Letter

Dark matter from $SU(6) \to SU(5) \times U(1)_N$

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Matter and dark matter are unified under the framework of $SU(6) \to SU(5) \times U(1)_N$. A dark-matter candidate is possible, not because it is stable, but because it has a very long lifetime, in analogy to that of the proton in theories of grand unification. A specific example is presented.

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I. INTRODUCTION

The existence of dark matter appears to be indisputable [1]. Instead of taking it for granted as an *ad hoc* addition to the Standard Model (SM) of quarks and leptons, a more fundamental question may be asked as to its relationship with visible matter. One possible answer is that both belong to the same organizing symmetry, such as $SO(10)$, but are distinguished by a marker symmetry such as $U(1)_\chi$ [2,3] in $SO(10) \to SU(5) \times U(1)_\chi$. Fermions and scalars which are odd and even under $U(1)_\chi$ belong to the visible sector, whereas fermions and scalars which are even and odd under $U(1)_\chi$ belong to the dark sector. They are distinguished by $(-1)^{Q_\chi+2j}$ where j is the particle's spin. The lightest dark particle is assumed to be neutral and is stable because of this odd-even symmetry.

Another possible answer is that there is no marker symmetry and both visible and dark matter coexist in multiplets of an organizing symmetry such as $SU(6)$ [4,5], but the dark-matter candidate itself has a very long lifetime, just as the proton has a very long lifetime in theories of grand unification. A specific complete model of $SU(6) \to SU(5) \times U(1)_N$ is presented here for the first time, where a dark fermion decays to SM particles through a superheavy gauge boson.

II. $SU(6)$ UNIFICATION OF VISIBLE AND DARK MATTER

Consider the extension of the well-known $SU(5)$ model [6] of grand unification to $SU(6)$ with $SU(6) \to SU(5) \times U(1)_N$. Although one family of fundamental fermions under $SU(5)$ is contained in the anomaly-free combination of 5^* and 10, the analogous case for $SU(6)$ [7,8] is two $6^* = (5^*, -1) + (1, 5)$ and one $15 = (10, 2) + (5, -4)$. Let

$$6^*_{F1} = \begin{pmatrix} d^c \\ d^c \\ d^c \\ e \\ \nu \\ N_1 \end{pmatrix}, \qquad 6^*_{F2} = \begin{pmatrix} D^c \\ D^c \\ D^c \\ E^- \\ E^0 \\ N_2 \end{pmatrix},$$

$$15_F = \begin{pmatrix} 0 & u^c & u^c & -u & -d & -D \\ -u^c & 0 & u^c & -u & -d & -D \\ u^c & -u^c & 0 & -u & -d & -D \\ u & u & u & 0 & -e^c & -E^+ \\ d & d & d & e^c & 0 & -\bar{E}^0 \\ D & D & D & E^+ & \bar{E}^0 & 0 \end{pmatrix}. \quad (1)$$

Their $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_N$ assignments are listed in Table I. Note that all are left handed.

The scalar sector consists of the following:

(i) (1) $84_S = (5,1) + (45,1) + (24,-5) + (10,7)$ which breaks the $U(1)_N$ of $SU(6)$ along $(1,1,0,-5)$ of $(24,-5)$ $[v_1]$ to $SU(3)_C \times SU(2)_L \times U(1)_Y$

(ii) (2) $21^*_S = (15^*,-2) + (5^*,4) + (1,10)$ which breaks the $U(1)_N$ of $SU(6)$ along (1,10) $[v_2]$ to $SU(5)$

(iii) (3) $35_S = (1,10) + (5,6) + (5^*,-6) + (24,0)$ which breaks $SU(6)$ along (1,1,0,0) $[v_3]$ from (24,0) to $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_N$

(iv) (4) 15^*_S which breaks $SU(2)_L \times U(1)_Y$ along $(1,2,-1/2,4)$ $[v_4]$ from $(5^*,4)$ to $U(1)_Q$

(v) (5) $84'_S = (5,1) + (45,1) + (24,-5) + (10,7)$ which breaks $SU(2)_L \times U(1)_Y$ along $(1,2,1/2,1)$ $[v_5]$ from (45,1) to $U(1)_Q$

The $SU(6)$ symmetry is first broken to $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_N$ by v_3 at a high scale. Subsequent breaking of $U(1)_N$ is by $v_{1,2}$ and $SU(2)_L \times U(1)_Y$ by $v_{4,5}$. In addition, the Z_2 discrete symmetry is imposed so that

TABLE I. Fermion content of $SU(6) \to SU(5) \times U(1)_N$ model.

Fermion	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$U(1)_N$
d^c	3^*	1	1/3	−1
(ν, e)	1	2	−1/2	−1
N_1	1	1	0	5
D^c	3^*	1	1/3	−1
(E^0, E^-)	1	2	−1/2	−1
N_2	1	1	0	5
(u, d)	3	2	1/6	2
u^c	3^*	1	−2/3	2
e^c	1	1	1	2
D	3	1	−1/3	−4
$(E^+, \bar{E}^0)$	1	2	1/2	−4

6^*_{F1} and $84'_S$ are odd and the other multiplets are even. This symmetry is obeyed by all dimension-four terms in the Lagrangian, but is softly broken by the trilinear scalar coupling $84_S \times 15^*_S \times 84'_S$, which contains the $v_1 v_4 v_5$ term.

III. FERMION MASSES

Because of the Z_2 symmetry, 6^*_{F1} is distinguished from 6^*_{F2}. Hence, only $6^*_{F2} \times 15_F$ transforms as 84_S and only $6^*_{F1} \times 15_F$ transforms as $84'_S$. Thus, $D^c D$ and $E^- E^+ + E^0 \bar{E}^0$ masses are proportional to v_1, whereas $d^c d$ and $e e^c$ masses are proportional to v_5. Similarly, both $6^*_{F1} \times 6^*_{F1}$ and $6^*_{F2} \times 6^*_{F2}$ transform as 21^*_S and 15^*_S, so that $N_{1,2}$ have Majorana masses proportional to v_2 and νN_1, $E^0 N_2$ masses to v_4. Finally, $15_F \times 15_F$ transforms as 15^*_S, with $u^c u$ masses proportional to v_4 as well.

Neutrinos obtain Majorana seesaw masses proportional to v_4^2/v_2, with N_1 acting as the usual right-handed neutrino in left-right models. This shows that $SU(6)$ may be used for seesaw neutrino masses in lieu of the customary $SO(10)$. The 3×3 mass matrix spanning $(N_2, E^0, \bar{E}^0)$ is of the form

$$\mathcal{M}_{NE} = \begin{pmatrix} f_N v_2 & f_{NE} v_4 & 0 \\ f_{NE} v_4 & 0 & f_E v_1 \\ 0 & f_E v_1 & 0 \end{pmatrix}. \tag{2}$$

IV. GAUGE BOSON MASSES AND INTERACTIONS

The gauge bosons belonging to the adjoint 35 representation of $SU(6)$ are superheavy with masses proportional to v_3 except for those corresponding to $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_N$. The breaking of $U(1)_N$ comes from $v_{1,2}$ and that of $SU(2)_L \times U(1)_Y \times U(1)_N$ comes from $v_{4,5}$. The charged $W^\pm$ mass is given by

$$m_W^2 = \frac{1}{2} g_L^2 (v_4^2 + v_5^2), \tag{3}$$

the massless photon is $A = (e/g_L) W_3 + (e/g_Y) B$, where B is the $U(1)_Y$ gauge boson and $g_Y/g_L = \tan\theta_W$ with $e^{-2} = g_L^{-2} + g_Y^{-2}$, whereas the 2×2 mass-squared matrix spanning (Z, Z_N), where $Z = W_3 \cos\theta_W - B \sin\theta_W$, is given by

$$\mathcal{M}^2_{ZZ_N} = \begin{pmatrix} (g_Z^2/2)(v_4^2 + v_5^2) & g_Z g_N (4v_4^2 - v_5^2) \\ g_Z g_N (4v_4^2 - v_5^2) & 2g_N^2 (25 v_1^2 + 100 v_2^2 + 16 v_4^2 + v_5^2) \end{pmatrix}, \tag{4}$$

where $g_Z^2 = g_L^2 + g_Y^2$. For simplicity, $v_5 = 2v_4$ may be assumed, so that Z and Z_N do not mix, thereby preserving all electroweak precision measurements involving the Z boson.

The gauge interactions of Z are given by

$$\mathcal{L}_Z = -g_Z Z_\mu j_Z^\mu = -g_Z Z_\mu (j_{3L}^\mu - \sin^2\theta_W j_Q^\mu) \tag{5}$$

and those of Z_N by

$$\begin{aligned} \mathcal{L}_N &= -g_N Z_{N\mu} j_N^\mu \\ &= -g_N Z_{N\mu} [2\bar{u}_L \gamma^\mu u_L - 2\bar{u}_R \gamma^\mu u_R + 2\bar{d}_L \gamma^\mu d_L + \bar{d}_R \gamma^\mu d_R - \bar{e}_L \gamma^\mu e_L - 2\bar{e}_R \gamma^\mu e_R - \bar{\nu}_L \gamma^\mu \nu_L + 5\bar{N}_{1L} \gamma^\mu N_{1L} \\ &\quad + 5\bar{N}_{2L} \gamma^\mu N_{2L} - 4\bar{D}_L \gamma^\mu D_L + \bar{D}_R \gamma^\mu D_R - \bar{E}_L^- \gamma^\mu E_L^- + 4\bar{E}_R^- \gamma^\mu E_R^- - \bar{E}_L^0 \gamma^\mu E_L^0 + 4\bar{E}_R^0 \gamma^\mu E_R^0]. \end{aligned} \tag{6}$$

As such, Z_N may be produced at the collider through its couplings to u and d quarks, and be discovered through its couplings to charged leptons. The present collider limit [9] is estimated to be a few TeV.

V. ELECTROWEAK SCALAR SECTOR

At the level of $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_N$, there are two scalar singlets $\eta_1 \sim (1, 1, 0, -5)$, $\eta_2 \sim (1, 1, 0, 10)$ and two scalar doublets $\Phi_4 = (\phi_4^0, \phi_4^-) \sim (1, 2, -1/2, 4)$, $\Phi_5 = (\phi_5^+, \phi_5^0) \sim (1, 2, 1/2, 1)$. Their Yukawa couplings with the fermions of (1) are

$$D^c D\eta_1^*, \qquad (E^- E^+ + E^0 \bar{E}^0)\eta_1^*, \qquad N_1 N_1 \eta_2^*, \qquad N_2 N_2 \eta_2^*, \qquad d^c(d\bar{\phi}_5^0 + u\phi_5^-), \tag{7}$$

$$u^c(u\bar{\phi}_4^0 + d\phi_4^+), \qquad e^c(e\bar{\phi}_5^0 + \nu\phi_5^-), \qquad N_1(\nu\bar{\phi}_4^0 + e\phi_4^+), \qquad N_2(E^0\bar{\phi}_4^0 + E^-\phi_4^+). \tag{8}$$

Note that these dimension-four terms all obey the imposed Z_2 symmetry discussed earlier under which d^c, (ν, e), and (ϕ_5^+, ϕ_5^0) are odd, and the others are even.

The Higgs potential is

$$\begin{aligned} V = {} & m_1^2\eta_1^*\eta_1 + m_2^2\eta_2^*\eta_2 + m_4^2\Phi_4^\dagger\Phi_4 + m_5^2\Phi_5^\dagger\Phi_5 + [\mu_1\eta_1\tilde{\Phi}_4^\dagger\Phi_5 + \mu_2\eta_1^2\eta_2 + \text{H.c.}] \\ & + \frac{1}{2}\lambda_1(\eta_1^*\eta_1)^2 + \frac{1}{2}\lambda_2(\eta_2^*\eta_2)^2 + \frac{1}{2}\lambda_4(\Phi_4^\dagger\Phi_4)^2 + \frac{1}{2}\lambda_5(\Phi_5^\dagger\Phi_5)^2 \\ & + \lambda_{12}(\eta_1^*\eta_1)(\eta_2^*\eta_2) + \lambda_{14}(\eta_1^*\eta_1)(\Phi_4^\dagger\Phi_4) + \lambda_{15}(\eta_1^*\eta_1)(\Phi_5^\dagger\Phi_5) + \lambda_{24}(\eta_2^*\eta_2)(\Phi_4^\dagger\Phi_4) \\ & + \lambda_{25}(\eta_2^*\eta_2)(\Phi_5^\dagger\Phi_5) + \lambda_{45}(\Phi_4^\dagger\Phi_4)(\Phi_5^\dagger\Phi_5) + \lambda'_{45}(\Phi_4^\dagger\Phi_5)(\Phi_5^\dagger\Phi_4). \end{aligned} \tag{9}$$

Note that the dimension-three μ_1 term breaks the Z_2 symmetry softly. Together with the μ_2 term, they ensure that there would be no extra accidental U(1) symmetry in V beyond $U(1)_Y$ and $U(1)_N$.

The minimum of V is determined by

$$0 = v_1(m_1^2 + \lambda_1 v_1^2 + \lambda_{12}v_2^2 + \lambda_{14}v_4^2 + \lambda_{15}v_5^2 + 2\mu_2 v_2) - \mu_1 v_4 v_5, \tag{10}$$

$$0 = v_2(m_2^2 + \lambda_2 v_2^2 + \lambda_{12}v_1^2 + \lambda_{24}v_4^2 + \lambda_{25}v_5^2) + \mu_2 v_1^2, \tag{11}$$

$$0 = v_4(m_4^2 + \lambda_4 v_4^2 + \lambda_{14}v_1^2 + \lambda_{24}v_2^2 + \lambda_{45}v_5^2) - \mu_1 v_1 v_5, \tag{12}$$

$$0 = v_5(m_5^2 + \lambda_5 v_5^2 + \lambda_{15}v_1^2 + \lambda_{25}v_2^2 + \lambda_{45}v_4^2) - \mu_1 v_1 v_4. \tag{13}$$

The 4×4 mass-squared matrix spanning $\sqrt{2}\text{Im}(\eta_1^0, \eta_2^0, \phi_4^0, \phi_5^0)$ is given by

$$\mathcal{M}_A^2 = \begin{pmatrix} \mu_1 v_4 v_5/v_1 - 4\mu_2 v_2 & -2\mu_2 v_1 & \mu_1 v_5 & \mu_1 v_4 \\ -2\mu_2 v_1 & -\mu_2 v_1^2/v_2 & 0 & 0 \\ \mu_1 v_5 & 0 & \mu_1 v_1 v_5/v_4 & \mu_1 v_1 \\ \mu_1 v_4 & 0 & \mu_1 v_1 & \mu_1 v_1 v_4/v_5 \end{pmatrix}. \tag{14}$$

Two zero eigenvalues appear, corresponding to $[v_1, -2v_2, -v_4 v_5^2/(v_4^2 + v_5^2), -v_4^2 v_5/(v_4^2 + v_5^2)]$ and $(0, 0, v_4, -v_5)$, becoming the longitudinal components of Z_N and Z, respectively. The remaining two massive pseudoscalar components span $(2v_2, v_1, 0, 0)$ and $[v_1, -2v_2, (v_1^2 + 4v_2^2)/v_4, (v_1^2 + 4v_2^2)/v_5]$ with 2×2 mass-squared matrix,

$$\begin{pmatrix} 4\mu_1 v_2^3 v_4^2 v_5^2 - \mu_2 v_1 (v_1^2 + 4v_2^2)^2 v_4 v_5 & 2\mu_1 v_1 v_2^2 v_4 v_5 \sqrt{v_4^2 v_5^2 + (v_1^2 + 4v_2^2)(v_4^2 + v_5^2)} \\ 2\mu_1 v_1 v_2^2 v_4 v_5 \sqrt{v_4^2 v_5^2 + (v_1^2 + 4v_2^2)(v_4^2 + v_5^2)} & \mu_1 v_1^2 v_2 [v_4^2 v_5^2 + (v_1^2 + 4v_2^2)(v_4^2 + v_5^2)] \end{pmatrix}, \tag{15}$$

divided by $v_1 v_2 (v_1^2 + 4v_2^2) v_4 v_5$. It shows explicitly that $\mu_1 = 0$ or $\mu_2 = 0$ implies one zero eigenvalue, and $\mu_1 = \mu_2 = 0$ implies two. In the limit $v_{4,5} \ll v_{1,2}$, it reduces to

$$\mathcal{M}_A^2 = \begin{pmatrix} -\mu_2 (v_1^2 + 4v_2^2)/v_2 & 0 \\ 0 & \mu_1 v_1 (v_4^2 + v_5^2)/v_4 v_5 \end{pmatrix}. \tag{16}$$

The 4×4 mass-squared matrix spanning $\sqrt{2}\mathrm{Re}(\eta_1^0, \eta_2^0, \phi_4^0, \phi_5^0)$ is given by

$$\begin{pmatrix} 2\lambda_1 v_1^2 + \mu_1 v_4 v_5/v_1 - 4\mu_2 v_2 & 2\lambda_{12} v_1 v_2 + 2\mu_2 v_1 & 2\lambda_{14} v_1 v_4 - \mu_1 v_5 & 2\lambda_{15} v_1 v_5 - \mu_1 v_4 \\ 2\lambda_{12} v_1 v_2 + 2\mu_2 v_1 & 2\lambda_2 v_2^2 - \mu_2 v_1^2/v_2 & 2\lambda_{24} v_2 v_4 & 2\lambda_{25} v_2 v_5 \\ 2\lambda_{14} v_1 v_4 - \mu_1 v_5 & 2\lambda_{24} v_2 v_4 & 2\lambda_4 v_4^2 + \mu_1 v_1 v_5/v_4 & 2\lambda_{45} v_4 v_5 - \mu_1 v_1 \\ 2\lambda_{15} v_1 v_5 - \mu_1 v_4 & 2\lambda_{25} v_2 v_5 & 2\lambda_{45} v_4 v_5 - \mu_1 v_1 & 2\lambda_5 v_5^2 + \mu_1 v_1 v_4/v_5 \end{pmatrix}. \tag{17}$$

Let $h = \sqrt{2}[v_4 \mathrm{Re}(\phi_4^0) + v_5 \mathrm{Re}(\phi_5^0)]/\sqrt{v_4^2 + v_5^2}$, then its mass is given by

$$m_h^2 = \frac{2\lambda_4 v_4^4 + 2\lambda_5 v_5^4 + 4\lambda_{45} v_4^2 v_5^2}{v_4^2 + v_5^2}. \tag{18}$$

It is the only linear combination of the four neutral scalar fields which has no $v_{1,2}$ contribution to its mass and acts as the SM Higgs boson in its interactions. The other three scalar bosons are much heavier and have suppressed mixing with h, assuming again $v_{4,5} \ll v_{1,2}$,

Consider now the linear combinations $S_1 = \sqrt{2}[v_1 \mathrm{Re}(\eta_1^0) + 2v_2 \mathrm{Re}(\eta_2^0)]/\sqrt{v_1^2 + 4v_2^2}$ and $S_2 = \sqrt{2}[2v_2 \mathrm{Re}(\eta_1^0) - v_1 \mathrm{Re}(\eta_2^0)]/\sqrt{v_1^2 + 4v_2^2}$. Then

$$m_{S_1}^2 = \frac{2\lambda_1 v_1^4 + 8\lambda_2 v_2^4 + 8\lambda_{12} v_1^2 v_2^2 + \mu_1 v_1 v_4 v_5}{v_1^2 + 4v_2^2}, \tag{19}$$

$$m_{S_2}^2 = -\frac{\mu_2}{v_2}(v_1^2 + 4v_2^2) + (8\lambda_1 + 2\lambda_2 - 8\lambda_{12})\frac{v_1^2 v_2^2}{v_1^2 + 4v_2^2} + \frac{4\mu_1 v_4 v_5 v_2^2}{v_1(v_1^2 + 4v_2^2)}, \tag{20}$$

$$m_{S_1 S_2}^2 = \frac{2v_1 v_2}{v_1^2 + 4v_2^2}[(2\lambda_1 - \lambda_{12})v_1^2 + (4\lambda_{12} - 2\lambda_2)v_2^2] + \frac{2\mu_1 v_4 v_5 v_2}{v_1^2 + 4v_2^2}. \tag{21}$$

This shows that if $\mu_2 \gg v_{1,2}$, then S_2 is much heavier than S_1 and their mixing is suppressed. This scenario is useful for the dark-matter phenomenology to be discussed later.

The remaining scalar $H = \sqrt{2}[v_5 \mathrm{Re}(\phi_4^0) - v_4 \mathrm{Re}(\phi_5^0)]/\sqrt{v_4^2 + v_5^2}$ has mass given by

$$m_H^2 = \frac{\mu_1 v_1 (v_4^2 + v_5^2)}{v_4 v_5} + \frac{(2\lambda_4 + 2\lambda_5 - 4\lambda_{45}) v_4^2 v_5^2}{v_4^2 + v_5^2}. \tag{22}$$

It mixes with h through the term

$$m_{hH}^2 = \frac{2v_4 v_5 [(\lambda_4 - \lambda_{45}) v_4^2 + (\lambda_{45} - \lambda_5) v_5^2]}{v_4^2 + v_5^2}. \tag{23}$$

This shows that $\mu_1, v_1 \gg v_{4,5}$ guarantees that H is heavier than h and their mixing is suppressed, as remarked earlier.

VI. DARK MATTER

Of the new particles beyond those of the SM, N_1 acts as the seesaw anchor of ν as already explained. It replaces the usually assumed right-handed neutrino. The others are the color-triplet fermions D of charge $-1/3$, the vectorlike electroweak doublet fermions (E^-, E^0), and the neutral singlet N_2 fermion. At the level of the SM extension to $U(1)_N$, as is clear from (7) and (8), this latter set of particles are distinguished from those of the SM by a discrete Z_2 symmetry under which they are odd. The lightest, presumably N_2, could then be dark matter. This is analogous to the stability of the proton from baryon number conservation in the SM. However, once it is realized that these particles are embedded into $SU(6)$, it is clear that N_2 must also decay, just as the proton.

The superheavy gauge bosons (5,6) and $(5^*, -6)$ of the adjoint vector 35 representation connect N_1 to (d^c, e, ν), N_2 to (D^c, E^-, E^0), and $(D, E^+, \bar{E}^0)$ to (u^c, u, d, e^c), as shown in (1). Together with (2), they allow the decay $N_2 \to e^+ W^-$ through the superheavy neutral gauge boson X^0 in (5,6) as shown in Fig. 1. This amplitude is proportional to $(v_4^2/v_2)(m_E/m_X^2)$, and is of similar magnitude to that of proton decay. This establishes the notion that dark-matter stability is akin to proton stability in the context of the unification of matter and dark matter, in parallel to that of quarks and leptons.

As for the heavy E leptons, it is clear that E^0 decays to $N_2 h$ from (8) and E^- decays to $E^0 W^-$. The heavy D quark decays through the scalar $(3, 1, -1/3, -4)$ component of $(5, -4)$ in 15_S or 21_S to $N_2 d N_1$, with N_1 decaying to νh. Note that these interactions do not violate the Z_2 symmetry separating matter from dark matter at low energy. They serve the purpose of allowing the heavier dark particles to decay to N_2 rapidly, assuming that the mediating scalars are not too heavy. Note also that proton decay is possible through scalar exchange as in $SU(5)$. Here it occurs through the mixing of $(3, 1, -1/3, -4)$ with $(3, 1, -1/3, 1)$ through the term $6_S \times 15_S^* \times 84_S$ and may be suppressed with a large mass for $(3, 1, -1/3, 1)$ as in $SU(5)$.

VII. RELIC ABUNDANCE AND DIRECT SEARCH

The relic abundance of the very long-lived N_2 is determined by its annihilation to scalar bosons which are in thermal equilibrium with SM particles. In particular, the

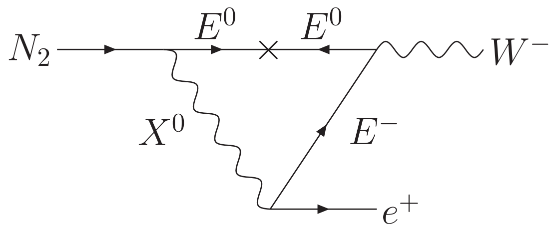

FIG. 1. Decay of N_2 through superheavy gauge boson X^0.

dominant process is shown in Fig. 2, assuming $m_{S_1} < m_{N_2} \ll m_{S_2}$. The $N_2N_2S_1$ coupling is $\sqrt{2}m_{N_2}/\sqrt{v_1^2 + 4v_2^2}$, and the $S_1S_1S_1$ coupling is dominated by $6\sqrt{2}\mu_2 v_1^2 v_2/(v_1^2 + 4v_2^2)^{3/2}$. Hence, the annihilation cross section at rest multiplied by relative velocity is

$$\sigma_{ann} \times v_{\rm rel} = \frac{9\mu_2^2 v_1^4 v_2^2}{128\pi m_{N_2}^2 (v_1^2 + 4v_2^2)^4} \sqrt{1 - \frac{m_{S_1}^2}{m_{N_2}^2}} \left(1 - \frac{m_{S_1}^2}{4m_{N_2}^2}\right)^{-2}. \tag{24}$$

As an example, let $v_1 = 2v_2 = 5$ TeV, $m_{N_2} = 1$ TeV, $m_{S_1} = 800$ GeV, and $\mu_2 = 14.7$ TeV, then $\sigma_{ann} \times v_{\rm rel} = 3 \times 10^{-26}$ cm^3/s, the canonical value for the correct dark-matter relic abundance of the Universe.

As for direct search, since N_2 is a Majorana fermion, it does not couple to the Z_N gauge boson at rest, so its only interaction with matter at underground experiments is through the SM Higgs boson. The mixing of S_1 with h comes from the term

$$m_{S_1 h}^2 = \frac{2v_1^2(\lambda_{14}v_4^2 + \lambda_{15}v_5^2) + 2v_2^2(\lambda_{24}v_4^2 + \lambda_{25}v_5^2) - 2\mu_1 v_1 v_4 v_5}{\sqrt{v_1^2 + 4v_2^2}\sqrt{v_4^2 + v_5^2}}. \tag{25}$$

Let $\theta = m_{S_1 h}^2/m_{S_1}^2$, then the coupling of N_2 to h is $m_N\theta\sqrt{2/(v_1^2 + 4v_2^2)}$, whereas h couples to quarks by $m_q/\sqrt{2(v_4^2 + v_5^2)}$. The spin-independent elastic scattering cross section of N_2 off a xenon nucleus per nucleon is given by

$$\sigma_0 = \frac{4}{\pi}\left(\frac{m_{N_2} m_{Xe}}{m_{N_2} + m_{Xe}}\right)^2 \left|\frac{54f_p + 77f_n}{131}\right|^2, \tag{26}$$

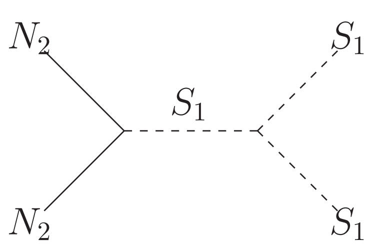

FIG. 2. N_2N_2 annihilation to S_1S_1.

where [10]

$$\frac{f_p}{m_p} = \left[0.075 + \frac{2}{27}(1 - 0.075)\right] \frac{m_{N_2}\theta}{m_h^2\sqrt{v_1^2 + 4v_2^2}\sqrt{v_4^2 + v_5^2}}, \tag{27}$$

$$\frac{f_n}{m_n} = \left[0.078 + \frac{2}{27}(1 - 0.078)\right] \frac{m_{N_2}\theta}{m_h^2\sqrt{v_1^2 + 4v_2^2}\sqrt{v_4^2 + v_5^2}}. \tag{28}$$

For $m_{N_2} = 1$ TeV, $v_1 = 2v_2 = 5$ TeV, $\sqrt{v_4^2 + v_5^2} = 174$ GeV, $m_h = 125$ GeV, the upper limit on θ from [11] $\sigma_0 < 10^{-45}$ cm^2 is 1.84×10^{-3}. Without fine-tuning, θ is of order $\sqrt{(v_4^2 + v_5^2)/(v_1^2 + 4v_2^2)} \sim 1/60$. Hence, a fine-tuning of order $1/10$ is required. This is possible by adjusting the value of μ_1 in the range of $v_{1,2}$ relative to other free parameters in (25).

VIII. GAUGE COUPLING UNIFICATION

The fermion content of this model is that of the SM extended by two complete $SU(5)$ fermion multiplets $(D^c, D^c, D^c, E^-, E^0)$, $(D, D, D, E^+, \bar{E}^0)$ and two fermion singlets $N_{1,2}$ per family. There are also two electroweak Higgs doublets $\Phi_{4,5}$ instead of just one in the SM, and two Higgs singlets. As far as the evolution of the known three gauge couplings, it follows the same pattern as the SM model with two Higgs doublets. It is well known that these gauge couplings do not unify under such circumstances. However, with the appropriate addition of some particle multiplets [12–15], unification of gauge couplings is possible at a high energy scale.

Consider the one-loop renormalization-group equation

$$\frac{1}{\alpha_i(M_1)} - \frac{1}{\alpha_i(M_2)} = \frac{b_i}{2\pi}\ln\frac{M_2}{M_1}, \tag{29}$$

where $\alpha_i = g_i^2/4\pi$ and the coefficients b_i are determined by the particle content between M_1 and M_2. In the SM with one Higgs doublet, these are given by

$$SU(3)_C\colon\ b_C = -11 + (4/3)N_F = -7, \tag{30}$$

$$SU(2)_L\colon\ b_L = -22/3 + (4/3)N_F + 1/6 = -19/6, \tag{31}$$

$$U(1)_Y\colon\ b_Y = (4/3)N_F + 1/10, \tag{32}$$

where $N_F = 3$ is the number of quark and lepton families and b_Y has been normalized by the well-known factor of $3/5$.

The contributions of the extra fermions and scalars of this model supply the following changes:

$$\Delta b_C = (2/3)N_F, \qquad \Delta b_L = (2/3)N_F + 1/6,$$
$$\Delta b_Y = (2/3)N_F + 1/10. \tag{33}$$

The further addition of the $\xi \sim (3, 1, -1/3, -4)$ scalar contributes $\Delta b_C = 1/6$ and $\Delta b_Y = 1/15$.

Gauge coupling unification may be achieved following Ref. [2], by adding a colored fermion octet $\Omega \sim (8, 1, 0, 0)$ with $\Delta b_C = 2$ as well as an electroweak fermion triplet $\Sigma \sim (1, 3, 0, 0)$ with $\Delta b_L = 4/3$, both from an assumed 35_F of $SU(6)$, and a scalar triplet $S \sim (1, 3, 0, -5)$ with $\Delta b_L = 2/3$ from the 84_S already present. The resulting evolution equations become

$$\frac{1}{\alpha_U} = \frac{1}{\alpha_C} + \frac{17/6}{2\pi}\ln\frac{M_U}{M_Z} + \frac{1/6}{2\pi}\ln\frac{M_\xi}{M_Z} + \frac{2}{2\pi}\ln\frac{M_\Omega}{M_Z} + \frac{2}{2\pi}\ln\frac{M_{Z_N}}{M_Z}, \tag{34}$$

$$\frac{1}{\alpha_U} = \frac{1}{\alpha_L} - \frac{1}{2\pi}\ln\frac{M_U}{M_Z} + \frac{4/3}{2\pi}\ln\frac{M_\Sigma}{M_Z} + \frac{2/3}{2\pi}\ln\frac{M_S}{M_Z} + \frac{2}{2\pi}\ln\frac{M_{Z_N}}{M_Z} + \frac{1/6}{2\pi}\ln\frac{M_\Phi}{M_Z}, \tag{35}$$

$$\frac{1}{\alpha_U} = \frac{3}{5\alpha_Y} - \frac{94/15}{2\pi}\ln\frac{M_U}{M_Z} + \frac{1/15}{2\pi}\ln\frac{M_\xi}{M_Z} + \frac{2}{2\pi}\ln\frac{M_{Z_N}}{M_Z} + \frac{1/10}{2\pi}\ln\frac{M_\Phi}{M_Z}, \tag{36}$$

where M_Φ is the mass of the heavier scalar doublet, and α_C, α_L, α_Y are evaluated at M_Z, with central values given by [9]

$$\alpha_C = 0.118, \qquad \alpha_L = (\sqrt{2}/\pi)G_F M_W^2 = 0.0340,$$
$$\alpha_Y = \alpha_L \tan^2\theta_W = 0.0102. \tag{37}$$

Eliminating α_U, the two conditions on the various intermediate masses are

$$34.318 = \ln\frac{M_U}{M_Z} + \frac{1}{23}\ln\frac{M_\xi}{M_Z} + \frac{12}{23}\ln\frac{M_\Omega}{M_Z} - \frac{8}{23}\ln\frac{M_\Sigma}{M_Z} - \frac{4}{23}\ln\frac{M_S}{M_Z} - \frac{1}{23}\ln\frac{M_\Phi}{M_Z}, \tag{38}$$

$$35.089 = \ln\frac{M_U}{M_Z} - \frac{1}{79}\ln\frac{M_\xi}{M_Z} + \frac{20}{79}\ln\frac{M_\Sigma}{M_Z} + \frac{10}{79}\ln\frac{M_S}{M_Z} + \frac{1}{79}\ln\frac{M_\Phi}{M_Z}. \tag{39}$$

Subtracting the two equations to eliminate M_U, and assuming $M_\Omega = M_\Sigma = M_S$, the condition

$$0.771 = -\frac{102}{(79)(23)}\ln\frac{M_\xi}{M_Z} + \frac{30}{79}\ln\frac{M_S}{M_Z} + \frac{102}{(79)(23)}\ln\frac{M_\Phi}{M_Z} \tag{40}$$

is obtained. This is satisfied, for example, with $M_\Phi = 500$ GeV, $M_S = 1$ TeV, and $M_\xi = 5.9$ TeV, resulting in $M_U = 6.57 \times 10^{16}$ GeV and $\alpha_U = 0.0386$ for $M_{Z_N} = 3$ TeV. As for α_N, using the normalization factor of 1/60, it may now be deduced from

$$\frac{1}{\alpha_U} = \frac{1}{60\alpha_N} - \frac{227/30}{2\pi}\ln\frac{M_U}{M_Z} + \frac{4/15}{2\pi}\ln\frac{M_\xi}{M_Z} + \frac{5/12}{2\pi}\ln\frac{M_S}{M_Z} + \frac{187/36}{2\pi}\ln\frac{M_{Z_N}}{M_Z} + \frac{17/180}{2\pi}\ln\frac{M_\Phi}{M_Z}, \tag{41}$$

yielding $\alpha_N = 2.61 \times 10^{-4}$.

IX. CONCLUSIONS

Following up on the notion [5] that dark matter is a long-lived particle just as the proton in a grand unified theory by extending $SU(5)$ to $SU(6)$, a specific complete model is presented. The SM fermions are augmented per family by one heavy D quark, one heavy vectorlike lepton doublet (E^0, E^-), and two singlets $N_{1,2}$, as shown in (1). A discrete Z_2 symmetry is imposed on the dimension-four terms of the Lagrangian, but is softly and spontaneously broken in the scalar sector. The resulting theory has the following features. The heavy Majorana fermion N_1 acts as the right-handed neutrino in allowing the corresponding observed neutrino to have a small seesaw mass, doing this in the context of $SU(6)$ instead of $SO(10)$. The heavy Majorana fermion N_2 is a dark-matter candidate with interactions consistent with the correct relic abundance and may be observed in underground direct-search experiments through its induced coupling to the SM Higgs boson. However, N_2 is not protected by a symmetry and decays through a superheavy gauge boson in $SU(6)$, just as the proton is not absolutely stable and decays through a superheavy gauge boson in $SU(5)$. The pattern of symmetry breaking is assumed to be $SU(6) \to SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_N$ at a scale of order 10^{16} GeV. Gauge coupling unification is possible and demonstrated with an explicit example. The heavier D and (E^0, E^-) particles decay rapidly to N_2 through scalars at an intermediate mass. The Z_N gauge boson is potentially observable at a few TeV.

By incorporating dark matter as an essential component of grand unification, it is shown that just as the conservation of baryon number and lepton parity in the SM is violated in the unification of quarks and leptons, the conservation of dark parity at low energy is violated in the unification of matter and dark matter. The longevity of dark matter is then linked to the longevity of the proton, as a natural explanation of the former's existence.

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