Neutron interferometry and tests of short-range modifications of gravity

J. M. Rocha and F. Dahia

Departamento de Física, Universidade Federal da Paraíba, Caixa Postal 5008, 58059-970, João Pessoa, Paraíba, Brazil

(Received 9 March 2021; accepted 20 May 2021; published 7 June 2021)

We consider tests of short-distance modifications of gravity based on neutron interferometry in the scenario of large extra dimensions. Avoiding the noncomputability problem in the calculation of the internal gravitational potential of extended sources, typical of models with zero-width brane, we determine the neutron optical potential associated with the higher-dimension gravitational interaction between the incident neutron and a material medium in the context of thick brane theories. Proceeding this way, we identify the physical quantity of the extra dimension model that the neutron interferometry is capable of constraining. We also consider interferometric experiments in which the phase shifter is an electric field, as in the test of the Aharonov-Casher effect. We argue that this experiment, with this nonbaryonic source, can be viewed as a test of the short-range behavior of post-Newtonian parameters that measure the capacity of the pressure and the internal energy for producing gravity.

DOI: 10.1103/PhysRevD.103.124014

I. INTRODUCTION

Braneworld scenarios [1–4], according to which our ordinary four-dimensional spacetime is embedded in a higher-dimensional ambient space, have motivated many recent studies on the modifications of gravity in short-distance scale [5].

Compared to the Kaluza-Klein pioneering work on extra dimension formally based on general relativity, these new higher-dimensional theories, among which the ADD model [1] is a prototype, are distinguished by a peculiar feature. They are based on the fundamental assumption that matter and fields are confined in the 3-brane (the ordinary three-dimensional space) while gravity has access to all directions [1–4]. The dilution of the gravitational field through the whole ambient space would be the reason for the feebleness of gravity in comparison to the strength of the localized fields and, therefore, could be the explanation for the hierarchy problem, whose resolution was originally the main motivation for some braneworld models [1,2].

In these scenarios, the effects of extra dimensions on the gravitational field may become significant in a length scale R much greater than the scale in which the confined fields would feel them directly. Thus, extra dimensions with a size much larger than the Planck length are phenomenologically feasible since gravity, differently from what happens to the other fields, is being empirically tested in submillimeter domains only recently [5,6].

An important prediction of the existence of large extra dimensions according to these braneworld models is a strong amplification of the gravitational field in short distances ($r \ll R$), implying that the theory could be experimentally checked. This possibility has motivated the search for empirical signs of the supposed hidden dimensions in laboratory tests from areas such as spectroscopy [7–17], neutron interferometry [18–22], torsion balance experiments [6], and Casimir effect tests [23–25]. Besides these table-top experiments, there are important tests from astrophysics [26–28] and high-energy colliders [29,30].

Regarding the laboratory tests, modifications of gravity are usually parametrized by means of an additional Yukawa-like potential energy of the form $(\alpha GMm/r)\exp(-r/\lambda)$, where M and m are the masses of interacting particles and G is the Newtonian gravitational constant. Experiments from diverse areas put upper bounds on the amplification factor α in different ranges of the length scale parameter λ [5].

The Yukawa parametrization is very useful because it may encompass modifications of gravity with different theoretical origins [31–33]. The ADD model predicts a correction of the same type for the external gravitational potential produced by a particle in the domain of large distances $r \gg R$ [34]. In this case, the parameter α is proportional to the number of hidden dimensions δ. The exact relation depends on the topology of the supplementary space and on the length scale at which the stabilization of its volume takes place [34,35]. In its turn, in short distances, the gravitational potential is expected to exhibit a power-law behavior, i.e., it should be proportional to $(R/r)^{\delta+1}$ [1].

It is important to remark that both Yukawa and the power-law approximations are valid for pointlike particles. In configurations where the particles' wave functions overlap, the internal gravitational potential of the source should be considered. It happens that inside an extended source, the gravitational potential is not computable in a scenario of zero-width brane with $\delta \geq 2$ [13,29,36].

One way to circumvent this difficulty is to consider a thick brane model [37–40], which allows us, for instance, to estimate the influence of hidden dimensions in the energy shift of $S-states$ in the hydrogenlike atoms [13,41].

Experiments based on the interferometry of neutrons constitute important tests of nonstandard gravity and provide some of the most stringent upper bounds on the Yukawa parameter α in the range between 10^{-12} m and 10^{-9} m [21].

Schematically, in this kind of experiment, incident neutrons are divided into two beams that follow spatially separated paths and are then recombined to form an interference pattern in the detected intensity of the neutron flux [42]. A quantum mechanical phase difference is acquired when the partial beams are subjected to different physical interactions along the two paths as they have to pass through material plates or to cross regions with electromagnetic fields, according to the aim of the experiment.

In general, most analyses are based on the Yukawa parametrization. However, when the neutron is in contact with the medium, its phase is affected by the neutron optical potential which depends on the internal gravitational potential of the material. Thus, in order to estimate the effects of hidden dimensions on the neutron interferometry, we calculate the internal potential of a solid phase shifter in the context of a thick brane scenario, thus avoiding the divergence problems related to models with zero-width brane. More specifically, in Sec. II, we determine the forward scattering length of the anomalous gravitational interaction between the incident neutron and the nucleus of the material in the leading order, identifying, thus, the physical quantity of the extra dimension model that the neutron interferometry is able to constraint. As expected, this quantity depends on a parameter related to the localization of the matter in the brane, but, as we shall see, it also depends on the nuclear model of the atomic nuclei that constitute the material.

One way to surpass this limiting aspect is to consider experiments in which the phase shifter is a nonbaryonic source. An example is the experiment conceived to test the Aharanov-Casher (AC) effect [43], which can be described as a version of the Aharonov-Bohm effect for an electric neutral particle [42,43]. In the AC experiment [44], the phase difference arises from the interaction between the neutron's magnetic moment and the electric field as the beams cross the interior of electrostatic chambers.

According to the general relativity theory, all kinds of energy are capable of curving spacetime. So, in that experiment, incident neutrons also interact with the gravitational field produced by the electric field. In Sec. III, we calculate the additional phase shift due to this interaction and discuss the possibility of extracting independent constraints for short-distance modifications of gravity from the AC experiment.

These interferometric bounds can be considered in a more general context of metric theories and their post-Newtonian parameters [45]. As we shall see, the constraints put from this nonbaryonic source can be seen as limits for short-distance deviations of two post-Newtonian parameters that measure the capacity of the internal energy and pressure to bend spacetime in comparison to the rest mass of matter.

For a restricted class of metric theories, the interferometric constraints obtained here can be compared to the bounds extracted from the MTV-G experiment [46–48] and from the spectroscopy [48] concerning short-distance modifications of the post-Newtonian γ-parameter (related to the curvature of spatial sections of the spacetime). As we shall see, neutron interferometry establishes the most stringent bounds for this parameter in the length scale between 1.4×10^{-7} m and 10^{-4} m.

II. INTERNAL POTENTIAL OF AN EXTENDED SOURCE IN THICK BRANE SCENARIO

According to the ADD-model, the spacetime has a certain number (δ) of compact spacelike extra dimensions. The background spacetime is flat and contains a supplementary space with a finite volume, $(2\pi R)^{\delta}$. Matter and all the standard model fields are confined in the brane. Hence, the energy-momentum distribution of the localized fields may be described by a tensor of this kind [29]:

$$T_{AB}=\eta_{A}^{\mu}\eta_{B}^{\nu}T_{\mu\nu}(x)f(z). \tag{1}$$

Here we are adopting the following notations: Greek indices run from 0 to 3, and capital Latin indices go from 0 to $3+\delta$. The ordinary spacetime coordinates are represented by x, while z indicates coordinates of the compact space. The tensor η_{AB} is the Minkowski metric. For an idealized zero-width brane, $f(z)$ is a Dirac delta distribution, but, in the case of a thick brane, $f(z)$ is some regularization of that singular distribution.

The confined fields are the source of a gravitational field in the bulk that obeys a higher-dimensional version of Einstein equations. In the weak-field regime, the metric is approximately given by $g_{AB}=\eta_{AB}+h_{AB}$, where the tensor h_{AB}, which describes small perturbations in the geometry, satisfy the linearized Einstein equations:

$$\Box h_{AB}=-\frac{16\pi G_D}{c^4}\bar{T}_{AB}. \tag{2}$$

Here the symbol $\Box$ is the D'Alembertian operator associated with the Minkowski metric with a signature $(-,+,\cdots,+)$ and $\bar{T}_{AB}=[T_{AB}-(\delta+2)^{-1}\eta_{AB}T_{C}^{C}]$. It is important to remark that the above equation is valid in the harmonic gauge that is defined by the condition:

$$\partial_A\left(h^{AB} - \frac{1}{2}\eta^{AB}h_C^C\right) = 0. \tag{3}$$

The gravitational constant G_D of the higher-dimension theory, that appears in equation (2), should satisfy the relation $G_D = G(2\pi R)^\delta$ to recover the conventional results of general relativity at large distances [1,29]. Another requisite is the asymptotic stabilization of the volume of the supplementary space [49].

On the other hand, in short distance, the dominant term of the solution is independent of the topology of the supplementary space. In the static regime, the solution of Eq. (2), in this approximation order, is given by:

$$h_{AB}(\vec{X}) = \frac{16\pi\Gamma(\frac{\delta+3}{2})G_D}{(\delta+1)2\pi^{(\delta+3)/2}c^4}\left(\int \frac{\bar{T}_{AB}(\vec{X}')}{|\vec{X}-\vec{X}'|^{1+\delta}}d^{3+\delta}X'\right), \tag{4}$$

where $\vec{X}$ and $\vec{X}'$ are spatial coordinates of the ambient space. By ignoring the topology, we are taking a lower estimate of the potential strength. To illustrate this, consider a torus topology, as an example. As a consequence of the periodicity along each transversal direction of the brane that is implied by this topology, the resulting potential is mathematically equivalent to a superposition of potentials produced by a net of images of the source regularly spread in unfolded extra dimensions [34]. Therefore, by considering only the term (4), we are not taking into account the contribution of all images.

In the context of an interferometry experiment, Eq. (4) gives the higher-dimensional gravitational potential produced by the phase shifter, which can be a material plate or an electric field (see Fig. 1), as we are going to consider later. In its turn, the coupling of the incident neutron with that gravitational field can be extracted, in the ray optic approximation, from the Lagrangian of a test particle with mass m that is moving in the brane:

$$L = mc(g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu)^{1/2}, \tag{5}$$

where $\dot{x}^\mu$ means the derivative of the particle's coordinates with respect to its proper time. As the motion is restricted to the brane, $g_{\mu\nu}$ is the induced metric in $z=0$. For a nonrelativistic test particle, as the slow neutron in the experiment, it follows from (5) that the gravitational interaction is described by the potential energy $U_G = m\varphi$, where $\varphi = -h_{00}c^2/2$ is the modified gravitational potential calculated from (4).

A. Baryonic source

Crossing a material medium, the neutron interacts with the atomic nucleus via the anomalous gravitational force according to the large extra dimension scenario. Each nucleus can be treated as a nonrelativistic source with

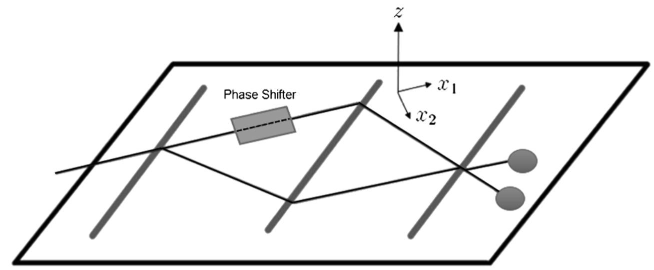

FIG. 1. This picture shows schematically an interferometric experiment as seen from the higher-dimensional perspective. The z-axis represents the extra-dimensional directions. The variables x_1, x_2 and x_3 (not visualized) correspond to coordinates of the perceived three-dimensional world. The incident neutrons move (black thin lines) in the brane center, which is illustrated by the plane $z=0$. The beams follow two different paths after being split by silicon crystal (the first thick grey line at left). Due to the gravitational interaction of the neutron with the phase shifter (a material medium or an electric field), the beams acquires a phase difference that produces an interference pattern as the two beams are recombined at the third silicon crystal and follow to the detectors (disks).

an energy-momentum tensor approximately given by $T_{\mu\nu} = \rho_N u_\mu u_\nu$, where ρ_N is the proper baryonic mass of the nucleus and u^μ is its four-velocity. In the rest frame of the medium, $u^\mu = c\delta_0^\mu$ in the first approximation. Thus, it follows that the gravitational potential of a single nucleus evaluated in a point $\vec{x}$ in the brane is given by:

$$\varphi(\vec{x}) = -\hat{G}_D\left(\int \frac{\rho_N(\vec{x}')f(z)}{(|\vec{x}-\vec{x}'|^2+z^2)^{\frac{1+\delta}{2}}}d^3x'd^\delta z\right), \tag{6}$$

where, for convenience, we have defined $\hat{G}_D = 4\Omega_\delta G_D$ and $\Omega_\delta = \Gamma(\frac{\delta+3}{2})/(\delta+2)\pi^{(\delta+1)/2}$. As we have already mentioned, if $f(z)$ is a Dirac delta distribution, the potential is not computable in any interior point (i.e., for $|\vec{x}| < R_N$, the nuclear radius) in the case of a codimension greater than one. However, an estimate of the internal potential ($\varphi_{\rm int}$) can be determined by considering that the brane has a thickness and that the baryonic mass of the nucleus is distributed along the extra dimensions according to some regular function $f(z)$, such as a Gaussian function centered at $z=0$. In the leading order, it is possible to show that $\varphi_{\rm int}(\vec{x})$ is proportional to the baryonic mass density distribution $\rho_N(\vec{x})$ of the nucleus [13]. Therefore, this internal interaction cannot be distinguished from the strong interaction between the neutron and the nucleus, which is described by a semiempirical potential of Wood-Saxel type [18,50].

An instructive, although nonrigorous, way to estimate the magnitude order of the potential (6) is to consider that, due to the mass distribution in the extra dimensions, the neutron and the source are separated in the $z-$direction by an effective distance σ, whose exact value depends on $f(z)$.

At this distance, only a fraction of the source inside a 3-ball of radius σ in the brane parallel directions contributes significantly to the potential. Thus, inside the nucleus, the gravitational potential will be proportional to $G_D(\sigma^3\rho_N)/\sigma^{\delta+1}$, therefore, $\varphi_{\rm int}(\vec{x})$ is proportional to the density distribution $\rho_N(\vec{x})$, in this approximation as mentioned above.

Perfect silicon crystal interferometers use a slow neutron with a wavelength (λ_n) of Angstrom order. Thus, the nuclear interaction, whose effective action is restricted to the nucleus size, can be approximated by the Fermi pseudopotential. The average of this potential energy in the medium is given by [42]:

$$U_F = \frac{2\pi\hbar^2 N}{m} b, \tag{7}$$

where the parameter b is the forward scattering length and N is the atomic density of the material. The effects of the so-called neutron optical potential (7) on the phase shift of the neutron beam can be determined empirically. According to the available data, the extracted value of b is roughly proportional to $A^{1/3}$, where A is the atomic mass of the nucleus [21]. If there are hidden dimensions and the gravitational interaction between the neutron and the nucleus is described by the hypothetical potential (6), then the measured value of b should contain a small contribution ($b_G^{\rm int}$), associated with the influence of the internal gravitational potential $\varphi_{\rm int}$. However, $b_G^{\rm int}$ would be indiscernible from the nuclear scattering length because of the reasons we have pointed out previously. The incident neutron may also have other interactions with the atom, which can influence the parameter b, but they are weaker than the nuclear force and can be ignored here, considering our purposes.

The external part of the potential (6) also give a contribution ($b_G^{\rm ext}$) to the scattering length which, in principle, could be differentiated from the effective nuclear scattering length (b_N). Thus, we could write $b = b_N + b_G^{\rm ext}$. In order to estimate $b_G^{\rm ext}$, let us calculate explicitly $\varphi_{\rm ext}(\vec{x})$, the gravitational potential outside the nucleus. As we shall see, in braneworld models, the exterior gravitational potential depends on internal characteristics of the nucleus (its radius, for instance) even when we assume a spherically symmetric distribution in the ordinary space. To illustrate this, let us consider the case of five extra dimensions ($\delta = 5$). For the sake of simplicity, let us model the nucleus as a 3-sphere (in the ordinary three-dimensional space) with a certain radius R_N and a uniform mass density. As we are calculating the potential outside the nucleus, then we can also consider a deltalike confinement of matter in the brane in $z = 0$, i.e., we can take $f(z)$ as a Dirac delta distribution with support on $z = 0$. It follows from (6) that the potential of the single nucleus with total mass M is given by:

$$\varphi_{\rm ext}(\vec{x}) = -\hat{G}_D \int_{|\vec{x}'|<R_N} \frac{\rho_N}{|\vec{x}-\vec{x}'|^6} d^3x' = -\frac{\hat{G}_D M}{(r^2 - R_N^2)^3}, \tag{8}$$

where $\rho_N = M/(4\pi R_N^3/3)$ and $r = |\vec{x}|$. Around the nucleus, the external potential (8) is quite different from the potential produced by a pointlike mass and even diverges at R_N. However, as the expression (8) is obtained in the thin-brane limit, then the validity of this approximation is restricted to $r > R_N + \sigma$, where σ is the length scale in which the idealized model of delta-like confinament fails. The parameter σ is of the order of the brane thickness (smaller than 10^{-18} m [1]). It is interesting to observe that the external potential evaluated at $R_N + \sigma$ is of the same order of the internal potential, $(\hat{G}_D\rho)/\sigma^3$ for $\delta = 5$, as we should expect.

Based on these considerations, we could estimate $b_G^{\rm ext}$, in the Born approximation [50], by taking

$$b_G^{\rm ext} = \frac{m}{2\pi\hbar^2} \int_{R_N+\sigma}^{\infty} m\varphi_{\rm ext}(\vec{x}) d^3\vec{x}. \tag{9}$$

In the leading order, we find:

$$b_G^{\rm ext} = -\frac{m}{2\pi\hbar^2}\left(\hat{G}_D \frac{\pi m M}{4R_N\sigma^2}\right). \tag{10}$$

In general, for codimensions $\delta > 3$, we have similar results which can be summarized in a single formula. Writing G_D in terms of the Newtonian constant G and R (the compactification scale of the hidden dimensions), we obtain:

$$b_G^{\rm ext} = -\frac{m}{2\pi\hbar^2}\left(\vartheta \frac{R^\delta}{R_N\sigma^{\delta-3}} GMm\right) \tag{11}$$

where ϑ is a coefficient whose value depends on the number of hidden dimensions. For comparison purposes, it is interesting to also calculate b_G considering an anomalous gravitational potential in the Yukawa form. In this case, we obtain $b_G = -(m/2\pi\hbar^2)(4\pi\alpha\lambda^2 GMm)$. From these results, we see that the expression (11) can be rewritten in a notation that resembled the Yukawa parametrization, provided we formally take $\lambda = R$ and reinterpret the amplification factor as $\alpha = \vartheta R^{\delta-2}/(4\pi R_N\sigma^{\delta-3})$ Thus, when experimental bounds from neutron interferometry are expressed in terms of the Yukawa parametrization, which is usual in the literature, the Yukawa parameter should be interpreted in this special form in the context of large extra dimensions. It is interesting to mention that the Yukawa amplification factor, according to the new interpretation, can reach much higher values in comparison to the standard α, which is just proportional to number of extra dimensions (δ) as predicted by the original ADD theory for large distances [34,35].

In principle, this $b_G^{\rm ext}$ calculated above could be distinguished from nuclear scattering length due to its peculiar dependence on the atomic mass. Indeed, following the method described in Ref. [21], we could check whether the data, collected from different types of materials, are compatible with an extra scattering length that is proportional to M/R_N and, therefore, to $A^{2/3}$. However, it is important to highlight that b_G^{ext} is influenced by the behavior of the external potential φ_{ext}, which depends on details of the mass distribution of the nucleus. Equation (11) corresponds to the simplest nuclear model, characterized by a uniform distribution inside the nucleus. Thus, as the density mass distribution is not precisely known, our ability to establish clear constraints on the parameters of the extra dimensions theory from the analysis of b_G^{ext} is limited.

B. Nonbaryonic source

It is possible to obtain empirical bounds that do not depend on nuclear models, considering a nonbaryonic phase shifter as a source for the gravitational field inside the interferometer. In the experiment designed to test the Aharonov-Casher effect [44], neutron beams pass through the interior of capacitors, accumulating phase shifts due to the interaction between the electric field ($\vec{E}$) and the neutron's magnetic moment ($\vec{\mu}$) imposed by the spin-orbit coupling $(\mu/mc)\vec{\sigma}\cdot(\vec{E}\times\vec{p})$, where $\vec{\sigma}$ stands for the Pauli matrices.

In contact with the electric field, the incident neutron does not interact via nuclear force, but it interacts gravitationally. Indeed, according to general relativity, the energy and the stress of the electric field inside the capacitor produce a gravitational field which affects the motion of the neutron. The standard theory predicts a negligible effect that is not detectable within the current precision of the instruments. However, in the context of modified gravitational theory, such as the large extra dimension models, the expected amplification of gravity in short distances could be tested without being masked by the nuclear interaction, even in the case when the length scale of the anomalous interaction is smaller than the nuclear size.

In the mentioned experiment, the field is approximately uniform in the region between the electrodes, and its direction, let us say x_2, is perpendicular to the direction of the neutron beam (x_1-axis), see Fig. 1. In SI units, the stress-energy tensor of the electromagnetic field is given by:

$$T_{\mu\nu}^{(EM)} = \epsilon_0 c^2\left(F_{\mu\lambda}F_{\nu}^{\ \lambda} - \frac{1}{4}\eta_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}\right), \qquad (12)$$

where $F_{\mu\nu}$ is the electromagnetic tensor and ϵ_0 is the electric permittivity of the free space. Inside the capacitor, the non-null components are $F_{20} = -F_{02} = E/c$, where E is the field strength. Therefore:

$$T_{\mu\nu}^{(EM)} = \frac{1}{2}\epsilon_0 E^2\begin{pmatrix} +1 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & +1 \end{pmatrix}. \qquad (13)$$

This tensor describes an anisotropic stress distribution with an energy density $u = \epsilon_0 E^2/2$. In the orthogonal directions of the field $\vec{E}$, the effective pressures measure $P_1 = P_3 = P_\perp = u$. But, in the parallel direction, the field configuration gives rise to a tension $P_2 = P_\parallel = -u$. The average pressure satisfies the usual state equation of radiation, $\hat{P} = u/3$.

This electromagnetic field produces a gravitational potential ($\chi = -h_{00}c^2/2$) that can be calculated from (4). The 00-component of the reduced energy-momentum can be written as $\bar{T}_{00} = \frac{\delta+1}{\delta+2}u + \frac{3}{\delta+2}\hat{P}$. As the pressure is of the same order as the electromagnetic energy density, its contribution to the gravitational potential χ cannot be ignored in this experiment. Taking into account the equation of state, $\hat{P} = u/3$, it follows that the gravitational interaction of the electromagnetic field with the incident neutron can be described by the potential:

$$\chi(\vec{x}) = -\frac{(\delta+2)}{(\delta+1)}\frac{\hat{G}_D}{c^2}\left(\int\frac{u(\vec{x}')f(z)}{(|\vec{x}-\vec{x}'|^2+z^2)^{\frac{1+\delta}{2}}}d^3x'd^\delta z\right), \qquad (14)$$

which is greater than the potential produced by a nonrelativistic source with the same energy density by a factor $(\delta+2)/(\delta+1)$. This is related to the fact that pressure, which also produces gravity, is of the same order of the energy density in the case of an electromagnetic field, as mentioned before, but it is negligible in comparison to the rest energy of nonrelativistic matter.

In the zero-width brane idealization, the potential $\chi(\vec{x})$ diverges in any point $\vec{x}$ where u is non-null, in the case of $\delta \geq 2$. However, this internal potential can be calculated in a thick brane scenario, admitting that the confinement of the electric field in the brane is described by any regular and normalized distribution $f(z)$.

In order to simplify the calculation of the internal potential, let us assume the realistic hypothesis that R is much smaller than the distance between the electrodes ($d = 1.54$ mm). Then, in any interior point $\vec{x}$, away from the capacitor's boundary, the major contribution for the potential comes from a portion of the source contained in a region $B_3(R)$, which corresponds to a spherical neighborhood of radius R in the ordinary three-dimensional space centered at $\vec{x}$. For $\delta > 2$, in the leading order, we get from (14):

$$\chi(\vec{x}) = -\frac{4\pi\zeta\hat{G}_D}{\varepsilon^{\delta-2}}\frac{1}{2}\frac{\epsilon_0 E^2}{c^2}, \qquad (15)$$

where the coefficient ζ, in terms of the gamma function (Γ), is:

$$\zeta = \frac{\sqrt{\pi}}{8}\frac{\delta(\delta+2)\Gamma(\frac{\delta-2}{2})}{(\delta+1)\Gamma(\frac{\delta+1}{2})}. \quad (16)$$

The next correction term has a relative order of ε/R, at least. The potential χ also depends on the parameter ε that is defined as:

$$\frac{1}{\varepsilon^{\delta-2}} = \frac{2}{\delta}\int \frac{f(z)}{z^{\delta-2}} d^{\delta}z, \quad (17)$$

which is the average of the function $1/z^{\delta-2}$ with respect to the distribution $f(z)$. As pointed out in Ref. [51], in the leading order, the gravitational field produced by localized sources inside the brane does not depend on many details of the confinement mechanism, but on a specific statistical moment (the expected value of $z^{2-\delta}$) of the field distribution in the supplementary space.

The part of the source which lies outside the region $B_3(R)$ gives a contribution for the resulting potential that has a relative order of $(\varepsilon/R)^{\delta-2}$.

In the next section, we are going to discuss the effect of this potential on the incident neutron.

III. CONSTRAINTS FROM THE NEUTRON INTERFEROMETRY

For the sake of simplicity, let us consider a minor change of the AC experiment. Let us admit that only one of the partial beams passes through the region filled with the electric field, while the other beam is shielded (see Fig. 1). Inside the capacitor, the neutron will interact with the electric field through the gravitational anomalous interaction too. This extra interaction will provide an additional phase factor to the beam's wave function. Considering that it is a short-range interaction, the second beam will not be affected by the anomalous potential. Thus, the relative phase between the two paths will be given by [42]:

$$\Delta\Phi = \frac{1}{\hbar}\int_C \Delta\vec{p}\cdot d\vec{s}, \quad (18)$$

where $\hbar$ is the reduced Planck constant and $\Delta\vec{p}$ is the variation of the neutron linear momentum caused by the anomalous gravitational interaction in relation to the linear momentum of the free neutron. The integration is performed along the path of the first beam inside the capacitor. In this approximation, we are neglecting the contribution from the potential outside the medium.

The variation of the linear momentum can be determined from the energy conservation and can be expressed in terms of the wavelength of the incident neutron. Along the direction of motion (x_1-axis, inside the capacitor), we find $\Delta p_1 = m^2\chi\lambda_n/h$, where χ is constant and is given by (15) in the leading order. Therefore, the phase difference acquired by the beam after traversing the capacitor of length L is

$$\Delta\Phi = \frac{m^2}{h^2}\left(\frac{4\pi^2\zeta\hat{G}_D}{\varepsilon^{\delta-2}}\epsilon_0\frac{E^2}{c^2}\right)\lambda_n L. \quad (19)$$

As we have pointed out before, interestingly, the result (19) can be formally derived from a Yukawa parametrization too. If we take $\lambda = R$ and reinterpret the dimensionless Yukawa parameter as $\alpha = (2\pi)^{\delta}\zeta\Omega R^{\delta-2}/\varepsilon^{\delta-2}$, then the formula above could be rewritten as $\Delta\Phi = G\alpha\lambda^2 m^2\epsilon_0 E^2\lambda_n L/h^2c^2$, i.e., in the same form that would be obtained from an anomalous gravitational field described by the Yukawa parametrization. Thus, considering that the Yukawa parametrization can describe modifications of gravity with other physical origins besides hidden dimensions, then in order to be more generic as possible from the phenomenological point of view, we are going to express our results in terms of the Yukawa parameters hereafter.

In the AC experiment [44], the field strength is $E = 30$ kV/mm, $L = 2,53$ cm, and the neutron wavelength is $\lambda_n = 1.477$ Å. The predicted phase shift $\Phi_{AC} = 1.50$ mrad is compatible with the measurements within an error of the order of $\eta \sim 10^{-3}$ rad [44]. Therefore, additional effects from any hypothetical interaction could not be greater than η.

In principle, the gravitational phase shift, which depends on E^2, can be distinguished from the AC shift, which is proportional to E. Thus, it seems reasonable to expect that we can get constraints for the anomalous gravitational interaction from the analysis of an experiment of this kind. In order to make an estimate, let us assume that the empirical procedure of testing the modified gravity produced by the capacitor electric field has a precision of the same order of η. Thus, it follows from (19) that the corresponding Yukawa parameter should satisfy the upper limit:

$$\alpha\lambda^2 < 0.26\times 10^{20}\ \mathrm{m}^2\left(\frac{\eta}{10^{-3}\ \mathrm{rad}}\right)\left(\frac{\text{Å}}{\lambda_n}\right)\left(\frac{\mathrm{cm}}{L}\right) \times\left(\frac{30\ \mathrm{kV/mm}}{E}\right)^2, \quad (20)$$

which is valid for $\lambda < 10^{-4}$ m. This bound is very weak compared to the traditional constraints based on baryonic sources [21]. For example, according to Ref. [21], the analysis from data of forward scattering length imposes the constraint: $\alpha\lambda^2 < 2.150\times 10^5$ m^2. When the analysis also includes data from the total and differential cross section of the neutron-nucleus interaction, it establishes an even more stringent bound: $\alpha\lambda^2 < 482$ m^2 valid for 1 nm $< \lambda <$ 1 mm [50,52]. However, we should emphasize that, unlike the other bounds, the condition (20) can be extended to length scales smaller than nuclear size, since it is extracted from a nonbaryonic source. Moreover, we should highlight that the restriction (20) imposes bounds on post-Newtonian potentials (since in this experiment the neutrons probe the gravitational field produced by an electromagnetic field), while the baryonic bounds constrain modifications of the Newtonian potential. Thus, in this sense, the baryonic bounds and the constraint (20) establishes empirical bounds on different physical quantities. We shall discuss this point in more detail in the next section.

IV. POST-NEWTONIAN POTENTIALS

Strictly speaking, the potential χ cannot be interpreted as a modification of the Newtonian potential since its source is not the matter's rest mass. Instead, it should be considered as a short-distance deviation of a post-Newtonian correction potential, given that the electric energy acting as a source of a gravitational field has no correspondence in the Newtonian theory.

In the weak-field regime, alternative metric theories can be distinguished by means of parameters that work as effective gravitational couplings related to post-Newtonian potentials. In the standard PPN formalism, there are ten parameters [45]. Two of them, β_3 and β_4 (following the notation of Ref. [53]), are of special importance here. They measure how much gravity is produced by internal non-baryonic energy and by pressure in comparison to the general relativity predictions, respectively.

The post-Newtonian potential χ is influenced by a combination of the energy density (u) and the average pressure ($\hat{P}$) associated with the electric field. Thus, in accordance with this more general formalism, in which a broader class of metric theories could be considered (not only extra dimensions models), the parameter α should be replaced, in the constraint equation (20), by a combination of two Yukawa parameters associated with short-distance modifications of the post-Newtonian parameters β_3 and β_4.

In general, the post-Newtonian parameters are independent, and their values are to be determined from phenomenology. However, for a restricted class of metric theories that satisfy the full global conservation laws and that are free of preferred spatial position, $\beta_3 = 1$, automatically [45,53]. This implies that all kinds of energy have the same capacity of curving spacetime. On the other hand, the gravitational coupling associated with the pressure is not fixed in this class of theories, but should satisfy the relation [45,53]: $\beta_4 = \gamma$, where γ is another post-Newtonian parameter, that is associated with the curvature of the pure spatial sections of the spacetime.

The prediction of GR theory, $\gamma = 1$, has been confirmed by tests such as the Cassini experiment that investigated the time-delay and the deflection of radio waves under the influence of the gravitational field of the Sun [54]. The empirical value, $\gamma = 1 + (2.1 \pm 2.3) \times 10^{-5}$, is the most stringent bound for that parameter in the length scale of the solar radius [54].

In the microscopic domain, this parameter γ has been investigated too, by examining possible effects of an anomalous gravitational field produced by a nucleus on the precession of the electron's spin (see, for example, [46,48]). The analysis is based on the gravitational spin-orbit coupling [55], which is described by a Hamiltonian that can be appropriately written as [48]:

$$H_{Gso} = \frac{1}{m_e c^2}\frac{1}{r}\left(\frac{1}{2}\frac{d\varphi}{dr} + \frac{d\tilde{\varphi}}{dr}\right)(\vec{S}\cdot\vec{L}), \qquad (21)$$

where m_e is the mass of the electron, $\vec{S}$ and $\vec{L}$ corresponds to its spin and angular momentum, respectively. The function φ is the modified version of the Newtonian potential produced by the nucleus and $\tilde{\varphi}$ corresponds to the Post-Newtonian potential associated with the spatial components of the metric, which is related to PPN-parameter γ. By using the Yukawa parametrization, the post-Newtonian potential can be written as $\tilde{\varphi} = \gamma(r)GM/r$, where $\gamma(r) = (1 + \alpha e^{-r/\lambda})$ is the short-range modification of that parameter.

Thus, experiments that are sensitive to the spin precession of the electron can probe the behavior of the PPN-parameter γ in the microscopic domain. An example is the MTV-G experiment, which investigates the evolution of the spin of electrons that are scattered by heavy nuclei [46,47]. The absence of any anomalous signal in the empirical data establishes some bounds on short-distance deviations of the parameter γ [46,48].

Other constraints on γ were extracted from the spectroscopy of the hydrogen atom in Ref [48]. The fine structure of P-states is influenced by the spin-orbit coupling H_{Gso}. As discussed in Ref. [48], the analysis of $2P_{1/2} - 2P_{3/2}$ transition put independent constraints on the short-range behavior of that post-Newtonian parameter too.

Here we are arguing that the constraint (20), extracted from neutron interferometry, establishes upper bounds on the PPN-parameters β_3 and β_4. It happens that, for a class of metric theories, $\beta_3 = 1$ and $\beta_4 = \gamma$. In this case, the coefficient α in (20) can be viewed effectively as the Yukawa parameter related to the short-distance modifications of γ, i.e., $\gamma(r) = (1 + \alpha e^{-r/\lambda})$. With that motivation, in Fig. 2 we compare the constraints from the neutron interferometry (equation (20), obtained here) with the MTV-G and spectroscopic bounds previously determined in the literature [46,48].

The neutron interferometry yields the strongest upper limits for amplification of the post-Newtonian $\gamma -$ parameter in the range 1.4×10^{-7} m *and* 10^{-4} m. Clearly, these restrictions are very weak in comparison to the traditional

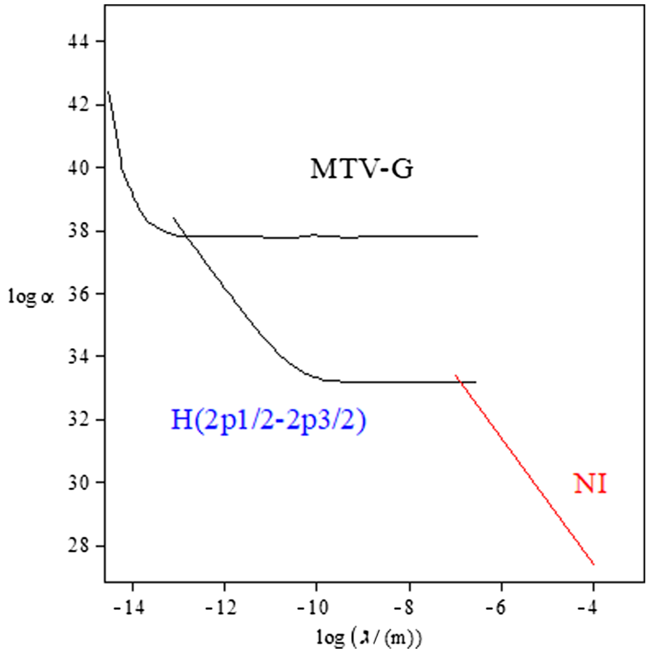

FIG. 2. Neutron interferometric (NI) bounds (obtained in this paper) on the Yukawa parameter α related to the short-range modifications of the Post-Newtonian $\gamma-$ parameter in comparison to the MTV-G and spectroscopy limits (extracted from [46–48]). The region above the lines are excluded.

bounds on modifications of the Newtonian gravitational constant G at short distances. In the mentioned range, the most stringent constraints are given by Casimir experiments (for instance, $\alpha < 2 \times 10^9$ at $\lambda = 1\ \mu$m, see Ref. [24]) and by torsion balance tests ($\alpha \leq 1$ at $\lambda = 56\ \mu$m, see Ref. [6]). In these experiments, the tested gravitational fields are produced by baryonic sources, while in the NI experiment considered here, neutrons probe the gravitational field produced by an electromagnetic field.

Therefore, as we have mentioned in the previous section, the constraints shown in Fig. 2 determines empirical restriction on certain post-Newtonian parameters, whereas the baryonic bounds impose limits on the modifications of the Newtonian potential. It turns out that, within the general context of the metric theories, the post-Newtonian and Newtonian potentials are phenomenologically independent physical quantities, as pointed out in the PPN formalism. Thus, it is interesting to investigate them separately.

V. FINAL REMARKS

Large extra dimensions theories are often cited as motivation in the search for modifications of gravity on short-distance scales in several laboratory tests. However, the thin brane models cannot be probed by experiments based on neutron interferometry straightforwardly, since the internal potential of a material sample, playing the role of a phase shifter, is not computable in scenarios where the brane has no thickness and the number of codimensions is greater than one.

Therefore, when interferometric constraints are expressed in terms of Yukawa parameter α, it is not clear what physical quantity related to the higher-dimensional theory could, in fact, be bound by the empirical data.

The ADD model predicts that, far from a pointlike source, the Newtonian potential is corrected by an additional Yukawa term. Considering a supplementary space with a flat torus background and a model with massless radion, the Yukawa parameter is $\alpha = 2\delta$ [34]. This interpretation is valid, for example, in torsion-balance tests, since the bodies used in the experiment are spatially separated. Thus, taking into account the finite size effects of the interacting bodies, constraints on the compactification radius of large extra-dimensional models can be extracted from the torsion-balance experiments employing the Yukawa parametrization [6].

However, in the interferometry experiment, the neutron is in contact with the material (the phase shifter) in a certain interval of its path. Therefore, the Yukawa parametrization is not appropriate to describe the effects of hidden dimensions on the neutron's phase factor. By the way, if we inadvertently employ the above interpretation for α with the purpose of analyzing the interferometric bounds, the upper limits we obtain for the number of hidden dimensions are practically irrelevant.

Inside the material, the power-law parametrization would be the adequate one to study modifications of gravity in the context of the braneworld scenario. However, it leads to divergence problems in the calculation of the internal gravitational potential.

This difficulty can be circumvented in the context of a thick brane scenario. Indeed, considering that the localized fields have a regular distribution inside the thick brane, we have calculated the forward scattering length, b_G, associated with the higher-dimensional gravitational interaction between the neutron and the nucleus. As we have seen, the part of b_G that can be distinguished from the nuclear scattering length, b_G^{ext}, is proportional to ratio $R^\delta/(R_N\sigma^{\delta-3})$, which involves the compactification radius R, the nucleus radius R_N and σ, a parameter of the order the brane thickness that is associated with a statistical moment of the distribution that describes the localized field inside the brane.

The explicit determination of b_G^{ext} allows us to recognize the higher-dimensional quantity that is subjected to the empirical constraints put by neutron interferometry. However, the parameter b_G^{ext} depends on the nuclear model. Hence, in order to obtain constraints free of this dependence, we were led to consider experiments where the phase shifter is nonbaryonic, as in the experiment that measures the Aharonov-Casher shift.

In the context of PPN formalism, the AC experiment can be seen as a test of the behavior of post-Newtonian

$\gamma -$ parameter on the short-length scale for a class of metric theories. In comparison to other empirical constraints, extracted from the MTV-G experiment and from the $2P_{3/2} - 2P_{1/2}$ transition in the hydrogen atom, we find that the limits imposed by neutron interferometry on deviations of that parameter are the most stringent in the length scale between 1.4×10^{-7} m and 10^{-4} m.

ACKNOWLEDGMENTS

J. M. Rocha thanks CNPq for financial support.

[1] N. Arkani-Hamed, S. Dimopoulos, and G. R. Dvali, Phys. Lett. B **429**, 263 (1998).
[2] I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, and G. R. Dvali, Phys. Lett. B **436**, 257 (1998).
[3] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999).
[4] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 4690 (1999).
[5] J. Murata and S. Tanaka, Classical Quantum Gravity **32**, 033001 (2015).
[6] D. J. Kapner, T. S. Cook, E. G. Adelberger, J. H. Gundlach, B. R. Heckel, C. D. Hoyle, and H. E. Swanson, Phys. Rev. Lett. **98**, 021101 (2007).
[7] F. Luo and H. Liu, Chin. Phys. Lett. **23**, 2903 (2006); Int. J. Theor. Phys. **46**, 606 (2007).
[8] Z.-G. Li, W.-T. Ni, and A. P. Patón, Chin. Phys. B **17**, 70 (2008).
[9] Z. Li and X. Chen, arXiv:1303.5146.
[10] L. B. Wang and W. T. Ni, Mod. Phys. Lett. A **28**, 1350094 (2013).
[11] Z. Wan-Ping, Z. Peng, and Q. Hao-Xue, Open Phys. **13**, 96 (2015).
[12] E. J. Salumbides, A. N. Schellekens, B. Gato-Rivera, and W. Ubachs, New J. Phys. **17**, 033015 (2015).
[13] F. Dahia and A. S. Lemos, Phys. Rev. D **94**, 084033 (2016).
[14] D. Grandon and A. Vega, arXiv:1903.00587.
[15] M. S. Safronova, D. Budker, D. DeMille, D. F. J. Kimball, A. Derevianko, and C. W. Clark, Rev. Mod. Phys. **90**, 025008 (2018).
[16] V. V. Nesvizhevsky and K. V. Protasov, Classical Quantum Gravity **21**, 4557 (2004).
[17] E. J. Salumbides, W. Ubachs, and V. I. Korobov, J. Mol. Spectrosc. **300**, 65 (2014).
[18] A. Frank, P. van Isacker, and J. Gomez-Camacho, Phys. Lett. B **582**, 15 (2004).
[19] Yu. N. Pokotilovski, Phys. At. Nucl. **69**, 924 (2006).
[20] G. L. Greene and V. Gudkov, Phys. Rev. C **75**, 015501 (2007).
[21] V. V. Nesvizhevsky, G. Pignol, and K. V. Protasov, Phys. Rev. D **77**, 034020 (2008).
[22] W. M. Snow *et al.*, Phys. Rev. D **101**, 062004 (2020).
[23] S. K. Lamoreaux, Phys. Rev. Lett. **78**, 5 (1997).
[24] A. O. Sushkov, W. J. Kim, D. A. R. Dalvit, and S. K. Lamoreaux, Phys. Rev. Lett. **107**, 171101 (2011).
[25] V. B. Bezerra, G. L. Klimchitskaya, V. M. Mostepanenko, and C. Romero, Phys. Rev. D **83**, 075004 (2011).
[26] S. Cullen and M. Perelstein, Phys. Rev. Lett. **83**, 268 (1999).
[27] S. Hannestad and G. G. Raffelt, Phys. Rev. D **67**, 125008 (2003); **69**, 029901(E) (2004).
[28] S. Vagnozzi and L. Visinelli, Phys. Rev. D **100**, 024020 (2019).
[29] G. E. Giudice, R. Rattazzi, and J. D. Wells, Nucl. Phys. **B544**, 3 (1999).
[30] CMS Collaboration, Phys. Lett. B **755**, 102 (2016).
[31] W.-M. Yao *et al.*, J. Phys. G **33**, 1 (2006).
[32] E. Aelberger, J. Gundlach, B. Heckel, S. Hoedl, and S. Schlamminger, Prog. Part. Nucl. Phys. **62**, 102 (2009).
[33] K. S. Stelle, Gen. Relativ. Gravit. **9**, 353 (1978).
[34] A. Kehagias and K. Sfetsos, Phys. Lett. B **472**, 39 (2000).
[35] E. G. Adelberger, B. R. Heckel, and A. E. Nelson, Annu. Rev. Nucl. Part. Sci. **53**, 77 (2003).
[36] F. del Aguila, M. Perez-Victoria, and J. Santiago, J. High Energy Phys. 10 (2006) 056.
[37] V. Rubakov and M. Shaposhnikov, Phys. Lett. **125B**, 136 (1983).
[38] D. B. Kaplan, Phys. Lett. B **288**, 342 (1992).
[39] S. Randjbar-Daemi and M. Shaposhnikov, Phys. Lett. B **492**, 361 (2000).
[40] N. Arkani-Hamed and M. Schmaltz, Phys. Rev. D **61**, 033005 (2000).
[41] F. Dahia and A. S. Lemos, Eur. Phys. J. C **76**, 435 (2016).
[42] H. Rauch and S. A. Werner, *Neutron Interferometry: Lessons in Experimental Quantum Mechanics, Wave-Particle Duality, and Entanglement*, 2nd ed. (Oxford University Press, Oxford, 2015).
[43] Y. Aharonov and A. Casher, Phys. Rev. Lett. **53**, 319 (1984).
[44] A. Cimmino, G. I. Opat, A. G. Klein, H. Kaiser, S. A. Werner, M. Arif, and R. Clothier, Phys. Rev. Lett. **63**, 380 (1989).
[45] C. M. Will, *Theory and Experiment in Gravitational Physics*, revised ed. (Cambridge University Press, Cambridge, England, 1993).
[46] S. Tanaka, Y. Nakaya, K. Ninomiya, H. Nishio, J. Onishi, R. Openshaw, M. Pearson, Y. Totsuka, and J. Murata, Phys. Part. Nucl. Lett. **45**, 237 (2014).
[47] S. Tanaka, Y. Nakaya, K. Ninomiya, H. Nishio, J. Onishi, R. Openshaw, M. Pearson, Y. Totsuka, and J. Murata, J. Phys. Conf. Ser. **453**, 012018 (2013).
[48] A. S. Lemos, G. C. Luna, E. Maciel, and F. Dahia, Classical Quantum Gravity **36**, 245021 (2019).

[49] I. Antoniadis, K. Benakli, A. Laugier, and T. Maillard, Nucl. Phys. **B662**, 40 (2003).
[50] H. Leeb and J. Schmiedmayer, Phys. Rev. Lett. **68**, 1472 (1992).
[51] F. Dahia, E. Maciel, and A. S. Lemos, Eur. Phys. J. C **78**, 526 (2018).
[52] H. Abele, Prog. Part. Nucl. Phys. **60**, 1 (2008).
[53] C. W. Misner, K. S. Thorne, and J. A. Wheeler, in *Gravitation* (W. H. Freeman and Co., San Francisco, 1973), pp. 1066–1095.
[54] B. Bertotti, L. Iess, and P. Tortora, Nature (London) **425**, 374 (2003).
[55] E. Fischbach, B. S. Freeman, and W.-K. Cheng, Phys. Rev. D **23**, 2157 (1981).