Noncovariance of "covariant polymerization" in models of loop quantum gravity

Martin Bojowald
Institute for Gravitation and the Cosmos, The Pennsylvania State University, 104 Davey Lab, University Park, Pennsylvania 16802, USA

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It is possible to implement a certain form of modified gravity inspired by loop quantization through nonbijective canonical transformations. The canonical nature might suggest that such modifications are guaranteed to preserve general covariance. Here, however, we show that a dedicated space-time analysis is still required, even in the case of a bijective canonical transformation. In addition, a complete global analysis is presented for a recent proposal of a nonbijective transformation, showing that it does not preserve general covariance and that the only novel physical effect introduced by the modification is the presence of certain time-reversal hypersurfaces between classical space-time regions. These results provide further insights into the physical interpretation of modified dynamics in models of loop quantum gravity.

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I. INTRODUCTION

Models of loop quantum gravity attempt to implement quantum-geometry effects by using certain modifications of the classical equations of canonical gravity. The canonical nature, as usual, implies that general covariance is not manifest and must be tested by dedicated means. Several no-go results for general covariance and slicing independence in such models have recently been derived, using setups relevant for cosmology [1] and black holes [2,3]. The only known way to realize covariance in models of loop quantum gravity is through a deformed version [4,5] that implies signature change at high density or curvature when applied to modifications commonly used in loop quantum cosmology or loop quantum black holes [4,6–12]. (Signature change may be avoided in some cases, but it would require nonstandard modifications such as complex connections [13–16], Euclidean-type gravity [17,18] or nonbouncing background solutions [19].)

It is therefore important to explore possible alternative modifications. In this context, the recent paper [20] suggests applying a nonbijective canonical transformation to the classical theory, hoping that the modified model will be close enough to the classical system to preserve covariance, yet different enough to be considered a modification because the transformation is not bijective. As we will show in this paper, covariance is a subtle issue even in this case and must be derived. Once this task has been completed, it can be seen that the modifications are not compatible with general covariance or slicing independence in a global space-time structure. The equations suggested in [20] therefore do not show how models of loop quantum gravity could be made consistent with general covariance, and they do not provide counter examples to the no-go results of [1,2].

Our analysis of general covariance makes use of effective line elements, as defined in [21]. A proper effective line element provides a geometrical interpretation of solutions of a modified theory of gravity. For the line element to have a proper geometrical meaning, it must be invariant under coordinate changes. But modified equations of a model may well change the gauge transformations imposed on basic fields, in particular if the model is formulated canonically and does not make use of space-time tensors. Therefore, the existence of suitable metric components constructed from the basic fields of the modified theory such that they form an invariant line element is, in general, not guaranteed. Even if metric components exist, their relationship with the basic fields is usually modified, compared with the classical relationship, in order to account for modified gauge transformations.

In [20] and elsewhere in the literature, however, the simple classical relationship between metric components and basic fields is mistakenly assumed to hold also in the presence of modifications. A derivation of proper effective line elements then corrects the resulting understanding of space-time structure, and it reveals the global geometry implied by solutions of the modified theory. As a result, solutions of [20] are simply concatenations of classical space-time regions, separated by time-reversal hypersurfaces. These hypersurfaces, derived in more detail in Sec. III B below, are implicitly defined by time derivatives of canonical fields changing sign in a discontinuous manner. Their presence makes it possible for extrinsic curvature to remain bounded. However, they can be defined only using noninvariant quantities, thus violating covariance on a global level.

In addition to the suggestion made in [20], we will also consider the case of a bijective canonical transformation. Such a transformation should, of course, exactly preserve physical properties of the classical theory, including general covariance. Nevertheless, we will see that space-time structure in such a "modified" canonical theory is nontrivial and requires a dedicated analysis before physical conclusions can be drawn. The model therefore provides an instructive example: Even though it is unable to imply new physics, a careless analysis might wrongly suggest new effects such as singularity resolution. These lessons will then be applied to the model proposed in [20]. They are also relevant more broadly in a large number of models of loop quantum gravity in which line elements have been used for modified theories without confirming their geometrical validity [3].

The results of the present paper demonstrate the importance of considering properly defined effective line elements to express solutions of equations of motion in modified canonical theories of gravity. They also underline the highly nontrivial nature of covariance in models of loop quantum gravity, which turns out to be violated even by the minimal modifications suggested in [20], based on a canonical transformation from the classical theory.

II. SPACE-TIME ANALYSIS

The aim of this paper is to present a detailed space-time analysis of the model introduced in [20] and related examples. Since the model is canonical, we use methods of canonical gravity (see [22,23] for details).

A. Variables and transformations

Canonical gravity of spherically symmetric models is described by line elements of the form [24]

$$\mathrm{d}s^2 = -N^2\mathrm{d}t^2 + q_{xx}(\mathrm{d}x + M\mathrm{d}t)^2 + q_{\varphi\varphi}(\mathrm{d}\vartheta^2 + \sin^2\vartheta\mathrm{d}\varphi). \tag{1}$$

The spatial part is determined by two functions, q_{xx} and $q_{\varphi\varphi}$, depending on the radial position x as well as time t, while the lapse function N and shift vector M, also depending on x and t, describe its extension to space-time. In spherically symmetric models of loop quantum gravity [25,26], one usually replaces metric components with components E^x and E^φ of a densitized triad, such that

$$q_{xx} = \frac{(E^\varphi)^2}{|E^x|}, \qquad q_{\varphi\varphi} = |E^x|. \tag{2}$$

In what follows it will be sufficient to assume $E^x > 0$, fixing the orientation of space.

The triad components are, up to constant factors, canonically conjugate to components of extrinsic curvature, K_x and K_φ, such that

$$\{K_x(x_1), E^x(x_2)\} = 2G\delta(x_1, x_2), \tag{3}$$

$$\{K_\varphi(x_1), E^\varphi(x_2)\} = G\delta(x_1, x_2), \tag{4}$$

with Newton's constant G. Extrinsic curvature depends on time and space derivatives of the densitized triad (as well as lapse and shift) in a way that may be modified in models of loop quantum gravity. We will not need the precise relationships but only use the canonical structure.

Depending on the time gauge, equations of motion for the basic phase-space variables are generated by combinations of the Hamiltonian constraint, $H[N]$, and the diffeomorphism constraint, $D[M]$. We will not need the precise form of these expressions either but only refer to their nature as gauge generators of deformations of spatial hypersurfaces in space-time. These transformations correspond to classical space-time [27] provided the constraints obey Dirac's hypersurface-deformation brackets [28], in particular

$$\{H[N_1], H[N_2]\} = -D[E^x(E^\varphi)^{-2}(N_1N_2' - N_1'N_2)]. \tag{5}$$

The presence of a phase-space dependent structure function, $E^x(E^\varphi)^{-2}$, implies that the structure of space-time is sensitive to modifications of the constraints.

As shown in [29], the structure function can be eliminated in an equivalent constrained system obtained by suitable combinations of H and D. This construction has also been used in the recent analysis of [20]. However, based on [27], the behavior of hypersurface deformations and therefore of general covariance and slicing independence requires a bracket of the form (5) for the generators of normal deformations of spatial hypersurfaces. Discussions of covariance therefore cannot avoid referring to this relationship, especially in attempted modifications.

The main ingredient in models of loop quantum gravity is a substitution of (almost) periodic functions of connection or extrinsic-curvature components for the classical quadratic dependence in the Hamiltonian constraint. If this substitution is done only in these places, and in a careful way relating different substitution functions to one another, the bracket (5) in vacuum is modified by a new factor of the structure function such that the structure of space-time is nonclassical [30–32]. (See [33,34] for an analogous result in the cosmological context.) In the presence of a scalar field, no such substitution is known that preserves the form of (5) even if one accepts modifications of the structure function [35].

The authors of [20] suggest that this difficulty may be overcome if one uses a canonical transformation instead of substitution. For the gravitational variables, they propose to transform from the pair (K_φ, E^φ) to a new pair $(\tilde{K}_\varphi, \tilde{E}^\varphi)$ such that

$$K_\varphi = \frac{\sin(\ell \tilde{K}_\varphi)}{\ell}, \qquad E^\varphi = \frac{\tilde{E}^\varphi}{\cos(\ell \tilde{K}_\varphi)}. \tag{6}$$

The pair (K_x, E^x) remains unchanged. There is a similar transformation for a scalar matter field, which we do not use explicitly here because (6) is sufficient for a discussion of space-time structure: The scalar field does not appear in the structure function of (5).

Expressed in terms of the new variables, the Hamiltonian constraint depends on $\tilde{K}_\varphi$ through a periodic function, as in standard modifications, while the dependence of E^φ on $\tilde{K}_\varphi$ leads to new modifications in metric functions not considered before. The hope is that these new modifications may preserve general covariance because the model is obtained by a canonical transformation from a covariant theory. At the same time, only a bounded range of K_φ is realized for an infinite range of $\tilde{K}_\varphi$, which could introduce new physical effects and help with the resolution of singularities.

B. Bijective canonical transformation

The model of [20] is based on a canonical transformation of the classical theory which is not bijective, and therefore need not be completely equivalent to classical gravity. It may therefore be considered a modified version of spherically symmetric general relativity. The case of a bijective canonical transformation, by contrast, could be deemed too trivial to be worthy of attention because it cannot lead to new physics. It is nevertheless instructive to see how a dedicated space-time analysis would proceed if we were faced with a proposed modified theory without knowing that it is simply obtained by a bijective canonical transformation from classical general relativity.

The setup is therefore as follows: We are given a canonical theory with canonical pairs $(\tilde{K}_\varphi, \tilde{E}^\varphi)$ and (K_x, E^x) and perhaps some matter fields, as well as a consistent set of diffeomorphism and Hamiltonian constraints in these variables. The consistent constraints have been derived by applying a bijective canonical transformation

$$K_\varphi = f(\tilde{K}_\varphi), \qquad E^\varphi = \frac{\tilde{E}^\varphi}{\mathrm{d}f/\mathrm{d}\tilde{K}_\varphi} \tag{7}$$

to the constraints of classical spherically symmetric gravity in canonical form, where f is a monotonic function such that $f(\tilde{K}_\varphi) \approx \tilde{K}_\varphi$ for $\tilde{K}_\varphi$ sufficiently small compared with some reference scale. Given these conditions, f may well be such that the full range of K_φ is mapped to a finite range of $\tilde{K}_\varphi$ in which case the transformation would be bijective provided the new variable $\tilde{K}_\varphi$ is always restricted to this finite range. In spite of the underlying equivalence with classical gravity, one could therefore claim that consistent constraints imply new physics and that singularities are resolved because curvature $(\tilde{K}_\varphi)$ remains bounded, all while preserving general covariance.

More generally, we could assume a bijective two variable transformation

$$K_\varphi = f_1(\tilde{K}_\varphi, \tilde{E}^\varphi), \qquad E^\varphi = f_2(\tilde{K}_\varphi, \tilde{E}^\varphi) \tag{8}$$

such that $\{f_1, f_2\} = G$. It would not be straightforward to reconstruct this transformation if we were just given the resulting constraints. How would we then spot possible erroneous claims of new physics and show that the theory is, in fact, completely equivalent to spherically symmetric general relativity?

To some extent, the situation is comparable to the task of telling that a "new" solution of general relativity has just been obtained from a well-known one by a coordinate transformation. Like a canonical transformation, a coordinate transformation, if incompletely analyzed, could also suggest bounded curvature if it maps a finite space-time region that does not include singularities into a full infinite range of a new coordinate. In this case, there are standard methods to analyze the global meaning of solutions, for instance by checking geodesic completeness to determine whether an infinite range of some coordinate amounts to an infinite geometric distance, or just to some finite interval.

At this point, however, the two examples of a canonical transformation and a coordinate transformation start to differ conceptually. While any coordinate transformation preserves space-time structure and covariance, a canonical transformation need not do so. In particular, a coordinate transformation gives us an unambiguous new metric to be used for a geometrical derivation. But a canonical transformation, without further analysis, does not tell us whether some new field $\tilde{E}^\varphi$ can indeed be used in a metric component just like the original E^φ, or whether the new $\tilde{K}_\varphi$ is indeed a curvature component with the same geometrical meaning as K_φ. At this point, at the latest, we should become suspicious of claims about eliminated singularities in a bijectively transformed theory because a bounded $\tilde{K}_\varphi$ does not necessarily imply bounded curvature. How do we turn our suspicion into a proof that the singularity claims are incorrect?

C. Effective line elements

A canonical space-time analysis gives us a clear answer to the questions posed in the preceding subsection. Solutions of a modified canonical theory of gravity are not necessarily geometrical, that is, one cannot simply assume that inserting some $\tilde{E}^\varphi$ instead of E^φ in (2) results in a well-defined space-time line element of the form (1) with the same lapse N and shift M as used in the relevant equations of motion. Any line element $\mathrm{d}s^2 = g_{\alpha\beta}\mathrm{d}x^\alpha \mathrm{d}x^\beta$, by definition, has to be invariant with respect to a combination of coordinate transformations of $\mathrm{d}x^\alpha$ and gauge transformations of the canonical metric components.

While $\mathrm{d}x$ and $\mathrm{d}t$ in (1) still transform like standard coordinate differentials after applying a canonical transformation such as (6), (7) or (8), the new field $\tilde{E}^\varphi$ does not have the same (gauge) transformation behavior as the classical E^φ because the transformation depends on $\tilde{K}_\varphi$ which, like K_φ, is not a space-time scalar. Therefore, using a modified $\tilde{E}^\varphi$ in q_{xx} for (1) implies that modified metric components no longer transform in a way dual to coordinate differentials, and the line element is not invariant. Geometrical derivations from such an expression are meaningless because they depend on coordinate choices. (One could try to modify the transformations of $\mathrm{d}x$ and $\mathrm{d}t$ to compensate for the modified gauge transformations of $\tilde{E}^\varphi$, for instance by using nonclassical manifolds. However, no such manifold structure is known for the specific modifications discussed here. For the example of noncommutative manifolds from the perspective of hypersurface deformations, see [36].)

As shown in [21], it is sometimes possible to apply a field redefinition to canonical fields in a modified theory so as to bring their gauge transformations to a form required for an invariant effective line element. In the present case, one can use methods introduced in nnn to find a suitable field redefinition of $\tilde{E}^\varphi$, which can be summarized as follows: A field E^φ that, together with its conjugate K_φ, appears in the Hamiltonian and diffeomorphism constraints of a canonical theory plays the role of a metric component as in (2) if and only if the Poisson bracket of two Hamiltonian constraints equals (5). In the bijectively transformed theory, however, this bracket is replaced by

$$\{H[N_1], H[N_2]\} = -D[E^x f_2(\tilde{K}_\varphi, \tilde{E}^\varphi)^{-2}(N_1 N_2' - N_1' N_2)], \qquad (9)$$

or

$$\{H[N_1], H[N_2]\} = -D[(\mathrm{d}f/\mathrm{d}\tilde{K}_\varphi)^{-2} E^x (\tilde{E}^\varphi)^{-2}(N_1 N_2' - N_1' N_2)] \qquad (10)$$

in the simpler one-variable transformation. Therefore, using $\tilde{E}^\varphi$ in (2) does not yield a legitimate metric component, and $\tilde{K}_\varphi$ is not a component of extrinsic curvature.

In order to derive the correct space-time structure and a meaningful metric, we should find a suitable function $\tilde{\tilde{E}}^\varphi$ of $\tilde{E}^\varphi$ and $\tilde{K}_\varphi$ in terms of which the Poisson bracket of two Hamiltonian constraints takes on the classical form (5). It is easy to see that $\tilde{\tilde{E}}^\varphi = E^\varphi$ is just the classical field in (9) or (10). Completing this substitution to a canonical transformation then leads us back to the classical K_φ from $\tilde{K}_\varphi$, and inserting this transformation in the constraints tells us that the theory is nothing but classical. [The canonical conjugate K_φ of some function E^φ on the phase space $(\tilde{K}_\varphi, \tilde{E}^\varphi)$ is not uniquely determined because any function of E^φ could be added to K_φ while maintaining the nature of a canonical conjugate. However, this freedom is eliminated by the boundary condition that $K_\varphi \approx \tilde{K}_\varphi$ for $\tilde{K}_\varphi$ small with respect to some scale used in the model.] At this point, we would have debunked any potential claims of new physics and singularity resolution.

Our example is artificial and deals with a trivial modification of classical general relativity. It is nevertheless instructive because it shows the importance of a dedicated analysis of space-time structure in canonical terms. It is also relevant because arguments comparable to some ingredients of our example have often been made in models of loop quantum gravity. These models deal with actual modifications of classical gravity and there is a possibility for new physics to emerge. But also in this case, it is often, and incorrectly, assumed that some field $\tilde{E}^\varphi$ that shows some semblance to the classical E^φ can be used to define a meaningful metric component using (2). This geometrical interpretation is possible only if $\tilde{E}^\varphi$ is such that the Poisson bracket of two Hamiltonian constraints equals (5) where E^φ is simply replaced by $\tilde{E}^\varphi$, without introducing any multiplicative factor or other modifications of the structure function. Unfortunately, this condition is rarely realized in models of loop quantum gravity, which often do not even check that the Poisson bracket of two Hamiltonian constraints remains closed after modifications.

III. POLYMERIZED MODELS

In the case of [20], it is clear that the bracket of two Hamiltonian constraints remains closed after applying a nonbijective canonical transformation. Moreover, the modification is nontrivial because the canonical transformation used in this case, given by (6), is not bijective. As seen in the preceding subsection, however, a dedicated space-time analysis is necessary to interpret the theory even in the case of a bijective transformation. It should then certainly be performed also in the nonbijective case, but this has not been attempted in [20]. It is therefore unclear whether physical statements suggested there are correct.

The modified theory has Hamiltonian constraints such that

$$\{H[N_1], H[N_2]\} = -D[\cos^2(\ell\tilde{K}_\varphi) E^x (\tilde{E}^\varphi)^{-2}(N_1 N_2' - N_1' N_2)], \qquad (11)$$

with a modified structure function, obtained by simply applying the canonical transformation to (5). Since the modification introduces new zeros of the structure function at $\ell\tilde{K}_\varphi = \frac{1}{2}(2n+1)\pi$ with integer n, it eliminates some contributions of the diffeomorphism constraint from the right-hand side. The presence of structure functions implies that generators of hypersurface deformations form a Lie algebroid [37–39] over phase space, labeling independent contributions from the constraints. New zeros in the structure function introduced by the transformation mean that the algebroid gains new Abelian subalgebroids by restriction to the zero-level sets of the structure function. The algebraic structure is therefore inequivalent to its classical form. (The authors of [20] claim that the modification "preserves the constraint algebra," which presumably refers to a partial Abelianization of the generators as in [29]. However, as shown in [35], such a reformulation of the constraints is not sufficient for a discussion of general covariance and space-time structure.)

An algebraic structure inequivalent to that determined by the classical constraints implies that its relationship to standard hypersurface deformations is not obvious. Covariance is therefore nontrivial in the modified system. The nonbijective nature of the canonical transformation employed now to obtain the modification is precisely the reason why there are additional zeros in the modified structure function of hypersurface-deformation brackets. According to [20], the nonbijective nature of the transformation might provide a chance for the modified theory to describe new physical effects, but it is also the reason why covariance is no longer obvious even though the modification has been obtained by canonically transforming a covariant theory. (The claim "It has the advantage that it is a canonical transformation from the original variables. That means that it preserves the constraint algebra and the covariance of the theory, which previous choices did not." of [20] is therefore unjustified.) In the presence of modified hypersurface deformations with an inequivalent algebraic structure, covariance has to be derived by a careful analysis of generic solutions and their geometrical meaning, using effective line elements.

A. Local solutions

Local solutions for $\tilde{E}^\varphi$ and $\tilde{K}_\varphi$ can be derived without explicitly solving modified equations of motion because they can simply be obtained by applying a local (in phase space) inverse of the canonical transformation (6) to a classical solution in canonical form. Starting at small ℓK_φ for the classical solution, any modified local solution $\tilde{K}_\varphi$ remains valid until ℓK_φ reaches the values ± 1, the local extrema of $\sin(\ell \tilde{K}_\varphi)$ where the canonical transformation is no longer invertible.

If one were to solve modified equations directly for $(\tilde{K}_\varphi, \tilde{E}^\varphi)$, starting with some initial values, it would be possible to cross regions where $\ell \tilde{K}_\varphi = \pm \frac{1}{2}\pi$, again corresponding to the first local extrema of $\sin(\ell \tilde{K}_\varphi)$ close to small ℓK_φ. Such an extension of the local solution is no longer a simple local inverse of the canonical transformation, and presumably gives rise to "novel phenomena" that are, according to [20], introduced by the modification.

However, a solution in the range where $\ell \tilde{K}_\varphi > \frac{1}{2}\pi$ (the case of $\ell \tilde{K}_\varphi < -\frac{1}{2}\pi$ being analogous) and $\ell \tilde{K}_\varphi < \frac{3}{2}\pi$, can again be interpreted as a local inverse of (6), but one that makes use of a different branch of the arcsine compared with the initial region at $|\ell \tilde{K}_\varphi| < \frac{1}{2}\pi$. The canonical transformation therefore provides a classical analog in any range of $\ell \tilde{K}_\varphi$ that excludes the values $\frac{1}{2}(2n+1)\pi$ with integer n. While the analogous K_φ is always bounded thanks to (6), there is no upper limit on $\ell \tilde{K}_\varphi$ beyond which classical analogs would no longer exist.

We have obtained a direct correspondence between local solutions in the classical and modified theories. The next question we have to address is whether physics or geometry in the modified theory should be based on the field $\tilde{K}_\varphi$ and its conjugate $\tilde{E}^\varphi$, or on their local classical analogs K_φ and E^φ. This question is relevant for the application presented in [20], in which critical collapse is studied numerically by evaluating a "black hole mass." Unfortunately, [20] does not specify how this mass is obtained, but presumably it refers to a mass parameter extracted in the usual way from a line element, constructed from $\tilde{E}^\varphi$ rather than E^φ in the modified theory. We therefore have to analyze how a meaningful line element can be obtained in the modified theory. As discussed in Sec. II C, a meaningful effective line element requires specific transformation properties to hold for its coefficients.

B. Global structure

Using local inverses of the canonical transformation, we have obtained local solutions in canonical form, resulting in evolutions of $\tilde{K}_\varphi$ and $\tilde{E}^\varphi$ depending on some time coordinate implicitly determined by lapse and shift. Such a solution of equations of motion in a modified theory is not necessarily geometrical. As in our example of a bijective canonical transformation, using methods of [40], a field redefinition of $\tilde{E}^\varphi$ is necessary before we can apply effective line elements. Not surprisingly, this field redefinition is again an application of the canonical transformation (6), mapping $\tilde{E}^\varphi$ back to E^φ which clearly has the correct transformation behavior for a well-defined line element to result from (2) and (1).

Methods of effective line elements therefore show that physics and geometry in the modified theory should be based on the classical analogs found in the previous subsection, and not on the modified solutions $\tilde{K}_\varphi$ and $\tilde{E}^\varphi$. In any region in which (6) is locally invertible, the modified theory simply describes a transformed version of classical gravity. Any potential for new physical effects is restricted to subsets of measure zero in phase space and (generically) space-time. In order to understand their meaning, we have to determine how different regions of classical analogs may be connected in an effective space-time picture of global form.

So far, we have obtained formal piecewise solutions for the canonical fields $\tilde{K}_\varphi$ and $\tilde{E}^\varphi$ as well as effective line elements that faithfully describe their geometrical meaning, based on field redefinitions. The final question is how these piecewise solutions can be glued back together to obtain a global space-time picture. Such a gluing cannot be based on classical matching conditions because they would simply lead to a global classical solution that does not respect the boundedness of K_φ implied by (6).

Given a solution for $\tilde{K}_\varphi$ and $\tilde{E}^\varphi$, a classical analog and an effective line element is obtained by applying the canonical transformation (6). Since the transformation is not bijective, different ranges of $\tilde{K}_\varphi$ may correspond to the same classical geometry. If we first restrict ourselves to ranges of $\tilde{K}_\varphi$ in which the transformation is invertible, the corresponding phase-space region corresponds, via the effective line element, to a region in space-time which generically is incomplete because it is cut off at fixed values of K_φ. A global solution therefore requires an extension through the hypersurfaces on which $\ell\tilde{K}_\varphi = \frac{1}{2}(2n+1)\pi$ with integer n.

It is easy to see how different regions are connected if we first focus on two neighbors, such as the low-curvature region, called region I where $|\ell\tilde{K}_\varphi| < \frac{1}{2}\pi$, and a region II where $\frac{1}{2}\pi < \ell\tilde{K}_\varphi < \frac{3}{2}\pi$. For a transition from region I to region II to happen, $\dot{\tilde{K}}_\varphi > 0$ when $\ell\tilde{K}_\varphi = \frac{1}{2}\pi$, which by continuity extends to a region around the transition hypersurface. Since K_φ is a continuous function of $\tilde{K}_\varphi$, it approaches the same value at the transition hypersurface from both regions, given by $\ell K_\varphi = 1$. Applying (6), we see that the corresponding analog solutions K_φ behave like time reversed versions in a neighborhood of the transition hypersurface; $\dot{K}_\varphi = \ell\cos(\ell\tilde{K}_\varphi)\dot{\tilde{K}}_\varphi$ has opposite signs on the two sides of the transition hypersurface because $\cos(\ell\tilde{K}_\varphi)$ has opposite signs in the two regions while $\dot{\tilde{K}}_\varphi > 0$ as we already saw.

For the same reason, E^φ has opposite signs on the two sides and, unlike K_φ, is not continuous because it goes through infinity if $\tilde{E}^\varphi$ remains finite. (The classical equations of motion imply that K_φ is proportional to $\dot{E}^x$ rather than $\dot{E}^\varphi$, such that it may remain regular while E^φ grows without bounds.) Therefore, the time derivative of the absolute value $|E^\varphi|$, which is relevant for q_{xx} in (2), has opposite signs on the two sides; The second term in

$$|E^\varphi|^\bullet = \mathrm{sgn}(E^\varphi)\left(\frac{\dot{\tilde{E}}^\varphi}{\cos(\ell\tilde{K}_\varphi)} + \ell\frac{\tilde{E}^\varphi}{\cos^2(\ell\tilde{K}_\varphi)}\sin(\ell\tilde{K}_\varphi)\dot{\tilde{K}}_\varphi\right) \sim \ell\frac{\mathrm{sgn}(E^\varphi)}{\cos^2(\ell\tilde{K}_\varphi)}\tilde{E}^\varphi K_\varphi\dot{\tilde{K}}_\varphi, \tag{12}$$

is dominant near the hypersurface and enjoys the required sign property. The geometry in region II can therefore be interpreted as a time-reversed classical solution compared with the time direction in region I. (It is not necessarily a time reversal of the same solution as in region I because E^φ is not continuous across the transition hypersurface.)

Applying this result to all transitions, we see that a global solution of the modified theory is a concatenation of infinitely many classical regions with alternating orientations of time. In each region, the geometry is indistinguishable from a classical solution. The only new physics therefore resides in the time reversals, which make it possible for K_φ to remain bounded.

C. Noncovariance

In each local region, the geometry is coordinate and slicing independent provided the changes of coordinates and slicings are sufficiently "small" such that they do not leave the range of K_φ relevant for the region. (We can apply slicing independence only in the classical analogs, where the correct version (5) of hypersurface deformations holds.) Globally, space-time in this model could be covariant only if the reversal surfaces were covariantly defined, but this is not the case; They refer to fixed values of $\ell K_\varphi = \pm 1$, and K_φ is not a space-time scalar.

Choosing a different slicing in a classical analog in general shifts the positions of time reversal surfaces. A complete solution for $\tilde{K}_\varphi$ and $\tilde{E}^\varphi$ therefore violates slicing independence, even after it has locally been mapped to a suitable effective line element. For instance, in a vacuum solution there would be no time reversals outside the horizon in a Schwarzschild slicing, but there are other exterior slicings in which ℓK_φ can be large and trigger time reversal in the modified geometry. Even with minimal modifications introduced by the model, general covariance is violated.

IV. CONCLUSIONS

We have presented a detailed analysis of space-time structure in models obtained by bijective or nonbijective canonical transformations of classical gravity. Although the bijective case is completely equivalent to classical gravity, a space-time analysis is nontrivial because the equivalence may be hidden if complicated canonical transformations are applied. Our discussion showed that basic fields of a modified theory, in general, cannot be identified directly with metric components that play the same role as their classical counterparts.

While such a model would be considered trivial from the perspective of modified gravity, it is nevertheless instructive because it highlights the subtle nature of space-time structure in canonical theories. In particular, the importance of identifying suitable metric components or effective line elements constructed from the basic fields of a canonical modified theory remains highly relevant if the theory is genuinely modified. The nontrivial nature of such identifications has often been overlooked in models of loop quantum gravity.

We applied our detailed construction of effective line elements that consistently describe the space-time geometry of solutions to the modified theory introduced in [20]. This model uses a nonbijective canonical transformation and is therefore inequivalent to classical gravity. However, we have shown that the only new physical effect is the introduction of time-reversal surfaces connecting classical space-time regions. This observation corrects the claim "As the canonical transformation is not invertible in the whole of phase space it still allows to have the usual novel phenomena that loop quantizations introduce in regions where one expects general relativity not to be valid, like close to singularities." made in [20]. Locally, general relativity is valid in all regions of the modified theory, without any novel phenomena that have been claimed previously in loop quantizations. Our constructions also show that effective geometries described by the model depend only on the local extrema of the function $K_\varphi(\tilde{K}_\varphi)$. The specific sine function, usually motivated by expressions of holonomies used in loop quantum gravity, does not matter at all.

Even though the modifications are obtained by a canonical transformation of a covariant theory, their global solutions violate covariance precisely at those places where "novel phenomena" happen. This outcome heightens the covariance crisis of loop quantum gravity; Even a minor modification of the classical equations, inspired by loop quantum gravity but implemented by a canonical transformation, is in conflict with the requirement of general covariance.

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