Vortices penetrating two-flavor quark-hadron continuity

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(Received 19 March 2021; accepted 28 April 2021; published 1 June 2021)

Alice and Boojums are both representative characters created by Lewis Carroll. We show that they possibly meet in cores of rotating neutron stars. Recent studies of quark-hadron continuity suggest that neutron superfluid matter can connect smoothly to two-flavor symmetric quark matter at high densities. We study how this can be maintained in the presence of the vortices. In the neutron matter, quantized superfluid vortices arise. In the two-flavor dense quark matter, vortices carrying color magnetic fluxes together with fractionally quantized superfluid circulations appear as the most stable configuration, and we call these as the non-Abelian Alice strings. We show that three integer neutron superfluid vortices and three non-Abelian Alice strings of different color magnetic fluxes with total color flux canceled out are joined at a junction called a Boojum.

DOI: 10.1103/PhysRevD.103.114003

I. INTRODUCTION

Neutron stars, particularly pulsars, provide us with a unique opportunity to study states of matter under extreme conditions: highest known baryon density in the universe, rapid rotation, strong magnetic fields, etc (see, e.g., Refs. [1–3] for reviews). In the present work, we address the combined effect of high-density and rapid rotation; namely the quantum vortices that appear in the neutron superfluid and the color-superconducting quark matter.

There are several ongoing observations of neutron stars such as the ones with two-solar-mass [4–6], probed by gravitational wave detectors [7,8], and investigated in the NICER mission [9,10], etc., and they have led to a flurry of studies about neutron stars in diverse fields of research. Among such efforts, numerous articles focus on the equation of state (EoS). The recent development in this direction was constructing a phenomenological hybrid EoS with a smooth crossover for the hadron-to-quark phase transition [1,11–14] rather than a first-order phase transition as conventionally done in many literatures. The crossover construction enables neutron stars to have a sizable quark core inside consistently with the constraints put by observations. Such construction should not be regarded as an exotic alternative because there can indeed be a possibility that a substantial quark core inside a heavy neutron star is realized as suggested by the model-independent analysis [15]. The plausible ground state of such cold dense quark matter is color superconductor [16,17] with various patterns of the diquark pairing being known such as color-flavor locked (CFL) phase [18] in three-flavor symmetric matter and two-flavor superconducting (2SC) phase [19,20] in two-flavor symmetric matter.

The idea of crossover construction of the EoS can be traced back to the concept of *quark-hadron continuity* between the CFL phase and the hyperon superfluid phase [21] (see also [22–26]). The both color-superconducting and hadronic superfluid phases share the same symmetry breaking patterns and low-lying excitations, so these phases can be connected continuously. The recent breakthrough is an extension to the case with rapid rotation under which superfluid vortices are created in hadronic and quark matter [27–31]. In the CFL phase, superfluid vortices appear [32,33], but each superfluid vortex is unstable against a decay into a set of more stable vortices [34,35]. The most stable vortices are non-Abelian semi-superfluid vortices carrying color magnetic fluxes and fractionally quantized $1/3$ circulation of the Abelian superfluid vortices [34–38]. Thus, an important question was raised in Ref. [27] how these non-Abelian vortices penetrate into hyperonic matter; it was suggested in Ref. [27] that one non-Abelian vortex in the CFL phase should be connected to one superfluid vortex in the hyperon matter. This work stimulated the discussion whether there is a discontinuity between the vortices in two phases [29] or not [30] from the viewpoint of topological order based on a Wilson loop linking a vortex.

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Aharonov-Bohm (AB) phases of quasiparticle encircling vortices provide us with further insight into this problem by comparing these in the hyperonic and CFL phases. It was shown in Ref. [28] that three non-Abelian vortices, which carry different color magnetic fluxes with total color canceled out, must joint at one point to three integer vortices in the hyperonic matter. Such a junction point of vortices is called a colorful *Boojum* [39,40]; originally, the similar structures found in helium superfluids were named Boojums by Mermin [42,43] and have been predicted to occur in ^{3}He superfluids [45] in particular at the A-B phase boundary [46,47], liquid crystals [48], Bose-Einstein condensates [49], and quantum field theory [50]. Moreover, in neutron stars, Boojums were suggested to explain pulsar glitch phenomena, which is a sudden speed-up in rotation [51].

Thus far, we have discussed quark-hadron continuity within the ideal three-flavor symmetric setup in the limit of strange (s) quark mass degenerate with up (u) and down (d) quark masses. In the realistic setup, however, the hadronic matter is dominated by neutrons, which are composed of two-flavor u and d valence quarks, and the s-quark masses are heavy so that they do not participate in condensation. Neutrons are superfluids, for which pairing in $^3P_2$ channel are responsible [52–55] (see also Refs. [56,57] and references therein for recent studies). By contrast, the conventional 2SC phase does not exhibit the property of superfluidity because $\mathrm{U}(1)_B$ symmetry remains unbroken in this phase. It was, however, proposed recently in Refs. [58] that the two-flavor color superconductor can be superfluids if we take into account the $^3P_2$ pairing of d-quarks in addition to the 2SC pairing, and this novel phase was named $2\mathrm{SC} + \langle dd\rangle$ phase. The consideration of the superfluidity led to the two-flavor counterpart of the quark-hadron continuity, i.e., the $2\mathrm{SC} + \langle dd\rangle$ phase and $^3P_2$ neutron superfluid phase are continuously connected. The two-flavor continuity is indeed consistent with the crossover construction of the EoS and more natural in the sense that we use the hadronic EoS dominated by nucleons in the most of the cases; see Ref. [14] for an explicit crossover construction of the EoS within the two-flavor setup.

Then, a natural question arises that if Boojums are also present in the two-flavor quark-hadron continuity. The most stable vortices in the two-flavor dense QCD are "*non-Abelian* Alice strings" [59], which are the non-Abelian counterpart of the so-called Alice strings [60–63]. These vortices have $\mathrm{U}(1)_B$ fractional windings along with the color-magnetic fluxes. Three non-Abelian Alice strings are more energetically favorable than a single superfluid vortex, so that the latter decays into the former. Quasiparticles winding around a non-Abelian Alice string pick up nontrivial (color nonsinglet) AB phases, unlike those of CFL vortices with color singlet AB phases.

In this work, we show that in the two-flavor quark-hadron continuity picture, three Alice strings with red, blue, green color magnetic fluxes in two-flavor quark matter must join at a junction point to three integer vortices in $^3P_2$ neutron matter, forming a colorful Boojum.

II. TWO-FLAVOR DENSE MATTER

We give a brief synopsis of the two-flavor hadronic and quark matter at high density [58].

The hadronic phase that we consider is a neutron $^3P_2$ superfluid with the order parameter operator [52,53]

$$\hat{A}^{ij} = \hat{n}^T \mathcal{C}\gamma^i \nabla^j \hat{n}, \tag{1}$$

with a neutron field operator $\hat{n}$ and the charge conjugation operator $\mathcal{C}$. Here, the Roman letters $(i, j, \ldots)$ denote spatial coordinates, and the matrices γ^i and spatial derivatives ∇^j account for spin and angular momentum contributions in the $^3P_2$ pairing, respectively. We assume that neutrons made out of u- and d-quarks can be described as a quark-diquark system $\hat{n} = \epsilon^{\alpha\beta\gamma}(\hat{u}_\alpha^T \mathcal{C}\gamma^5 \hat{d}_\beta)\hat{d}_\gamma$, with the Greek letters $(\alpha, \beta, \ldots)$ denoting color indices.

On the other hand, the relevant quark matter is the $2\mathrm{SC} + \langle dd\rangle$ phase, where the expectation value of $\hat{A}^{ij}$ can be taken, within the mean field approximation, as

$$A^{ij} = \langle \hat{A}^{ij}\rangle \simeq (\Phi_{2\mathrm{SC}})^\alpha (\Phi_{2\mathrm{SC}})^\beta (\Phi_{dd})^{ij}_{\alpha\beta}, \tag{2}$$

$$\Phi_{2\mathrm{SC}} \equiv \langle \epsilon^{\alpha\beta\gamma} \hat{u}_\beta^T \mathcal{C}\gamma^5 \hat{d}_\gamma\rangle, \qquad \Phi_{dd} \equiv \langle \hat{d}_\alpha^T \mathcal{C}\gamma^i \nabla^j \hat{d}_\beta\rangle. \tag{3}$$

Here, the spatial indices i, j of Φ_{dd} are suppressed, and hereafter an appropriate tensor structure is implied. Note that quantities with (without) hat symbols denote operators (condensates). The two condensates $\Phi_{2\mathrm{SC}}$ and Φ_{dd} in Eq. (2) account for the color superconductivity of the quark matter: $\Phi_{2\mathrm{SC}}$ is the so-called 2SC condensate, while the novel feature here is represented by Φ_{dd}, which is the diquark condensate of d-quarks in the $^3P_2$ channel. The symmetry of QCD relevant to this work is $G_{\mathrm{QCD}} = \mathrm{SU}(3)_\mathrm{C} \times \mathrm{U}(1)_\mathrm{B} \ni (U, e^{i\theta_\mathrm{B}})$ acting on quark fields $\hat{q}$ as $\hat{q} \to e^{i\theta_\mathrm{B}} U\hat{q}$ as a column vector belonging to the fundamental representation 3 of $\mathrm{SU}(3)_\mathrm{C}$, under which the diquark condensates (3) transform as

$$\Phi_{2\mathrm{SC}} \to e^{2i\theta_\mathrm{B}} U^* \Phi_{2\mathrm{SC}}, \qquad \Phi_{dd} \to e^{2i\theta_\mathrm{B}} U \Phi_{dd} U^T. \tag{4}$$

Since A^{ij} is nonzero in the both phases, the local order parameters indeed cannot distinguish these two phases, implying the quark-hadron continuity [58].

III. VORTICES IN TWO-FLAVOR DENSE MATTER

We classify the vortices that appear in the two-flavor neutron and quark matter. First, let us discuss $^3P_2$ neutron superfluid vortices. In terms of the $^3P_2$ order parameter in

Eq. (1), a single integer vortex behaves at large distance as [54,64–66]

$$A^{ij}(\varphi) \sim e^{i\varphi} A^{ij}(\varphi = 0). \tag{5}$$

In the weak coupling limit, the $^3P_2$ superfluid is in the nematic phase [55] with $A^{ij}(\varphi = 0) \sim \mathrm{diag}(s, s, 1 - s)$ with a real parameter s, which is actually determined by the temperature and magnetic field [56,57]. At higher magnetic field, half-quantized vortices are the most stable [67], but we do not consider such a case for simplicity.

Next, let us discuss vortices in the $2\mathrm{SC} + \langle dd \rangle$ phase [59]. The simplest vortex is what we call a $\mathrm{U}(1)_B$ superfluid or an Abelian vortex, given by

$$\Phi_{dd}(\varphi) = f_0(r) e^{i\varphi} \Delta_{dd} \mathbf{1}_3 \sim e^{i\varphi} \Delta_{dd} \mathbf{1}_3$$
$$\Phi_{2\mathrm{SC}} = h(r) \frac{e^{i\varphi}}{\sqrt{3}} \Delta_{2\mathrm{SC}} (1, 1, 1)^T \tag{6}$$

with the boundary condition $f_0(0) = h(0) = 0$ and $f_0(\infty) = h(\infty) = 1$ for the profile functions $f_0(r)$ and $h(r)$, and (r, φ) being the polar coordinates. We have also assumed a unitary gauge fixing for the diquark condensates. This vortex carries a unit quantized circulation in $\mathrm{U}(1)_B$ as encoded in the factor $e^{i\varphi}$, and thus is created under rotation because of the superfluidity. This is topologically stable due to $\pi_1[\mathrm{U}(1)_B] = \mathbb{Z}$ but is dynamically unstable against decay into three non-Abelian Alice strings introduced below. The $^3P_2$ order parameter in Eq. (2) behaves at large distance as

$$A^{ij} \sim e^{3i\varphi} A^{ij}(\varphi = 0), \tag{7}$$

corresponding to three integer vortices in the $^3P_2$ neutron matter, compared with Eq. (5).

Next, we present a non-Abelian Alice string as the most stable vortex, behaving at spatial infinity as

$$\Phi_{dd}(\varphi) \sim e^{i\varphi/3} U(\varphi) \Phi_{dd}(\varphi = 0) U^T(\varphi), \tag{8}$$

$$U(\varphi) = \mathcal{P} \exp\left(ig \int_0^{\varphi} \mathbf{A} \cdot d\ell \right). \tag{9}$$

The full vortex ansatz is of the form of

$$\begin{aligned}
\Phi_{dd}(\varphi) &= \Delta_{dd} \mathrm{diag}(f(r) e^{i\varphi}, g(r), g(r)),\\
U(\varphi) &= e^{i(\varphi/6)\mathrm{diag}(2,-1,-1)},\\
A_i &= -\frac{a(r)}{6g} \frac{\epsilon_{ij} x^j}{r^2} \mathrm{diag}(2, -1, -1),\\
\Phi_{2\mathrm{SC}} &= (\Delta_{2\mathrm{SC}}, 0, 0)^T,
\end{aligned} \tag{10}$$

for the red color magnetic flux (r),

$$\begin{aligned}
\Phi_{dd}(\varphi) &= \Delta_{dd} \mathrm{diag}(g(r), f(r) e^{i\varphi}, g(r)),\\
U(\varphi) &= e^{i(\varphi/6)\mathrm{diag}(-1,2,-1)},\\
A_i &= -\frac{a(r)}{6g} \frac{\epsilon_{ij} x^j}{r^2} \mathrm{diag}(-1, 2, -1),\\
\Phi_{2\mathrm{SC}} &= (0, \Delta_{2\mathrm{SC}}, 0)^T,
\end{aligned} \tag{11}$$

for the green one (g), and

$$\begin{aligned}
\Phi_{dd}(\varphi) &= \Delta_{dd} \mathrm{diag}(g(r), g(r), f(r) e^{i\varphi}),\\
U(\varphi) &= e^{i(\varphi/6)\mathrm{diag}(-1,-1,2)},\\
A_i &= -\frac{a(r)}{6g} \frac{\epsilon_{ij} x^j}{r^2} \mathrm{diag}(-1, -1, 2),\\
\Phi_{2\mathrm{SC}} &= (0, 0, \Delta_{2\mathrm{SC}})^T,
\end{aligned} \tag{12}$$

for the blue one (b), with the boundary conditions for the profiles $f(0) = g'(0) = a(0) = 0$, $f(\infty) = g(\infty) = a(\infty) = 1$. These carry $1/6$ quantized color-magnetic fluxes, $\mathcal{F} = \mathcal{F}_0/6$, and $1/3$ quantized circulations.

For all the three cases, the $^3P_2$ order parameter given in Eq. (2) behaves at large distance as

$$A^{ij}(\varphi) \sim e^{i\varphi} A^{ij}(\varphi = 0), \tag{13}$$

coinciding with Eq. (5) of one integer vortex in the $^3P_2$ neutron matter.

IV. VORTEX CONTINUITY AND BOOJUMS

As mentioned earlier, the idea of the vortex continuity was originally proposed in Ref. [27]. Their discussion of the vortex continuity was based on quantity called the Onsager-Feynman circulation whose definition is given by $C = \oint \boldsymbol{v} \cdot d\ell = 2\pi n/\mu$ with n and μ being the winding number and chemical potential of the condensate, respectively. The circulations of vortices in the hadronic and quark phase are turned out to be identical, so, it led to the observation that a single hadronic vortex would be smoothly connected to a single non-Abelian vortex. One can also calculate the circulation for our case: The circulation of a neutron $^3P_2$ vortex is $C_{nn} = \pi/\mu_\mathrm{B}$ with μ_B being the baryon chemical potential. The circulation of a non-Abelian Alice string is given by $C_\mathrm{NA} = \pi/3\mu_\mathrm{q} = \pi/\mu_\mathrm{B}$, where $\mu_\mathrm{q} = \mu_\mathrm{B}/3$ is the quark chemical potential. The circulations in the both phases coincide with each other, $C_{nn} = C_\mathrm{NA}$. Equivalently, the expressions for the neutron superfluid order parameter in Eqs. (5) and (13) coincide with each other. Thus, at a glance one might think that a single non-Abelian Alice string would be connected to a single integer $^3P_2$ vortex. It is, however, *not* true as shown below.

In order to investigate how vortices are connected, we employ the (generalized) AB phases of quarks encircling around vortices. When the neutron field $\hat{n}$ encircles around a single integer vortex given in Eq. (5), it receives a phase factor (generalized AB phase) $\exp(i\varphi/2)$ at angle φ, and after the complete encirclement it obtains $\exp(i\pi) = -1$. Then, we next take quarks as probe particles and calculate the AB phase that the quarks receive. Since we have assumed the neutron operator as $\hat{n} = \epsilon^{\alpha\beta\gamma}(\hat{u}_\alpha^T C\gamma^5 \hat{d}_\beta)\hat{d}_\gamma$, light quarks $\hat{q} = \hat{u}, \hat{d}$ obtain the phase $\Gamma_{nn}^{u,d}(\varphi) \equiv \exp(i\varphi/6)$ when they encircle the neutron vortex at angle φ:

$$\hat{q}(\varphi = 0) \to \hat{q}(\varphi) \sim \Gamma_{nn}^{u,d}(\varphi)\hat{q}(\varphi = 0). \tag{14}$$

Thus, at the quark level, the generalized AB phase forms a $\mathbb{Z}_6$ group. The heavy quark field $\hat{s}$ receives no phase around the vortex, i.e., $\Gamma_{nn}^s(\varphi) \equiv 1$. We explain our notation that Γ_{nn}^ψ is the generalized AB phase around neutron $^3P_2$ vortex probed by particle $\psi = \hat{u}, \hat{d}, \hat{s}$.

Let us turn to quark matter and calculate phase factors of quarks around the Alice string. For any $\varphi \neq 0$, the light quark field ($\hat{q} = \hat{u}, \hat{d}$) and heavy quark field ($\hat{s}$) are given by a holonomy action as

$$\hat{q}(\varphi) \sim e^{i\theta_{\mathrm{B}}(\varphi)} U(\varphi)\hat{q}(\varphi = 0), \tag{15}$$

$$\hat{s}(\varphi) \sim U(\varphi)\hat{s}(\varphi = 0), \tag{16}$$

respectively, where $U(\varphi)$ is defined in Eq. (9). Here, we use the following shorthand notation for the generalized AB phase. The field $\hat{\psi}(= \hat{q}, \hat{s}, \ldots)$ of color $\beta(= r, g, b)$ encircling around the flux tubes of the $\alpha(= r, g, b)$ color magnetic flux acquires the AB phase denoted by $\Gamma_{\alpha\beta}^\psi$:

$$\hat{\psi}_\beta(\varphi = 0) \to \hat{\psi}_\beta(\varphi) \sim \Gamma_{\alpha\beta}^\psi(\varphi)\hat{\psi}_\beta(\varphi = 0), \tag{17}$$

where the index β is not contracted. The AB phases of a heavy quark encircling flux tubes are

$$\Gamma_{\alpha\beta}^s(\varphi) = \begin{array}{c} \\ r \\ g \\ b \end{array} \begin{array}{c} \begin{array}{ccc} r & g & b \end{array} \\ \begin{pmatrix} re^{+i\varphi/3} & e^{-i\varphi/6} & e^{-i\varphi/6} \\ ge^{-i\varphi/6} & e^{+i\varphi/3} & e^{-i\varphi/6} \\ be^{-i\varphi/6} & e^{-i\varphi/6} & e^{+i\varphi/3} \end{pmatrix} \end{array}, \tag{18}$$

where, as explicitly indicated above, the row ($\alpha = r$, g, b) denotes the color of the flux tubes, and the column ($\beta = r$, g, b) denotes the colors of the heavy (s) quark encircling them. Thus, the heavy quark field s forms a $\mathbb{Z}_6$ group around the Alice string.

When the light quarks u, d encircle the Alice string, they also receive $\mathrm{U}(1)_B$ transformation $e^{+i\varphi/6}$ as well as the AB phase that they have in common with those of the s-quarks. Therefore, the generalized AB phases are

$$\Gamma_{\alpha\beta}^{u,d}(\varphi) = e^{i\varphi/6}\Gamma_{\alpha\beta}^s(\varphi) = \begin{pmatrix} e^{i\varphi/2} & 1 & 1 \\ 1 & e^{i\varphi/2} & 1 \\ 1 & 1 & e^{i\varphi/2} \end{pmatrix}. \tag{19}$$

Thus, the light quarks u, d form a $\mathbb{Z}_2$ group around the Alice string.

From the above calculations of generalized AB phases around the vortices, one can immediately conclude that one integer vortex in the $^3P_2$ phase cannot be connected to one Alice string with any color flux. We can check this by the fact that the AB phases of the light quarks do not match between the hadron and two-flavor quark matters:

$$\Gamma_{nn}^{u,d}(\varphi) \neq \Gamma_{\alpha\beta}^{u,d}(\varphi) \quad \text{for any } \alpha, \qquad \beta = r, g, b \tag{20}$$

and the same for heavy quarks:

$$\Gamma_{nn}^s(\varphi) \neq \Gamma_{\alpha\beta}^s(\varphi) \quad \text{for any } \alpha, \qquad \beta = r, g, b. \tag{21}$$

Only possibility is that a bundle of three integer vortices in the $^3P_2$ neutron matter can be connected to a bundle of three Alice strings with different color fluxes r, g, b. In this case, the (generalized) AB phases for both phases completely coincide:

$$[\Gamma_{nn}^{u,d}(\varphi)]^3 = \Gamma_{r\beta}^{u,d}(\varphi)\Gamma_{g\beta}^{u,d}(\varphi)\Gamma_{b\beta}^{u,d}(\varphi) \quad \text{for } \beta = r, g, b \tag{22}$$

for the light quarks u, d of the color β, and

$$[\Gamma_{nn}^s(\varphi)]^3 = \Gamma_{r\beta}^s(\varphi)\Gamma_{g\beta}^s(\varphi)\Gamma_{b\beta}^s(\varphi) \quad \text{for } \beta = r, g, b \tag{23}$$

for the heavy quark s of the color β. We thus reach the picture of Boojum illustrated in Fig. 1.

So far we have assumed that two-flavor dense QCD is in the so-called deconfined phase in which non-Abelian Alice strings can exist. On the other hand, if it is in the confined phase, non-Abelian Alice strings must be confined either to doubly quantized non-Abelian strings around which all AB

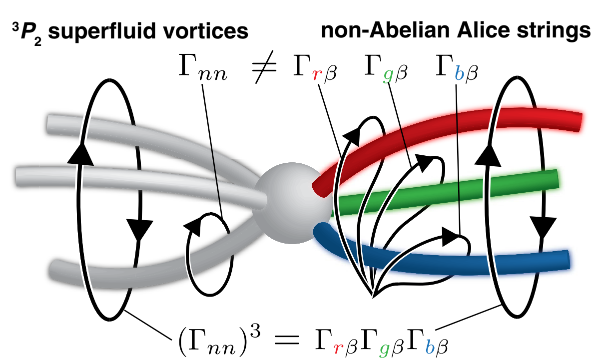

FIG. 1. A schematic figure of the Boojum. Three $^3P_2$ neutron vortices in the hadronic phase are joined to three non-Abelian Alice strings in the color-superconducting phase. We also show the Aharonov-Bohm phase around each vortices.

phases are color singlet, or to $\mathrm{U}(1)_{\mathrm{B}}$ Abelian strings [68]. In the context of quark-hadron continuity in Fig. 1, three non-Abelian Alice strings in the two-flavor quark matter is confined to one $\mathrm{U}(1)_{\mathrm{B}}$ Abelian string. Thus, three integer vortices in the $^3P_2$ neutron superfluid are combined to one Abelian string in the two-flavor quark matter.

The mismatch between the AB phases of a single hadronic vortex and a single non-Abelian vortex tells us that if we try to connect them, it might lead to the discontinuity. The Boojums are therefore needed to maintain the continuity. Our discussion have been carried out on the level of the operator in this work. There can be a possibility that the AB phase may receive the nontrivial contributions and the phase may be dynamically screened if we take the expectation value under the vacuum, which may lead to the different result, but we believe our present study already captures the important feature and the further nonperturbative analysis is the beyond the scope of this work.

V. SUMMARY

In summary, we have found that the Boojums between the non-Abelian Alice strings in the two-flavor quark matter (the $2\mathrm{SC} + \langle dd \rangle$ phase) and the $^3P_2$ neutron vortices in the hadronic matter. As previously suggested in the three-flavor case, Boojum structures are ubiquitous in quark-hadron continuity. Neutron stars are rapidly rotating object so that they are accompanied typically by about 10^{17} of vortices. The vortices are believed to play a crucial role in pulsar glitches, and it would be a interesting and important question how our work influence such events.

The problem of the quark-hadron continuity is not only of the phenomenological relevance, but also tightly related with the fundamental problem of the gauge theory, particularly, the phase structure of gauge theory with fundamental Higgs field. The idea that the Higgs phase and the confinement phase is indistinguishable, which is commonly referred to as Fradkin-Shenker theorem [69–71], is a baseline for the idea of quark-hadron continuity, and is still under a debate nowadays [31]. Since the nonperturbative studies of gauge theories are still limited at present, we believe that the quark-hadron continuity in the bulk matter and vortices from the neutron star phenomenology should serve an important clues.

ACKNOWLEDGMENTS

We thank Shigehiro Yasui for a discussion at the early stage of this work. This work is supported in part by Grant-in-Aid for Scientific Research, JSPS KAKENHI Grants No. 20J10506 (Y. F.) and No. JP18H01217 (M. N.).

[1] G. Baym, T. Hatsuda, T. Kojo, P. D. Powell, Y. Song, and T. Takatsuka, Rep. Prog. Phys. **81**, 056902 (2018).

[2] V. Graber, N. Andersson, and M. Hogg, Int. J. Mod. Phys. D **26**, 1730015 (2017).

[3] T. Kojo, AAPPS Bull. **31**, 11 (2021).

[4] P. Demorest, T. Pennucci, S. Ransom, M. Roberts, and J. Hessels, Nature (London) **467**, 1081 (2010); E. Fonseca *et al.*, Astrophys. J. **832**, 167 (2016); Z. Arzoumanian *et al.* (NANOGrav Collaboration), Astrophys. J. Suppl. Ser. **235**, 37 (2018).

[5] J. Antoniadis *et al.*, Science **340**, 1233232 (2013).

[6] H. T. Cromartie *et al.*, Nat. Astron. **4**, 72 (2020).

[7] B. P. Abbott *et al.* (Virgo and LIGO Scientific Collaborations), Phys. Rev. Lett. **119**, 161101 (2017).

[8] B. Abbott *et al.* (LIGO Scientific and Virgo Collaborations), Astrophys. J. Lett. **892**, L3 (2020).

[9] T. E. Riley *et al.*, Astrophys. J. Lett. **887**, L21 (2019).

[10] M. Miller *et al.*, Astrophys. J. Lett. **887**, L24 (2019).

[11] K. Masuda, T. Hatsuda, and T. Takatsuka, Astrophys. J. **764**, 12 (2013); Prog. Theor. Exp. Phys. **2013**, 073D01 (2013).

[12] T. Kojo, P. D. Powell, Y. Song, and G. Baym, Phys. Rev. D **91**, 045003 (2015).

[13] G. Baym, S. Furusawa, T. Hatsuda, T. Kojo, and H. Togashi, Astrophys. J. **885**, 42 (2019).

[14] T. Kojo, D. Hou, J. Okafor, and H. Togashi, arXiv:2012.01650 [Phys. Rev. D (to be published)].

[15] E. Annala, T. Gorda, A. Kurkela, J. Nättilä, and A. Vuorinen, Nat. Phys. **16**, 907 (2020).

[16] D. Bailin and A. Love, Phys. Rep. **107**, 325 (1984).

[17] M. G. Alford, A. Schmitt, K. Rajagopal, and T. Schäfer, Rev. Mod. Phys. **80**, 1455 (2008).

[18] M. G. Alford, K. Rajagopal, and F. Wilczek, Nucl. Phys. **B537**, 443 (1999).

[19] M. G. Alford, K. Rajagopal, and F. Wilczek, Phys. Lett. B **422**, 247 (1998).

[20] R. Rapp, T. Schäfer, E. V. Shuryak, and M. Velkovsky, Phys. Rev. Lett. **81**, 53 (1998).

[21] T. Schäfer and F. Wilczek, Phys. Rev. Lett. **82**, 3956 (1999).

[22] M. G. Alford, J. Berges, and K. Rajagopal, Nucl. Phys. **B558**, 219 (1999).

[23] K. Fukushima, Phys. Rev. D **70**, 094014 (2004),

[24] T. Hatsuda, M. Tachibana, N. Yamamoto, and G. Baym, Phys. Rev. Lett. **97**, 122001 (2006); N. Yamamoto, M. Tachibana, T. Hatsuda, and G. Baym, Phys. Rev. D **76**, 074001 (2007).

[25] T. Hatsuda, M. Tachibana, and N. Yamamoto, Phys. Rev. D **78**, 011501 (2008),

[26] A. Schmitt, S. Stetina, and M. Tachibana, Phys. Rev. D **83**, 045008 (2011),

[27] M. G. Alford, G. Baym, K. Fukushima, T. Hatsuda, and M. Tachibana, Phys. Rev. D **99**, 036004 (2019).

[28] C. Chatterjee, M. Nitta, and S. Yasui, Phys. Rev. D **99**, 034001 (2019); J. Phys. Soc. Jpn. Conf. Proc. **26**, 024030 (2019).
[29] A. Cherman, S. Sen, and L. G. Yaffe, Phys. Rev. D **100**, 034015 (2019).
[30] Y. Hirono and Y. Tanizaki, Phys. Rev. Lett. **122**, 212001 (2019); J. High Energy Phys. 07 (2019) 062.
[31] A. Cherman, T. Jacobson, S. Sen, and L. G. Yaffe, Phys. Rev. D **102**, 105021 (2020).
[32] M. M. Forbes and A. R. Zhitnitsky, Phys. Rev. D **65**, 085009 (2002).
[33] K. Iida and G. Baym, Phys. Rev. D **66**, 014015 (2002).
[34] E. Nakano, M. Nitta, and T. Matsuura, Phys. Rev. D **78**, 045002 (2008); Prog. Theor. Phys. Suppl. **174**, 254 (2008).
[35] M. G. Alford, S. Mallavarapu, T. Vachaspati, and A. Windisch, Phys. Rev. C **93**, 045801 (2016).
[36] A. Balachandran, S. Digal, and T. Matsuura, Phys. Rev. D **73**, 074009 (2006).
[37] M. Eto and M. Nitta, Phys. Rev. D **80**, 125007 (2009); M. Eto, E. Nakano, and M. Nitta, Phys. Rev. D **80**, 125011 (2009); M. Eto, M. Nitta, and N. Yamamoto, Phys. Rev. Lett. **104**, 161601 (2010).
[38] M. Eto, Y. Hirono, M. Nitta, and S. Yasui, Prog. Theor. Exp. Phys. **2014**, 012D01 (2014).
[39] M. Cipriani, W. Vinci, and M. Nitta, Phys. Rev. D **86**, 121704 (2012).
[40] See Refs. [41] for earlier works on a similar structure.
[41] D. Blaschke, D. M. Sedrakian, and K. M. Shahabasian, Astron. Astrophys. **350**, L47 (1999); D. M. Sedrakian, K. M. Shahabasyan, D. Blaschke, and M. K. Shahabasyan, Astrophysics (Engl. Transl.) **51**, 544 (2008).
[42] N. D. Mermin, in *Surface Singularities and Superflow in 3He-A, Quantum Fluids and Solids*, edited by S. B. Tickey, E. D. Adams, and J. W. Dufty (Plenum, New York, 1977), p. 322.
[43] Originally the boojum is a particular variety of the fictional animal species called *snarks* created by Lewis Carroll in his nonsense poem “The Hunting of the Snark.” See [44] for the story how Mermin made “boojum” an internationally accepted scientific term.
[44] N. C. Mermin, *Boojums All the Way Through: Communicating Science in a Prosaic Age.* (Cambridge University Press, Cambridge, England, 1990).
[45] G. E. Volovik, *The Universe in a Helium Droplet, International Series of Monographs on Physics* (Clarendon Press, Oxford, 2003).
[46] R. Blaauwgeers, V. B. Eltsov, G. Eska, A. P. Finne, R. P. Haley, M. Krusius, J. J. Ruohio, L. Skrbek, and G. E. Volovik, Phys. Rev. Lett. **89**, 155301 (2002).
[47] D. I. Bradley, S. N. Fisher, A. M. Guenault, R. P. Haley, J. Kopu, H. Martin, G. R. Pickett, J. E. Roberts, and V. Tsepelin, Nat. Phys. **4**, 46 (2008).
[48] J. M. Carlson, S. A. Langer, and J. P. Sethna, Europhys. Lett. **5**, 327 (1988).
[49] K. Kasamatsu, H. Takeuchi, and M. Nitta, J. Phys. Condens. Matter **25**, 404213 (2013); K. Kasamatsu, H. Takeuchi, M. Nitta, and M. Tsubota, J. High Energy Phys. 11 (2010) 068; Phys. Rev. A **85**, 053639 (2012); **88**, 013620 (2013).
[50] M. Eto, Y. Isozumi, M. Nitta, K. Ohashi, and N. Sakai, J. Phys. A **39**, R315 (2006); Y. Isozumi, M. Nitta, K. Ohashi, and N. Sakai, Phys. Rev. D **71**, 065018 (2005).
[51] G. Marmorini, S. Yasui, and M. Nitta, arXiv:2010.09032.
[52] M. Hoffberg, A. Glassgold, R. Richardson, and M. Ruderman, Phys. Rev. Lett. **24**, 775 (1970).
[53] R. Tamagaki, Prog. Theor. Phys. **44**, 905 (1970); T. Takatsuka and R. Tamagaki, Prog. Theor. Phys. **46**, 114 (1971); T. Takatsuka, Prog. Theor. Phys. **47**, 1062 (1972); T. Takatsuka and R. Tamagaki, Prog. Theor. Phys. Suppl. **112**, 27 (1993).
[54] R. W. Richardson, Phys. Rev. D **5**, 1883 (1972).
[55] J. Sauls and J. Serene, Phys. Rev. D **17**, 1524 (1978).
[56] T. Mizushima, K. Masuda, and M. Nitta, Phys. Rev. B **95**, 140503 (2017); T. Mizushima, S. Yasui, and M. Nitta, Phys. Rev. Research **2**, 013194 (2020).
[57] S. Yasui, C. Chatterjee, and M. Nitta, Phys. Rev. C **99**, 035213 (2019); S. Yasui, C. Chatterjee, M. Kobayashi, and M. Nitta, Phys. Rev. C **100**, 025204 (2019).
[58] Y. Fujimoto, K. Fukushima, and W. Weise, Phys. Rev. D **101**, 094009 (2020); Y. Fujimoto, Nucl. Phys. **A1005**, 121757 (2021).
[59] Y. Fujimoto and M. Nitta, Phys. Rev. D **103**, 054002 (2021).
[60] A. S. Schwarz, Nucl. Phys. **B208**, 141 (1982).
[61] M. G. Alford, K. Benson, S. R. Coleman, J. March-Russell, and F. Wilczek, Phys. Rev. Lett. **64**, 1632 (1990), **65**, 668(E) (1990); Nucl. Phys. **B349**, 414 (1991); M. G. Alford, K.-M. Lee, J. March-Russell, and J. Preskill, Nucl. Phys. **B384**, 251 (1992).
[62] U. Leonhardt and G. Volovik, Pis’ma Zh. Eksp. Teor. Fiz. **72**, 66 (2000).
[63] C. Chatterjee and M. Nitta, J. High Energy Phys. 09 (2017) 046; Eur. Phys. J. C **77**, 809 (2017); Phys. Rev. D **101**, 085002 (2020); M. Nitta, J. High Energy Phys. 03 (2021) 276.
[64] P. Muzikar, J. Sauls, and J. Serene, Phys. Rev. D **21**, 1494 (1980).
[65] J. Sauls, D. Stein, and J. Serene, Phys. Rev. D **25**, 967 (1982).
[66] K. Masuda and M. Nitta, Phys. Rev. C **93**, 035804 (2016); C. Chatterjee, M. Haberichter, and M. Nitta, Phys. Rev. C **96**, 055807 (2017); Y. Masaki, T. Mizushima, and M. Nitta, Phys. Rev. Research **2**, 013193 (2020).
[67] K. Masuda and M. Nitta, Prog. Theor. Exp. Phys. **2020**, 013D01 (2020).
[68] Y. Fujimoto and M. Nitta, arXiv:2103.15185.
[69] K. Osterwalder and E. Seiler, Ann. Phys. (N.Y.) **110**, 440 (1978).
[70] E. H. Fradkin and S. H. Shenker, Phys. Rev. D **19**, 3682 (1979).
[71] T. Banks and E. Rabinovici, Nucl. Phys. **B160**, 349 (1979).