Coulomb effects in the decays $\Upsilon(4S) \to B\bar{B}$

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A simple exactly solvable model is proposed for describing the decays $\Upsilon(4S) \to B^0\bar{B}^0$ and $\Upsilon(4S) \to B^+B^-$. Our predictions agree with available experimental data. Using this model, we analyze the Coulomb effects in the spectra of these decays. It is shown that the frequently used assumption of factorization of Coulomb effects is not fulfilled. The Coulomb interaction leads to the difference in the positions and heights of the peaks corresponding to the charged and neutral modes. As a result, the ratio of probability of $\Upsilon(4S) \to B^+B^-$ decay and $\Upsilon(4S) \to B^0\bar{B}^0$ decay is a nontrivial function of energy.

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I. INTRODUCTION

The study of resonances of mass M_R, which slightly exceeds the particle-antiparticle pair production threshold $M_{\rm th}$, is a very important task. Such a study makes it possible to investigate in detail the effects of strong interaction in the region where perturbation theory is not applicable. Of particular interest is the case when $M_R - M_{\rm th}$ is of the order of the resonance width Γ_R, since in this case the shape of the resonance curve becomes very nontrivial. In addition, the low relative velocity of a pair of charged particles makes the influence of Coulomb effects very important. In particular, these conditions correspond to the decays of $\Upsilon(4S) \to B^0\bar{B}^0$ and $\Upsilon(4S) \to B^+B^-$.

For resonance $\Upsilon(4S)$, we have $M_R = 10579.4 \pm 1.4$ MeV, $M_R - M_{\rm th} = 20.1 \pm 1.4$ MeV for neutral mesons and 20.7 ± 1.4 MeV for charged mesons, $\Gamma_R = 20.5 \pm 2.5$ MeV, and the sum of the probabilities W_c and W_n of the decays $\Upsilon(4S) \to B^+B^-$ and $\Upsilon(4S) \to B^0\bar{B}^0$, respectively, is almost 100%. Note that the mass difference $M_{B^0} - M_{B^+} \approx 0.3$ MeV $\ll \Gamma_R$, and in the first approximation this difference can be neglected.

Currently, there are a number of experimental works devoted to the decays of the $\Upsilon(4S)$ meson [1–10]. The parameter $t = \pi\alpha/v \approx 0.37$, which determines the magnitude of the Coulomb effects, is not small (here v is the velocity of the B meson, α is the fine-structure constant, $\hbar = c = 1$). Therefore, it would be possible to estimate the magnitude of the Coulomb effects using the Sommerfeld-Gamow-Sakharov factor, $W_c/W_n \sim t/[1 - \exp(-t)] \approx 1.2$. However, the value of this ratio given in the PDG tables [11] is much smaller, $W_c/W_n = 1.058 \pm 0.024$. Various theoretical approaches have been proposed to estimate Coulomb effects in $\Upsilon(4S) \to B\bar{B}$ decays with completely different qualitative predictions of the magnitude of these effects [12–18]. In these works, either the Breit-Wigner approximation, or the Flatté formula, or the description of final-state interaction of mesons in the one-loop approximation have been applied. In addition, to take into account the electromagnetic interaction, the hypothesis was used, according to which the cross section for the production of charged mesons is equal to the cross section for the production of neutral mesons, multiplied by the Sommerfeld-Gamow-Sakharov factor and by the ratio of phase spaces corresponding to decays into charged and neutral mesons. All these approaches have not succeeded to describe experimental data, and at present there is no consensus regarding the nature of the $\Upsilon(4S)$ resonance.

The appearance of experimental data for the resonance shape of $\Upsilon(4S)$ [8,9] allows one to make progress in understanding Coulomb effects. Unfortunately, these experimental data contain information only on the shape of the spectral line for the sum $W_c + W_n$, and the ratio W_c/W_n is measured only at the energy corresponding to the maximum of this sum. However, there is a question whether the positions of peaks in the spectrum of charged and neutral B mesons coincides, and how the Coulomb interaction affects the shape and position of these peaks.

In our paper, we have suggested a simple exactly solvable model that describes the available experimental data and allows us to answer, at least qualitatively, the questions

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mentioned above. Of course, at present it is not known much about the exact interaction Hamiltonian of B mesons. However, for our purposes, this is not a big problem. This is similar to the situation with the phenomenological description of the quarkonium spectra, where completely different analytical representations of the potentials are used, and all of them describe both the quarkonium spectra and the wave functions well enough. Therefore, we expect our predictions to clarify the physics of near-threshold resonances and explain the meaning of observed effects.

II. THEORETICAL APPROACH

Technically, the method for solving the problem under discussion is similar to that developed for calculating the cross section of e^+e^- annihilation into proton-antiproton and neutron-antineutron pairs near the pair production thresholds [19–21]. The main difference of these two problems, in addition to different spins, angular momenta, and a significant mass difference of the proton and neutron, is the high probability of meson production near the nucleon-antinucleon pair production threshold. Therefore, in $\Upsilon(4S) \to B\bar{B}$ decay we can use the usual potential instead of an optical potential, as in the case of nucleon-antinucleon pair production.

In our problem a $B\bar{B}$ pair is produced in a state with an orbital angular momentum $l = 1$. At small distances, a pair of $b\bar{b}$ quarks is produced in a state with zero isospin, which, as a result of hadronization, transforms into a superposition of interacting B^+B^- and $B^0\bar{B}^0$ mesons. Due to the electromagnetic interaction of charged mesons, a state with isospin zero is admixed with a state of isospin one. As a result, the probabilities of charged and neutral meson pair production are different. Of course, the difference between the masses of B^+ and B^0 also leads to the isospin violation, but this difference is very small (~0.3 MeV), and can be ignored in the first approximation. The Coulomb interaction of b quarks at small distances also results in an isospin symmetry violation. However, as already mentioned above, the magnitude of the Coulomb effects is determined by the parameter $t = \pi\alpha/v$, where $v = \sqrt{(M_R - 2m_b)/m_b} \sim 0.7$, and m_b is the b-quark mass. Therefore, $t \sim 0.03 \ll 1$ and the Coulomb interaction of b quarks at small distances is insignificant. Hence, the effect of isospin symmetry violation is mainly related to the Coulomb interaction of mesons in the final state.

Following [21], consider the radial wave function $\Psi^T(r) = (U^{(c)}(r), U^{(n)}(r))$ of a $B\bar{B}$ pair, where $U^{(c)}(r)$ corresponds to a pair of charged B mesons, and $U^{(n)}(r)$ corresponds to a pair of neutral B mesons, T means transpose. It is convenient to pass from the wave function $\Psi^T(r)$ to the wave function $\psi^T(r) = kr\Psi^T(r) = (u^{(c)}(r), u^{(n)}(r))$, where $k = \sqrt{M_B E}$, M_B is the mass of the meson, and E is the energy of the $B\bar{B}$ pair, counted from $M_{th} = 2M_B$. The function $\psi(r)$ satisfies the equation

$$\left[-\frac{1}{M_B}\frac{\partial^2}{\partial r^2} + \frac{2}{M_B r^2} + V(r) + V_{\text{ex}}(r)\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} - \frac{\alpha}{r}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} - E\right]\psi(r) = 0,$$
$$V(r) = \frac{1}{2}[\mathcal{V}_1(r) + \mathcal{V}_0(r)], \qquad V_{\text{ex}}(r) = \frac{1}{2}[\mathcal{V}_1(r) - \mathcal{V}_0(r)], \tag{1}$$

where $\mathcal{V}_1(r)$ and $\mathcal{V}_0(r)$ are the potentials of meson interaction in the states with isospin one and zero, respectively. The potential $V_{\text{ex}}(r)$ leads to the transitions $B^+B^- \leftrightarrow B^0\bar{B}^0$.

It is necessary to find two solutions $\psi_i(r)$ of (1) with asymptotics at large distances

$$\psi_1^T(r) = \frac{1}{2i}(S_{11}\chi_c^+ - \chi_c^-, S_{12}\chi_n^+),$$
$$\psi_2^T(r) = \frac{1}{2i}(S_{21}\chi_c^+, S_{22}\chi_n^+ - \chi_n^-). \tag{2}$$

Here S_{ij} are some functions of energy and

$$\chi_c^\pm = \exp\{\pm i[kr - \pi/2 + \eta_k \ln(2kr) + \sigma_k]\},$$
$$\chi_n^\pm = \exp[\pm i(kr - \pi/2)], \qquad \sigma_k = \frac{i}{2}\ln\frac{\Gamma(2 + i\eta_k)}{\Gamma(2 - i\eta_k)},$$
$$\eta_k = \frac{M_B\alpha}{2k}, \tag{3}$$

where $\Gamma(x)$ is the Euler Γ function.

The probabilities W_c and W_n of decays $\Upsilon(4s) \to B^+B^-$ and $\Upsilon(4s) \to B^0\bar{B}^0$, respectively, are

$$W_c = Nk\left|\frac{\partial}{\partial r}U_1^{(c)}(0) - \frac{\partial}{\partial r}U_1^{(n)}(0)\right|^2,$$
$$W_n = Nk\left|\frac{\partial}{\partial r}U_2^{(c)}(0) - \frac{\partial}{\partial r}U_2^{(n)}(0)\right|^2, \tag{4}$$

where N is some constant. This constant is the same for both channels since it is determined by the physics of small distances, while all other factors appear due to the interaction at large distances. Recall that the superscripts (c) and (n) in the functions $U(r)$ denote the first and second components of the radial wave function $\Psi(r)$, and the subscripts 1 and 2 in these functions correspond to the first and second solutions of the wave equation. As a model potential, we choose $V(r) = -V_0\theta(a - r)$ and $V_{\text{ex}}(r) = g\delta(r - a)$, where $\theta(x)$ is the Heaviside function, $\delta(x)$ is the Dirac δ function, V_0, g, and a are some parameters. Of course, potentials can be written in very different forms. We have chosen the simplest model, which, on the one hand, allows a simple analytical solution, and on the other hand, is sufficient to describe the available experimental data. Since we were interested in the effect of charge-exchange interaction at large distances, we chose the position of delta function at the point $r = a$. Using this potential model, it is easy to obtain an analytical solution, which simplifies the analysis of the influence of Coulomb effects on the probability of pair production. We are confident that, at least qualitatively, our predictions correspond to the actual experimental situation.

For $r < a$, the solutions are regular at the point $r = 0$ and have the form

$$
\begin{aligned}
u_{1,2}^{(c)}(r) &= A_{1,2}\mathcal{F}(y), \qquad u_{1,2}^{(n)}(r) = B_{1,2}f(y), \\
\mathcal{F}(y) &= \frac{C_q}{3}y^2 e^{-iy}F(i\eta_q + 2, 4, 2iy), \qquad f(y) = \frac{\sin y}{y} - \cos y, \\
C_q &= \sqrt{\frac{2\pi\eta_q(1+\eta_q^2)}{1-\exp(-2\pi\eta_q)}}, \quad y = qr, \quad q = \sqrt{M_B(E+V_0)}.
\end{aligned} \tag{5}
$$

Here $F(b, c, z)$ is the confluent hypergeometric function of the first kind, $A_{1,2}$ and $B_{1,2}$ are some coefficients. Note that the index q in the functions C_q and η_q indicates the momentum dependence (and correspondingly the energy dependence) of these functions.

For $r > a$ the solutions are

$$
\begin{aligned}
u_1^{(c)}(r) &= \frac{1}{2i}[S_{11}H^+(k,x) - H^-(k,x)], \qquad u_1^{(n)}(r) = \frac{1}{2i}S_{12}h^+(x), \\
u_2^{(c)} &= \frac{1}{2i}S_{21}H^+(k,x), \qquad u_2^{(n)}(r) = \frac{1}{2i}[S_{22}h^+(x) - h^-(x)], \\
H^+(k,x) &= 4i\exp[ix + i\sigma_k - \pi\eta_k/2]x^2\mathcal{U}(2 - i\eta_k, 4, -2ix), \\
H^-(k,x) &= -4i\exp[-ix - i\sigma_k - \pi\eta_k/2]x^2\mathcal{U}(2 + i\eta_k, 4, 2ix), \\
h^+(x) &= \left(\frac{1}{x} - i\right)e^{ix}, \qquad h^-(x) = \left(\frac{1}{x} + i\right)e^{-ix}, \\
x &= kr, \qquad k = \sqrt{M_B E}.
\end{aligned} \tag{6}
$$

Here $\mathcal{U}(b, c, z)$ is the confluent hypergeometric function of the second kind. The following relations hold:

$$
\mathcal{F}(y) = \frac{1}{2i}[H^+(q,y) - H^-(q,y)], \qquad f(y) = \frac{1}{2i}[h^+(y) - h^-(y)].
$$

Using the continuity of the function $\psi(r)$ at the point $r = a$ and the condition

$$
\psi'(a+0) - \psi'(a-0) = M_B g\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\psi(a),
$$

we find the coefficients A_i and B_i. As a result, we obtain the probabilities W_c and W_n

$$
\begin{aligned}
W_c &= b\frac{kq^2}{M_B^3}\left|\frac{q}{D}\{C_q[kh^{+\prime}(ka)f(qa) - qh^+(ka)f'(qa)] - M_B g h^+(ka)\mathcal{F}(qa)\}\right|^2, \\
W_n &= b\frac{kq^2}{M_B^3}\left|\frac{q}{D}\{[kH^{+\prime}(ka)\mathcal{F}(qa) - qH^+(ka)\mathcal{F}'(qa)] - C_q M_B g H^+(ka)f(qa)\}\right|^2, \\
D &= [kh^{+\prime}(ka)f(qa) - qh^+(ka)f'(qa)][kH^{+\prime}(ka)\mathcal{F}(qa) - qH^+(ka)\mathcal{F}'(qa)] \\
&\quad - M_B^2 g^2 h^+(ka)H^+(ka)f(qa)\mathcal{F}(qa),
\end{aligned} \tag{7}
$$

where b is some constant and $Z'(x) \equiv \partial Z(x)/\partial x$. Recall that the constant b is the same for both channels since it is determined by the physics of small distances. It is the ratio R of the cross section $e^+e^- \to B\bar{B}$ to the Born cross section $e^+e^- \to \mu^+\mu^-$, that is usually presented in the experimental papers. Taking into account slow energy dependence of the cross section of $\mu^+\mu^-$ production in the energy region considered, we choose the constant b to reproduce this ratio R. The expressions (7) are exact within the model under consideration and are very convenient for analyzing various effects.

III. DISCUSSION OF THE RESULTS

It turned out that our predictions are rather sensitive to the value of parameter g. However, comparison with experimental data shows that g is very small. Below we put $g = 0$, so the expressions (7) become much simpler

$$W_c = b\frac{kq^2}{M_B^3}\left|\frac{qC_q}{kH^{+\prime}(ka)\mathcal{F}(qa) - qH^+(ka)\mathcal{F}'(qa)}\right|^2,$$
$$W_n = b\frac{kq^2}{M_B^3}\left|\frac{q}{kh^{+\prime}(ka)f(qa) - qh^+(ka)f'(qa)}\right|^2. \quad (8)$$

Our analysis shows that the observed resonance $\Upsilon(4S)$ is related to a low-energy state in the p wave having the energy $E_R = M_R - 2M_B$ much smaller than the potential V_0. Therefore, the potential V_0 can be chosen in the form

$$V_0 = \frac{(n\pi)^2}{M_B a^2} - \tilde{E}_R, \quad n = 3, 4, 5, 6..., \quad (9)$$

where $\tilde{E}_R \approx 22$ MeV is a parameter close to the value of the resonance energy E_R. It only slightly depends on a and is almost independent of n. It turned out that for any a in the interval 2 fm $\le a \le 2.5$ fm and for $n \ge 3$, the curves for W_c and W_n, described by Eqs. (8) and (9), have similar shapes (up to the general scale b), so that the ratio W_c/W_n conserves. Below we use the values $V_0 = 269$ MeV, $a = 2.5$ fm, $b = 23$, $g = 0$. These values correspond to $n = 5$.

The dependence of W_c and W_n on E (8) is shown in Fig. 1. The solid curve corresponds to W_c and the dotted curve corresponds to W_n. It is seen that there are two peaks with the different positions and heights, the distance between peaks is ~2 MeV. This is a consequence of the Coulomb interaction, since in the absence of this interaction the peaks would coincide (recall that we did not take into account the small mass difference of B^+ and B^0).

Naturally, taking into account the electromagnetic interaction results in a different energy dependence of the decay probabilities in the charged and neutral modes. This is why the positions of the peaks in different modes are also different and cannot be identified as the mass of a single resonance. Within our approach, it is not too difficult to take into account the small mass difference between charged and neutral B mesons. This difference also shifts the positions of the peaks (~0.6 MeV). However, this effect is essentially less significant than that of the Coulomb interaction. Besides, it is not important for the energy dependence of the ratio W_c/W_n. Therefore, for the sake of simplicity in discussing the nature of the $\Upsilon(4S)$ resonance, in this work we do not take into account this mass difference.

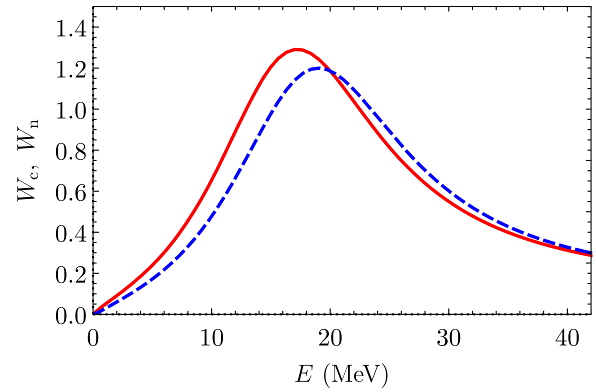

FIG. 1. Energy dependence of W_c (solid curve) and W_n (dotted curve).

The width of each peak is approximately 17 MeV, and the width of $W_{\rm tot} = W_c + W_n$ is about 20 MeV, which is a consequence of the different positions of the peaks W_c and W_n. The energy dependence of $W_{\rm tot}$ is shown in Fig. 2 by a dashed curve, and $W_{\rm tot}$ averaged over the beam-energy spread of PEP-II is shown by a solid curve. The same figure shows experimental data from Ref. [9], which are based on Ref. [8] and take into account radiative corrections and radiative return. Assuming Gaussian distribution with $\Delta = 4.6$ MeV, averaging was carried out according to the formula

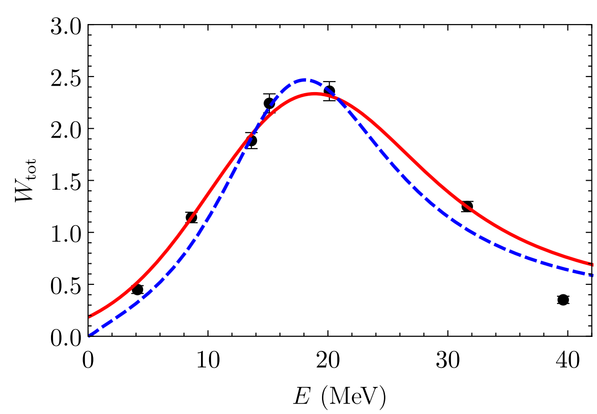

FIG. 2. Energy dependence of the probability $W_{\rm tot} = W_c + W_n$ (dashed curve) and $\langle W_{\rm tot}\rangle$ (solid curve). The dots show the experimental data that take into account the radiative corrections and the radiative return [9].

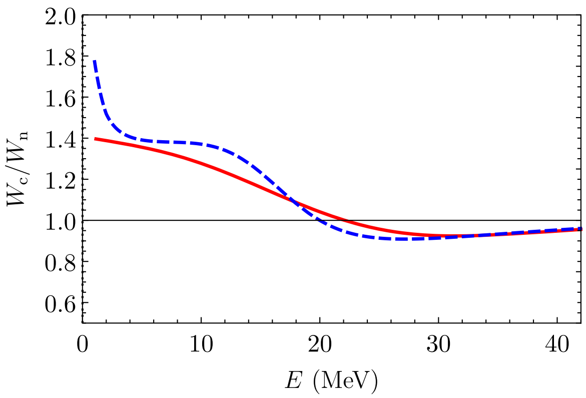

FIG. 3. Energy dependence of the ratio W_c/W_n (dashed curve) and $\langle W_c\rangle/\langle W_n\rangle$ (solid curve).

$$\langle W(E)\rangle = \int_0^\infty W(E')\exp\left[-\frac{(E-E')^2}{2\Delta^2}\right]\frac{dE'}{\sqrt{2\pi}\Delta}. \quad (10)$$

One can see very good agreement between the predictions and the experimental data everywhere, except for the region above 35 MeV, which is most likely due to the close threshold of $B^*\bar{B}$ and $B\bar{B}^*$ pair production.

The energy dependence of the ratio W_c/W_n is shown in Fig. 3. The dotted curve corresponds to Eq. (8) while the solid curve is the ratio of probabilities averaged over the beam-energy spread. It is seen that the ratio W_c/W_n strongly depends on E, and near the maximum of $W_{\rm tot}$ the function $W_c/W_n - 1$ passes through zero. The ratio W_c/W_n increases rapidly at lower energy and can reach 1.4 at $E \sim 10$ MeV.

The assumption that $W_c = C_k^2 W_n$ is very often used to describe Coulomb effects, where C_k is given by Eq. (5) with the replacement $q \to k$. Note that the function C_k^2 is nothing but the well-known Sommerfeld-Gamow-Sakharov factor corresponding to a p wave. The energy dependence of the ratio $W_c/C_k^2 W_n$ is shown in Fig. 4. It is seen that the hypothesis of factorization of the Coulomb effects does not work. Note that $W_n \propto k^3$ and $W_c \to$ const $\neq 0$ at $E \to 0$ so that $W_c/C_k^2 W_n$ tends to a constant, but this constant is not equal to unity. The violation of the factorization of Coulomb effects was first noted in [21] when describing the production of nucleon-antinucleon pairs in e^+e^- annihilation near the threshold.

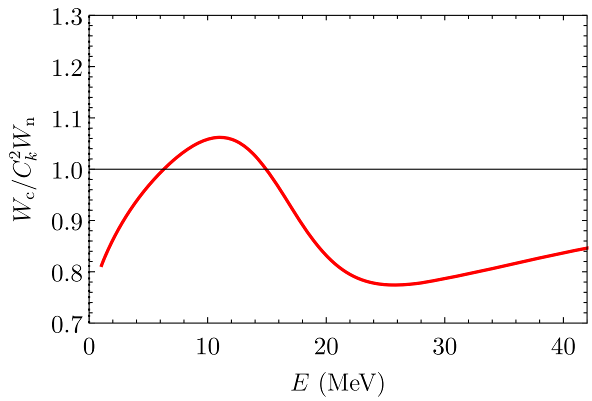

FIG. 4. The energy dependence of $W_c/C_k^2 W_n$.

IV. CONCLUSION

In our work, we have suggested a simple description of the decay probabilities $\Upsilon(4S) \to B^+B^-$ and $\Upsilon(4S) \to B^0\bar{B}^0$. Our results are in good agreement with the available experimental data. We predict the existence of two peaks whose positions and heights differ due to Coulomb effects. Moreover, the ratio W_c/W_n is a nontrivial function of energy, which increases rapidly as the energy decreases with respect to the peak position. It is also shown that the frequently used assumption of factorization of the Coulomb corrections is not in agreement with the exact results. We believe that our approach can also be applied to describe other resonances having the energies close to the thresholds of decay into corresponding mesons. Such resonances include, for example, $X(3872)$, whose nature is still widely discussed. However, in this case the mass difference of charged and neutral mesons is more important than the effect of the Coulomb interaction.

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[1] J. P. Alexander *et al.* (CLEO Collaboration), Phys. Rev. Lett. **86**, 2737 (2001).
[2] S. B. Athar *et al.* (CLEO Collaboration), Phys. Rev. D **66**, 052003 (2002).
[3] B. Aubert *et al.* (*BABAR* Collaboraion), Phys. Rev. D **65**, 032001 (2002).
[4] N. C. Hastings *et al.* (Belle Collaboraion), Phys. Rev. D **67**, 052004 (2003).
[5] B. Aubert *et al.* (*BABAR* Collaboraion), Phys. Rev. D **69**, 071101 (2004).
[6] B. Aubert *et al.* (*BABAR* Collaboraion), Phys. Rev. Lett. **95**, 042001 (2005).

[7] B. Aubert *et al.* (*BABAR* Collaboraion), Phys. Rev. D **72**, 032005 (2005).
[8] B. Aubert *et al.* (*BABAR* Collaboraion), Phys. Rev. Lett. **102**, 012001 (2009).
[9] X.-K. Dong, X.-H. Mo, P. Wang, and C.-Z. Yuan, Chin. Phys. C **44**, 083001 (2020).
[10] R. Mizuk *et al.* (Belle Collaboraion), arXiv:2104.08371.
[11] P. A. Zyla *et al.* (Particle Data Group Collaboration), Prog. Theor. Exp. Phys. (**2020**), 083C01.
[12] D. Atwood and W. J. Marciano, Phys. Rev. D **41**, 1736 (1990).
[13] G. P. Lepage, Phys. Rev. D **42**, 3251 (1990).
[14] N. Byers and E. Eichten, Phys. Rev. D **42**, 3885 (1990).
[15] R. Kaiser, A. V. Manohar, and T. Mehen, Phys. Rev. Lett. **90**, 142001 (2003).
[16] M. B. Voloshin, Mod. Phys. Lett. A **18**, 1783 (2003).
[17] M. B. Voloshin, Yad. Fiz. **68**, 804 (2005), https://elibrary.ru/item.asp?id=9157608 [Phys. At. Nucl. **68**, 771 (2005)].
[18] W. H. Liang, N. Ikeno, and E. Oset, Phys. Lett. B **803**, 135340 (2020).
[19] V. F. Dmitriev, A. I. Milstein, and S. G. Salnikov, Yad. Fiz. **77**, 1234 (2014) [Phys. At. Nucl. **77**, 1173 (2014)].
[20] V. F. Dmitriev, A. I. Milstein, and S. G. Salnikov, Phys. Rev. D **93**, 034033 (2016).
[21] A. I. Milstein and S. G. Salnikov, Nucl. Phys. **A977**, 60 (2018).