Hubble sinks in the low-redshift swampland

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Local determinations of the Hubble constant H_0 favor a higher value than Planck based on cosmic microwave background and Λ cold dark matter (ΛCDM). Through a model-independent expansion, we show that low redshift ($z \lesssim 0.7$) data comprising baryon acoustic oscillations, cosmic chronometers, and Type Ia supernovae have a preference for quintessence models that lower H_0 relative to ΛCDM. In addition, we confirm that an exponential coupling to dark matter cannot alter this conclusion in the same redshift range. Our results leave open the possibility that a coupling in the matter-dominated epoch, potentially even in the dark ages, may yet save H_0 from sinking in the string theory swampland.

I. INTRODUCTION

Last century, Allan Sandage framed cosmology as the search for two numbers [1]. Fifty years later, it is widely accepted that the deceleration parameter q_0 is negative [2,3], and the Hubble constant H_0 is known to within 10% [4–8], making it one of the best measured quantities in late-time cosmology. At the lower end of this H_0 window, one finds the Planck determination based on the cosmological model ΛCDM and the cosmic microwave background (CMB) [4]. In contradistinction, a host of local determinations favor higher values [5–8], resulting in Hubble tension [9], an intriguing discrepancy between early and late Universe determinations of H_0.

The convergence of different experiments suggests that a high local H_0 is legit (to lose one experiment, may be regarded as a misfortune; to lose all looks like carelessness). One logical possibility, advocated by the swampland program within string theory [10] is that we replace Λ with quintessence [11,12], the simplest alternative in effective field theory (EFT). Against this backdrop, our goal is to ascertain if H_0 is raised or lowered relative to ΛCDM by the swampland.

To date the swampland [13] has led to an intriguing web of conjectures (see [14,15] for reviews), which remarkably impinge on the real world. The most consequential precludes de Sitter vacua [16] and is in conflict with ΛCDM [10]. Subsequent to-and-fro discussion led to a refinement [17,18] (also [19]), thus allowing the conjecture to coexist with the Higgs's potential [20]. Nevertheless, the de Sitter conjecture is constrained by cosmological data [21–23], and it was highlighted early on that Hubble tension may be an issue [24].

Here, we return to the Hubble tension thread within the context of generic quintessence models [11,12]. Recall that "quintessence" may be defined as a canonically coupled scalar with scalar potential $V > 0$ and an equation of state (EOS) for dark energy that is bounded below $w \geq -1$. However, the latter can be relaxed within EFT by coupling dark matter to dark energy through "coupled quintessence" [25–27]. While it is known that a coupling allows one to effectively reduce w and hence to raise H_0 [28,29], in contrast, specific uncoupled quintessence models lower H_0 [30–33] (see also Table I [34]) [35], thus implying that H_0 is in the swampland. In this paper we show that this is more generally true.

In contrast to models with fixed potentials, e.g., [38–42], here we exploit perturbation at low redshift to work in a model-independent way. In short, we construct a large class of "bottom-up" potentials, which we confront with direct measurements of the Hubble parameter $H(z)$ inferred from cosmic chronometers [43–49] and baryon acoustic oscillations (BAOs) [50,51]. In addition, we include low-z BAO measurements by 6dF [52] ($z = 0.106$) and SDSS-MGS surveys [53] ($z = 0.15$), and determinations of $E(z) \equiv H(z)/H_0$ from type Ia supernovae [54] (derived from [55]).

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Finally, we allow for an exponential potential, which is the dark-matter-quintessence coupling of choice in string theory.

Our findings are simply expressed. Quintessence models prefer a lower H_0 value than ΛCDM. Moreover, neither a H_0 prior [5] nor an exponential coupling to dark matter— provided it is constrained by galaxy warps [56,57]—can change the conclusion in the redshift range. Models where the coupling is turned on earlier are still viable, but only at higher redshifts, namely within the matter-dominated era $(0.4 \lesssim z)$. Furthermore, if the coupling is turned on in the dark ages beyond $z \approx 6$ [34], a proper test of such a scenario may rest upon future developments in 21-cm cosmology [58,59]. Nevertheless, this loophole aside, low-redshift observations place the de Sitter conjecture [16] at odds with local determinations of H_0, since the class of models championed by the conjecture only exacerbates Hubble tension.

II. SETUP

We consider flat Friedmann-Lemaitre-Robertson-Walker spacetime and the following coupled quintessence equations:

$$H^2 = \frac{1}{3}\left(V + \frac{1}{2}\dot{\phi}^2 + f\frac{\rho_c}{a^3} + \frac{\rho_b}{a^3}\right), \quad (1)$$

$$0 = \ddot{\phi} + 3H\dot{\phi} + \partial_\phi V + \frac{\rho_c}{a^3}\partial_\phi f, \quad (2)$$

where dots denote time derivatives, a is the scale factor, and $H \equiv \frac{\dot{a}}{a}$ is the Hubble parameter. In addition, $f(\phi)$ is a coupling between the dark matter density ρ_c and the quintessence field ϕ, while ρ_b denotes the baryonic matter density. Observe that setting $f = 1$, we recover uncoupled quintessence with matter density $\rho_m = \rho_c + \rho_b$. These equations allow an effective EOS $w_{\text{eff}} < -1$ when $f < 1$ [27]. Concretely, we consider $f = e^{-C(\phi-\phi_0)}$ where ϕ_0 and $C \geq 0$ are constant. Finally, observe that we have set $M_p = 1$ for simplicity.

We emphasize that the basic ingredients of equations (1) and (2), i.e., a canonically coupled scalar, a potential, and an exponential coupling are ubiquitous in low-energy effective descriptions of string theory, e.g., [60–66], thus, the basic building blocks of the swampland are in place. Now, to solve these equations, we recall the usual definition of the scale factor in terms of redshift $a = (1 + z)^{-1}$, normalized so that $a = 1$ today. Using the chain rule, one establishes that $d/dt = -(1 + z)Hd/dz$, and it is easy to recast the Friedmann (1) and scalar equation (2) in terms of redshift. We consider the following expansion for the scalar,

$$\phi - \phi_0 = \alpha z + \beta z^2 + \gamma z^3 + \ldots, \quad (3)$$

around its value today ϕ_0 at $z = 0$, where α, β, and γ are constants. At small z, $\phi - \phi_0$ is small, and we can further expand the potential:

$$V = V_0 + V_0'(\phi - \phi_0) + \frac{1}{2}V_0''(\phi - \phi_0)^2 + \ldots, \quad (4)$$

where we have defined $V_0 \equiv V(\phi_0)$, $V_0' \equiv V'(\phi_0)$, etc. The Hubble parameter to third order in redshift is

$$H = H_0(1 + h_1 z + h_2 z^2 + h_3 z^3 + \ldots), \quad (5)$$

where h_i may be expressed in terms of the parameters as

$$h_1 = \frac{1}{2}\alpha^2 + \frac{3}{2}\Omega_{m0},$$

$$h_2 = \frac{1}{8}\alpha^4 + \frac{1}{4}\alpha^2 + \alpha\beta - \frac{3}{4}\alpha C\Omega_{c0} + \frac{3}{8}\Omega_{m0}(4 - 3\Omega_{m0}),$$

$$h_3 = \frac{1}{48}\alpha^6 + \frac{1}{16}\alpha^4(\Omega_{m0} + 2) + \alpha\gamma$$

$$+ \frac{1}{16}\alpha^2\Omega_{m0}(9\Omega_{m0} - 2) + \frac{1}{2}\alpha\beta\left(\alpha^2 + \Omega_{m0} + \frac{4}{3}\right)$$

$$+ \frac{2}{3}\beta^2 + \frac{1}{16}\Omega_{m0}(8 - 36\Omega_{m0} + 27\Omega_{m0}^2)$$

$$+ \left(\frac{1}{8}\alpha^3 + \frac{9}{8}\alpha\Omega_{m0} - \alpha - \frac{1}{2}\beta\right)\Omega_{c0}C + \frac{\alpha^2}{4}C^2\Omega_{c0}. \quad (6)$$

Evidently, when $\alpha = \beta = \gamma = 0$, we recover the ΛCDM cosmology, while at $z = 0$, we have $H(z = 0) = H_0$. Observe that we have employed the usual definitions $\Omega_{c0} = \rho_c/(3H_0^2)$, $\Omega_{b0} = \rho_b/(3H_0^2)$, $\Omega_{m0} = \Omega_{b0} + \Omega_{c0}$, to define the baryonic Ω_{b0} and dark matter density Ω_{c0} today.

Note that $\alpha = \beta = \gamma = 0$ is our reference model. Since we are working perturbatively around $z = 0$, we first establish the range of validity of the expansion. To see this, we note that when $\Omega_{m0} \approx 0.3$, namely the canonical Planck value [4], the corresponding values for h_i are $h_1 \approx 0.45$, $h_2 \approx 0.35$, $h_3 \approx -0.007$. Concretely, if we demand that our expansion is within 1% of the exact analytic result, $E(z) = \sqrt{1 - \Omega_{m0} + \Omega_{m0}(1 + z)^3}$, then the linear, quadratic and cubic approximations are good to $z \approx 0.18$, $z \approx 0.76$, and $z \approx 0.82$, respectively.

The potential and its derivatives at $z = 0$ can be expressed in terms of α, β, γ:

$$V_0 H_0^{-2} = 3\Omega_{\phi 0} - \frac{1}{2}\alpha^2, \quad (7)$$

$$V_0' H_0^{-2} = -\alpha(h_1 - 2) - 2\beta + 3\Omega_{c0}C, \quad (8)$$

$$V_0'' H_0^{-2} = -(h_1^2 - 2h_1 + 2h_2 - 2) - 6\frac{\beta h_1}{\alpha}$$

$$- \frac{6\gamma}{\alpha} - 3\Omega_{c0}\left(C^2 - 3\frac{C}{\alpha}\right), \quad (9)$$

where for a given coupling C, we have a one-to-one correspondence between (α, β, γ) and (V_0, V_0', V_0''). Note also that $\Omega_{\phi 0}$ is the dark energy density today $\Omega_{\phi 0} = 1 - \Omega_{m0}$. The effective EOS can easily be worked out,

$$w_{\text{eff}} = -1 + \frac{\alpha^2}{3\Omega_{\phi 0}} + \frac{z}{\Omega_{\phi 0}^2}\left[\frac{\alpha^4}{3}(\Omega_{\phi 0} - 1) + \frac{\alpha^2}{3}\Omega_{\phi 0}(5 - 3\Omega_{\phi 0}) + \frac{4}{3}\alpha\beta\Omega_{\phi 0} - \alpha C \Omega_{c0}\Omega_{\phi 0}\right]$$

$$+ \frac{z^2}{\Omega_{\phi 0}^3}\left[\frac{\alpha^6}{6}(\Omega_{\phi 0}^2 - 3\Omega_{\phi 0} + 2) + \frac{\alpha^4}{6}\Omega_{\phi 0}(17\Omega_{\phi 0} - 14 - 3\Omega_{\phi 0}^2) + 2\alpha^3\beta\Omega_{\phi 0}(\Omega_{\phi 0} - 1) + \frac{\alpha^2}{3}\Omega_{\phi 0}^2(10 - 9\Omega_{\phi 0})\right.$$

$$\left. + \frac{4}{3}\alpha\beta\Omega_{\phi 0}^2(5 - 3\Omega_{\phi 0}) + \frac{4}{3}\beta^2\Omega_{\phi 0}^2 + 2\alpha\gamma\Omega_{\phi 0}^2 - C\Omega_{c0}\Omega_{\phi 0}\left(\frac{1}{2}\alpha^3(\Omega_{\phi 0} - 3) + (3\alpha + \beta)\Omega_{\phi 0}\right) + \frac{1}{2}\alpha^2 C^2\Omega_{c0}\Omega_{\phi 0}^2\right]$$

$$+ \dots. \tag{10}$$

Interestingly, regardless of the value of C, $w_{\text{eff}} > -1$ at $z = 0$ and a crossing into the phantom regime ($w_{\text{eff}} < -1$) happens only at higher redshift. Note that when $C = 0$, we simply denote w_{eff} as w.

III. METHODOLOGY

We begin by generating 10^6 triples (α, β, γ), in a normal distribution about $(0,0,0)$ with a uniform standard deviation, which we take to be $\sigma = 0.1$ [67]. We impose a conservative redshift cutoff, $z_{\max} = 0.7$, and impose

$$|\alpha| \gtrsim z_{\max}|\beta|, \qquad |\beta| \gtrsim z_{\max}|\gamma|, \qquad |\phi - \phi_0| \lesssim 1, \tag{11}$$

to ensure that perturbation makes sense; i.e., the higher order numbers are smaller. Note that these cuts can be implemented without establishing a best-fit value of Ω_{m0} and we are not imposing slow roll.

For the triples surviving these primary cuts, we perform two-parameter fits of the data [43–50,52–54,68] and demand that

$$V_0 \gtrsim |V_0' \cdot (\phi - \phi_0)| \quad |V'| \gtrsim \frac{1}{2}|V_0'' \cdot (\phi - \phi_0)|, \tag{12}$$

once again to ensure a valid expansion. In addition, we impose the nominal restriction $\Omega_{m0} \gtrsim 0.2$, which is in line with weak lensing results [69,70], and impose the 2σ constraints from Fig. 1, which we have inferred from a Markov chain Monte Carlo (MCMC) exploration of cosmic chronometer, BAO, and Pantheon data below $z < 0.7$. Observe that, in contrast to [21], our bounds come from low redshift data, which means they are less constraining, and for uncoupled models, we impose $w(z) > -1$.

It is worth emphasizing that our expansion is expected to be valid to $z \approx 0.7$, but if we include distance moduli or angular diameter distance measurements, since they are integrated quantities, the approximation holds only for $z \lesssim 0.3$. This allows us to easily accommodate isotropic BAO at $z = 0.106$ [52] and $z = 0.15$ [53], but for higher redshift BAO we are restricted to $D_H(z) \equiv c/H(z)$ and its covariance matrix. For this reason, there is extra information in Fig. 1. and this leads to additional constraints. Finally, since we have a large parameter space, e.g.,

$(H_0, \Omega_{m0}, \alpha, \beta, \gamma, C)$, one can expect any best-fit values to be within 1σ of ΛCDM, which ultimately favors the standard model. However, since our interest here is exploring the effect on H_0 of changing the underlying potential (theory), we simply scan over α, β, γ, and C.

Once the cuts are imposed, we are left with a large class of "bottom-up" quintessence models, which we further divide into cosmologies where H_0 increases/decreases relative to the ΛCDM model, i.e., $\alpha = \beta = \gamma = 0$. For reference, we record the best-fit values in this case,

$$H_0 = 68.89^{+1.20}_{-1.15} \text{ km s}^{-1} \text{ Mpc}^{-1},$$
$$\Omega_{m0} = 0.30^{+0.02}_{-0.02}. \tag{13}$$

Note that the values are pretty consistent with Planck [4], and due to our inflated errors this represents a $\sim 3.3\sigma$ discrepancy with the highest local H_0 determination based on cepheid-calibrated supernovae [5].

Once the coupling is introduced, the dark matter particles are subject to a long-range attractive force with an effective Newton's constant $G_{\text{eff}} = G_N(1 + \frac{1}{2}C^2)$. While the CMB constrains the conformal coupling to be $C \lesssim 0.1$ [38,40], if the coupling is turned on at late times; e.g., [34] this
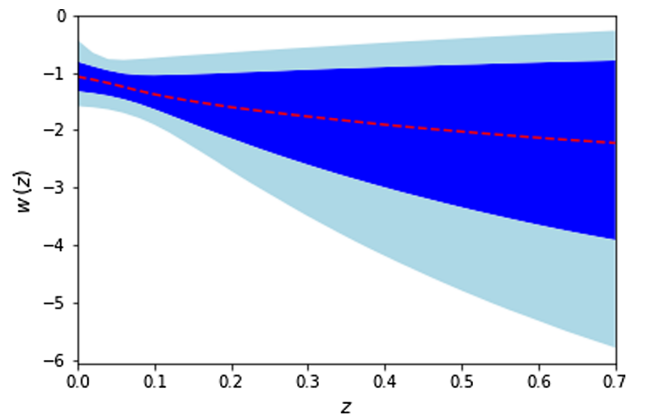

FIG. 1. Bounds on the Chevallier-Polarski-Linderr (CPL) parametrization [36,37], $w(z) = w_0 + w_a z/(1 + z)$, which are constructed directly from the chains of an MCMC exploration of cosmic chronometer, BAOs, and supernovae data restricted to $z \leq 0.7$.

constraint can be evaded. Nevertheless, stronger constraints exist for galaxies up to redshift $z \sim 0.05$ [56,57], whereby $C \lesssim 0.05$. This number will be constrained further in future, but here we settle on the value $C = 0.1$. This relaxes the constraint of [56,57] slightly, but it still preserves its integrity as a strong observational bound. Finally, since baryonic matter is uncoupled, we instead adopt the Planck prior $\Omega_{b0} = 0.05$ and only fit H_0 and Ω_{c0}.

IV. RESULTS

In Fig. 2 we present a plot of the χ^2 as a function of the best-fit H_0 for uncoupled models, where a dashed line denotes the χ^2 for the ΛCDM model, $\alpha = \beta = \gamma = 0$. From the plot it is clear that deviations from ΛCDM that lower H_0 are favored by the data. In contrast, H_0 may increase, but this worsens the fit to the data. Interestingly, we find no exceptions to this statement in 87,675 random models and we have checked that adding a H_0 prior [5] throughout does not change our conclusions.

In Fig. 3 we present the same plot but for coupled quintessence models with $C = 0.1$. We are again making a comparison to the ΛCDM model. Concretely, here we first identified triples of (α, β, γ), i.e., potentials where $w > -1$, thus ensuring that the uncoupled model is a bona fide quintessence model, before adding the coupling. Once the coupling is added, we can enter the $w < -1$ region of parameter space and we repeat the fitting procedure. Thus, the configurations presented in Fig. 3 are a subset of the Fig. 2 configurations but subject to a coupling. In contrast to the uncoupled case, the range of H_0 is extended, but since C is small, the effect is not so great. One could choose a larger coupling, but this would bring us into conflict with [56,57]. Finally, we have checked that removing the poorer quality cosmic chronometer data do not change our conclusions.
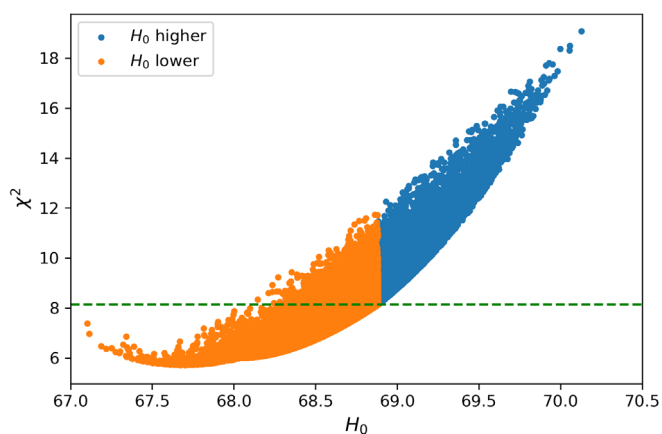
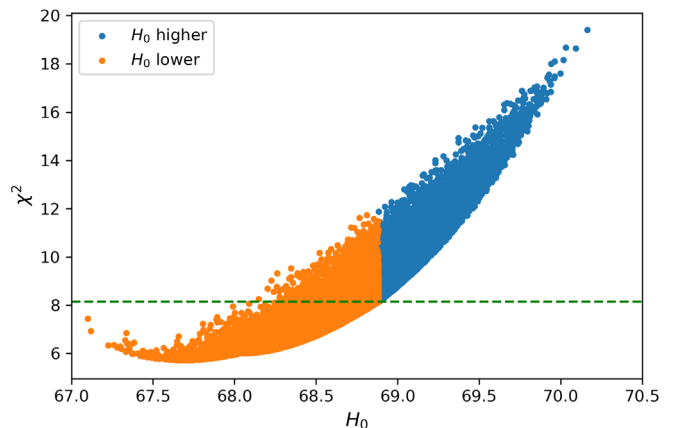

FIG. 3. Distribution of 80,732 surviving models as a function of χ^2 versus H_0 for the coupling $C = 0.1$. Here all models that raise H_0 also raise the χ^2, whereas 46,665 models lower H_0 and χ^2.

Finally, we can make some remarks on the universal constants c, c' from the de Sitter conjecture [16–18]. We find that models that lower H_0 occur at smaller values of $|V'|/V$ relative to models that raise H_0. Ostensibly, this means that the de Sitter conjecture may (theoretically) favor models that raise H_0. That being said, such models have exclusively $V'' > 0$, whereas models that lower H_0 allow $V'' < 0$. For this reason, the refined de Sitter conjecture [17,18] favors models with lower H_0 than ΛCDM. Note, this is a statement that does not rely on χ^2, but still makes use of fits to data.

V. DISCUSSION

With an eye on the de Sitter swampland conjecture [16], we have performed a scan over (coupled) quintessence models at low redshift. Since α, β, γ are in one-to-one correspondence with V_0, V_0', V_0'', our best-fit values for H_0 probe different models (potentials) in the vicinity of ΛCDM. Within our assumptions and the data employed we arrive at the conclusion that H_0 decreases relative to ΛCDM in any quintessence model with an exponential coupling. Such models are expected to arise from string theory constructions, e.g., [62,63], but explicit constructions are especially challenging [71,72].

Our analysis here employs Taylor expansion around $z = 0$, so we are simply assuming analyticity. This allows us to describe a large class of single field quintessence models with a canonically normalized kinetic term. Nevertheless, to turn any of these models into viable cosmologies, one needs to ensure that there exists a high redshift completion to CMB. In fact, modulo the fact that truncating the $w_0 w_a$CDM or CPL model [36,37] at linear order in $(1 - a)$ is guaranteed to constrain the potential (see comments in the Appendix) [73], the results of [32] suggest little will change for uncoupled quintessence models that admit a high redshift completion to CMB. So, uncoupled

FIG. 2. Distribution of our 87,675 models, which survive our cuts, as a function of χ^2 versus H_0. The dashed green line highlights the χ^2 for ΛCDM and all models that raise H_0 also increase the χ^2 value. In contrast 53,744 models lower both H_0 and χ^2.

quintessence is clearly at odds with high local determinations of H_0 [5–8]. In short, if Hubble tension in ΛCDM is now a "problem" or "crisis," quintessence simply instills a sense of panic.

This leaves a coupling between quintessence and dark matter as a potential loophole. Here we have focused on the exponential coupling, which is the most natural from string theory, and imposed strict bounds that are derived from the absence of differential fifth forces in galaxy warps [56,57]. We find that the coupling is too small to induce any effect within the redshift range. Indeed, it is clear from Eq. (6) that the $O(z)$ term in the Hubble parameter is unaffected by the coupling, so the coupling is suppressed near $z \approx 0$. With such a small coupling, any interaction between quintessence and dark matter will have to act over an extended redshift range, in other words beyond the dark-energy-dominated regime, to make progress on Hubble tension. In particular, we may have to embrace a coupling that acts in the dark ages [34], which is currently beyond the scope of experiment, but interestingly, this may quickly change [59].

APPENDIX: COMMENT ON THE CPL MODEL

In this Appendix, we review the analysis of Scherrer [74], before taking it further using a combination of CMB, BAO, and supernovae data. Recall that the CPL model [36,37] can be expected to correspond to a specific class of potentials when $w(z) \geq -1$ [74]. More concretely, one can solve the potential in terms of the scale factor [74],

$$V(a) = V_0 \frac{(1 - w_0 - w_a + w_a a)}{(1 - w_0)} a^{-3(1 + w_0 + w_a)} e^{3w_a(a-1)},$$
(A1)

where we have normalized the potential so that $V(a = 1) = V_0$. Now, to determine ϕ as a function of a, one just needs to perform the following integral:

$$\phi = \int \frac{\sqrt{3(1 + w_0 + w_a - w_a a)}}{\sqrt{1 + (\rho_{m0}/\rho_{\phi 0})a^{3(w_0 + w_a)}e^{3w_a(1-a)}}} \frac{da}{a},$$
(A2)

where following [74], we adopt $\rho_{m0}/\rho_{\phi 0} = 3/7$. Note that ϕ is only defined up to a constant, but to fix it we assume $\phi(a = 1) = 0$. As can be seen from Figs. 1 or 2 of [74], CPL models are not generic and they correspond to a specific class of quintessence models.

Here, we will take the observation a little further in light of the results of Vagnozzi *et al.* [32]. There, given that they are fitting CMB data at $z \approx 1090$, the authors impose the conditions,

$$w_0 \geq -1, \qquad w_0 + w_a \geq -1,$$
(A3)

which will guarantee that $w(z) \geq -1$ for all z. So, here we will take the CMB + BAO + Pantheon MCMC chains from Planck analysis of the CPL model [4] and impose the constraints (A3). This then leaves one with 400 odd pairs of (w_0, w_a), which given the preference of the data for the cosmological constant are all pretty close to $(-1, 0)$. We next integrate (A2) from $a = 0.00092$ ($z = 1090$) to $a = 1$ ($z = 0$). This provides us a mapping between ϕ and a for each CPL model, i.e., each (w_0, w_a) pair. One can then work outwards from $\phi = 0$, corresponding to $a = 1$, to determine V/V_0 as a function of ϕ. In practice, we bin V/V_0 into bins of $\Delta\phi = 0.01$ and extract out the mean and 1σ, 2σ intervals in each bin, before applying a spline to these values. The end result is shown in Fig. 4.

There are a number of take-home points from the plot. First, for reasons that we have explained above, the quintessence potential corresponding to the CPL model is pretty constrained. This is even more apparent when one integrates (A2) all the way to $a \approx 0$. This is in line with
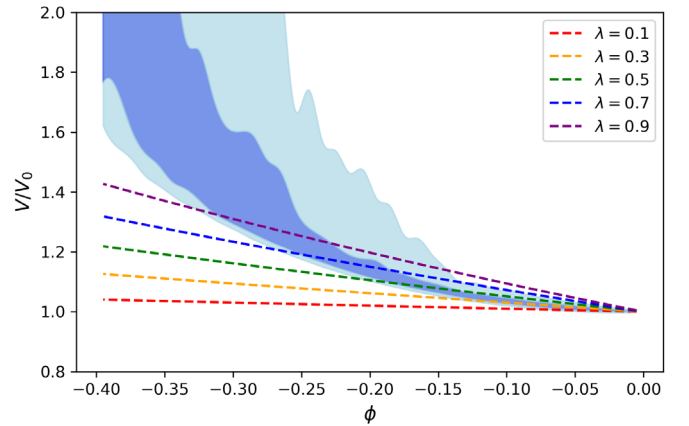

FIG. 4. The 1 and 2σ intervals for the normalized potential V/V_0 corresponding to CPL models that are consistent with CMB, BAO and Pantheon. For illustrative processes we plot the simplest quintessence model, $V/V_0 = e^{-\lambda\phi}$ for different values of λ.

earlier comments in Ref. [74]. Second, CMB+BAO +Pantheon data have a clear preference for the cosmological constant Λ, so it is no surprise that the potential is extremely flat as one approaches $\phi = 0$ ($a = 1$). Finally, it is worth noting that if one interprets this even in terms of the simplest quintessence model, one may come to the erroneous conclusion that they are ruled out.

This is not the case, it is just simply that the quintessence potentials corresponding to the CPL model are indeed very special, so special that $V = V_0 e^{-\lambda\phi}$ falls outside the class for certain ranges of ϕ, or the corresponding ranges in a. That being said, it should be easy to show that our results are consistent with [21] in the small ϕ regime, i.e., at late times.

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