Letter

Redshift drift as a model independent probe of dark energy

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It is well known that a positive value of redshift drift is a signature of violation of the strong energy condition within general relativistic Friedmann-Lemaître-Robertson-Walker (FLRW) universe models. In the Lambda Cold Dark Matter (ΛCDM) paradigm of cosmology, founded on the FLRW model ansatz, the violation of the strong energy condition is realized by a dark energy component with "equation of state" equal to minus unity. This dark energy component, which is unexplained in the standard model of particle physics, facilitates the acceleration of cosmological length scales. Such an acceleration is needed to provide a consistent fit of the general relativistic FLRW universe models to available data. It remains a possibility that dark energy, and the associated violation of the strong energy condition, is an artefact of the FLRW model ansatz imposed for the interpretation of data, rather than due to a fundamental cosmological constant of nature. Here we show—without making assumptions on the metric tensor of the Universe—that redshift drift is a promising direct probe of violation of the strong energy condition within the theory of general relativity. We discuss our results in relation to upcoming Lyman-α forest measurements of redshift drift.

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I. INTRODUCTION

In general relativistic theory energy conditions are physically motivated constraints, which can be applied to the energy momentum tensor of space-time. Energy conditions are important tools for constraining the possible solutions of the Einstein field equations and for deriving general theorems about the nature of gravitating systems. The Penrose and Hawking singularity theorems [1,2] use energy conditions to arrive at physical scenarios where the developments of singularities are unavoidable. In particular, the strong energy condition is a central assumption in the focusing theorem, which states the conditions under which a matter congruence develops singularities in finite proper time. The strong energy condition stipulates that the inequality $R_{\mu\nu}n^{\mu}n^{\nu} \geq 0$ is satisfied everywhere, where $R_{\mu\nu}$ is the Ricci-curvature tensor of the space-time and n^{μ} is any timelike vector field, and physically amounts to the statement that gravity between massive particles is universally attractive [3].

Within the Friedmann-Lemaître-Robertson-Walker (FLRW) framework of cosmology the strong energy condition is considered abandoned by observations [4,5]. The most direct evidence for violation of the strong energy condition, comes from the observed acceleration of space when interpreting data from supernovae of type Ia within the FLRW class of models [6–8].

A number of studies have suggested different mechanisms which can potentially result in the (apparent) acceleration of cosmological length scales *without* violation of the strong energy condition. Studies of the effect of the "backreaction" of cosmic structure on the evolution of large scale volume sections [9,10], suggest that emergent acceleration effects can appear as a consequence of the differential expansion between regions with high concentrations of gravitationally bound structures and fast expanding void regions [11–13]. Furthermore, the possibility of significant local acceleration of length scales in the frame of nonideal observers in FLRW space-times due to a small peculiar 4-acceleration of the observers has been pointed out in [14,15]. In addition, the possibility of inferring of a positive acceleration of space when interpreting distance–redshift data in a Universe with local inhomogeneities and anisotropies within the FLRW framework of *exact* homogeneity and isotropy has been described in [16].

Measurements of the drift of redshift in proper time of the observer [17–19] are promising probes of the expansion history of the Universe. In the conventionally studied FLRW universe models, a positive value of redshift drift is a signature of dark energy [20]. The direct detection of the time-evolution of redshift—expected within one to a few decades of observation time with facilities such as CODEX and the Square Kilometer Array (SKA) [21–23]—allows for model independent determination of kinematic properties of the Universe.

So far most theoretical studies of redshift drift have been done within the FLRW class of models—though

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see [24–31] for theoretical and numerical studies of the redshift drift signal in certain types of Lemaître-Tolman-Bondi, Bianchi I, and Szekeres models. Redshift drift has been proposed as a discriminator between the Lambda Cold Dark Matter (ΛCDM) concordance model and Lemaître-Tolman-Bondi void models [24,25] and a particular axially symmetric quasispherical Szekeres model [26]. Fully general treatments of redshift drift, making no assumption about the metric or the congruence of observers, have been considered in [32,33]. To the best of my knowledge, so far no model independent tests have been proposed for the upcoming measurements of redshift drift signals.

As is evident from the study of backreaction effects over cosmic volume sections [34,35], the lowest order description of a *statistically* homogeneous and isotropic general relativistic space-time need not be that of an FLRW model universe solution.[1] The reason is that inhomogeneities and anisotropies on small and intermediate length scales can contribute with noncanceling effects in the evolution of large scale integrated volume sections, thus systematically affecting the large scale or "monopole" description of the space-time. The same principle might be expected to hold true in general for observations in a universe exhibiting some level of *statistical* homogeneity and isotropy (irrespective of the magnitude of backreaction effects over cosmic volumes): Local inhomogeneities and anisotropies will in general contribute with accumulated effects along null rays and systematic effects from the position of the observer, resulting in a "monopole" description of the observed signal with nontrivial contributions from small and intermediate scale inhomogeneities. It has been detailed how local structures alter measurements of redshift drift in a general space-time setting [33]. Such effects from local inhomogeneity are not *a priori* expected to be subdominant—also not in space-times with a notion of statistical homogeneity and isotropy—and the size of the effects must ultimately be determined by data. This in turn raises the question if redshift drift as a probe of dark energy—or more generally the strong energy condition—is valid in universe models that are not subject to the FLRW idealization. In this paper we propose a model independent test of the strong energy condition by redshift drift measurements. The test is simple, despite its general application, and relies on lower bound measurements of the redshift drift signal.

A. Notation and conventions

Units are used in which $c = 1$. Greek letters $\mu, \nu, \ldots$ label space-time indices in a general basis. Einstein notation is used such that repeated indices are summed over. The signature of the space-time metric $g_{\mu\nu}$ is $(-+++)$ and the connection ∇_μ is the Levi-Civita connection. Round brackets () containing indices denote symmetrization in the involved indices and square brackets [] denote anti-symmetrization. Bold notation $\boldsymbol{V}$ for the basis-free representation of vectors V^μ is used occasionally.

[1]See [36,37] for a debate on the accuracy of the FLRW approximation and the significance of cosmological backreaction on different scales.

II. REDSHIFT DRIFT IN A GENERAL SPACE-TIME

In this section we review the expression for redshift drift for a generic space-time congruence of physical observers and emitters—for details, see[2] [33]. We consider a general space-time congruence of observers and emitters (henceforth referred to as the "observer congruence") with worldlines generated by the 4-velocity field $\boldsymbol{u}$ and parametrized by the proper time function τ. The redshift drift of light rays generated by the 4-momentum field $\boldsymbol{k}$ and passing from an emitter placed at space-time point $\mathcal{E}$ to an observer situated at space-time point $\mathcal{O}$ can be written

$$\left.\frac{dz}{d\tau}\right|_{\mathcal{O}} = (1+z)\mathfrak{H}_{\mathcal{O}} - \mathfrak{H}_{\mathcal{E}} + \mathcal{S}_{\mathcal{E}\to\mathcal{O}}, \tag{1}$$

with

$$\mathcal{S}_{\mathcal{E}\to\mathcal{O}} = E_{\mathcal{E}} \int_{\lambda_{\mathcal{E}}}^{\lambda_{\mathcal{O}}} d\lambda \mathcal{I}, \qquad \mathcal{I} \equiv -k^\nu \nabla_\nu \left(\frac{e^\mu \nabla_\mu E}{E^2} \right), \tag{2}$$

where the function λ satisfies $k^\mu \nabla_\mu \lambda = 1$, and is an affine parameter along each null line. The redshift z and photon energy function E associated with the light rays are given by

$$z \equiv \frac{E_{\mathcal{E}}}{E_{\mathcal{O}}} - 1, \qquad E \equiv -k^\mu u_\mu, \tag{3}$$

and the change of the photon energy E along a given null ray is given by

$$\mathfrak{H} \equiv -\frac{k^\mu \nabla_\mu E}{E^2} = \frac{1}{3}\theta - e^\mu a_\mu + e^\mu e^\nu \sigma_{\mu\nu}. \tag{4}$$

The spatial unit vector $\boldsymbol{e}$ describes the spatial propagation direction of the null ray relative to an observer comoving with $\boldsymbol{u}$, and is defined by the decomposition

$$k^\mu = E(u^\mu - e^\mu). \tag{5}$$

The function $\mathfrak{H}$ is an observationally natural generalization of the Hubble parameter of FLRW space-time: $\mathfrak{H}_{\mathcal{O}}$ plays the role of the proportionality constant between redshift and distance in the generalized Hubble law valid for arbitrary space-times in the $\mathcal{O}(z)$ vicinity of the observer [16]. The variables θ, $\sigma_{\mu\nu}$ and a^μ describe the expansion, shear, and

[2]For another interesting representation of redshift drift in general space-time models, see [32].

4-acceleration of the observer congruence. Together with the vorticity tensor $\omega_{\mu\nu}$, they describe the kinematics of the observer congruence

$$\begin{aligned} \nabla_\nu u_\mu &= \frac{1}{3}\theta h_{\mu\nu} + \sigma_{\mu\nu} + \omega_{\mu\nu} - u_\nu a_\mu, \\ \theta &\equiv \nabla_\mu u^\mu, \qquad \sigma_{\mu\nu} \equiv h^\beta_{\langle\nu} h^\alpha_{\mu\rangle} \nabla_\beta u_\alpha, \\ \omega_{\mu\nu} &\equiv h^\beta_\nu h^\alpha_\mu \nabla_{[\beta} u_{\alpha]}, \qquad a^\mu \equiv \dot{u}^\mu, \end{aligned} \tag{6}$$

where $h^\nu_\mu \equiv u_\mu u^\nu + g^\nu_\mu$ is the spatial projection tensor defined in the frame of the 4-velocity field $\boldsymbol{u}$ and where triangular brackets $\langle\rangle$ denote traceless symmetrization in the involved indices of a tensor in three dimensions.[3] The operator $\dot{} \equiv u^\mu \nabla_\mu$ denotes the derivative in proper time along flow lines of $\boldsymbol{u}$. From geometrical identities, the evolution of the kinematic variables θ, $\sigma_{\mu\nu}$, and $\omega_{\mu\nu}$ along the observer flow lines can be expressed as

$$\begin{aligned} \dot{\theta} = &-\frac{1}{3}\theta^2 - \sigma_{\mu\nu}\sigma^{\mu\nu} + \omega_{\mu\nu}\omega^{\mu\nu} \\ &- u^\mu u^\nu R_{\mu\nu} + D_\mu a^\mu + a_\mu a^\mu, \end{aligned} \tag{7}$$

$$\begin{aligned} \dot{\sigma}_{\mu\nu} = &-\frac{2}{3}\theta\sigma_{\mu\nu} - \sigma_{\alpha\langle\mu}\sigma^\alpha_{\nu\rangle} + \omega_{\alpha\langle\mu}\omega^\alpha_{\nu\rangle} + 2a^\alpha\sigma_{\alpha(\nu}u_{\mu)} \\ &+ D_{\langle\mu}a_{\nu\rangle} + a_{\langle\mu}a_{\nu\rangle} - u^\rho u^\sigma C_{\rho\mu\sigma\nu} - \frac{1}{2}h^\alpha_{\langle\mu}h^\beta_{\nu\rangle}R_{\alpha\beta}, \end{aligned} \tag{8}$$

$$\dot{\omega}_{\mu\nu} = -\frac{2}{3}\theta\omega_{\mu\nu} + 2\sigma^\alpha_{[\mu}\omega_{\nu]\alpha} - D_{[\mu}a_{\nu]} - 2a^\alpha\omega_{\alpha[\mu}u_{\nu]}, \tag{9}$$

where $R_{\mu\nu}$ is the Ricci tensor of the space-time and $C_{\mu\nu\rho\sigma}$ is the Weyl tensor. The operator D_μ is the covariant spatial derivative[4] defined on the 3-dimensional space orthogonal to $\boldsymbol{u}$. The integrand in (2) is conveniently expressed in terms of the (differentiated) kinematic variables of the observer congruence in the following series expansion [33]:

$$\begin{aligned} \mathcal{I} = &\mathcal{I}^o + e^\mu \mathcal{I}^{\boldsymbol{e}}_\mu + d^\mu \mathcal{I}^{\boldsymbol{d}}_\mu + e^\mu e^\nu \mathcal{I}^{\boldsymbol{ee}}_{\mu\nu} + e^\mu d^\nu \mathcal{I}^{\boldsymbol{ed}}_{\mu\nu} \\ &+ e^\mu e^\nu e^\rho \mathcal{I}^{\boldsymbol{eee}}_{\mu\nu\rho} \end{aligned} \tag{10}$$

with coefficients

$$\begin{aligned} \mathcal{I}^o &\equiv -\frac{1}{3}(4\omega^{\mu\nu}\omega_{\mu\nu} + D_\mu a^\mu + a^\mu a_\mu) - d^\mu d_\mu, \\ \mathcal{I}^{\boldsymbol{e}}_\mu &\equiv \frac{1}{3}D_\mu\theta + \frac{1}{3}\theta a_\mu + \frac{2}{5}D_\nu\sigma^\nu{}_\mu + \frac{2}{5}a^\nu\sigma_{\mu\nu} - 2a^\nu\omega_{\mu\nu}, \\ \mathcal{I}^{\boldsymbol{d}}_\mu &\equiv -2a_\mu, \\ \mathcal{I}^{\boldsymbol{ee}}_{\mu\nu} &\equiv -(4\omega_{\alpha\mu}\sigma^\alpha{}_\nu + 4\omega_{\alpha\langle\mu}\omega^\alpha{}_{\nu\rangle} + D_{\langle\nu}a_{\mu\rangle} + a_{\langle\mu}a_{\nu\rangle}), \\ \mathcal{I}^{\boldsymbol{ed}}_{\mu\nu} &\equiv 4(\sigma_{\mu\nu} - \omega_{\mu\nu}), \\ \mathcal{I}^{\boldsymbol{eee}}_{\mu\nu\rho} &\equiv D_{\langle\rho}\sigma_{\mu\nu\rangle} + a_{\langle\mu}\sigma_{\nu\rho\rangle}, \end{aligned} \tag{11}$$

where $d^\mu \equiv h^\mu{}_\nu e^\alpha \nabla_\alpha e^\nu$ denotes the spatially projected "4-acceleration" of $\boldsymbol{e}$. The magnitude of $\boldsymbol{d}$ can be seen as a measure of the failure of $\boldsymbol{e}$ to define an axis of local rotational symmetry, and is thus a quantification of the local departure from isotropy [38]. In deriving (11), it has been assumed that the null congruence is irrotational, such that $\nabla_{[\alpha}k_{\nu]} = 0$. Note that all coefficients in (11) with more than one space-time index are traceless.

III. MODEL INDEPENDENT TEST OF THE STRONG ENERGY CONDITION

For the purpose of examining the strong energy condition, it is convenient to rewrite the expression for redshift drift (1), such that the Ricci curvature of the space-time appears explicitly in the formula. For this purpose the following identity:

$$(1+z)\mathfrak{H}_{\mathcal{O}} - \mathfrak{H}_{\mathcal{E}} = E_{\mathcal{E}}\int_{\lambda_{\mathcal{E}}}^{\lambda_{\mathcal{O}}} d\lambda \mathcal{A}, \qquad \mathcal{A} \equiv k^\nu \nabla_\nu\left(\frac{\mathfrak{H}}{E}\right) \tag{12}$$

will be useful. Combining (1), (2), and (12) gives

$$\left.\frac{dz}{d\tau}\right|_{\mathcal{O}} = E_{\mathcal{E}}\int_{\lambda_{\mathcal{E}}}^{\lambda_{\mathcal{O}}} d\lambda \Pi, \qquad \Pi \equiv \mathcal{I} + \mathcal{A}. \tag{13}$$

In the FLRW limit, the integrand Π reduces to the well known length scale acceleration "$\ddot{a}/a$", where a is the uniform FLRW scale factor. The function $-\mathcal{A}/\mathfrak{H}^2$ enters in the "Hubble law" for generic space-times [16] as an effective deceleration parameter, replacing the FLRW deceleration parameter in the series expansion of luminosity distance in redshift—see [16] for details. In a similar spirit as for $\mathfrak{H}$ and $\mathcal{I}$, the function $\mathcal{A}$ can be written as a truncated multipole expansion

$$\begin{aligned} \mathcal{A} = &\mathcal{A}^o + e^\mu \mathcal{A}^{\boldsymbol{e}}_\mu + e^\mu e^\nu \mathcal{A}^{\boldsymbol{ee}}_{\mu\nu} + e^\mu e^\nu e^\rho \mathcal{A}^{\boldsymbol{eee}}_{\mu\nu\rho} \\ &+ e^\mu e^\nu e^\rho e^\kappa \mathcal{A}^{\boldsymbol{eeee}}_{\mu\nu\rho\kappa} \end{aligned} \tag{14}$$

with coefficients

[3] For two indices we have that the traceless parts of symmetric spatial tensors $T_{\mu\nu} = T_{(\mu\nu)} = h^\alpha_\mu h^\beta_\nu T_{(\alpha\beta)}$ are given by $T_{\langle\mu\nu\rangle} = T_{\mu\nu} - \frac{1}{3}h_{\mu\nu}T$. Analogously for a tensor with three indices satisfying $T_{\mu\nu\rho} = T_{(\mu\nu\rho)} = h^\alpha_\mu h^\beta_\nu h^\gamma_\rho T_{(\alpha\beta\gamma)}$, we have $T_{\langle\mu\nu\rho\rangle} = T_{\mu\nu\rho} - \frac{1}{5}(T_\mu h_{\nu\rho} + T_\nu h_{\rho\mu} + T_\rho h_{\mu\nu})$. For four indices we have $T_{\langle\mu\nu\rho\kappa\rangle} = T_{\mu\nu\rho\kappa} - \frac{1}{7}(T_{\langle\mu\nu\rangle}h_{\rho\kappa} + T_{\langle\mu\rho\rangle}h_{\nu\kappa} + T_{\langle\mu\kappa\rangle}h_{\nu\rho} + T_{\langle\nu\rho\rangle}h_{\mu\kappa} + T_{\langle\nu\kappa\rangle}h_{\mu\rho} + T_{\langle\rho\kappa\rangle}h_{\mu\nu}) - \frac{1}{5}Th_{(\mu\nu}h_{\rho\kappa)}$. We have used the short hand notations $T_{\mu\nu} \equiv h^{\rho\kappa}T_{\mu\nu\rho\kappa}$, $T_\mu \equiv h^{\nu\rho}T_{\mu\nu\rho}$, and $T \equiv h^{\mu\nu}T_{\mu\nu}$.

[4] The acting of D_μ on a tensor field $T_{\nu_1,\nu_2,\ldots,\nu_n}{}^{\gamma_1,\gamma_2,\ldots,\gamma_m}$ is defined as: $D_\mu T_{\nu_1,\nu_2,\ldots,\nu_n}{}^{\gamma_1,\gamma_2,\ldots,\gamma_m} \equiv h_{\nu_1}{}^{\alpha_1}h_{\nu_2}{}^{\alpha_2}\ldots h_{\nu_n}{}^{\alpha_n}h_{\beta_1}{}^{\gamma_1}h_{\beta_2}{}^{\gamma_2}\ldots h_{\beta_m}{}^{\gamma_m}h_\mu{}^\sigma\nabla_\sigma T_{\alpha_1,\alpha_2,\ldots,\alpha_n}{}^{\beta_1,\beta_2,\ldots,\beta_m}$.

$$\begin{aligned}
\mathcal{A}^{o} &\equiv -\frac{1}{3}u^{\mu}u^{\nu}R_{\mu\nu} + \frac{2}{3}D_{\mu}a^{\mu} - \frac{3}{5}\sigma^{\mu\nu}\sigma_{\mu\nu} + \frac{1}{3}\omega^{\mu\nu}\omega_{\mu\nu},\\
\mathcal{A}^{\boldsymbol{e}}_{\mu} &\equiv -\frac{2}{3}\theta a_{\mu} + a^{\nu}\sigma_{\mu\nu} + a^{\nu}\omega_{\mu\nu}\\
&\quad -\frac{1}{3}D_{\mu}\theta - \frac{2}{5}D_{\nu}\sigma^{\nu}{}_{\mu} - h^{\nu}_{\mu}\dot{a}_{\nu},\\
\mathcal{A}^{\boldsymbol{ee}}_{\mu\nu} &\equiv 3a_{\langle\mu}a_{\nu\rangle} - 2\sigma_{\alpha\mu}\omega^{\alpha}{}_{\nu} - \frac{9}{7}\sigma_{\alpha\langle\mu}\sigma^{\alpha}{}_{\nu\rangle} + \omega_{\alpha\langle\mu}\omega^{\alpha}{}_{\nu\rangle}\\
&\quad + 2D_{\langle\mu}a_{\nu\rangle} - u^{\rho}u^{\sigma}C_{\rho\mu\sigma\nu} - \frac{1}{2}h^{\alpha}{}_{\langle\mu}h^{\beta}{}_{\nu\rangle}R_{\alpha\beta},\\
\mathcal{A}^{\boldsymbol{eee}}_{\mu\nu\rho} &\equiv -D_{\langle\rho}\sigma_{\mu\nu\rangle} - 5a_{\langle\mu}\sigma_{\nu\rho\rangle},\\
\mathcal{A}^{\boldsymbol{eeee}}_{\mu\nu\rho\kappa} &\equiv 3\sigma_{\langle\mu\nu}\sigma_{\rho\kappa\rangle},
\end{aligned} \tag{15}$$

where $R_{\mu\nu}$ is the Ricci curvature of the space-time, and where we have used the deviation Eqs. (7) and (8). In the derivation of (14) we have used the definition (4) to write $\mathcal{A} = k^{\mu}\nabla_{\mu}(\mathfrak{H})/E + \mathfrak{H}^2$ together with the identity

$$\frac{k^{\nu}\nabla_{\nu}e^{\mu}}{E} = (e^{\mu}-u^{\mu})\mathfrak{H} - e^{\nu}\left(\frac{1}{3}\theta h^{\mu}{}_{\nu} + \sigma^{\mu}{}_{\nu} + \omega^{\mu}{}_{\nu}\right) + a^{\mu}. \tag{16}$$

By combining the multipole coefficients in (11) and (15) of the same order, we finally have that the integrand in (13) can be expressed as

$$\begin{aligned}
\Pi &= \Pi^{o} + e^{\mu}\Pi^{\boldsymbol{e}}_{\mu} + d^{\mu}\Pi^{\boldsymbol{d}}_{\mu} + e^{\mu}e^{\nu}\Pi^{\boldsymbol{ee}}_{\mu\nu} + e^{\mu}d^{\nu}\Pi^{\boldsymbol{ed}}_{\mu\nu}\\
&\quad + e^{\mu}e^{\nu}e^{\rho}\Pi^{\boldsymbol{eee}}_{\mu\nu\rho} + e^{\mu}e^{\nu}e^{\rho}e^{\kappa}\Pi^{\boldsymbol{eeee}}_{\mu\nu\rho\kappa}
\end{aligned} \tag{17}$$

with coefficients

$$\begin{aligned}
\Pi^{o} &\equiv -\frac{1}{3}u^{\mu}u^{\nu}R_{\mu\nu} + \frac{1}{3}D_{\mu}a^{\mu} - \frac{1}{3}a^{\mu}a_{\mu}\\
&\quad - d^{\mu}d_{\mu} - \frac{3}{5}\sigma^{\mu\nu}\sigma_{\mu\nu} - \omega^{\mu\nu}\omega_{\mu\nu},\\
\Pi^{\boldsymbol{e}}_{\mu} &\equiv -\frac{1}{3}\theta a_{\mu} + \frac{7}{5}a_{\nu}\sigma^{\nu}{}_{\mu} - a^{\nu}\omega_{\mu\nu} - h^{\nu}_{\mu}\dot{a}_{\nu},\\
\Pi^{\boldsymbol{d}}_{\mu} &\equiv -2a_{\mu},\\
\Pi^{\boldsymbol{ee}}_{\mu\nu} &\equiv 2a_{\langle\mu}a_{\nu\rangle} - \frac{9}{7}\sigma_{\alpha\langle\mu}\sigma^{\alpha}{}_{\nu\rangle} - 3\omega_{\alpha\langle\mu}\omega^{\alpha}{}_{\nu\rangle} - 6\sigma_{\alpha\mu}\omega^{\alpha}{}_{\nu}\\
&\quad + D_{\langle\mu}a_{\nu\rangle} - u^{\rho}u^{\sigma}C_{\rho\mu\sigma\nu} - \frac{1}{2}h^{\alpha}{}_{\langle\mu}h^{\beta}{}_{\nu\rangle}R_{\alpha\beta},\\
\Pi^{\boldsymbol{ed}}_{\mu\nu} &\equiv 4(\sigma_{\mu\nu} - \omega_{\mu\nu}),\\
\Pi^{\boldsymbol{eee}}_{\mu\nu\rho} &\equiv -4a_{\langle\mu}\sigma_{\nu\rho\rangle},\\
\Pi^{\boldsymbol{eeee}}_{\mu\nu\rho\kappa} &\equiv 3\sigma_{\langle\mu\nu}\sigma_{\rho\kappa\rangle}.
\end{aligned} \tag{18}$$

The multipole coefficients (18) are given in terms of kinematic and dynamic variables associated with the observer congruence and the Ricci curvature tensor $R_{\mu\nu}$. While the coefficients in (11) and (15) contain spatial gradients of θ and $\sigma_{\mu\nu}$, these contributions cancel in (18), and the only spatial gradients that remain are of the 4-acceleration a^{μ}. The anisotropic function Π reduces to the FLRW scale factor acceleration "$\ddot{a}/a$" in the idealized isotropic and homogeneous limit. In the FLRW limit, the only nonzero contribution is the first term $-\frac{1}{3}u^{\mu}u^{\nu}R_{\mu\nu}$ of the monopole contribution Π^{o}—all of the remaining terms in (18) arise from the contributions of inhomogeneity and anisotropy along the null rays. These modifications of the FLRW law for redshift drift need not be small or cancel under the integral sign (13), and the measurements of redshift drift cannot *a priori* be expected to obey FLRW predictions in realistic universe models with structure.

A. The dominant monopole approximation

Let us consider the case where the monopole term Π^{o} is dominant in the integral expression for redshift drift (13), such that the contributions to the integral from the remaining terms in the series expansion (17) are small compared to the contributions from Π^{o}. This corresponds to the physical assumption that the systematic alignment of $\boldsymbol{e}$ and $\boldsymbol{d}$ with the fluid variables such as shear and 4-acceleration is weak over the length scales of photon propagation.[5] In this scenario, we have that all spatial directions of photon propagation can be treated on equal footing at lowest order, with the leading order expression for redshift drift

$$\left.\frac{dz}{d\tau}\right|_{\mathcal{O}} = E_{\mathcal{E}}\int_{\lambda_{\mathcal{E}}}^{\lambda_{\mathcal{O}}} d\lambda \Pi^{o}. \tag{19}$$

From (18) we see that the only potentially positive contributions to Π^{o} are from the terms $-\frac{1}{3}u^{\mu}u^{\nu}R_{\mu\nu}$ and $\frac{1}{3}(D_{\mu}a^{\mu} - a^{\mu}a_{\mu})$. The other terms entering the expression for Π^{o} are nonpositive and in general contribute with accumulated negative contributions to the measured redshift drift. In general relativistic theory, negative values of $u^{\mu}u^{\nu}R_{\mu\nu}$ are equivalent to violation of the strong energy condition. It is not surprising that positive values of $D_{\mu}a^{\mu}$ of sufficient magnitude can cause positive redshift drift signals in the observer's frame, since positive

[5]The cancellation of spatially projected traceless combinations of fluid kinematic variables has been argued to be a realistic scenario in space-times where a notion of statistical homogeneity and isotropy is present, and where structure is slowly evolving relative to the timescale it takes for photons to pass an approximate homogeneity scale [39,40]. This suggested cancellation has been shown to not hold true in general, exemplified by the systematic alignment of the propagation direction $\boldsymbol{e}$ of the null ray with the positive eigenvector of the shear tensor in a Tardis space-time [41] and in Swiss cheese models based on Lemaître-Tolman-Bondi and anisotropic Szekeres structures [42,43]. The level of accuracy of the dominant monopole approximation $\int_{\lambda_{\mathcal{E}}}^{\lambda_{\mathcal{O}}} d\lambda\Pi \approx \int_{\lambda_{\mathcal{E}}}^{\lambda_{\mathcal{O}}} d\lambda\Pi^{o}$ for light propagation over cosmological distances must be tested under various model assumptions.

values of $D_\mu a^\mu$ contribute positively to the local acceleration of length scales through Raychaudhuri's equation (7). Rewriting Π^o in terms of the local acceleration of length scales by using (7) yields $\Pi^o = \frac{1}{9}\theta^2(1+3\frac{\dot{\theta}}{\theta^2}) - \frac{2}{3}a^\mu a_\mu - d^\mu d_\mu - \frac{4}{15}\sigma^{\mu\nu}\sigma_{\mu\nu} - \frac{4}{3}\omega^{\mu\nu}\omega_{\mu\nu}$, and it is evident that all correction terms in Π^o to the local acceleration of length scales in the observer frame $\frac{1}{9}\theta^2(1+3\frac{\dot{\theta}}{\theta^2})$ are nonpositive. For general relativistic space-times in which integrated values of Π are dominated by the monopole contribution Π^o for light propagation over cosmological distances, the only physical mechanisms that might result in the detection of positive values for redshift drift are thus (i) a special 4-acceleration profile of the space-time congruence of observers yielding integrated positive values of $D_\mu a^\mu - a^\mu a_\mu$ along the detected null rays[6]; (ii) violation of the strong energy condition. This realization is the main result of this paper: *A measured positive value of redshift drift indicates that the strong energy condition is violated.* A positive detection of redshift drift is in principle possible without such a violation, but it requires a 4-acceleration profile of the observer congruence giving systematic contributions along the null rays through its gradient. Alternatively, contributions from systematic alignment of the direction variables $\boldsymbol{e}$ and $\boldsymbol{d}$ with the dynamic fluid variables $\Pi^{\boldsymbol{e}}_\mu$, $\Pi^{\boldsymbol{d}}_\mu$, $\Pi^{\boldsymbol{ee}}_{\mu\nu}$, etc., in (18) over the length scales of light-propagation can cause inaccuracy of the approximation (19).

In the monopole approximation (19), inhomogeneities tend to act with negative contributions to the redshift drift signal. We might thus in general expect the redshift drift signal to be negative in the absence of sources violating the strong energy condition—even for space-times exhibiting globally defined acceleration of large scale cosmological volume sections due to the backreaction[7] of cosmic structures. This expectation is consistent with the numerical findings in [30,31] for general relativistic inhomogeneous models without a cosmological constant, where the only example of positive redshift drift signals was obtained in an unphysical space-time scenario with a source of negative energy density violating the strong energy condition [31].

B. Applicability for Lyman-α forest measurements

A promising probe of redshift drift in the near future is the Lyman-α forest, i.e., the plethora of absorption lines observed in the spectra of quasars, resulting from the Lyman-α electron transition of the neutral hydrogen atom [21,46]. Since the Lyman-α forest is only observable from the ground for $z \gtrsim 1.7$, the first detections of redshift drift are expected to be at redshifts where the ΛCDM model predicts close to zero or negative values of redshift drift [22]—see Fig. 14 of [22] for a comparison between the predicted redshift drift in the concordance ΛCDM model and in a FLRW model with the same matter density profile but without dark energy. Sources with $z \sim 1$ where the redshift drift signal is predicted to be maximal within the ΛCDM model must be probed by other strategies than ground based Lyman-α forest measurements—see, e.g., [23,47].

Taking the ΛCDM estimates at face value, the regime $2 \lesssim z \lesssim 5$ probed by the Lyman-α forest [21,22] is not suitable for directly probing the strong energy condition. However, we might observationally infer lower bounds on the integral of $u^\mu u^\nu R_{\mu\nu}$ from the emitter to the source, given the above monopole approximation, and given that assumptions can be made on the 4-acceleration profile of the observer. For instance, if we assume a geodesic observer congruence we have that (19) implies $\frac{dz}{d\tau}|_{\mathcal{O}} \leq -\frac{1}{3}E_{\mathcal{E}}\int_{\lambda_{\mathcal{E}}}^{\lambda_{\mathcal{O}}} d\lambda u^\mu u^\nu R_{\mu\nu}$. A measured lower bound $\frac{d\hat{z}}{d\tau}|_{\mathcal{O},\min} \leq \frac{dz}{d\tau}|_{\mathcal{O}}$ will in this case imply $\overline{E_{\mathcal{E}}\int_{\lambda_{\mathcal{E}}}^{\lambda_{\mathcal{O}}} d\lambda u^\mu u^\nu R_{\mu\nu}} \leq -3\frac{d\hat{z}}{d\tau}|_{\mathcal{O},\min}$, where the overbar represents the appropriate statistical averaging of sources within a redshift bin. Thus $\frac{d\hat{z}}{d\tau}|_{\mathcal{O},\min}$ constrains the integrated amount of ordinary matter density that can be present without violating the strong energy condition. If $\frac{d\hat{z}}{d\tau}|_{\mathcal{O},\min}$ is negative but close to zero, we might conclude that the matter density probed by null rays from the emitters to the source is close to zero, resembling an almost vacuum Universe, *or* that the strong energy condition has been violated in the space-time regime probed. On the other hand, if a detected upper bound on redshift drift signals is below that of the ΛCDM predicted signal, this would give rise to a reassessment of the ΛCDM paradigm of cosmology. As suggested by the current framework, the accumulated contribution of inhomogeneities along the null rays might bias redshift drift signals towards lower values than anticipated in the homogeneous and isotropic FLRW model universes and act as an effective matter source.

IV. CONCLUSION

In FLRW universe models a positive detection of redshift drift implies a nonzero cosmological constant [17–19] and hence violation of the strong energy condition. In this paper we have considered the redshift drift signal in a general space-time, and written it in a form useful for examining the sign of redshift drift and its link to the strong energy condition in general relativistic universe models with no symmetry assumptions imposed on the metric. The redshift drift signal can be written in terms of a physically

[6] In general relativistic perfect fluid cosmologies, the 4-acceleration field is given by $a_\mu = -D_\mu(p)/(\epsilon+p)$, where ϵ is the energy density and p is the pressure associated with the perfect fluid description [44] (see also the generalization in [45] to arbitrary general relativistic space-times). Accumulated positive values of $D_\mu a^\mu - a^\mu a_\mu = -D^\mu D_\mu(p)/(\epsilon+p) + D^\mu(p)D_\mu(\epsilon)/(\epsilon+p)^2$ along null rays might in this case occur for specific pressure and energy density profiles.

[7] For overviews of backreaction in cosmological modeling, see, e.g., [34,35].

interpretable multipole series, where the coefficients are given in terms of kinematic variables and 4-acceleration of the observer congruence along with Ricci and Weyl curvature variables. The monopole contribution in this series represents the isotropic contribution common for all directions on the sky of the observer congruence. In a Universe where this monopole contribution is statistically dominant in the integral over the light path, and where the 4-acceleration profile of observers is not of a special form, a measured positive value of redshift drift is a direct signature of violation of the strong energy condition. We have discussed the applicability of the derived results for upcoming Lyman-α forest measurements of redshift drift.

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