Letter

Producing fully charm structures in the J/ψ-pair invariant mass spectrum

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Focusing on the novel phenomenon of several enhancement structures existing in the invariant mass spectrum of a J/ψ pair, which was reported by LHCb very recently, we find a new dynamical mechanism based on the special reactions, where all the possible combinations of a double charmonium directly produced by a proton-proton collision are transitioned into final-state $J/\psi J/\psi$. Under this framework, totally different from the popular tetraquark explanation, the LHCb experimental data of the line shape of the J/ψ-pair invariant mass spectrum can be well mimicked. Since the proposed dynamical mechanism is universal enough, we further predict the possible enhancement structures existing in the $\Upsilon(1S)$-pair invariant mass spectrum, which may provide a realistic approach to test the new mechanism proposed in this work and form a new task for the LHCb and CMS experiments.

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I. INTRODUCTION

Since 2003, benefiting from the accumulation of more and more experimental data, searches for exotic multiquark matter have become a hot issue in hadron physics and attracted more and more interest from theorists and experimentalists. Important for the progress of this topic are the observations of a number of charmonium-like XYZ states and the hidden-charm pentaquark P_c states in the relevant high-energy experiments (see Refs. [1–5] for more details). The study of these novel phenomena has indeed enlarged our knowledge for the nonperturbative quantum chromodynamics (QCD).

Very recently, the LHCb Collaboration measured the J/ψ-pair mass spectrum by using proton-proton data at center-of-mass energies of 7, 8, and 13 TeV, where a narrow structure around 6.9 GeV was observed with a significant signal of more than a standard deviation of 5.1σ [6]. Besides this, there exists an obvious broad structure ranging from the threshold of di-J/ψ to 6.8 GeV, and an underlying peak near 7.3 GeV. This novel phenomenon stimulated research for fully heavy tetraquark states, and people discussed the possible assignment of $X(6900)$ in the corresponding spectroscopy [7–19]. In fact, as early as 1981, the exotic hadrons composed of $cc\bar{c}\bar{c}$ were systematically studied by Chao for the first time [20], who predicted that fully charm tetraquark states are all above the threshold of strong decay into two charmonia. Later, a similar conclusion was obtained in Refs. [21,22]. On the other hand, in Ref. [23], the authors used an MIT bag model with the Born-Oppenheimer approximation for heavy quarks and found that there exists a stable fully charm tetraquark against breakup into $c\bar{c}$ pairs. In the following decades, the mass spectra for fully heavy quark states have still been hotly debated in various model schemes—for instance, the quark potential model [24–30], the QCD sum rule [31–33], the nonrelativistic effective field theory [34], the covariant Bethe-Salpeter equations [35], and other phenomenological methods [36–39]. Anyway, all the studies mentioned above support the existence of exotic fully heavy quark hadrons, although this conclusion was also questioned in some theoretical papers [40,41].

When checking the LHCb experimental data of the line shape of the J/ψ-pair invariant mass spectrum [6], we notice an interesting fact. In the vicinity of the reported

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enhancement structures in J/ψ-pair invariant mass spectrum, we may find the corresponding thresholds of charmonium pairs, such as $\eta_c(1S)\chi_{c1}(1P)$, $\chi_{c0}(1P)\chi_{c1}(1P)$, and $\chi_{c0}(1P)\chi'_{c1}(2P)$. What is more important is that these charmonium pairs are transitioned into the $J/\psi J/\psi$ final state, which seems to have a close relation to the LHCb observation [6]. Inspired by such a novel phenomenon, in this letter we propose a new dynamical mechanism which is totally different from the traditionally introduced fully charm tetraquark explanation. After performing a realistic calculation of the discussed charmonium pair transitted into di-J/ψ, we find that the experimental line shape including the observed enhancement structures of J/ψ-pair invariant mass spectrum can be well mimicked. This successful description of the LHCb experimental data not only provides a peculiar viewpoint to decode the nature of this new observation, but also follows a famous proverb, "What you see is not what you get."

To test the proposed dynamical mechanism in this letter, we further extend our theoretical framework to study the transition of possible combinations of a double bottomonium directly produced by a proton-proton collision into final-state $\Upsilon(1S)\Upsilon(1S)$, and we predict the possible enhancement structures existing in the $\Upsilon(1S)$-pair invariant mass spectrum, which are around 19.0, 19.3, and 19.7 GeV. Obviously, this will be a new task for the LHCb and CMS experiments.

II. DYNAMICAL MECHANISM

Actually, the Large Hadron Collider (LHC) is an ideal experimental platform to search for fully charm structures with the $cc\bar{c}\,\bar{c}$ configuration, where their production can be achieved by the hadronization of four charm quarks in the single-parton scattering (SPS) [42–49] process of $gg \to c\bar{c}c\bar{c} + X$. Then, it can dominantly decay into a pair of charmonium states like $\eta_c\eta_c$, $J/\psi J/\psi$, etc., through the so-called fall-apart decay mechanism [7,50,51]. Among the allowed final states, the $J/\psi J/\psi$ final state is the most promising candidate to search for fully charm structures above the production threshold, since a J/ψ particle can be effectively reconstructed by a $\mu^+\mu^-$ pair via a muon detector. On the other hand, more double-J/ψ events can be directly produced by the single-parton scattering (SPS) [42–49] and the double-parton scattering (DPS) [52–55] processes in high-energy proton-proton collisions, which usually correspond to a continuum contribution to the invariant mass spectrum of $J/\psi J/\psi$. Thus, it seems that several structures in the mass spectrum of di-J/ψ observed by LHCb are indeed good candidates of fully charm tetraquark states. The origin of resonance peak phenomena in hadron physics are, however, usually complicated more than our common understanding. Hence, the nature of the new structure $X(6900)$ observed by LHCb has to be judged carefully.

Various combinations of double charmonia can be directly produced by both the SPS and DPS processes. Then, the double charmonium allowed by quantum numbers from direct production can be transferred into the final states of $J/\psi J/\psi$. The relevant schematic diagrams are shown in Fig. 1, where the interaction among intermediate charmonium pairs and J/ψ pairs are absorbed into a vertex.

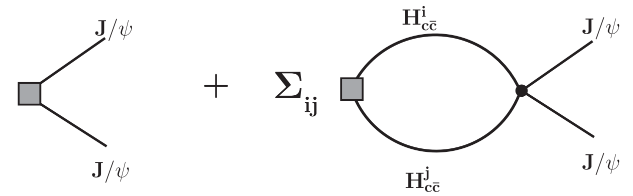

FIG. 1. The schematic diagrams for the production mechanism of a double charmonium $J/\psi J/\psi$, where $H^i_{c\bar{c}}$ stands for allowed intermediate charmonium states, such as η_c, J/ψ, χ_{cJ} with $J = 0$, 1, 2, etc. Here, the gray rectangle corresponds to direct production of a double charmonium in hadron collisions.

Starting from an S-wave interaction between intermediate charmonium pairs, the production amplitude of double J/ψ by the rescattering mechanism in Fig. 1 becomes proportional to the scalar two-point loop integral, whose analytical form can be given in the rest frame of di-J/ψ by

$$
\begin{aligned}
&L_{ij}(m_{J/\psi J/\psi})\\
&=\int \frac{dq^4}{(2\pi)^4}\frac{e^{-(2\vec{q})^2/\alpha^2}}{(q^2-m_i^2+i\epsilon)((P-q)^2-m_j^2+i\epsilon)}\\
&\simeq \frac{i}{4m_im_j}\left\{\frac{-\mu\alpha}{\sqrt{2}(2\pi)^{3/2}}+\frac{\mu\sqrt{2\mu m_0}(\mathrm{erfi}[\frac{\sqrt{8\mu m_0}}{\alpha}]-i)}{2\pi/e^{-\frac{8\mu m_0}{\alpha^2}}}\right\}, \qquad (1)
\end{aligned}
$$

where m_i (m_j) is the mass of an intermediate charmonium state $H^i_{c\bar{c}}$ ($H^j_{c\bar{c}}$) marked in Fig. 1, and $\mu = (m_im_j)/(m_i + m_j)$ and $m_0 = m_{J/\psi J/\psi} - m_i - m_j$. The value $P = (m_{J/\psi J/\psi}, 0, 0, 0)$ stands for the four-momentum of a double-J/ψ system, and erfi(x) represents the imaginary error function. Here, an exponential form factor $e^{-(2\vec{q})^2/\alpha^2}$ is introduced to avoid the ultraviolet divergence of a scalar two-point loop integral, and α is a cutoff parameter. It is worth emphasizing that the analytical expression in Eq. (1) is based on nonrelativistic approximations. Such nonrelativistic treatments should be proper in our dynamical mechanism, since the typical velocity of intermediate charmonium states should be relatively small in the considered di-J/ψ invariant mass region.

For the one-loop rescattering processes in Fig. 1, there exists a square-root branch point, $\sqrt{m_{J/\psi J/\psi} - m_i - m_j}$, where an integral singularity at the threshold of $m_i + m_j$ appears at the on-shell of two intermediate charmonium states. The threshold singularity causes a cusp exactly at the corresponding threshold in the invariant mass distribution of $m_{J/\psi J/\psi}$. Here, according to the sequence of threshold magnitudes and the criterion of quantum number conservation, $H^i_{c\bar{c}}H^j_{c\bar{c}} = J/\psi J/\psi$, $\eta_c\chi_{cJ}$, $J/\psi h_c$, $\chi_{cJ}\chi_{cJ}$, and $\chi_{c0}\chi'_{c1}$, with $J = 0$, 1, 2, are selected. In this work, without any special emphasis, η_c, h_c, χ_{cJ}, and χ'_{c1} refer to $\eta_c(1S)$, $h_c(1P)$, $\chi_{cJ}(1P)$, and $\chi'_{c1}(2P)$, respectively, and $\chi'_{c1}(2P) = X(3872)$ [56–60]. This selection covers the energy region from 6.194 to 7.400 GeV, in which there exist several relatively clear peaks measured by LHCb [6]. It is worth emphasizing that the direct production rates of η_c, $X(3872)$, and P-wave charmonium states χ_{cJ} with $J = 0$, 1, 2 in high-energy proton-proton collisions have been proven to be comparable with that of the J/ψ particle by both experiments [61,62] and theoretical calculations from nonrelativistic QCD (NRQCD) [63–71]. Combined with an S-wave coupling on the production vertex of their combinations, it is fully possible that the resonance-like shapes from the rescattering processes are responsible for the recent LHCb data without introducing any hadronic resonances of the configuration of $cc\bar{c}\,\bar{c}$. In the following, in order to verify the above idea, we will focus on the experimental line shape of an invariant mass spectrum of di-J/ψ by LHCb.

The direct production of double charmonium from SPS and DPS processes in high-energy proton-proton collisions usually behaves like a continuous distribution in its invariant mass spectrum, whose complete estimation is very complicated and parameter dependent in theoretical approaches [42–47,72–76]. So, in this work, referring to the treatment of the experimental analysis of LHCb [6], the invariant mass distribution of direct production of a double charmonium $H^i_{c\bar{c}}H^j_{c\bar{c}}$ with an S wave can be parametrized as

$$\mathcal{A}^2_{\rm direct} = g^2_{\rm direct} e^{c_0 m_{ij}} \frac{1}{8\pi} \frac{\sqrt{\lambda(m^2_{ij}, m^2_i, m^2_j)}}{m^2_{ij}}, \qquad (2)$$

where $\lambda(x,y,z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$ is the Källen function, and m_{ij} is the corresponding invariant mass. For the rescattering processes with two types of intermediate charmonium pairs, $H^i_{c\bar{c}}H^j_{c\bar{c}} = J/\psi J/\psi, \chi_{cJ}\chi_{cJ}, \chi_{c0}\chi'_{c1}$ and $H^i_{c\bar{c}}H^j_{c\bar{c}} = \eta_c\chi_{cJ}, J/\psi h_c$ in Fig. 1, the line shapes on the invariant mass spectrum of $m_{J/\psi J/\psi}$ are given by

$$\mathcal{A}^2_{ij}(m_{J/\psi J/\psi}) = g^2_{ij} L^2_{ij}(m_{J/\psi J/\psi}) \frac{e^{c_0 m_{J/\psi J/\psi}} p_{J/\psi}}{m_{J/\psi J/\psi}} \qquad (3)$$

and

$$\mathcal{A}'^2_{ij}(m_{J/\psi J/\psi}) = g^2_{ij} L^2_{ij}(m_{J/\psi J/\psi}) \frac{e^{c'_0 m_{J/\psi J/\psi}} p^3_{J/\psi}}{m_{J/\psi J/\psi}}, \qquad (4)$$

respectively, where $p_{J/\psi}$ is the momentum of a final state J/ψ. It is worth noting that the system parities of the above two types of rescattering processes are $P = +1$ and $P = -1$, respectively. As indicated in Eq. (2), a form factor in the form $e^{c_0 m_{ij}}$ is introduced to depict the energy dependences of double-charmonium production in the pp collision. In the rescattering process, all the charmonium pairs transit into di-J/ψ via loops; thus, the invariant mass m_{ij} should be equivalent to $m_{J/\psi J/\psi}$ and can be factored out from the loop integral in Eq. (1). Moreover, the parameter c_0 could be different for every single rescattering channel. Here, for simplicity, we only introduce two kinds of parameters: c_0 and c'_0 in the rescattering amplitudes corresponding to different P parities.

Thus, the total line shape for the invariant mass distribution of producing a double J/ψ in high-energy proton-proton collisions can be written as

$$\mathcal{A}^2 = \left|\mathcal{A}_{\rm direct}(m_{J/\psi J/\psi}) + \sum_{mn} e^{i\phi^{mn}} \mathcal{A}_{mn}(m_{J/\psi J/\psi})\right|^2 + \left|\mathcal{A}'_{\rm direct}(m_{J/\psi J/\psi}) + \sum_{mn} e^{i\phi^{mn}} \mathcal{A}'_{mn}(m_{J/\psi J/\psi})\right|^2, \qquad (5)$$

where ϕ^{mn} is the phase between direct contribution and the corresponding rescattering process. Here, we introduce a background term $\mathcal{A}'_{\rm direct} = (\frac{g'^2_{\rm direct}\lambda(m^2_{J/\psi J/\psi}, m^2_{J/\psi}, m^2_{J/\psi})^{\frac{1}{2}} e^{c'_0 m_{J/\psi J/\psi}} p^2_{J/\psi}}{8\pi m^2_{J/\psi J/\psi}})^{\frac{1}{2}}$ for the rescattering amplitude $\mathcal{A}'_{mn}$ with $P = -1$, which corresponds to a direct production of the P-wave double J/ψ.

III. LINE SHAPE OF J/ψ-PAIR INVARIANT MASS SPECTRUM

With the above preparations, we can directly study the LHCb data based on our proposed dynamical mechanism. In the LHCb data, two significant structures appear in the vicinities of 6.5 and 6.9 GeV. By checking the thresholds of a pair of charmonia, one can find that the thresholds of $\chi_{c1}\eta_c$ and $\chi_{c0}\chi_{c1}$ are about 6.495 and 6.925 GeV, respectively. Therefore, we fit the experimental line shape of the invariant mass spectrum of di-J/ψ by including $\chi_{c1}\eta_c$ and $\chi_{c0}\chi_{c1}$ rescattering contributions. In the present scenario, there are eight free parameters against 100 LHCb data. The fitted line shapes are presented in Fig. 2, and the values of the fitting parameter are listed in Table I. The corresponding value of $\chi^2/\text{d.o.f.}$ is estimated to be 1.54. Such a good fitting result based on an appropriate parameter number means that our proposed dynamical model could successfully provide a fantastic explanation to $X(6900)$ and an underlying fully charm structure at 6.5 GeV.

Although the overall line shape of LHCb's data can be described by introducing two intermediate channels of $\chi_{c1}\eta_c$ and $\chi_{c0}\chi_{c1}$, we still notice that there might be more underlying structures involved in the di-J/ψ mass spectrum. Therefore, for completeness of the theoretical analysis, we will continue to investigate the rescattering contributions from more intermediate double-charmonia channels. From our analysis, one can find that there exist eleven predicted threshold cusps at the di-J/ψ energy region from 6.194 to 7.400 GeV, which far exceed the number of visible obvious peak structures observed by

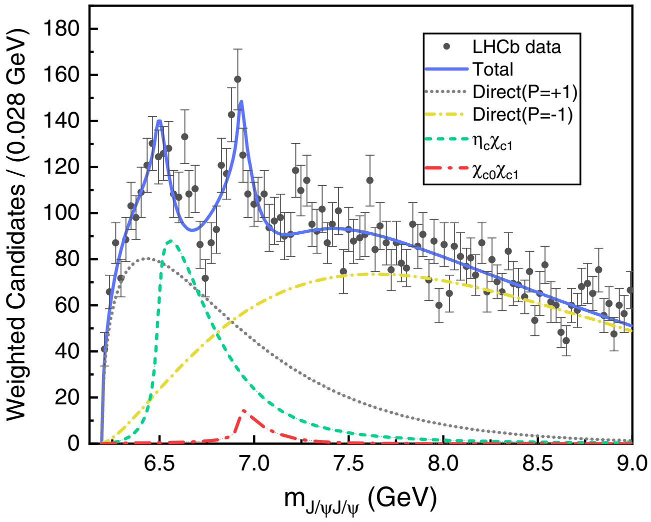

FIG. 2. The minimum parameter fit to the line shape of the LHCb data [6] based on a dynamical rescattering mechanism.

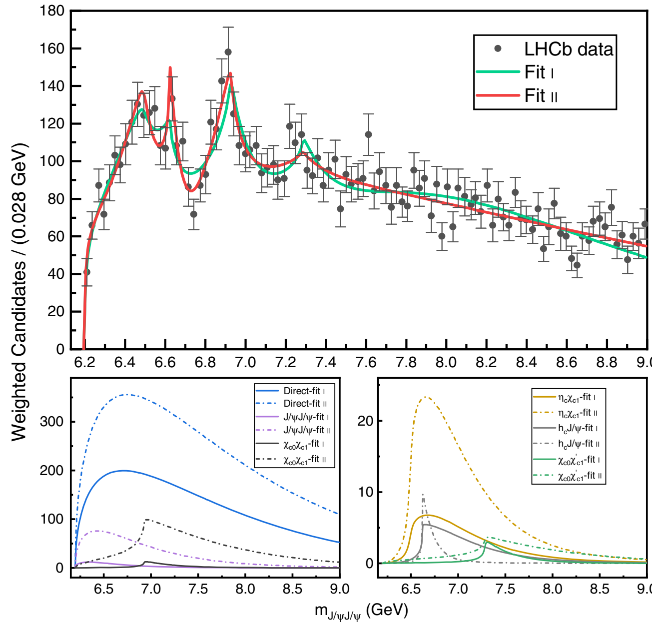

FIG. 3. The complete fit of our line shape to the LHCb data [6] based on a dynamical rescattering mechanism. Here, two fitting schemes—Fit I and Fit II—are introduced, which correspond to the same and independent cutoffs α for different combinations of intermediate charmonium states selected in the fitting procedure, respectively.

LHCb and will bring some difficulties in our theoretical analysis. Fortunately, due to the property of an approximate mass degeneracy among three P-wave charmonium χ_{cJ} states, we find that the threshold cusps are actually and mainly concentrated in five energy positions: i.e., $(6.45 \sim 6.58)$, 6.64, $(6.87 \sim 7.00)$, $(7.03 \sim 7.13)$, and 7.32 GeV. With present experimental precision, it is difficult to distinguish the individual signals from these close peaks, where their contributions may overlap and behave like one peak structure. Thus, in the realistic analysis of experimental LHCb data, we only consider the rescattering processes from $J/\psi J/\psi$, $\eta_c\chi_{c1}$, $J/\psi h_c$, $\chi_{c0}\chi_{c1}$, and $\chi_{c0}\chi'_{c1}$, where $\eta_c\chi_{c1}$ and $\chi_{c0}\chi_{c1}$ are representative channels which approximately contain all of the contributions from $\eta_c\chi_{cJ}$ and $\chi_{c0}\chi_{cJ}$, respectively, with $J = 0, 1, 2$. Here, it is worth emphasizing that the widths of some of charmonium states such as η_c are not small, and their width effects may be important for the square of the loop integral, which can be included by replacing $m_{i(j)}$ in Eq. (1) with $(m_{i(j)} - i\Gamma_{i(j)}/2)$. To further reduce the fitting parameters, we notice that there is no evident structure in the energy region between 7.03 and 7.13 GeV of the LHCb data with present precision; thus, we exclude the contributions from $\chi_{c1}\chi_{c1}$, $\chi_{c1}\chi_{c2}$, and $\chi_{c2}\chi_{c2}$.

In Fig. 3, our complete theoretical fit to the experimental line shape of invariant mass spectrum vs $m_{J/\psi J/\psi}$ in high-energy proton-proton collisions is given based on the dynamical rescattering mechanism in Fig. 1. The relevant fitting parameters are listed in Table II. In the fitting procedure, the schemes of Fit I and Fit II are adopted, which, as can be seen from Table II, correspond to the same and independent cutoff parameters α for different combinations of intermediate charmonium states, respectively. It can be seen that the line shape of the LHCb data can be well described in both Fit I and Fit II, but the scheme of Fit II locally performs better on peak structures. This means that the impacts of α on the line shape of the cusp are relatively small, and the dependence of the cutoff parameter will not change our conclusions. Anyway, in our fitting, three relatively obvious peak structures near 6.5, 6.9, and 7.3 GeV can be reproduced, which directly correspond to the three rescattering channels $\eta_c\chi_{c1}$, $\chi_{c0}\chi_{c1}$, and $\chi_{c0}\chi'_{c1}$. As for the channel of $J/\psi J/\psi$, it provides an effect of threshold enhancement, which explains the line shape behavior of experimental data near the threshold of $m_{J/\psi J/\psi}$. Furthermore, the peak position of $J/\psi h_c$ at 6.64 GeV precisely points out a jumping point in the LHCb data, as shown in Fig. 3, whose verification can be treated as an interesting experimental topic for LHCb and CMS in the future under more accumulated experimental data. It is obvious that the above evidence provides a strong support to the nonresonant nature of several visible

TABLE I. The parameters for reproducing the line shape of LHCb data with $\chi_{c1}\eta_c$ and $\chi_{c0}\chi_{c1}$ rescattering contributions. In this scenario, we set the parameter $\alpha_{\chi_{c1}\eta_c} = \alpha_{\chi_{c0}\chi_{c1}} = \alpha$.

Parameter	Value	Parameter	Value
c_0	-1.99 ± 0.01	c'_0	-1.02 ± 0.04
$\lvert g'_{\rm direct}/g_{\rm direct}\rvert$	0.0232 ± 0.0037	$\lvert g_{\eta_c\chi_{c1}}/g'_{\rm direct}\rvert$	198 ± 46
$\lvert g_{\chi_{c0}\chi_{c1}}/g_{\rm direct}\rvert$	55.9 ± 8.2	$\phi_{\eta_c\chi_{c1}}$	3.22 ± 0.07
$\phi_{\chi_{c0}\chi_{c1}}$	1.87 ± 0.16	α	1.40 ± 0.10

TABLE II. The parameters for reproducing the line shape of LHCb data in two fitting schemes of Fit I and Fit II.

Parameters	Fit I	Fit II
c_0 (GeV^{-1})	-1.52 ± 0.02	-1.45 ± 0.01
c_0' (GeV^{-1})	-0.946 ± 0.058	-1.05 ± 0.01
$\lvert g'_{\mathrm{direct}}/g_{\mathrm{direct}}\rvert$	0.0767 ± 0.0204	0.137 ± 0.042
$\lvert g_{J/\psi J/\psi}/g_{\mathrm{direct}}\rvert$	8.53 ± 3.64	14.0 ± 1.4
$\lvert g_{\eta_c\chi_{c1}}/g'_{\mathrm{direct}}\rvert$	91.6 ± 75.4	112 ± 28
$\lvert g_{J/\psi h_c}/g'_{\mathrm{direct}}\rvert$	69.7 ± 16.1	109 ± 8
$\lvert g_{\chi_{c0}\chi_{c1}}/g_{\mathrm{direct}}\rvert$	33.3 ± 8.2	38.5 ± 7.6
$\lvert g_{\chi_{c0}\chi'_{c1}}/g_{\mathrm{direct}}\rvert$	25.8 ± 10.6	19.0 ± 4.3
$\phi_{J/\psi J/\psi}$ (rad)	1.53 ± 0.51	3.16 ± 0.19
$\phi_{\eta_c\chi_{c1}}$ (rad)	2.69 ± 0.20	2.80 ± 0.15
$\phi_{J/\psi h_c}$ (rad)	4.40 ± 0.33	2.95 ± 0.24
$\phi_{\chi_{c0}\chi_{c1}}$ (rad)	2.14 ± 0.18	2.89 ± 0.20
$\phi_{\chi_{c0}\chi'_{c1}}$ (rad)	2.00 ± 0.33	3.23 ± 0.20
$\alpha_{J/\psi J/\psi}$ (GeV)	1.71 ± 0.01	2.30 ± 0.21
$\alpha_{\eta_c\chi_{c1}}$ (GeV)	1.71 ± 0.01	1.20 ± 0.21
$\alpha_{J/\psi h_c}$ (GeV)	1.71 ± 0.01	1.20 ± 0.03
$\alpha_{\chi_{c0}\chi_{c1}}$ (GeV)	1.71 ± 0.01	1.73 ± 0.26
$\alpha_{\chi_{c0}\chi'_{c1}}$ (GeV)	1.71 ± 0.01	5.20 ± 0.05
χ^2/d.o.f.	1.41	1.25

structures in the measurements of LHCb. Additionally, when focusing on individual contributions in theoretically describing the line shape of LHCb data, it is easily found that the direct contribution of double-J/ψ production is dominant for the whole invariant mass distribution, which is reasonable because of loop suppression of the rescattering mechanism.

Moreover, in the present dynamical mechanism, the structure near 6.5 GeV dominantly results from the rescattering channel of $h_c\chi_{c1}$ and the interferences with other channels; thus, the P-parity of this structure should be odd. Similarly, the dominant contribution of the structure near 6.9 GeV comes from the rescattering channel of $\chi_{c0}\chi_{c1}$ and the corresponding interferences, which indicate that the P-parity of this structure should be even. The P-parities of these two structures are opposite from the tetraquark interpretations in Ref. [7]. Thus, the concrete quantum numbers of these observed fully charmed structures determined in future precise experimental measurements may provide an effective way to distinguish different theoretical interpretations.

After studying the new observation in the invariant mass spectrum of a J/ψ pair based on our nonresonant theoretical framework, we want to suggest an accessible way to further test the role of dynamical rescattering processes in the production of a J/ψ pair. Similarly to the case of di-J/ψ, such rescattering processes should exist in di-Υ production. In Fig. 4, the theoretical predictions for the line shapes on the invariant mass spectrum of $m_{\Upsilon\Upsilon}$ by the dynamical rescattering mechanism are given, where for reference, the same parameter values of α and c_0 used in the Fit I scheme for studying the production of a double J/ψ are taken. In this case, the combinations of a double bottomonium $\Upsilon(1S)\Upsilon(1S)$, $\eta_b\chi_{bJ}$, $\Upsilon(1S)h_b$, and $\chi_{bJ}\chi_{bJ}$ with $J = 0$, 1, 2 are involved, which cover the energy region from 18.92 to 20 GeV. Here, Υ, η_b, h_b, and χ_{bJ} refer to $\Upsilon(1S)$, $\eta_b(1S)$, $h_b(1P)$, and $\chi_{bJ}(1P)$, respectively. From Fig. 4, we can see that all of the above rescattering processes can produce peak line shapes near their mass thresholds. Here, considering that the constituent mass of a b quark is roughly 3 times heavier than a c quark, the mass splittings among P-wave bottomonium states caused by the spin-dependent interaction are very small compared with those of a charmonium system because of the suppression of $1/m_Q^2$ from the spin-dependent interaction. Thus, applying the input of resonance parameters from PDG [77] (we refer to the theoretical estimates of Ref. [78] for the unknown widths), the maximum positions of the channels $\eta_b\chi_{bJ}$ together with Υh_b, and $\chi_{bJ}\chi_{bJ}$ with $J = 0$, 1, 2 are found to be clustered in the short energy intervals of 19.28 to 19.36 GeV and 19.73 to 19.83 GeV, respectively, and the channel $\Upsilon\Upsilon$ precisely leads to a cusp peak at 19.0 GeV. This means that three prominent peak structures near 19.0, 19.3, and 19.7 GeV should be observed in the invariant mass spectrum of $m_{\Upsilon\Upsilon}$, where a double Υ can be reconstructed by $\mu^+\mu^-\mu^+\mu^-$, similar to the search for double-J/ψ events.

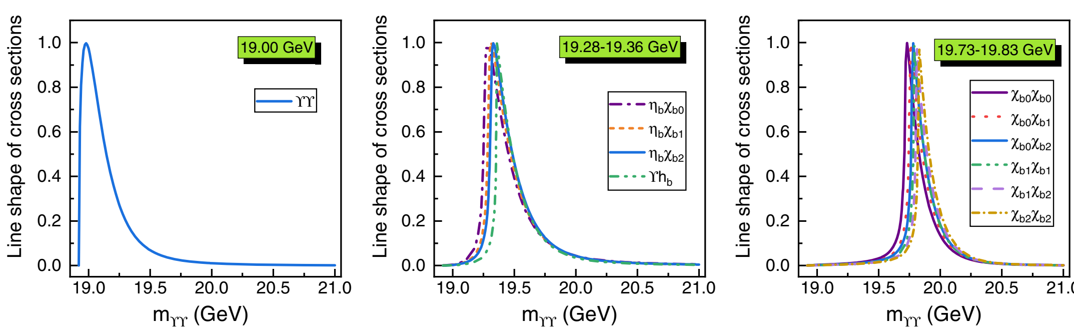

FIG. 4. The theoretical predictions for the peak line shapes from the dynamical rescattering mechanism in the invariant mass spectrum of $m_{\Upsilon\Upsilon}$, in which the channels of $\Upsilon(1S)\Upsilon(1S)$, $\eta_b\chi_{bJ}$, and $\Upsilon(1S)h_b$, $\chi_{bJ}\chi_{bJ}$ with $J = 0$, 1, 2 are considered. Here, the maximums are normalized to 1.

In fact, although many theoretical calculations for the spectroscopy of fully charm tetraquark states including $J^{PC} = 0^{++}$, 1^{-+}, and 2^{++} can explain the measured mass of $X(6900)$, there are obvious differences on the predictions for the mass spectrum of the corresponding partner of fully bottom tetraquark states [8–11,27–38,79–82]. On the other hand, because the heavy quark symmetry is assured, the mass difference among different combinations of intermediate heavy quarkonium states are almost independent of the heavy quark flavor. Thus, the measurements of the production of $\Upsilon\Upsilon$ in high-energy proton-proton collisions may be available to test the rescattering contributions and identify the nature of $X(6900)$ and other underlying structures, which should provide a good chance for LHCb and CMS.

IV. CONCLUSION

The LHCb Collaboration brought us some surprising results on the measurements of the invariant mass distribution of di-J/ψ production, where there exist three obvious peaks between 6.194 and 7.4 GeV [6]. Although the explanation of fully charm tetraquark states for these structures is straightforward, their origins should still be investigated carefully, because the peak phenomenon can also be produced by some special dynamical effects in addition to the resonance.

In this letter, we have proposed a nonresonant dynamical mechanism to understand several new structures observed by LHCb. Our idea is based on a reaction that different combinations of double charmonium directly produced in high-energy proton-proton collisions are transferred into final states of $J/\psi J/\psi$, which has been found to produce an obvious cusp at the corresponding mass threshold of a double charmonium. By fitting the experimental data by a line shape in the invariant mass spectrum of a J/ψ pair, three obvious peak structures near 6.5, 6.9, and 7.3 GeV are well reproduced, which naturally correspond to three rescattering channels $\eta_c\chi_{c1}$, $\chi_{c0}\chi_{c1}$, and $\chi_{c0}\chi'_{c1}$. Furthermore, we have predicted the peak line shape in the invariant mass spectrum of a Υ pair resulting from the similar rescattering processes between a double bottomonium, where three peak structures near 19.0, 19.3, and 19.7 GeV could be detected in experiments, which are related to the rescattering channels of $\Upsilon\Upsilon$, $\eta_b\chi_{bJ}$ together with Υh_b, and $\chi_{bJ}\chi_{bJ}$ with $J = 0, 1, 2$, respectively. These predictions should be helpful to indirectly confirm the nature of the newly observed structures by LHCb.

In fact, the double J/ψ or Υ is still an ideal final state to search for exotic fully heavy tetraquark structures. Because our present knowledge for inner structures of tetraquark states is quite poor, it is hard to precisely know their masses just by calculations from a variety of phenomenological models. However, there is a unique advantage for a peak behavior by the rescattering mechanism—i.e., their peak positions are very clear. Thus, combined with the fact that the degrees of freedom in a general four-body system are very considerable, which can result in a very rich mass spectrum, it is worth expecting to search for possible $QQ\bar{Q}\bar{Q}$ structures in which the corresponding peak positions are different from the threshold masses of rescattering channels. We believe that precise measurements of LHCb and CMS in the future should have a chance to discover definite candidates of fully heavy tetraquark states that may not be explained by our rescattering mechanism, which will open a new chapter for understanding the complicated nonperturbative behavior of QCD.

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