{X}}^2 + \partial_Z^2) - \frac{1}{4M_0}\partial_{y^I}^2 + U(\rho, Z), \\ U(\rho, Z) &= M_0 + \frac{M_0}{6}\rho^2 + \frac{N_c^2}{5M_0}\frac{1}{\rho^2} + \frac{M_0}{3}Z^2, \end{aligned} \tag{8}$$

where $M_0 = 8\pi^2\kappa$ is the classical soliton mass [12], and y_I is related to the orientation coordinates by $y_I = \rho a_I$. By solving the Schrödinger equations for the collective coordinates derived from the Hamiltonian (8), we find baryon wave functions. They are labeled by their momentum $\vec{p}$ and quantum numbers $(l, I_3, s_3, n_\rho, n_z)$, where $l/2$ is the equal isospin and spin values; I_3, s_3 are the third components of the isospin and spin; and n_ρ, n_z are the quanta for oscillations along the radial and z-directions. For the spin up proton ($I_3 = 1/2, s_3 = 1/2$) with a finite momentum $\vec{p}$, the wave functions of ground and Roper resonance are given as [12,16]

$$\psi_N \propto e^{i\vec{p}\cdot\vec{X}} R_N(\rho)\psi_Z(Z)(a_1 + ia_2),$$
$$\psi_{N^*(1440)} \propto e^{i\vec{p}\cdot\vec{X}} R_{N^*}(\rho)\psi_Z(Z)(a_1 + ia_2), \tag{9}$$

where

$$R_N(\rho) = \rho^{-1+2\sqrt{1+N_c^2/5}} e^{-\frac{M_0}{\sqrt{6}}\rho^2},$$
$$R_{N^*}(\rho) = \left(\frac{2M_0}{\sqrt{6}}\rho^2 - 1 - 2\sqrt{1+\frac{N_c^2}{5}}\right) R_N(\rho), \tag{10}$$

$$\psi_Z(Z) = e^{-\frac{M_0}{\sqrt{6}}Z^2}. \tag{11}$$

We note that the wave function for the z oscillation is the lowest ($n_z = 0$) for both the nucleon and Roper resonance. Thus the only difference between them is in the radial part, $R_N(\rho)$ and $R_{N^*}(\rho)$.

B. The asymptotic solution of the instanton

The BPST instanton that we have summarized in the previous subsection is only an approximate solution in the large λ limit where the instanton size is small. This can be used for the computation of baryon masses. However, for the computation of currents which are defined at $|z| \to \infty$ such a solution is not suited. As shown in Ref. [16] we need to find the solution that is properly extended to the large $|z|$ region to obtain the well-defined currents. In this paper, we simply summarize the final result of such a solution;

$$\hat{A}_0 = -\frac{1}{2a\lambda} G(\vec{x}, z; \vec{X}, Z),$$
$$\hat{A}_i = \frac{1}{2a\lambda}\left[\dot{X}^i + \frac{\rho^2}{2}\left(\frac{\chi^a}{2}\left(\epsilon^{iaj}\frac{\partial}{\partial X^j} - \delta^{ia}\frac{\partial}{\partial Z}\right) + \frac{\dot{\rho}}{\rho}\frac{\partial}{\partial X^i}\right)\right] \times G(\vec{x}, z; \vec{X}, Z),$$
$$\hat{A}_z = \frac{1}{2a\lambda}\left[\dot{Z} + \frac{\rho^2}{2}\left(\frac{\chi^a}{2}\frac{\partial}{\partial X^a} + \frac{\dot{\rho}}{\rho}\frac{\partial}{\partial Z}\right)\right] H(\vec{x}, z; \vec{X}, Z), \tag{12}$$

$$A_0 = 4\pi^2\rho^2 i\boldsymbol{a}\dot{\boldsymbol{a}}^{-1} G(\vec{x}, z; \vec{X}, Z) + 2\pi^2\rho^2 \boldsymbol{a}\tau^a \boldsymbol{a}^{-1}\left(\dot{X}^i\left(\epsilon^{iaj}\frac{\partial}{\partial X^j} - \delta^{ia}\frac{\partial}{\partial Z}\right) + \dot{Z}\frac{\partial}{\partial X^a}\right) \times G(\vec{x}, z; \vec{X}, Z),$$
$$A_i = -2\pi^2\rho^2 \boldsymbol{a}\tau^a \boldsymbol{a}^{-1}\left(\epsilon^{iaj}\frac{\partial}{\partial X^j} - \delta^{ia}\frac{\partial}{\partial Z}\right) G(\vec{x}, z; \vec{X}, Z),$$
$$A_z = -2\pi^2\rho^2 \boldsymbol{a}\tau^a \boldsymbol{a}^{-1}\frac{\partial}{\partial X^a} H(\vec{x}, z; \vec{X}, Z), \tag{13}$$

where the index i runs 1–3. In these equations, G and H are given by

$$G(\vec{x}, z; \vec{X}, Z) = \kappa \sum_{n=1}^{\infty} \psi_n(z)\psi_n(Z) Y_n(|\vec{x} - \vec{X}|),$$
$$H(\vec{x}, z; \vec{X}, Z) = \kappa \sum_{n=1}^{\infty} \phi_n(z)\phi_n(Z) Y_n(|\vec{x} - \vec{X}|).$$

The function $\psi_n(z)$ are the solutions of the eigenvalue equation

$$-h(z)^{-1}\partial_z(k(z)\partial_z\psi_n) = \lambda_n\psi_n(z), \tag{14}$$

with the eigenvalue λ_n [10], and

$$\phi_0(z) = \frac{1}{\sqrt{\kappa\pi}}\frac{1}{k(z)},$$
$$\phi_n(z) = \frac{1}{\sqrt{\lambda_n}}\partial_z\psi_n(z), \tag{15}$$

$$Y_n(r) = -\frac{1}{4\pi}\frac{e^{-\sqrt{\lambda_n}r}}{r}, \qquad r = |\vec{x}|. \tag{16}$$

C. Currents

Now we are ready to calculate the axial current. Following Ref. [16], the chiral current is derived from the coupling with the external gauge field $\delta\mathcal{A}_\alpha$ which is defined by

$$\mathcal{A}_\alpha(x^\mu, z) = \mathcal{A}_\alpha^{cl}(x^\mu, z) + \delta\mathcal{A}_\alpha(x^\mu, z). \tag{17}$$

They are related to the left and right gauge fields in the four dimensional space at $z \to \pm\infty$,

$$\delta\mathcal{A}_\mu(x^\nu, z \to +\infty) = \mathcal{A}_{L\mu}(x^\nu),$$
$$\delta\mathcal{A}_\mu(x^\nu, z \to -\infty) = \mathcal{A}_{R\mu}(x^\nu).$$

Substituting this field into the action, the coefficients of the first order in $\mathcal{A}_{L\mu}, \mathcal{A}_{R\mu}$ is identified with the left and right currents $\mathcal{J}_L^\mu, \mathcal{J}_R^\mu$ with the sign properly taken into account,

$$\kappa\int d^4x[2\mathrm{tr}(\delta\mathcal{A}^\mu k(z)\mathcal{F}_{\mu z}^{cl})]_{z=-\infty}^{z=+\infty},$$
$$= -2\int d^4x \mathrm{tr}(\mathcal{A}_{L\mu}\mathcal{J}_L^\mu + \mathcal{A}_{R\mu}\mathcal{J}_R^\mu), \tag{18}$$

where

$$\mathcal{J}_L^\mu = -\kappa(k(z)\mathcal{F}_{\mu z}^{cl})|_{z=+\infty},$$
$$\mathcal{J}_R^\mu = +\kappa(k(z)\mathcal{F}_{\mu z}^{cl})|_{z=-\infty}. \tag{19}$$

The vector and axial currents are then obtained by

$$\mathcal{J}_V^\mu = \mathcal{J}_L^\mu + \mathcal{J}_R^\mu,$$
$$\mathcal{J}_A^\mu = \mathcal{J}_L^\mu - \mathcal{J}_R^\mu = -\kappa[\psi_0(z)k(z)\mathcal{F}^{cl}_{\mu z}]^{z=+\infty}_{z=-\infty}, \qquad (20)$$

with $\psi_0(z) = (2/\pi)\arctan z$.

When the instanton oscillates along the z direction in a narrow range in the large λ limit, the metrices are approximated as $h(Z) \simeq k(Z) \simeq 1$. Then, substituting (13) for (20) gives the following form ($r \equiv |\vec{x} - \vec{X}|$)

$$J_A^i(r;\vec{X},Z,\rho,\vec{a}) = -2\pi^2\kappa\rho^2 \boldsymbol{a}\tau^a\boldsymbol{a}^{-1} \times ((\partial_i\partial_a - \delta^{ia}\partial_j^2)H^A - \epsilon^{iaj}\partial_j G^A) \qquad (21)$$

where

$$G^A(r;\vec{X},Z) = [\psi_0(z)k(z)\partial_z G]^{z=+\infty}_{z=-\infty} = -\sum_{n=1}^{\infty} g_{a^n}\psi_{2n}(Z)Y_{2n}(r), \qquad (22)$$

$$H^A(r;\vec{X},Z) = [\psi_0(z)k(z)H]^{z=+\infty}_{z=-\infty} = -\frac{1}{2\pi^2}\frac{1}{k(Z)}\frac{1}{r} - \sum_{n=1}^{\infty}\frac{g_{a^n}}{\lambda_{2n}}\partial_Z\psi_{2n}(Z)Y_{2n}(r), \qquad (23)$$

$$g_{a^n} = \lambda_{2n}\kappa\int dz h(z)\psi_{2n}\psi_0. \qquad (24)$$

To go further, it is convenient to present the Fourier transform in the momentum space, (in what follows the dependence on the collective coordinates $\vec{X}, Z, \rho, \vec{a}$ are suppressed)

$$\tilde{J}_A^\mu(\vec{q}) = \int d^3x e^{-i\vec{q}\cdot\vec{x}} J_A^\mu(r). \qquad (25)$$

We obtain the following form:

$$\tilde{J}_A^{cj}(\vec{q}) = e^{-i\vec{q}\cdot\vec{X}} 2\pi^2\kappa\rho^2 \mathrm{tr}(\tau^c\boldsymbol{a}\tau^a\boldsymbol{a}^{-1}) \times \left(\delta_{aj} - \frac{q_a q_j}{\vec{q}^2}\right)\sum_{n\geq 1}\frac{g_{a^n}\partial_Z\psi_{2n}(Z)}{\vec{q}^2 + \lambda_{2n}}. \qquad (26)$$

This current is regarded as an operator in terms of the dynamical variable $\vec{X}$, Z, ρ and $\vec{a}$, which is used when taking the matrix elements by the corresponding wave functions.

III. DECAY PROPERTIES OF ROPER RESONANCE

Now, we investigate the decay properties of the Roper resonance, in particular the one pion emission decay $N^*(1440) \to \pi N$. Because the Roper resonance has a very large width causing uncertainties in the Breit-Wigner fitting, we refer to the result of the pole analysis. Following the PDG table [18], we quote the following nominal values

$$M_{N^*} = 1360\text{–}1380(\sim 1370)\ \mathrm{MeV},$$
$$\Gamma_{\mathrm{total}} = 160\text{–}190(\sim 175)\ \mathrm{MeV}, \qquad (27)$$

and the branching ratio of the one pion decay

$$N^* \to N\pi\colon\ 55\text{–}75\%. \qquad (28)$$

Using the lower and upper bounds for the total decay width and branching ratio, we find the partial decay width of the one pion decay

$$\Gamma_{N^*\to\pi N} \sim 90\text{–}140\ \mathrm{MeV}. \qquad (29)$$

A. Axial coupling g_A

The axial coupling $g_A^{NN^*}$ for the transition $N^*(1440) \to N + \pi$ is defined as follows:

$$\int d^3x \langle N, s_3' I_3' | J_A^{ai} | N^*, s_3, I_3\rangle \times 2 = \frac{2}{3} g_A^{NN^*}(\sigma^i)_{s_3',s_3}(\tau^a)_{I_3',I_3}. \qquad (30)$$

The factor $2/3$ on the right-hand side is needed in the chiral limit [14]. Using (26) and (9), we obtain

$$g_A^{NN^*}(\vec{q}) = \frac{8\pi^2\kappa}{3}\langle R_{N^*}|\rho^2|R_N\rangle\sum_{n=1}\frac{g_{a_n}\langle\partial_Z\psi_{2n}(Z)\rangle}{\vec{q}^2 + \lambda_{2n}} \qquad (31)$$

where $\langle\partial_Z\psi_{2n}(Z)\rangle$ stands for the expectation value using the wave functions of Z. The matrix element of ρ^2 can be computed and the result is

$$\langle R_{N^*}|\rho^2|R_N\rangle = \left(1 + 2\sqrt{1 + \frac{N_c^2}{5}}\right)^{-1/2}\langle R_N|\rho^2|R_N\rangle = \frac{\sqrt{5}}{2N_c}\left(1 + 2\sqrt{1 + \frac{N_c^2}{5}}\right)^{1/2}\rho_{cl}^2 \qquad (32)$$

with ρ_{cl} being the classical instanton size given by

$$\rho_{cl}^2 = \frac{N_c}{8\pi^2\kappa}\sqrt{\frac{6}{5}}. \qquad (33)$$

We note that the transition matrix element for $N^*(1440) \to N + \pi$ is related to the nucleon matrix element, an interesting feature of the present model associated with the collective nature of baryons. The axial coupling constant is then defined at $\vec{q} = 0$, $g_A^{NN^*} = g_A^{NN^*}(\vec{0})$. Using the relation

$$\sum_{n=1} \frac{g_{a_n} \partial_Z \psi_{2n}(Z)}{\lambda_{2n}} = \frac{2}{\pi} \frac{1}{k(Z)}, \tag{34}$$

$g_a^{NN^*}$ can be expressed in a compact form:

$$g_A^{NN^*} = \frac{16\pi\kappa}{3} \langle R_{N^*} | \rho^2 | R_N \rangle \left\langle \frac{1}{k(Z)} \right\rangle. \tag{35}$$

In the above equations, $\langle \cdots \rangle$ stands for the expectation value using the wave functions of Z.

There are two parameters of this model, M_{KK} and κ. Following Adkins *et al.* [14] they are determined to reproduce the mass splitting of the nucleon and delta, and the pion decay constant $f_\pi = 64.5$ MeV,

$$M_{KK} = 488 \text{ MeV}, \qquad \kappa = 0.0137. \tag{36}$$

Then, the prediction of the present model for $g_A^{NN^*}$ is

$$g_A^{NN^*} = 0.402. \tag{37}$$

B. Decay width

The decay width of $N^*(1440) \to N + \pi$ can be computed by the formula

$$\Gamma_{N^*(1440)\to N+\pi} = \frac{1}{2M_{N^*}} \int \frac{d^3 p_N}{(2\pi)^3 2E_N} \frac{d^3 p_\pi}{(2\pi)^3 2E_\pi} \times (2\pi)^4 \delta^4(p_N + p_\pi) |t_{fi}|^2, \tag{38}$$

where the amplitude t_{fi} is given by the Lagrangian

$$L = i \frac{M_N + M_{N^*}}{2f_\pi} g_A^{NN^*} \bar{\psi}_{N^*} \gamma_5 \vec{\tau} \cdot \vec{\pi} \psi_N + \text{H.c.}, \tag{39}$$

as follows

$$\begin{aligned} t_{fi} &= \langle N(-\vec{q}) \pi(\vec{q}) | L | N^*(\vec{0}) \rangle \\ &= \sqrt{2M_{N^*}} \sqrt{M_N + E_N} \\ &\quad \times \frac{M_N + M_{N^*}}{2f_\pi} \frac{g_A^{NN^*}}{E_N + M_N} \langle s_3' | \vec{\sigma} \cdot \vec{q} | s_3 \rangle. \end{aligned} \tag{40}$$

Here we have expressed the effective πNN^* coupling $g_{\pi NN^*}$ in terms of the axial coupling by using the Goldberger-Treiman relation,

$$g_A^{NN^*} = \frac{f_\pi g_{\pi NN^*}}{(M_N + M_{N^*})/2}. \tag{41}$$

Hence we obtain

$$\Gamma_{N^*(1440)\to N+\pi} = \frac{q}{4\pi} \frac{M_N + E_N}{M_{N^*}} \left(\frac{M_N + M_{N^*}}{2f_\pi} \frac{g_A^{NN^*} q}{E_N + M_N} \right)^2. \tag{42}$$

where

$$q = \frac{(M_{N^*}^2 - (M_N + m_\pi)^2)^{1/2} (M_{N^*}^2 - (M_N - m_\pi)^2)^{1/2}}{2M_{N^*}} \tag{43}$$

Using $M_N = 940$ MeV, $M_{N^*} = 1370$ MeV, $m_\pi = 140$ MeV (pion mass), $q = 342$ MeV, we find

$$\Gamma_{N^*(1440)\to N+\pi} = 64 \text{ MeV}. \tag{44}$$

In this computation the value of $g_A^{NN^*}$ at $\vec{q} = 0$ is used. By considering the form factor effect, the $g_A^{NN^*}$ value at $\vec{q} = 342$ MeV becomes about 13% smaller, and hence $\Gamma_{N^*(1440)\to N+\pi} \sim 55$ MeV. If we use $M_{N^*} = 1440$ MeV and $q = 398$ MeV [18], we find 101 MeV for (44) and 84 MeV for the finite q. These estimations show that there is ambiguity in comparison with actual experimental data due to uncertainties in the exact resonance point.

These values are smaller than the experimental value (29). This is because the axial coupling $g_A^{NN^*}$ is small, which is a common feature of the solitonic picture of baryons. In fact, the nucleon g_A^{NN} is computed in a similar manner as for $g_A^{NN^*}$ by using the nucleon wave function $R_N(\rho)$. The result is

$$g_A^{NN} = 0.837. \tag{45}$$

This value is significantly smaller than the experimental value $g_A^{NN} = 1.25$. The small g_A is a common problem of the solitonic description of baryons. One possible resolution to recover the experimental value $g_A^{NN} = 1.25$ is to take into account $1/N_c$ corrections (Ref. [19] and references there). Here, however, we do not discuss this anymore. On the other hand, it is interesting to observe the relation between the axial couplings of the nucleon and that of the Roper-nucleon transition. Inspection of Eq. (32), we find

$$g_A^{NN} / g_A^{NN^*} = \left(1 + 2\sqrt{1 + \frac{N_c^2}{5}} \right)^{1/2} = 2.08. \tag{46}$$

We emphasize that this relation does not include any model parameters (except for $N_c = 3$), and so a model independent relation. Experimentally, if we use the partial decay width $\Gamma_{N^*\to\pi N} \sim 110$ MeV, we find the ratio

$$g_A^{NN} / g_A^{NN^*} = 1.25/0.77 \sim 1.62, \tag{47}$$

which agrees well with the present model prediction within ~20% accuracy, whose agreement is better than the absolute value.

IV. DISCUSSIONS AND SUMMARY

In this paper, we have studied one pion emission decay of the Roper resonance, $N^*(1440) \to N + \pi$ in the Sakai-Sugimoto model of the Holographic QCD. Baryons are described as collective states of instantons of the five-dimensional Yang-Mills theory. We have then employed the currents as defined in Ref. [16], and computed the matrix elements. As a result we have obtained a model independent relation (46) with a finite value of the axial coupling and hence a finite decay width for the Roper-nucleon decay, although its absolute value is somewhat small as compared to the experimental data.

The present picture of baryons as instantons with collective dynamics is very much the same as the Skyrmion picture, baryons as chiral solitons. In contrast, it is very much different from the conventional quark model one, where baryons are described by single particle states of the constituent quarks. As anticipated, the quark model gave only a tiny decay rate for the Roper resonance when the leading term in $1/m$ expansion of the quark-pion interaction is used, which has been the widely adopted prescription. At this order the suppression occurs due to the selection rule that forbids the transition in the long wavelength limit. For this problem a resolution has been recently proposed by including higher order terms of $1/m^2$ [4]. In the quark model, however, the model independent relation between $g_A^{NN^*}$ and g_A^{NN} is not derived. In this respect, such model independent relations would be helpful to further investigate the nature of nucleon resonances.

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