Letter

Remarks on nuclear matter: How an ω_0 condensate can spike the speed of sound, and a model of $Z(3)$ baryons

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I make two comments about nuclear matter. First, I consider the effects of a coupling between the $O(4)$ chiral field, $\vec{\phi}$, and the ω_μ meson, $\sim + \vec{\phi}^2 \omega_\mu^2$; for any net baryon density, a condensate for ω_0 is unavoidably generated. I assume that with increasing density, a decrease of the chiral condensate and the effective ω_0 mass gives a stiff equation of state (EOS). In order to match that onto a soft EOS for quarkyonic matter, I consider an $O(N)$ field at large N, where at nonzero temperature quantum fluctuations disorder, any putative pion "condensates" into a quantum pion liquid (QπL) [R. D. Pisarski *et al.*, Phys. Rev. D **102,** 016015 (2020)]. In this paper, I show that the QπL persists at zero temperature. If valid qualitatively at $N = 4$, the ω_0 mass goes up sharply and suppresses the ω_0 condensate. This could generate a spike in the speed of sound at high density, which is of relevance to neutron stars. Second, I propose a toy model of a $Z(3)$ gauge theory with three flavors of fermions, where $Z(3)$ vortices confine fermions into baryons. In $1 + 1$ dimensions, this model can be studied numerically with present techniques, using either classical or quantum computers.

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I. INTRODUCTION

To determine the equation of state (EOS) for nuclear matter, effective models can be used for baryon densities, n_B, up to and above that for nuclear saturation, n_{sat} [1–10]. Neutron stars probe densities $n_B > n_{\mathrm{sat}}$. In the past few years, the observation of neutron stars with masses above two solar masses [11,12] and astronomical observations, especially of their mergers in binary systems [13–32], has provided a wealth of data. Many models apply above n_{sat} [33–101], but to date, consensus is lacking.

In this paper, I make two comments about nuclear matter. The first is a suggestion as to how ω_0 [1–3,5] and pion [102–109] condensates can affect the EOS and generate nonmonotonic behavior for the speed of sound as the density increases well above n_{sat}. The second is a toy model in which fermion fields, analogous to quarks, are confined into baryons by a $Z(3)$ gauge field.

II. SPIKING THE SPEED OF SOUND

In the past few years, astronomical observations of neutron stars [11–32] have provided a significant insight into the nuclear EOS at densities above n_{sat}. This includes quantities such as their mass, radius, and tidal deformability. The EOS is given by the pressure, p, as a function of the energy density, e. Analyses with piecewise polytropic EOS are useful [18–21].

However, a more sensitive probe of the EOS is given by the speed of sound squared: $c_s^2 = \partial p/\partial e$. Free, massless fermions have $c_s^2 = 1/3$, which is termed soft. In contrast, several studies of neutron stars find that it is *essential* for the nuclear EOS to have a region in which the EOS is stiff, where c_s^2 is significantly larger than $1/3$ [22–32].

For example, consider the analysis of Drischler *et al.* [26], who extrapolate up from n_{sat} using chiral effective field theory. To obtain neutron stars with masses above two solar masses, they find that there is a region of density in which the EOS is stiff: if there is a neutron star of 2.6 solar masses, at some n_B, $c_s^2 \sim 0.55$. To agree with small tidal deformability from GW170817, though, the EOS of nuclear matter must be soft until $n_B \sim 1.5$–$1.8 n_{\mathrm{sat}}$. That is, there is a "spike" in the speed of sound, with a relatively narrow peak at a density significantly above n_{sat}: see, e.g., Fig. 1 of Greif *et al.* [23].

As the density $n_B \to \infty$, by asymptotic freedom the EOS approaches that of an ideal gas of dense quarks and gluons, and so is soft. In quantum chromodynamics (QCD), corrections to the quark EOS have been computed in part up to four loop order [110–114]. However, perturbation theory is only useful down to densities much larger than n_{sat}. At very high densities, excitations near the Fermi surface are dominated by color superconductivity [33,34,36].

Going down in density, nuclear matter becomes quarkyonic [48–60]. The free energy is close to that of QCD perturbation theory, but the excitations near the Fermi surface are confined, and so baryonic. A quarkyonic regime is inescapable for a $SU(N_{\rm color})$ gauge theory as $N_{\rm color} \to \infty$, as then quark loops are suppressed by $\sim 1/N_{\rm color}$. This does not seem to be special to large $N_{\rm color}$, though. In lattice gauge theory, when $N_{\rm color} \geq 3$, the sign problem prevents classical computers from computing at zero temperature and nonzero quark density [115]. Two colors, however, is free of the sign problem, and while it has unique features—notably, since baryons are bosons there is no Fermi sea—lattice simulations find a broad quarkyonic region [116–121]. This suggests the same applies to QCD, where $N_{\rm color} = 3$.

I assume that the quarkyonic EOS is soft. Bedaque and Steiner [122] have argued, from a variety of examples, *any* quasiparticle model is soft. While some authors propose that quarks can give a stiff EOS [28–30], for simplicity I do not.

To match a nuclear onto a quarkyonic EOS, McLerran and Reddy take a quark EOS up to some Fermi momentum k_{FQ}, which is then surrounded by a baryonic shell of width Δ, Fig. 1 of Ref. [54]. As the baryon density increases, k_{FQ} grows and Δ shrinks. Taking an ideal EOS for both quarks and baryons, an appropriate choice of the width Δ generates a spike in the speed of sound, Fig. 2 of Ref. [54].

At densities near k_{FQ}, though, *neither* equation of state is close to ideal. The quantum hadrodynamics (QHD) of Serot and Walecka [2,3,5] can be used for baryons, although to be capable of modeling a confined but chirally symmetric phase, all chiral partners of the nucleons and mesons must be included in a parity-doubled QHD (PdQHD) [63–70,75–79]. The quark EOS can be modeled by coupling quarks and gluons to a linear sigma model for mesons. Such a PdQHD was considered by Cao and Liao [78].

My purpose here is to discuss, in an entirely qualitative manner, of how an ω_0 condensate and strong fluctuations in a pion "condensate" [102–109] could affect the EOS in PdQHD. My discussion is admittedly speculative, because given the wealth of experimental data, it is not easy to describe the EOS of nuclear matter both near $n_{\rm sat}$ and at $n_B \gg n_{\rm sat}$.

In QHD, saturation results from a balance between repulsion from the ω_μ meson and attraction from the σ meson [2,3,5]. That the ω_μ meson could generate a stiff EOS was first noted by Zel'dovich [1]. Given the coupling of the ω_μ to a nucleon ψ as $\sim g_\omega \bar{\psi}\omega_\mu \gamma^\mu \psi$, then at any nonzero baryon density, $\langle \bar{\psi}\gamma^0\psi\rangle = n_B \neq 0$, a condensate for ω_0 is automatically generated [123],

$$\mathcal{L}^B_\omega = -g_\omega n_B \omega_0 + \frac{m_\omega^2 \omega_\mu^2}{2} \Rightarrow \langle \omega_0 \rangle = \frac{g_\omega}{m_\omega^2} n_B. \quad (1)$$

If only these terms matter, then the EOS is as stiff as possible, with the speed of sound equal to that of light, $c_s^2 = 1$. Son and Stephanov showed that QCD at nonzero isospin density provides a precise example of this [124].

Of course, in QHD, Eq. (1) is not the only term which matters. Integrating over nucleon loops at nonzero density, there is an infinite series of terms in ω_0 which are generated at $n_B \neq 0$, including those $\sim \omega_0^2$, $\sim \omega_0^3$, and so on. Similarly, the nucleon couples to the σ, whose properties also change with n_B. These effects have been computed to one loop order [2,3,5], but even for strong g_ω, do not dramatically alter the EOS.

The ω_μ Lagrangian is

$$\mathcal{L}_\omega = \frac{\mathcal{F}_{\mu\nu}^2}{4} + \frac{1}{2}(\tilde{m}_\omega^2 + \kappa^2 \vec{\phi}^2)\omega_\mu^2, \quad (2)$$

$\mathcal{F}_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu$ is the field strength for ω_μ, $\tilde{m}_\omega$ a mass term, and there is a quartic coupling $\sim \kappa^2$ between ω_μ and $\vec{\phi}$, where $\vec{\phi}$ is the $O(4)$ chiral field for two light flavors, $\vec{\phi} = (\sigma, \vec{\pi})$. The coupling κ^2 must be positive to ensure stability for large values of the ω_μ and $\vec{\phi}$ fields. (Incidentally, while the term $\sim \tilde{m}_\omega^2$ can be written in gauge invariant, unitarity form [125–130], that $\sim \kappa^2$ cannot [131].)

Although the coupling κ^2 violates vector meson dominance [90–92,132–140], this is relatively innocuous. For the ρ_μ meson, in vacuum, a similar term $\sim \kappa^2 \vec{\rho}_\mu^{\,2}$ just shifts the ρ_μ mass and does not alter its electromagnetic couplings. The mass of the ω_μ meson is $m_\omega^2 = \tilde{m}_\omega^2 + \kappa^2 \langle \vec{\phi}^2 \rangle$. Spontaneous symmetry breaking occurs in the QCD vacuum, $\langle \phi^i \rangle = (\sigma_0, 0)$, where for two flavors, $\sigma_0 = f_\pi$, the pion decay constant. Thus, in vacuum, in mean field theory, the mass squared of the ω_μ meson is $m_\omega^2 = \tilde{m}_\omega^2 + \kappa^2 \sigma_0^2$. Large $\tilde{m}_\omega$ favors small κ and vice versa. For massless pions, at tree level, $\tilde{m}_\omega = 0$ when $\kappa^2 = m_\omega / f_\pi \sim 8.4$.

Couplings similar to κ^2 have appeared before. In Refs. [7,35], a chirally asymmetric term $\sim \sigma^2 \omega_\mu^2$ was added to the Lagrangian, but the generalization to a chirally symmetric term is obvious. Reference [72] introduced mixing the ω_μ and $\vec{\rho}_\mu$ mesons, $\sim \vec{\rho}_\mu^{\,2} \omega_\mu^2$, and note the possibility of terms $\sim (\omega_\mu^2)^2$, etc. The implications of these terms for neutron stars were computed in Refs. [35,71–73,96]. References [69,70,74,75] denote κ^2 as h_1, but neglect it, as $\kappa^2 \sim 1/N_{\rm color}$ is small for a large number of colors. Reference [78] denotes κ^2 as g_{SV}; for massive pions, they find $\kappa \sim 9.0$, which seems large compared to the upper bound of $\kappa^2 < 8.4$ for massless pions.

As the density increases, chiral symmetry breaking becomes weaker and σ_0 decreases; m_ω decreases, at least as long as $\langle \vec{\phi}^2 \rangle = \sigma_0^2$. This is reminiscent of the scaling of Brown and Rho [37–47].

For the $\vec{\phi}$ Lagrangian, I take [102]

$$\mathcal{L}_\phi = \frac{(\partial_0\vec{\phi})^2}{2} + \frac{(\partial_i^2\vec{\phi})^2}{2M^2} + \frac{Z(\partial_i\vec{\phi})^2}{2} + \frac{m_0^2\vec{\phi}^2}{2} + \frac{\lambda(\vec{\phi}^2)^2}{4} \quad (3)$$

and work in the chiral limit, so there is no term linear in $\vec{\phi}$. Notice that the ω_μ meson does *not* appear [61,62,69,70,75–78]. This is because ω_μ corresponds to the $U(1)_B$ of baryon number, and with q the quark fields, $\phi \sim \bar{q}q$ is invariant under $U(1)_B$. Conversely, when $\vec{\rho}_\mu$ and $\vec{a}_1^\mu$ mesons are added, they do appear in Eq. (3), since $\vec{\phi}$ transforms nontrivially under $SU(2)_L \times SU(2)_R$. The ω_μ meson does interact with pions through anomalous interactions generated by the Wess-Zumino-Witten Lagrangian, such as $\omega_\mu \to 3\pi$ [62,69,70,74–82]. These interactions survive in the chirally symmetric phase and typically become $\omega \to \sigma\pi\pi\pi$, Eq. (15) of [62].

The anomalous interactions, though, all involve at least three derivatives, which for the ω_0 meson, are all spatial derivatives. This is why the coupling $\sim\kappa^2$ in Eq. (2) is so important, as the *only* renormalizable, nonderivative coupling which the ω_μ has with the chiral field $\vec{\phi}$.

This assumes that processes which violate the axial $U(1)_A$ symmetry survive at densities far above $n_{\rm sat}$, as indicated by an analysis using a dilute gas of instantons [141]. If at zero temperature and nonzero baryon density the axial $U(1)_A$ symmetry remains strongly broken by topologically nontrivial configurations even when the $O(4) = SU(2)_L \times SU(2)_R$ chiral symmetry is restored, then as the ω_μ meson is a chiral singlet, it need not become degenerate with its parity partner, which is presumably the $f_1(1285)$ [74]. This is unlike mesons which carry flavor, such as the ρ_μ and a_1^μ, which are degenerate in a chirally symmetric phase. Similarly, this is why I can restrict the chiral symmetry to be $O(4) = SU(2)_L \times SU(2)_R$ and not $U_A(1) \times SU(2)_L \times SU(2)_R$ [142].

Of course, the ω_μ meson interacts directly with nucleons [2,3,5,80–82]. The CBELSA/TAPS experiment found that the mass of the ω_μ does not shift significantly at nuclear densities [143,144], although its width is over 30 times larger than in vacuum [145–150]. This does not concur with QHD [2,3,5] nor Refs. [37–47], where the ω_μ mass decreases by $n_{\rm sat}$.

This does not exclude changes as the baryon density exceeds $n_{\rm sat}$. For the usual analyses of QHD [2,3,5], Refs. [37–47], and PdQHD [63–70,74–79], σ and ω_0 masses both decrease, as the balance between σ attraction and ω_0 repulsion gives a soft EOS.

My principal assumption is that for some $n_B > n_1 > n_{\rm sat}$, that one enters a region dominated by the ω_0 condensate. Notably, if Z decreases with increasing n_B, the effective mass squared of the σ increases as $\sim 1/Z$, while if $\kappa \neq 0$, the ω_0 becomes light as the chiral symmetry is restored. By Eq. (1), $\mathcal{L}_\omega^B = -g_\omega^2 n_B^2/(2m_\omega^2)$, and a heavy σ, with a light ω_0, could generate a stiff EOS for $n_B > n_1$.

Assuming that a light ω_0 gives a stiff EOS, then *how* can the ω_0 condensate evaporate to match onto a soft quarkyonic EOS? Presumably the couplings of the ω_μ with nucleons behave smoothly with density. That leaves the couplings of the ω_μ to the chiral field $\vec{\phi}$, but as demonstrated above, these are limited. This question does assume that a hadronic phase matches onto quarkyonic matter. It is possible to simply paste a stiff hadronic EOS onto a soft quark EOS through what is presumably a strongly first order transition. This is not consistent, however, with the analyses for either $N_{\rm color} \to \infty$ [48–60] or lattice results for $N_{\rm color} = 2$ [116–121], which indicate a quarkyonic regime. Nor why the nuclear EOS appears to be soft near $n_{\rm sat}$ and only stiff when $n_B \sim 1.5$–$1.8 n_{\rm sat}$ [26].

I stress that reducing the contribution of the ω_0 condensate at large chemical potential, μ, and low temperature, $\mu \gg T$, has *no* analogy to the more familiar case, at nonzero temperature and low density. When $T \gg \mu$, it is easy matching the EOS of hadronic matter, with a relatively few degrees of freedom, onto a quark-gluon plasma, with many. This is precise in the limit of a large number of colors, $N_{\rm color} \to \infty$, where the pressure in the hadronic phase is $\sim N_{\rm color}^0$ versus $\sim N_{\rm color}^2$ in the deconfined phase. Similarly, the contribution of the chiral condensate is only $\sim N_{\rm color}^1$ and decreases as T increases. In contrast, at $\mu \gg T$, the pressure is always $\sim N_c^1$, in both the hadronic and quark-gluon phases.

At nonzero density, the appearance of a condensate for ω_0 is special to the ω_μ meson: there is no other hadron which couples directly to the net baryon density. This assumes that the only net charge is for baryon number. When there is a net isospin charge, a condensate for the $\vec{\rho}_\mu$ meson is generated, $\sim\rho_0^3$. In this case, terms such as $\vec{\phi}^2\vec{\rho}_\mu^2$, among others [69,70,74,75], need to be included; further, couplings between the ω_μ and $\vec{\rho}_\mu$ mesons, $\sim\vec{\rho}_\mu^2\omega_\mu^2$, must be added [72].

It is then *very* difficult to fit the EOS of an ω_0 condensate onto that of cold quarks: either the coupling of the ω_0 becomes small, or the mass of the ω_0 becomes large. Since the coupling of the ω_0 is strong in vacuum, the former is most implausible. Thus, the mass of the ω_0 must increase, although this does not occur in mean field theory [151]. I now argue that the mass of the ω_0 increases sharply due to large quantum fluctuations.

Returning to the Lagrangian in Eq. (3), it is standard except for the term quartic in the spatial derivatives, $\sim 1/M^2$ [152]. Causality implies that only terms with two time derivatives enter. With the term $\sim 1/M^2$ to ensure stability, it is possible to allow the coefficient of the term with two spatial derivatives, Z, to be negative.

While in vacuum $Z = 1$ by Lorentz covariance, this is not true in a medium. If Z is negative, classically a condensate is generated, $\vec{\phi} = \sigma_0(\cos(k_c z), \sin(k_c z), 0, 0)$, where $k_c^2 = -ZM^2/2$. This is a pion condensate in the z direction [102–109,153]. In $1+1$ dimensions, such chiral spiral condensates are *ubiquitous* at low temperature and nonzero density [106–108], although in general the solutions are more involved. Given these examples, it is natural to assume that in QCD, at low temperature $Z < 0$ for some range in density above $n_{\rm sat}$.

Most discussions of a pion condensate use a nonlinear Lagrangian, in which the σ meson does not explicitly appear. The advantage of using a linear Lagrangian is that it is much easier studying how the symmetric phase is approached. Following Ref. [102], I generalize from $O(4)$ to $O(N)$, where the solution is direct as $N \to \infty$ [142,154].

The solution at large N is standard, and proceeds by introducing the a field $\xi = \vec{\phi}^2$, and a constraint field, ϵ, $\mathcal{L}_{\rm cons} = i\epsilon(\xi - \vec{\phi}^2)/2$. I only seek the solution for the symmetric phase, although the solution in the broken phase can also be determined [102]. Using this constraint, the $\vec{\phi}$ and ξ fields are integrated out to give the effective action

$$\mathcal{S}_{\rm eff} = \frac{N}{2}{\rm tr}\log\Delta^{-1} + \int d^4x\left(\frac{\epsilon^2}{4\lambda} + \frac{\tilde{m}_\omega^2\omega_\mu^2}{2} - g_\omega\omega_0\rho_B\right), \qquad (4)$$

Δ^{-1} is the inverse propagator for the $\vec{\phi}$ field, which in momentum space is $\Delta^{-1}(\omega, \vec{k}) = \omega^2 + E(k)^2$, where $E(k)^2 = (\vec{k}^2)^2/M^2 + Z\vec{k}^2 + m_{\rm eff}^2$. I expand about a stationary point in ϵ and ω_0, $\epsilon = i\hat{\epsilon} + \epsilon_q$ and $\omega_0 = \hat{\omega}_0 + \omega_0^q$, where ϵ_q and ω_0^q are quantum fluctuations. The effective mass $m_{\rm eff}^2 = m_0^2 + \hat{\epsilon} + \kappa^2\hat{\omega}_0^2$. To have a well-defined limit for large N, as $N \to \infty$ all terms in the action should scale as $\sim N$, so I take $\lambda, \kappa^2 \sim 1/N$, $g_\omega\rho_B, \hat{\omega}_0 \sim \sqrt{N}$, and $\tilde{m}_\omega^2, M^2, Z, m_0^2, \hat{\epsilon}, m_{\rm eff}^2 \sim N^0$. Remember that N is just a fictitious parameter and is not related to the number of colors or flavors.

Requiring that the effective action is a stationary point in ϵ_q and ω_0^q fixes $\hat{\epsilon}$ and $\hat{\omega}_0$,

$$\hat{\epsilon} = \lambda N {\rm tr}\Delta; \qquad \hat{\omega}_0 = \frac{g_\omega\rho_B}{\tilde{m}_\omega^2 + \kappa^2 N{\rm tr}\Delta}. \qquad (5)$$

The solution for general values of the parameters is involved, so to make a qualitative point I only consider the limit of $Z \to -\infty$, where classically the single mode condensate dominates. Instead, in perturbation theory, one finds that would be Goldstone bosons have a double pole at *non* zero momentum, about k_c [102]. Such a double pole generates a logarithmic infrared divergence at zero temperature and a power law divergence at nonzero temperature.

The solution at large N shows how these infrared divergences are avoided. As $Z \to -\infty$, at $N = \infty$ take $m_{\rm eff} \approx -ZM/2 + \delta m_{\rm eff}$; expanding $E(k)^2 \approx (k^2 - k_c^2)^2/M^2 - ZM\delta m_{\rm eff} + \cdots$ about $k = k_c$,

$$\delta m_{\rm eff} \approx \#\sqrt{-Z}M\exp\left(-\frac{2^{3/2}\pi^2}{\lambda N}(-Z)^{3/2}\right), \qquad (6)$$

where # is a positive, nonzero number.

It is worth contrasting this solution with that at nonzero temperature [102]. Then the integral over ω is a discrete sum, and the zero energy mode is the most important. It generates a power law divergence, with the solution $\delta m_{\rm eff} \approx 1/Z^4$, Eq. (58) of Ref. [102]. The statement in Ref. [102] that $\delta m_{\rm eff}$ vanishes at zero temperature is incorrect: it is just that $\delta m_{\rm eff}$ is suppressed exponentially in $1/\sqrt{-Z}$, instead of by a power. I refer to this disorder as a quantum pion liquid, QπL [155].

I have neglected the equation for $\hat{\omega}_0$ in Eq. (5). While $\delta m_{\rm eff}^2$ is very different at zero and nonzero temperature, though what matters there is the value of the loop integral. Since $\hat{\epsilon} \approx m_{\rm eff}^2 \sim Z^2M^2/4$, by Eq. (5) ${\rm tr}\Delta \approx Z^2M^2/(4\lambda N)$ when $Z \to -\infty$. As $\langle\vec{\phi}^2\rangle = N{\rm tr}\Delta$, the ω_0 mass increases sharply,

$$m_\omega^2 = \tilde{m}_\omega^2 + Z^2\frac{\kappa^2}{4\lambda}M^2, \qquad (7)$$

which by Eq. (5) suppresses the ω_0 condensate, $\hat{\omega}_0 \sim 1/Z^2$. Note that the presence of the coupling $\sim\kappa^2$ is essential for this to occur.

When Z is negative, classically one expects a pion condensate to form, but the solution at large N shows that instead a QπL forms. While this is rigorous at large N, as it arises from the double pole at $k_c \neq 0$ for the would be Goldstone modes, it is very likely that there is a QπL for *all* $N > 2$ [102]. I also assume that the quantum fluctuations are sufficiently strong so that a QπL forms even for massive pions.

My suggestion is thus the following. For $n_B > n_1 > n_{\rm sat}$, the theory enters a phase dominated by the ω_0 condensate, which stiffens the EOS. When $n_B > n_2$, it is approximately described by a QπL: both the σ and ω_0 are heavy, which suppresses $\hat{\omega}_0$. In total, the enhancement and then suppression of the ω_0 condensate generates a spike in the speed of sound.

Clearly, a detailed analysis is required to determine the dependence of the various parameters with density, or more properly for thermodynamics, with the baryon chemical potential, μ_B. This includes the μ_B dependence of the wave function renormalization constant Z, the mass parameter M (which is of some hadronic scale), m_0, λ, and so forth.

The most direct approach is to use PdQHD, with a self-consistent one loop approximation for the nucleons, the chiral fields $\vec{\phi}$, and the ω_0. While involved, I comment that it is far simpler to look for a QπL—which is just a nonmonotonic dispersion relation—than for a pion condensate, which is not spatially homogeneous [106].

As quantum computers are (very) far from computing the properties of cold, dense QCD [115], to proceed from first principles, requires the functional renormalization group (FRG) [90–92,133–140,156–159]. Reference [90] uses a chiral effective model up to $\sim 2n_{\rm sat}$, matching onto QCD perturbation theory with a Fierz complete FRG [137–139] at intermediate n_B. They find evidence for a spike in the speed of sound at $\sim 10n_{\rm sat}$ [90], which is much higher than Ref. [26]. The ultimate goal is to use the parameters determined by the FRG in vacuum [156–158] to compute the EOS for nuclear matter. Fu, Pawlowski, and Rennecke [159] find that $Z < 0$ at rather high T and $\mu_B \neq 0$, Fig. 21 of [159]. A complete FRG analysis should certainly see a quantum pion liquid, if it exists.

The pion is not an exact Goldstone boson, but I assume it is so light that the QπL phase wins over a pion condensate. The same may not be true for strange quarks [160]. When at some n_B the Fermi sea spills over to form one of strange quarks, if the pion Z is negative, by $SU(3)$ flavor symmetry that for kaons will be as well. As the strange quark is much heavier than up and down quarks, instead of a quantum kaon liquid, a kaon condensate *might* form [105]. This would be a crystal of real kinks, where $\langle \bar{s}s \rangle$ oscillates about a constant, nonzero value; Bringoltz [153] showed that this happens for the 't Hooft model in $1+1$ dimensions [161].

Admittedly my analysis is merely a sketch of how a spike in the speed of sound might arise in nuclear matter. It appears inescapable, though, that the interaction of the ω_0 and the chiral fields plays an essential role.

III. A MODEL OF $Z(3)$ BARYONS

Some properties of nuclear matter, such as those discussed above, are surely special to QCD. It would be useful, however, to have the simplest possible model which exhibits the confinement of some type of "quarks" into baryons. A $SU(N_{\rm color})$ gauge theory in $1+1$ dimensions [161] has baryons [153,162–164], but as $N_{\rm color} \to \infty$, there are $\sim N_{\rm color}^2$ degrees of freedom. There are also models in $1+1$ dimensions which are soluble about the conformal limit [107,108], but these do not generalize to higher dimensions.

An understanding of confinement from $Z(N_{\rm color})$ vortices in a $SU(N_{\rm color})$ gauge theory was proposed by 't Hooft [165,166]; for recent work, see [167–169] and references therein. I suggest discarding the non-Abelian degrees of freedom in $SU(N_{\rm color})$ to retain just those of $Z(N_{\rm color})$. A $Z(3)$ gauge theory is constructed following Krauss, Wilczek, and Preskill [170–172], with the Lagrangian

$$\frac{F_{\mu\nu}^2}{4} + |D_\mu^\chi \chi|^2 + m_\chi^2 |\chi|^2 + \lambda_\chi (|\chi|^2)^2 + \sum_{i=1}^{3} \bar{q}_i (D_\mu + m_q) q_i. \tag{8}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field strength for an Abelian gauge field, χ is a complex valued scalar, and there are three degenerate types, or "flavors" of fermions, q_i, with equal mass m_q. The q_i have unit charge, $D_\mu = \partial_\mu - ieA_\mu$, but I choose the scalar to have charge three, $D_\mu^\chi = \partial_\mu - 3ieA_\mu$.

I consider the case of $1+1$ dimensions first, and assume that the fermions are heavy. (Light fermions, $m_q \ll e$, may undergo spontaneous symmetry breaking, which because of the lack of Goldstone bosons in $1+1$ dimensions complicates the analysis [107,108,162–164] and is really secondary to my desire to construct a theory for nuclear matter.) If $m_\chi^2 < 0$, spontaneous symmetry breaking occurs and the photon becomes massive. For large distances, $> 1/(3e|m_\chi|)$, naively one expects that there is no interaction from the photons and the fermions propagate freely. Besides perturbative fluctuations, there are also vortices, which in two (Euclidean) dimensions are like pseudoparticles, localized at a given point. The vacuum is a superposition of vortices, where each vortex has an action $S_v \sim (m_\chi)^4/\lambda_\chi$. If χ had unit charge, the propagation of fermions is affected only when they are near a vortex and the vortices are relatively inconsequential.

When χ has charge three, however, a vortex can carry a $Z(3)$ charge, which greatly affects the propagation of the fermion. If a fermion of unit charge encircles a single vortex, it picks up an Aharonov-Bohm phase of $\exp(\pm 2\pi i/3)$. With a vacuum composed of an infinite number of vortices, these phases confine [173] the fermions entirely through these random phases, exactly analogous to how $Z(3)$ vortices in a $SU(3)$ gauge theory confine [168].

While a state such as q_1^3 is neutral under $Z(3)$, this vanishes, as q_1 is a fermion field which anticommutes with itself. This is different from QCD, where the antisymmetric tensor in color space can be used to form a baryon with one flavor, $\sim \epsilon^{abc} q_1^a q_1^b q_1^c$. Consequently, in a $Z(3)$ model to obtain (simple) baryons, it is necessary to take three flavors, so the baryon $\sim q_1 q_2 q_3$ is neutral under $Z(3)$. The mesons form an octet in flavor, which is (presumably) lighter than the singlet meson (plus higher excitations, of course).

In weak coupling, the action of a single vortex is small, vortices are dilute, and confinement occurs over large distances, $\sim \exp(-S_v)$. The fermions interact over distance $\sim 1/m_\chi$, but at long distances, only interact through the $Z(3)$ phases generated by the vortex ensemble in vacuum. These $Z(3)$ baryons are weakly bound over large distances, so that in any scattering experiment, it would be obvious that they have composite substructure. This is in contrast to QCD, where baryons have weak attraction at large distances, but a strong repulsive core at short distances.

That is, in QCD, it is hard prying the quarks out of a baryon. This would occur if the density of vortices is large. In the effective model above, this requires strong coupling, which cannot be studied analytically. However, this limit can be studied on the lattice and just produces a $Z(3)$ gauge theory [174] coupled to three flavors of degenerate fermions.

In $1+1$ dimensions, as for the $U(1)$ gauge theory [175,176], the $Z(N)$ gauge theory confines. On a lattice, classical computers have been used to study the properties in vacuum of $Z(2)$ [177–181] and $Z(3)$ [178,181] gauge theories with a single flavor. The behavior of a $U(1)$ theory with two flavors was computed at nonzero density in Ref. [182]. Thus, classical computers can be used to compute the properties of a $Z(3)$ gauge theory with three degenerate flavors at nonzero density. This can then provide a benchmark to compare against computing the free energy at nonzero density using quantum computers. The great advantage of a $Z(3)$ gauge group is that only two qubits are needed to describe a group element, as opposed to many more for any continuous gauge group.

In $2+1$ dimensions, the vortices sweep our lines in space-time and cylinders in $3+1$ dimensions. Assuming that $Z(3)$ vortices confine in QCD, these models should exhibit confinement as well. It would be interesting analyzing the behavior of $Z(3)$ nuclear matter at strong coupling as a counterpoint to that in QCD.

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