Heat release in accreting neutron stars

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Observed thermal emission from accreting neutron stars (NSs) in a quiescent state is believed to be powered by nonequilibrium nuclear reactions that heat the stellar crust (deep crustal heating paradigm). We derive a simple universal formula for the heating efficiency, assuming that an NS has a fully accreted crust. We further show that, within the recently proposed thermodynamically consistent approach to the accreted crust, the heat release can be parametrized by only one parameter—the pressure P_{oi} at the outer-inner crust interface (as we argue, this pressure should not necessarily coincide with the neutron-drip pressure). We discuss possible values of P_{oi} for a selection of nuclear models that account for shell effects, and we determine the net heat release and its distribution in the crust as a function of $P_{\text{o}i}$. We conclude that the heat release should be reduced by a factor of few in comparison to previous works.

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The crust of an accreting neutron star (NS) is driven out of thermodynamic equilibrium by the accretion process from the NS companion. Its composition is governed by exothermal nuclear reactions, which act to return it to the equilibrium. It is generally believed that the heat released in these reactions is responsible for the observed thermal luminosity of transiently accreting NSs [\[1](#page-4-0)–6] (deep crustal heating paradigm [\[7\]](#page-4-1)). Observations of such stars could shed light on the properties of superdense matter in their interiors [7–[12\].](#page-4-1)

Physics of accreted crust (AC) has been studied for about forty years since the pioneering works by Sato [\[13\]](#page-4-2) and Haensel and Zdunik [\[14,15\].](#page-4-3) During this time a number of AC models were developed. They are based on either detailed [\[16\]](#page-4-4) or simplified [\[17,18\]](#page-4-5) reaction networks, theoretical atomic mass tables, the liquid-drop approach [\[19,20\]](#page-4-6), or the density-functional theory [\[21\].](#page-4-7) The key question researchers want to answer: What is the AC composition and heat released in the crust per accreted nucleon? This question is usually addressed within the traditional approach, in which one follows the compositional changes in the given accreted fluid element as it is compressed under the weight of newly accreted material. In this approach all constituents (nuclei, electrons, unbound neutrons) in the fluid element move together, with one and the same velocity. Thus, the traditional approach completely ignores the possibility that unbound neutrons, presented in the inner crust, can redistribute themselves independently of nuclei in order to reduce the system energy. In [\[22\]](#page-4-8) we reveal crucial importance of this effect and show that neutrons in the inner crust must be in a hydrostatic/diffusion equilibrium (nHD) state, in which $\mu_n^{\infty} = \mu_n e^{\nu/2}$ = const, where μ_n is the neutron chemical potential and $e^{v/2}$ is the redshift factor. Equation of state (EOS) that respects the condition $\mu_n^{\infty} = \text{const}$ (hereafter nHD condition), turns out to be rather close to the catalyzed EOS of nonaccreting (ground state) inner crust, and is very different from the traditional AC EOSs [\[22\].](#page-4-8)

In this paper we further explore the consequences of nHD equilibrium. First, we present a universal formula [\(3\)](#page-1-0) showing that the deep crustal heat release Q^{∞} , as seen by a distant observer, for a fully accreted crust (FAC) is determined by EOS, but not by the details of nuclear transformations proceeding in the crust. Second, we calculate the net heat release and its distribution over the nHD crust.

II. GENERAL ENERGETICS II. GENERAL ENERGETICS

Consider an NS with AC and the mass M , and an NS with catalyzed crust, the mass M_{cat} , and the same total number of baryons, A_b (the cores of both NSs are assumed to be in the ground state). The energy excess stored in the NS with AC is $E^{ex} = M - M_{cat}$. This energy will be released if one waits sufficiently long for the AC to relax to catalyzed crust by means of nonequilibrium nuclear processes.

To calculate the increase in E^{ex} associated with the accretion process, let us add δA_b baryons (in the form of, e.g., H or He) with the average mass per baryon \bar{m}_b . This will increase M by an amount $\delta M = \bar{m}_b \delta A_b e^{\nu_s/2}$ [\[23,24\]](#page-4-9), where $e^{\nu_s/2}$ is the redshift factor at the NS surface. In turn, M_{cat} will increase, after adding the same δA_b baryons, by $\delta M_{\text{cat}} = \delta A_b \mu_{b,\text{cat}}^{\infty}$ [\[23,24\]](#page-4-9), where $\mu_{b,\text{cat}}^{\infty}$ is the redshifted baryon chemical potential in the catalyzed NS, which is constant throughout the star [\[22,25,26\].](#page-4-8) As a result,

$$
\delta E^{\text{ex}} = (\bar{m}_b e^{\nu_s/2} - \mu_{b, \text{cat}}^{\infty}) \delta A_b.
$$
 (1)

Generally, to determine what fraction of δE^{ex} goes into heat and what is stored in the (nonequilibrium) crust, one should study the kinetics of nuclear reactions there. However, in an important case of an NS with FAC a general formula for the heat release can be derived.

II. UNIVERSAL HEATING FORMULA FOR FACTO

In the process of accretion an NS eventually reaches the regime, in which crust EOS does not further change noticeably in time. We call such crust "fully accreted." Subsequent accretion of material onto the surface of FAC initiate nuclear reactions, which maintain the crust composition (see, e.g., [\[22\]](#page-4-8)). The heat release associated with these reactions can be calculated in analogy to the deriva-tion of Eq. [\(1\)](#page-0-0). Let us accrete δA_b baryons onto the surface of an NS with FAC. On the one hand, the energy of such a star will change by $\bar{m}_b \delta A_b e^{\nu_s/2}$; on the other hand, the star energy will vary by $\partial M/\partial A_b \partial A_b$, where the partial derivative is taken, by assumption, at fixed EOS of FAC. These two energies are not equal to one another; they differ by an amount of heat generated in the crust by nonequilibrium nuclear reactions, caused by accretion of δA_h baryons. Correspondingly, the total heat release per accreted baryon, redshifted to a distant observer, is given by

$$
Q_{\text{tot}}^{\infty} = \bar{m}_b e^{\nu_s/2} - \partial M / \partial A_b.
$$
 (2)

As shown in the Supplemental Material [\[27\]](#page-4-10) $\partial M/\partial A_b$ can be presented as $\partial M/\partial A_b = \mu_{b,\text{core}}^{\infty} + \mathcal{O}(Q_{\text{tot}}^{\infty}M_c/M)$, where $\mu_{b,\text{core}}^{\infty}$ is the redshifted baryon chemical potential in the NS core; $O(Q_{\text{tot}}^{\infty}M_c/M)$ is a small correction of the order of $Q_{\text{tot}}^{\infty}M_c/M$; and M_c is the crust mass.

In the upper layers of NSs (up to the density $\leq 10^9$ g cm⁻³) the accreted material fuse into heavy nuclei (ashes with average mass per baryon $\bar{m}_{b, \text{ash}}$) [\[6\]](#page-4-11). The respective energy, $Q_{\text{ash}}^{\infty} = (\bar{m}_b - \bar{m}_{b,\text{ash}})e^{\nu_s/2}$, is emitted from the surface without significant heating of the crust. The remaining part $Q^{\infty} \equiv Q^{\infty}_{tot} - Q^{\infty}_{ash}$ is released in the deep AC layers

$$
Q^{\infty} = \bar{m}_{b,\text{ash}} e^{\nu_s/2} - \partial M / \partial A_b
$$

= $\bar{m}_{b,\text{ash}} e^{\nu_s/2} - \mu_{b,\text{core}}^{\infty} + \mathcal{O}(Q_{\text{tot}}^{\infty} M_c / M).$ (3)

It can be interpreted as an excess of gravitational energy, which ashes have at the outer layers of the accreted crust plus some (typically small) excess of nuclear energy of the ashes with respect to the ground state composition (^{56}Fe) . Below, we neglect small corrections $\sim Q_{\text{tot}}^{\infty} M_c / M \ll$ 1 MeV in Eq. [\(3\),](#page-1-0) but, in principle, they can be calculated. Neglecting small energy carried away from the star in the form of neutrinos [\[18,20,21,28\]](#page-4-12), Q^{∞} represents the heat deposited in the crust.

Equation (3) is very general and can be applied to *any* FAC model. In the Supplemental Material [\[27\]](#page-4-10) we show

that it reproduces, in particular, the results of a traditional one-component model [\[14,15,19](#page-4-3)–21,29]. Equation [\(3\)](#page-1-0) can be further simplified for the nHD crust [see Eq. [\(4\)\]](#page-1-1), whose properties are briefly considered below.

IV. THE NHD CRUST: BASIC FEATURES

The thermodynamically consistent model of the inner crust should respect the nHD condition [\[22\]](#page-4-8). To account for this condition, one should substantially modify the traditional approach by self-consistently analyzing nuclear processes in the whole inner crust, allowing for redistribution of unbound neutrons over different crust layers and the core. Let us summarize the main properties of the nHD crust [\[22\]](#page-4-8), which will be used in what follows.

The outer-inner crust interface (oi interface) plays an important role. Above it unbound neutrons are absent and cannot travel between different layers; hence the traditional approach there is justified. Below the oi interface unbound neutrons must redistribute in order to meet the nHD condition. At first glance the position of this interface should coincide with the point where neutrons drip out of nuclei in the traditional AC model. However, as shown in [\[22\]](#page-4-8), it is not the case: unbound neutrons from the underlying layers can spread above this point if it is energetically favorable. Therefore, the position of the oi interface (parametrized by the pressure P_{oi}) should be considered as an additional parameter of the nHD crust model. In particular, P_{oi} should vary over time until an NS reaches the FAC state. In this state P_{oi} is fixed by the requirement that the total number of nuclei in the inner crust is almost constant during the accretion process (otherwise EOS should evolve in time, which contradicts the FAC definition).

The process that keeps constant the number of nuclei in the crust has been identified in [\[22\]](#page-4-8); it is related to a specific instability, which disintegrates nuclei in the inner crust at the same rate as they are provided by accretion onto the NS surface.

V. THERMODYNAMICALLY CONSISTENT
MODEL OF THE INNER CRUST: **HEAT RELEASE** HEAT RELEASE

According to the nHD condition $\mu_n^{\infty} = \text{const}$ in the inner crust and core. On the other hand, in the core $\mu_n = \mu_b$, while at the oi interface (from the inner crust side) $\mu_n = m_n$. In view of these facts, one has $\mu_{b,\text{core}}^{\infty} = m_n e^{\nu_{oi}/2}$, where $e^{\nu_{\alpha}/2}$ is the redshift at the oi interface. Now, expressing $\nu_s - \nu_{oi}$ using one of the Tolman-Oppenheimer-Volkoff equations [\[24\],](#page-4-13) Eq. [\(3\)](#page-1-0) can be rewritten as

$$
Q^{\infty} = e^{\nu_{\text{o}i}/2} \left[\bar{m}_{b,\text{ash}} \exp\left(\int_0^{P_{\text{o}i}} \frac{dP}{P + \epsilon} \right) - m_n \right], \quad (4)
$$

where $\epsilon = \epsilon(P)$ is the energy density and P is the pressure. In contrast to Eq. (3) this formula applies only to the nHD crust. It says that the heat release Q^{∞} , parametrized by the pressure P_{oi} , can easily be found provided that EOS in the outer crust (at $P < P_{oi}$) is known. Note that the outer crust can be modeled within the traditional approach, so that EOS there is relatively well established [\[16,20,21,30\]](#page-4-4). Another form of the expression for Q^{∞} is discussed in the Supplemental Material [\[27\]](#page-4-10), where we also present its independent microscopic derivation valid for the smoothed compressible liquid drop (CLD) model [\[22\]](#page-4-8).

To illustrate usefulness of the formula [\(4\),](#page-1-1) we calculate Q^{∞} for several nuclear models (BSk24, BSk25, BSk26: Hartree-Fock-Bogoliubov calculations [\[31\];](#page-5-0) FRDM12: finite-range droplet macroscopic model [\[32\]](#page-5-1)). The respective heat release Q, defined as $Q = Q^{\infty}e^{-\nu_{\text{o}i}/2}$, is shown by solid lines in Fig. [1](#page-2-0) as a function of P_{oi} (for simplicity, following [\[14,20,21\],](#page-4-3) pure $A = 56$ composition of the ashes is assumed). For the region of P_{oi} depicted in the figure, $Q \sim (0.2-1)$ MeV/nucleon, being almost a linear function of $P_{\text{o}i}$. It is by a factor of few smaller than the heat release \sim (1.5–2) MeV/nucleon, found in the traditional approach [\[16,20,21,33\].](#page-4-4) Note that the shell effects increase the heat release (for example, for the smoothed CLD model of [\[22\]](#page-4-8), which ignores them, we obtain $Q \approx 0.11$ MeV/nucleon; see Supplemental Material [\[27\]](#page-4-10)). A similar feature was pointed out for the traditional AC model in [\[21\]](#page-4-7).

To analyze the heat release distribution in the FAC we do the following. First, the net heat release in the outer crust (Q_o) and its distribution can easily be found in the traditional approach (see dots in Fig. [1](#page-2-0)). In turn, the heat release at the oi interface (Q_{oi}) is associated with exothermic neutron absorptions and electron emissions by nuclei crossing the oi interface. Neutrons, necessary for such absorptions are supplied by continuous upward neutron

FIG. 1. The heat release Q, Q_o, Q_{oi} , and Q_i vs P_{oi} for several nuclear models. BSk24, BSk25, BSk26: Hartree-Fock-Bogoliubov calculations [\[31\]](#page-5-0); FRDM12: finite-range droplet macroscopic model [\[32\]](#page-5-1). Arrows indicate the neutron drip pressure for catalyzed crust, $P_{\text{nd}}^{(\text{cat})}$. For each model, the pressure region above the neutron drip point for AC (calculated within the traditional approach) is shaded gray.

flow in the inner crust. The origin of the flow is the disintegration instability mentioned above. Neutrons, released in the course of this instability, redistribute in the inner crust and core in order to maintain the nHD equilibrium (for details see [\[22\]\)](#page-4-8). To find Q_{oi} , we slightly modified the reaction network of [\[18\]](#page-4-12) by allowing for absorptions of an arbitrary number of neutrons at the oi interface by incoming nuclei [in the spirit of the extended Thomas-Fermi plus Strutinsky integral (ETFSI) calcula-tions of [\[21\]](#page-4-7)]. The resulting Q_{oi} is shown in Fig. [1](#page-2-0) by long dashes. The remaining heat, $Q_i = Q - Q_o - Q_{oi}$, is released in the inner crust and is shown by dot-dashed lines.

VI. ^Poi DETERMINATION AND HEAT RELEASE DISTRIBUTION IN THE INNER CRUST

We demonstrate that Q_0 , Q_i , and Q_{oi} are fully determined by the pressure $P_{\text{o}i}$, if EOS in the outer crust is known. But how can we determine P_{oi} ? In Ref. [\[22\]](#page-4-8) we found $P_{\text{o}i}$ for the smoothed CLD model based on the SLy4 nuclear energy-density functional [\[34\]](#page-5-2). Here, to check the sensitivity of $P_{\text{o}i}$ to the shell effects, we determined it for the recently developed model [\[35\].](#page-5-3) The shell effects in this model are incorporated by adding tabulated shell energy corrections from [\[36,37\]](#page-5-4) on top of CLD energy density $(CLD + sh \text{ model in what follows})$. The resulting $CLD +$ sh model reproduces well ETFSI calculations for four modern energy-density functionals: BSk22, BSk24, BSk25, and BSk26. As in the most advanced calculations to date performed in the traditional approach [\[21\],](#page-4-7) we assume that the inner crust consists of nuclei of one particular species at each given density. A detailed discussion of our results based on the $CLD + sh$ model [\[35\]](#page-5-3) is presented in [\[38\].](#page-5-5) Here we describe our basic findings, obtained for pure $A = 56$ composition of the ashes and one of the CLD + sh models of [\[35\],](#page-5-3) corresponding to BSk24 functional. The results for other functionals are similar.

(i) As in the CLD model of $[22]$, for CLD + sh model [\[35\]](#page-5-3) there exists a pressure $P_{oi}^{(min)}$ such that for any $P_{oi} \geq$ $P_{\text{o}i}^{(\text{min})}$ the construction of the AC model is limited by the instability disintegrating nuclei into neutrons. The pressure $P_{\text{inst}}(P_{\text{o}i})$, at which this instability takes place, is a decreasing function of P_{oi} (see Supplemental Material [\[27\]](#page-4-10)). Thus, $P_{\text{inst}}(P_{\text{oi}}^{(\text{min})})$ is a maximum possible value of P_{inst} . Generally, disintegration of nuclei is accompanied by the energy release. It is interesting to note that for the $CLD + sh$ model [\[35\]](#page-5-3), the disintegration instability (for any $P_{\rm oil}$) occurs at $P_{\rm inst}$ smaller than the pressure at the crustcore boundary (where both P and μ_n must be matched). A part of the crust at $P > P_{\text{inst}}$ appears to be decoupled from the rest of the crust: the atomic nuclei in this "relic" part are not replaced during the accretion in the FAC regime (because all the upcoming nuclei disintegrate at $P = P_{\text{inst}}$). The relic part of the crust is formed during the transformation from the pristine catalyzed crust to FAC.

(ii) The shell effects complicate determination of P_{oi} in FAC. This happens because the relic part of the crust can be (at least, in principle) stabilized by the shell effects for a range of P_{oi} . The reason for that is largely unknown composition of nuclei in the relic region, which depends on the (highly uncertain) evolution preceding the FAC formation. Thus, unambiguous determination of $P_{\text{o}i}$ remains a task for the future. However, we numerically found that for $A = 56$ ashes and CLD + sh model [\[35\],](#page-5-3) $P_{\text{o}i}$ does not exceed $P_{\text{nd}}^{(\text{cat})}$: for higher P_{oi} the pressure P_{inst} becomes so low that FAC at $P < P_{\text{inst}}$ and NS core cannot be connected in a thermodynamically consistent way for any composition of the relic part of the crust [\[38\].](#page-5-5) The latter result is obtained assuming that shell corrections can be ignored for baryon densities larger than the proton drip density, which is chosen to be 0.073 fm⁻³, the same as in [\[36\]](#page-5-4).

Obviously, the lower bound on P_{oi} equals the minimal value of $P_{\text{o}i}$, at which the instability takes place, i.e., $P_{oi} = P_{oi}^{(min)}$. According to our calculations, for the $CLD + sh$ model considered here, we have $P_{\text{oi}}^{(\text{min})} \approx 0.91 P_{\text{nd}}^{(\text{cat})}, P_{\text{inst}}(P_{\text{oi}}^{(\text{min})}) \approx 0.267 \text{ MeV fm}^{-3}, \text{ and}$ $Q_i(P_{oi}^{(min)}) \approx 0.06 \text{ MeV/nucleon. Independently, } P_{oi}$ can be bounded from below by the condition $Q_i > 0$ (see Fig. [1](#page-2-0)): otherwise disintegration of nuclei is not energetically favorable and cannot proceed. The P_{oi} value corresponding to $Q_i = 0$ is denoted as $P_{oi}^{(0)}$. Clearly, it must be $P_{oi}^{(0)} \leq P_{oi}^{(min)}$. For the CLD + sh model $P_{oi}^{(0)} \approx 0.89 P_{nd}^{(cat)}$ and is close to $P_{oi}^{(min)}$, so that below we consider $P_{oi}^{(0)}$ as a universal lower bound on P_{oi} .

(iii) Assuming pure $A = 56$ composition of the ashes, the charge number of nuclei at the bottom of the outer crust is $Z = 20$. The shell effects stabilize Z at the value $Z = 20$ in almost the whole region $P \leq P_{\text{inst}}(P_{\text{oi}})$. This result is related to the local energy minimum at $Z = 20$ (which is the proton "magic number" in the inner crust [\[36\]](#page-5-4)) and does not depend on the choice of $P_{\text{o}i}$. Constancy of Z in the inner crust implies that almost all heat Q_i is released at the instability point $P = P_{\text{inst}}(P_{\text{oi}})$. (Let us note that the heat release in the inner crust cannot be associated with the change of the mass number A, which is treated as a continuous variable due to the presence of unbound neutrons [\[21,35,37\].](#page-4-7))

Table [I](#page-3-0) represents the heat release distribution in FAC for two values of P_{oi} : the lower bound $P_{oi}^{(0)} (\approx P_{oi}^{(min)})$, at which $Q_i = 0$ and for $P_{oi} = P_{nd}^{(cat)}$, which bounds P_{oi} from above. As in the case of Fig. [1,](#page-2-0) Table [I](#page-3-0) was calculated using the mass tables [\[31,32\]](#page-5-0), rather than the simplified $CLD + sh$ model based on ETFSI calculations. These two approaches lead to a bit different predictions (in particular, $P_{\text{nd}}^{(\text{cat})}$ differs by a few percent [\[36\]\)](#page-5-4), which explains why $P_{oi}^{(0)} =$ $0.92P_{\text{nd}}^{(\text{cat})}$ for BSk24 model in Table [I](#page-3-0) appears to be larger

TABLE I. Heat release distribution for the limiting values of $P_{\text{o}i}$. Q-values are in MeV/nucleon.

Model	$P_{\rm o}$	$P_{\rm oi}/P_{\rm nd}^{\rm (cat)}$	\mathcal{Q}_{o}	$\varrho_{\scriptscriptstyle{\text{oi}}}$	Q_i	Q
FRDM12	$P_{\rm o i}^{(0)}$	0.85	0.18	0.03	0.00	0.21
	$P_{\text{nd}}^{\text{(cat)}}$	1.00	0.18	0.03	0.32	0.53
BSk24	$P_{\rm o i}^{(0)}$	0.92	0.12	0.17	0.00	0.29
	$P_{\text{nd}}^{\text{(cat)}}$	1.00	0.12	0.15	0.19	0.46
BSk25	$P_{\rm o i}^{(0)}$	0.93	0.17	0.19	0.00	0.36
	$P_{\text{nd}}^{\text{(cat)}}$	1.00	0.17	0.17	0.17	0.51
BSk26	$P_{\rm o i}^{(0)}$	0.96	0.14	0.25	0.00	0.39
	$P^{(cat)}$ nd	1.00	0.14	0.13	0.20	0.47

than $P_{oi}^{(min)} = 0.91 P_{nd}^{(cat)}$, calculated employing CLD + sh model.

As follows from Fig. [1](#page-2-0), the minimal heat release Q occurs if P_{oi} equals the minimal value, at which disintegration instability takes place, $P_{oi} = P_{oi}^{(min)}$. It is interesting to point out that exactly this P_{oi} should be realized in FAC, if the Prigogine minimum entropy production theorem [\[39\]](#page-5-6) works for our problem.

VII. SUMMARY AND CONCLUSIONS

We present a universal formula [\(3\)](#page-1-0) for the heat release Q^{∞} by nonequilibrium nuclear reactions in the fully accreted NS crust. The formula is applicable to arbitrary composition of nuclear ashes and crust model. We further analyze the heat release in the outer crust Q_0 , in the inner crust Q_i , and at the oi interface, Q_{oi} , for the thermodynamically consistent FAC model respecting the nHD condition. We show that these quantities are parametrized by the pressure P_{oi} at the oi interface, provided that the nuclear mass model in the outer crust is specified [see Fig. [1](#page-2-0) and Eq. [\(4\)\]](#page-1-1). To calculate P_{oi} for FAC and determine the heat release distribution Q_i in the inner crust, we apply $CLD + sh$ model of [\[35\]](#page-5-3) with shell corrections. We demonstrate that for the ashes composed of ⁵⁶Fe, almost all heat Q_i is released at the instability point, where nuclei disintegrate into neutrons (at $P = P_{\text{inst}}$). This result does *not* depend on the actual value of P_{oi} . The charge number is fixed at the value $Z = 20$ in almost the whole region between the oi interface and the instability point.

We also argue that accounting for shell effects complicates the unambiguous determination of $P_{\text{o}i}$, which then depends on the way the FAC is formed. Our analysis indicates that P_{oi} for the employed CLD + sh model [\[35\]](#page-5-3) and ⁵⁶Fe ashes is bounded from below by $P_{oi}^{(min)}$ —the minimal possible value of $P_{\text{o}i}$, for which the disintegration instability can occur in the inner crust [\[22\].](#page-4-8) In turn, we also numerically found that P_{oi} does not exceed $P_{nd}^{(cat)}$ —the

neutron drip pressure in the catalyzed crust (see [\[38\]](#page-5-5) for more details). We emphasize that this upper bound on P_{oi} is model-dependent, being sensitive to the density profiles of the shell energies, which are still poorly known at large baryon densities [\[36,40\].](#page-5-4)

As a conservative estimate of P_{oi} in the FAC regime, we propose to take $P_{oi}^{(0)} \leq P_{oi} < P_{nd}^{(cat)}$, where $P_{oi}^{(0)}$ corresponds to $Q_i = 0$, and is just a bit smaller than $P_{oi}^{(min)}$ according to our calculations. The respective heat release distribution is shown in Table [I](#page-3-0) (see also Fig. [1](#page-2-0)). In particular, the deep crustal heating energy release $Q^{\infty} \sim (0.21 - 0.53)e^{\nu_{0i}/2}$ MeV/nucleon appears to be at least several times smaller than in the traditional approach, $Q^{\infty} \sim (1.5-2)e^{\nu_{0i}/2}$ MeV/nucleon. This fact calls for the reinterpretation of the existing observational data on thermal properties of transiently accreting NSs and should stimulate further work on developing realistic accreted crust models.

Concluding, we stress that the information provided in this paper is, in principle, sufficient to start modeling the thermal relaxation of x-ray transients, as well as their quiescent temperatures within the nHD approach. Such a modeling may help further constrain the pressure $P_{\text{o}i}$.

ACCEPT CHEMICS

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