Letter

When invariable cross sections change: The Electron-Ion Collider case

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In everyday research, it is tacitly assumed that scattering cross sections have fixed values for a given particle species, center-of-mass energy, and particle polarization. However, this assumption has been called into question after several observations of suppression of high-energy bremsstrahlung. This process will play a major role in experiments at the future Electron-Ion Collider, and we show how variations of the bremsstrahlung cross section can be profoundly studied there using the lateral beam displacements. In particular, we predict a very strong increase of the observed cross sections for large beam separations. We also discuss the relation of these elusive effects to other quantum phenomena occurring over macroscopic distances. In this context, spectacular and possibly useful properties of the coherent bremsstrahlung at the Electron-Ion Collider are also evaluated.

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A strong suppression of the bremsstrahlung process, $e^+e^- \to e^+e^-\gamma$, was observed in 1982 by the MD-1 experiment at the VEPP-4 collider in Novosibirsk, Russia, at a center-of-mass energy of 3.6 GeV [1]. Despite significant systematic errors, the evidence of increasing suppression with decreasing energy of bremsstrahlung photons was striking. Small lateral beam sizes and large impact parameters were proposed as the origin of this unexpected phenomenon [2,3], and a general framework for calculating effects due to finite beam sizes was developed, based on the quantum picture of colliding wave packets [4]. It has also been shown that similar effects occur for bremsstrahlung in electron-proton collisions [5,6]. In 1995, the MD effect was observed, at a 5σ confidence level, at the first ep collider HERA at DESY [7]. There, however, the effect was much smaller, only about 3%, as in the ZEUS experiment only much higher photon energies were measured and, in addition, the lateral beam sizes at HERA were much larger than 10 μm, the beam sizes at VEPP-4.

Despite these observations, the notion of effective cross section variations due to finite transverse beam sizes has been slowly gaining wide recognition. Somewhat paradoxically, in the case of bremsstrahlung, the beam-size effect increases with the energy of collisions as even bigger impact parameters start to play a role in this process. At the Large Electron-Positron (LEP) collider at CERN, the beam-size effect manifested itself in a spectacular way with a very beneficial extension of the beam lifetimes [8]. For the noncolliding LEP beams, the ultimate limit of beam lifetime of about 60 hours was due to the Compton scattering of thermal photons off the beam electrons and positrons—the process measured directly at HERA [7,9]. However, in the case of colliding beams the beam lifetimes at LEP were ultimately limited by the electron-positron bremsstrahlung (or, in other words, by the radiative Bhabha scattering), but the observed beam lifetime of about 20 hours was much longer than the predicted 14 hours based on the standard bremsstrahlung calculations. Nevertheless, to explain this bremsstrahlung suppression at LEP, an *ad hoc* cutoff mechanism due to large particle density was evoked [8], and the results from VEPP-4 were ignored.

It seems that this lack of comprehension might stem from deeply rooted convictions, particularly from the definition of a two-particle scattering cross section, which boils down to this very simple relation used repeatedly in physics:

$$R = L\sigma, \quad (1)$$

where R is the rate of events of a given binary process with the cross section σ (which can be obtained from the scattering amplitudes derived from the theory), and L is the

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factor called luminosity which depends on intensities of fluxes of colliding particles. In practice, this formula is extremely useful since it factorizes the setup-dependent L from the "universal" cross section, which for the given particle species depends only on the center-of-mass energy and particle polarizations. The problem is that such a factorization is not always correct. One possibility for breaking it is very well known: if the density of a beam of particles (or of a "target") becomes too large, one can no longer assume only binary interactions. But there is another possibility too, having nothing to do with the beam densities, but with a basic assumption used to arrive at such a factorization assumption that the momenta of both colliding particles are "completely fixed"; i.e., they can be described by pure and simple plane waves corresponding to beam sizes that are infinitely large laterally. In reality, this is never so and any real beam has a finite transversal size; therefore, instead of the plane wave description, the proper wave packet formalism, in principle, should always be used [10].

The bremsstrahlung process is so extraordinary due to extremely small momentum transfers between the radiating electron and the other charged particle, a proton in the case of electron-proton collisions (as in Fig. 1). In particular, it is kinematically allowable in this case for both incoming particles to experience no angular scattering, that is, to continue moving exactly along their initial directions, while the bremsstrahlung photon is emitted exactly in the direction of the electron momentum. It is precisely this configuration which provides the smallest virtuality Q^2 (see Fig. 1 for definitions of variables) of the exchanged photon:

$$|Q|_{\mathrm{min}} = m_e^2 m_p E_\gamma / (4 E_p E_e E_e'), \tag{2}$$

where m_e and m_p are the electron and proton masses, respectively.

At high energies, this minimal photon virtuality acquires exceedingly small values (for example, at HERA $Q^2_{\mathrm{min}} = 10^{-8}\ \mathrm{eV}^2$ for $E_\gamma = 1$ GeV), and that results in a typical size

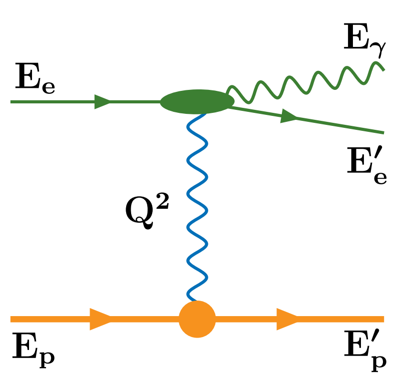

FIG. 1. Feynman diagram for electron-proton brems-strahlung. Respective energy variables of incoming and outgoing particles are shown, as are the virtuality of exchanged photon $Q^2 = -(P_p - P_p')^2$, where P_p and P_p' are four-momenta of the incoming and outgoing protons, respectively.

of transverse momentum transfers, $q_\perp$, of about $0.0001\ \mathrm{eV}/c$ only. This is because the bremsstrahlung differential cross section, with unintegrated scattering angles, is proportional to Q^{-4} (due to the propagator of exchanged photon); therefore, the photon virtualities close to Q^2_{min} dominate, and to a very good approximation $Q^2 = Q^2_{\mathrm{min}} + q_\perp^2$. It should be stressed again that even $q_\perp = 0$ is allowed in bremsstrahlung. If one analyses this process in the impact parameter space [11], these very small values of $q_\perp$ correspond to very large values of the impact parameter $b = \hbar / q_\perp$. That, in turn, explains why the original Bethe-Heitler cross section calculations in the Born approximation are so precise, despite our neglecting the size of the proton and its spin. In fact, the impact parameter in high-energy bremsstrahlung reaches highly macroscopic values in the above example from HERA corresponding to b of about 2 mm. And that precisely is the origin of the beam-size effect, or more generally of the observed modifications of bremsstrahlung cross sections. In deriving Eq. (1), it is assumed that both colliding beams can be properly represented by simple plane waves, which means that the assumed distribution of the impact parameter is uniform. However, when both colliding beams are strongly focused at the interaction point then that is no longer valid, as large impact parameters are suppressed and the bremsstrahlung differential cross section is "oversampled" at low impact parameters, where it is smaller. As a result, one observes an effective suppression of bremsstrahlung, increasing with decreasing photon energies, since the typical $q_\perp$ is proportional to E_γ; see Eq. (2).

In Fig. 2 the relative correction $(d\sigma_{\mathrm{corr}}/dy)/(d\sigma_{\mathrm{BH}}/dy)$ to the standard Bethe-Heitler cross section, that is, the

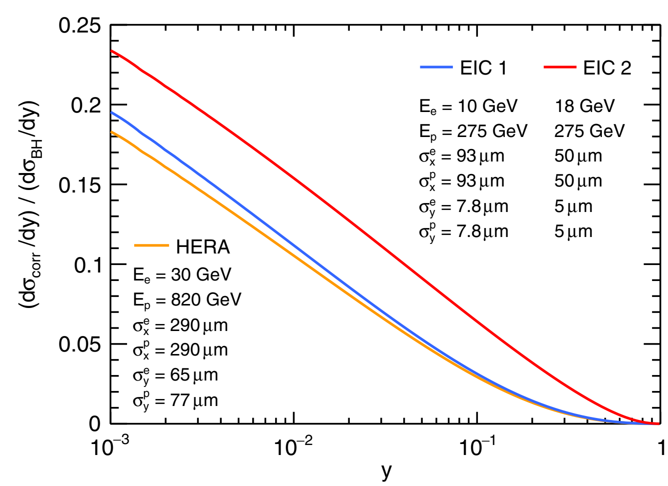

FIG. 2. Relative corrections to the standard Bethe-Heitler cross sections due to the beam-size effect. Relative suppression due to the beam-size effect $(d\sigma_{\mathrm{corr}}/dy)/(d\sigma_{\mathrm{BH}}/dy)$ is shown as a function of $y = E_\gamma / E_e$ for three cases of electron-proton bremsstrahlung. The corresponding beam energies and Gaussian lateral beam sizes at the interaction point are listed. The curves were obtained using Eqs. (6.7) and (6.5) in the Supplemental Material [13].

bremsstrahlung suppression due to beam-size effect, is shown as a function of $y = E_\gamma/E_e$, where it is assumed that $d\sigma_{\mathrm{obs}}/dy = d\sigma_{\mathrm{BH}}/dy - d\sigma_{\mathrm{corr}}/dy$. Three cases are considered: one at HERA and two at the future Electron-Ion Collider (EIC). Despite significantly larger beam energies, the effect at HERA is smaller than these at the EIC. This is because of much stronger beam focusing at the EIC; in particular, in the vertical direction "flat" beams (with a large aspect ratio of at least 10∶1) will be used there to avoid disruptive beam-beam effects, as the beam intensities will be much higher than those at HERA, which will result in the increase of luminosity by more than 2 orders of magnitude [12]. The EIC is in the early design phase and its nominal beam parameters are not yet fixed. The two EIC cases we discuss here are chosen for illustrative purposes. The EIC 1 set of parameters, taken from Ref. [12], corresponds to a "modest" case at low electron beam energy. In contrast, the EIC 2 "extreme" case assumes high electron beam energy and beam sizes slightly smaller than those considered nominal—the bremsstrahlung suppression will then be about 10% for 1 GeV photons, and still almost 1% at $y = 0.5$. The crossing angle at the EIC will be large but thanks to the crab crossing scheme the beams will effectively collide head on, as we assumed in calculations.

Very precise cross section measurements are essential for the success of the EIC scientific program. To achieve the requested 1% precision, an even more accurate determination of the collider luminosity is necessary. Equation (1) can be used to measure cross sections as well as to determine L if a certain σ is very well known and the corresponding R is accurately measured. As at HERA, the bremsstrahlung process will be used to precisely measure the EIC luminosity [17]; therefore, a thorough verification of the corrections due to the beam-size effect is mandatory. In principle, accurate measurements of bremsstrahlung angular distributions could provide interesting insights into the beam-size phenomenon [see Figs. S2 and S3 in the Supplemental Material [13]]. That is, however, not possible to perform, as the typical bremsstrahlung emission angles are much smaller than the angular divergences of beams at the EIC interaction point. Here, we propose a new, powerful, and straightforward way of conducting at the EIC unique studies of the role of large impact parameters in bremsstrahlung, and of the origin of the beam-size effect in particular.

We propose precisely measuring the bremsstrahlung spectra as a function of lateral beam displacements at the interaction point that is, by using the so-called van der Meer scans, usually performed (at the LHC, for example) to measure the lateral sizes of the "luminous region." Such scans result in strong changes of the collider luminosity L according to the formula, assuming Gaussian distributions of beam particles: $L(B) = L(0)\exp(-B/(\sigma_1^2+\sigma_2^2)/2)$, where B is the lateral displacement of one of the beams within the horizontal or vertical plane, σ_1 and σ_2 are the two Gaussian widths in a given plane, often equal, and $L(0)$ corresponds to the luminosity of nominal, head-on collisions. However, in the case of bremsstrahlung, its photon spectrum will be modified also in a very specific way. The lateral beam displacement will serve as a direct scanner of impact parameter after all.

To make specific predictions, we extended the well-developed framework of Ref. [11]: we also assume head-on beam collisions and Gaussian distributions of the beam particle coordinates in the transverse plane, $n(\vec{r}_\perp)$, but we allow for relative lateral beam displacements. In particular, we introduced a vertical beam displacement B of one of the colliding beams:

$$n(\vec{r}_\perp) = \frac{N}{2\pi\sigma_x\sigma_y}\exp\left(-\frac{x^2}{2\sigma_x^2} - \frac{(y-B)^2}{2\sigma_y^2}\right),$$

where $\vec{r}_\perp = (x, y, 0)$, N is the number of particles in a beam, and σ_x, σ_y are the standard deviations in the horizontal and vertical planes, respectively. This resulted in the replacement of the original equation (5.32) with the following formula:

$$G(\omega) = 2\int_0^\infty \frac{\rho_\perp}{\rho_m} K_1^2\left(\frac{\rho_\perp}{\rho_m}\right)\left[1 - \frac{e^{-v_+}}{\pi} \times \int_0^\pi e^{v_-\cos\varphi}\cosh\left(t_y \sin\left(\frac{\varphi}{2}\right)\right) d\varphi\right]\frac{d\rho_\perp}{\rho_m}, \qquad (3)$$

where G is the function which describes the number of missing equivalent photons of energy ω [see Eq. (5.14) in Ref. [11]], $\rho_m = E_p/(m_p\omega)$, $t_y = \rho_\perp B/a_y^2$, and $v_\pm = \rho_\perp^2(1 \pm a_y^2/a_x^2)/(4a_y^2)$, where $a_x^2 = \sigma_{x1}^2 + \sigma_{x2}^2$ and $a_y^2 = \sigma_{y1}^2 + \sigma_{y2}^2$ and σ_{x1}, σ_{y1} and σ_{x2}, σ_{y2} are, respectively, the horizontal and vertical beam sizes at the interaction point (that is, at $z = 0$) for two colliding beams; K_1 is the modified Bessel function of the second (third) kind.

For $B = 0$ the expression in the square brackets becomes $[1 - e^{-v_+} I_0(v_-)]$, as in Eq. (5.32) in Ref. [11], where I_0 is the modified Bessel function of the first kind.

In Fig. 3, we show how the relative cross section corrections will change for four values of y, in the case of vertical beam scans at the EIC. As expected, for large values of B the corrections become negative, which means that the bremsstrahlung cross section will strongly increase because the large impact parameters, for which the differential cross section is increasing, are oversampled in this case. In Fig. 4, we show how entire bremsstrahlung spectra are heavily distorted at the EIC for vertical beam displacements of only 20–30 μm. In these cases, the observed cross section will double for low photon energies. For even larger beam displacements, the observed increase will become almost a hundredfold; see Fig. 5, where we present the two-dimensional distributions of $(d\sigma_{\mathrm{corr}}/dy)/(d\sigma_{\mathrm{BH}}/dy)$ as a function of both $y = E_\gamma/E_e$ and the vertical beam displacement B. One should note, however, that even for large

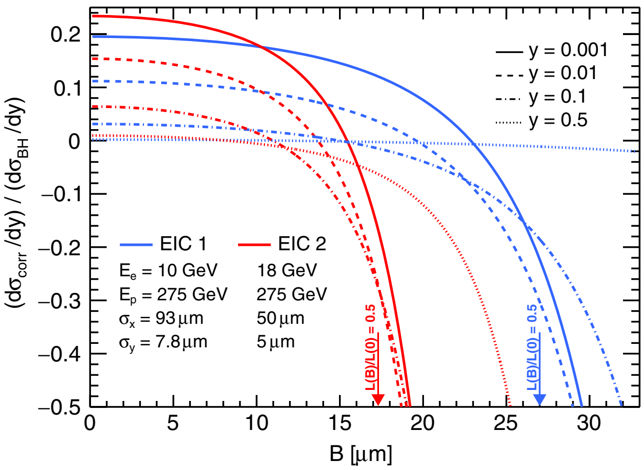

FIG. 3. Relative corrections to the standard Bethe-Heitler cross sections due to beam displacements at the EIC. The ratio $(d\sigma_{\mathrm{corr}}/dy)/(d\sigma_{\mathrm{BH}}/dy)$ is shown as a function of vertical beam displacement B for four values of y and two cases of electron-proton collisions at the EIC. Values of B corresponding to the 5% of the nominal luminosity are indicated, and the beam energies and Gaussian lateral beam sizes at the EIC interaction point are listed. The curves are obtained using Eqs. (6.7) and (6.5) in the Supplemental Material [13], with $G(\omega)$ given in Eq. (3).

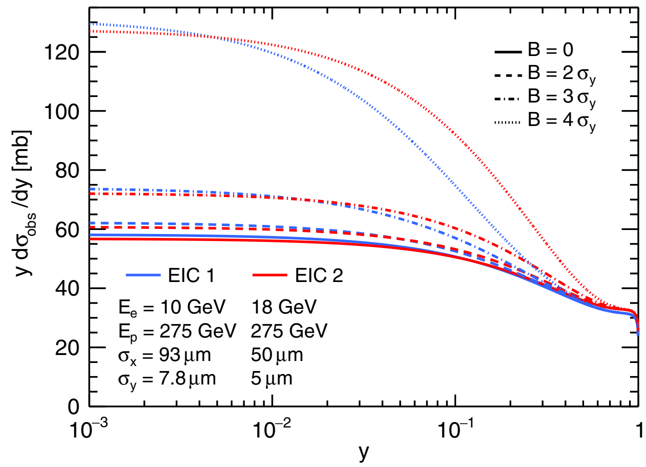

FIG. 4. The predicted spectra of ep bremsstrahlung at the EIC for several vertical beam displacements. The standard Bethe-Heitler cross section $d\sigma_{\mathrm{BH}}/dy$ is modified due to the beam-size effect and beam displacements B. The effective cross sections (multiplied by y for better visibility) are shown for two cases of electron-proton collisions at the EIC—the corresponding beam energies and Gaussian lateral beam sizes at the interaction point are listed. The vertical beam displacements $B = 2\sigma_y, 3\sigma_y$, and $4\sigma_y$ correspond, respectively, to 36%, 11%, and 2% of the nominal luminosity for $B = 0$. The curves are obtained using Eqs. (6.7) and (6.5) in the Supplemental Material [13], with $G(\omega)$ as given in Eq. (3).

values of B the suppression due to the beam-size effect is still significant for small photon energies. From the experimental point of view, it is preferable that such scans be done at (very) low beam intensities to avoid beam disruptions, during the scans, due to the strong beam-beam effects. Finally, similar effects are expected in the electron-ion collisions at the EIC, as the designed beam sizes in that case are close to those for electron-proton collisions.

A good understanding of the role of very large impact parameters in bremsstrahlung is mandatory for the precise luminosity determination at the future EIC, or for the proper explanation of the beam lifetimes at the KEK B-factory [18], but it might also be relevant in nonaccelerator physics. For example, it has been proposed that rapidly spinning neutron stars are the principal source of ultrahigh-energy cosmic rays. It is interesting to observe, if only for illustrative purposes, that the impact parameter for a 50 EeV electron radiating a 0.5 EeV photon reaches the size of the neutron star itself. According to the results we obtained, radiation of such a photon can be either strongly suppressed or amplified, depending on the geometrical aspects of particle collisions.

Some new effects due to very small transverse momentum transfers were discussed above, but there are also bremsstrahlung phenomena related to small longitudinal momentum transfers $q_{\parallel}$, such as the famous Landau-Pomeranchuk-Migdal effect [19]. In this case, basic assumptions about binary collisions do not hold, as these small transfers correspond to large coherence lengths (often called the formation length in this context) of the process, resulting in the bremsstrahlung suppression in dense materials if over such a longitudinal distance the incoming electrons undergo significant angular scattering [20–22]. In high-energy electron-proton collisions, this coherence length l_c, calculated in the laboratory reference frame, is given by the formula $l_c = \hbar/q_{\parallel} = 4\hbar c\gamma_e^2/E_\gamma$, where $\gamma_e = E_e/m_e$. It was realized that if that coherence length is significantly larger than the length σ_z of an opposite bunch, then the coherent bremsstrahlung takes place and beam electrons interact with the total field of all particles in the opposite bunch. The photon energy for which $l_c = \sigma_z$ is called the critical energy E_c.

Such a coherent effect results in a spectacular increase by many orders of magnitude of the bremsstrahlung cross section for $E_\gamma \ll E_c$. For these low energies, the photon spectrum (per single bunch crossing) can be simplified as follows [23]: $dN_\gamma = N_0 dE_\gamma/E_\gamma$, where $N_0 = 8\alpha N_e(r_e N_p/\sigma_x)^2/(9\sqrt{3})$ when assuming flat beams with identical transverse beam sizes, α is the fine structure constant, r_e is the electron classical radius, N_e and N_p are the electron and proton bunch populations, respectively, and σ_x is the horizontal beam size. As expected, the coherent bremsstrahlung per bunch collision scales with $N_e N_p^2$, but it also has a strong dependence on σ_x. At the EIC, owing to high beam intensities and small beam sizes, it will manifest itself in a unique and spectacular way. For example, one obtains $N_0 = 3.1(0.28) \times 10^{10}$ for these two sets of EIC parameters: $E_e = 18(10)$ GeV, $N_e = 6.2(17.2) \times 10^{10}$, $N_p = 2.05(0.69) \times 10^{11}$, $\sigma_x = 50(93)\,\mu$m,

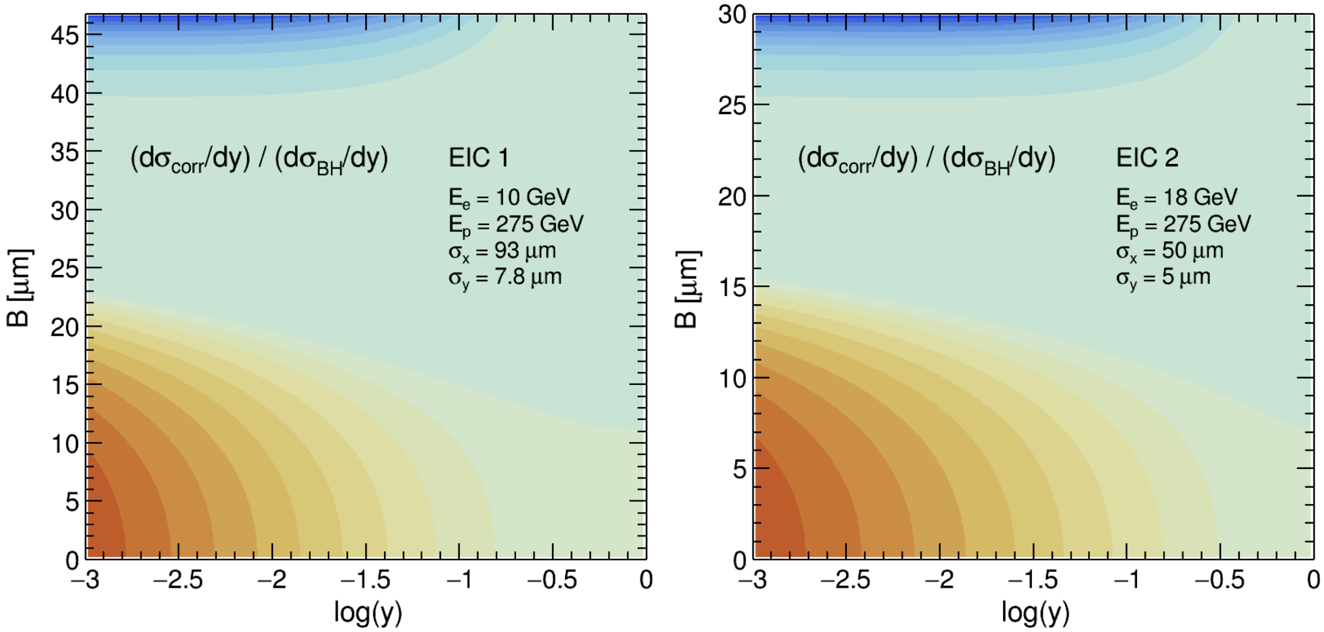

FIG. 5. Relative corrections to the standard Bethe-Heitler cross sections, due to both the beam-size effect and vertical beam displacements, as a function of B and y. The ratios $(d\sigma_{\text{corr}}/dy)/(d\sigma_{\text{BH}}/dy)$ are shown as a function of the vertical beam displacement B and the logarithm of the relative photon energy $y = E_\gamma/E_e$ for the two sets of EIC parameters: EIC 1 and EIC 2. The corresponding beam energies and Gaussian lateral beam sizes at the interaction point are listed. Shown are ten equidistant (in the third dimension) contours for the values above zero (displayed in brown) and ten equidistant contours for values below zero (displayed in blue). For the EIC 1 case, the distribution extends in the third dimension between approximately −84 and +0.2, whereas for the EIC 2 case this range spans approximately from −80.5 to +0.24.

and $\sigma_z = 6$ cm. The corresponding critical energy $E_c = 16.3$ and 5 keV, whereas the total radiated power in this process is about 900 and 100 W, respectively. In coherent bremsstrahlung, the photon energies are relatively small but, on average, every single beam electron will emit more than one such x-ray photon during every single bunch crossing. The radiation extends down to the "visible window"—there, the coherent bremsstrahlung signal at the EIC will manifest itself as a flash of blue light lasting a small fraction of 1 ns, repeated up to 10^8 times per second. For a human eye, at a spot where it hits the beam pipe, its peak illuminance will reach 10 000 lux. In the electron-ion collisions, the radiated power will be smaller, as the impact of larger ion charges is more than compensated for by much smaller bunch populations or the ion beam currents.

These results demonstrate that, on the one hand, unique studies of the coherent bremsstrahlung will also be possible at the EIC, as well as its potential applications for the beam diagnostics, but, on the other hand, at the highest electron energy this coherent process will require a special treatment due to large radiated power, which in addition will almost double for significant vertical beam displacements [23] such as those discussed above.

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