Production of *D*-wave states of $\bar{b}c$ quarkonium at the LHC

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The hadronic production of *D*-wave states of $\bar{b}c$ quarkonium is studied. The relative yield of such states is estimated for kinematic conditions of LHC experiments.

I. INTRODUCTION

Recently 2*S* excitations of B_c mesons have been discovered by CMS [1–3] in $B_c\pi^+\pi^-$ spectrum. The observation has been confirmed by LHCb experiment [4]. Thus, in the opinion of the authors of [5], it has opened a new era in the spectroscopy of ordinary quarkonia. The excellent experimental results, the long known theoretical prediction, that *D* states can also decay to $B_c\pi^+\pi^-$ with probability $\sim20\%$ [6] (see also [5]), and the earlier study of *D*-wave quarkonia production within fragmentation approximation [7] stimulate us to estimate the cross section of *D*-wave B_c states in hadronic interactions. It is worth to note, that the relative $B_c(2S)$ yield $\sigma(B_c(2S)/\sigma(B_c)$ published by CMS ($\sim8\%$) is in a good agreement with our prediction ($\sim10\%$) [8,9]. That is why we hope, that our prediction for relative yield of *D*-wave states obtained within analogous technique will fairly good describe the future experimental observation of the discussed states.

The article is organized as follows: Section II is devoted to the calculation technique description; in Sec. III the relative yield of *D*-wave B_c meson state is estimated for kinematic conditions of the LHC experiments; in Sec. IV we make conclusions on the possibility observation of such states at LHC; in the Appendix we provide some information about *D*-wave B_c masses and wave functions.

II. CALCULATION TECHNIQUE

To estimate the production amplitude of *D*-wave B_c states we use the analogous technique as for *S* and *P* waves,

namely, we perform calculations within the color singlet model neglecting the internal velocities of quarks inside quarkonium (see for details [10–24]):

$$A \sim \int d^3q \Psi^*(\boldsymbol{q}) \Bigg\{ T(p_i,\boldsymbol{q})|_{\boldsymbol{q}=0} + q^\alpha \frac{\partial}{\partial q^\alpha} T(p_i,\boldsymbol{q})|_{\boldsymbol{q}=0} + \frac{1}{2} q^\alpha q^\beta \frac{\partial^2}{\partial q^\alpha \partial q^\beta} T(p_i,\boldsymbol{q})|_{\boldsymbol{q}=0} + \cdots \Bigg\}, \tag{1}$$

where T is the amplitude of four heavy quark gluonic production with momenta p_i in the leading-order approximation, which is contributed by 36 Feynman diagrams; $\boldsymbol{q}$ is the quark three-momentum in the B_c meson rest frame, and $\Psi(\boldsymbol{q})$ is the B_c meson wave function.

For *D*-wave states the first two terms in (1) are equal to zero, and therefore an amplitude is proportional to the second derivative of the wave function at origin $R''(0)$ and to the second derivatives of T over $\boldsymbol{q}$. The amplitudes for the spin singlet A^{j_z} ($J = 2, j_z = l_z$) and for the spin triplet A^{Jj_z} ($J = 1, 2, 3; j_z = s_z + l_z$) can be expressed as follows (see also [7], where the *D*-wave B_c production was studied in the fragmentation approach):

$$A^{j_z} = \frac{1}{2}\sqrt{\frac{15}{8\pi}}R''_D(0)\epsilon^{\alpha\beta}(j_z)\frac{\partial^2 M(\boldsymbol{q})}{\partial q^\alpha \partial q^\beta}\Bigg|_{\boldsymbol{q}=0}, \tag{2}$$

$$A^{Jj_z} = \frac{1}{2}\sqrt{\frac{15}{8\pi}}R''_D(0)\Pi^{J,\alpha\beta\rho}(j_z)\frac{\partial^2 M_\rho(\boldsymbol{q})}{\partial q^\alpha \partial q^\beta}\Bigg|_{\boldsymbol{q}=0}, \tag{3}$$

where

$$\Pi^{J,\alpha\beta\rho}(j_z) = \sum_{l_z,s_z}\epsilon^{\alpha\beta}(l_z)\epsilon^\rho(s_z) \cdot C^{Jj_z}_{s_z l_z}, \tag{4}$$

ϵ^ρ and $\epsilon^{\alpha\beta}$ are vector and polarization tensors and $C^{Jj_z}_{s_z l_z}$ are Clebsch-Gordan coefficients.

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The states with a definite spin value are constructed by operators

$$\mathcal{P}(0,0) = \frac{1}{\sqrt{2}}\{v_+(p_{\bar{b}}+k)\bar{u}_+(p_c-k) - v_-(p_{\bar{b}}+k)\bar{u}_-(p_c-k)\} \tag{5}$$

and

$$\mathcal{P}(1,s_z) = \begin{cases} \mathcal{P}(1,1) = v_-(p_{\bar{b}}+k)\bar{u}_+(p_c-k) \\ \mathcal{P}(1,0) = \frac{1}{\sqrt{2}}\{v_+(p_{\bar{b}}+k)\bar{u}_+(p_c-k) + v_-(p_{\bar{b}}+k)\bar{u}_-(p_c-k)\} \\ \mathcal{P}(1,-1) = v_+(p_{\bar{b}}+k)\bar{u}_-(p_c-k) \end{cases} \tag{6}$$

The spinors in (5) and (6) are expressed as follows:

$$v_{\lambda_1}(p_{\bar{b}}+k) = \left(1 - \frac{\not{k}}{2m_b}\right)v_{\lambda_1}(p_{\bar{b}}), \tag{7}$$

$$\bar{u}_{\lambda_2}(p_c-k) = \left(1 - \frac{\not{k}}{2m_c}\right)\bar{u}_{\lambda_2}(p_c), \tag{8}$$

where $p_{\bar{b}} = \frac{m_b}{m_b+m_c}P_{B_c}$, $p_c = \frac{m_c}{m_b+m_c}P_{B_c}$ and $k(\boldsymbol{q})$ is a Lorentz boost of four-vector $(0,\boldsymbol{q})$ to the system where the B_c momentum is equal to P_{B_c}.

Amplitudes and their derivatives have been calculated numerically. To simplify the calculations we square and summarize amplitudes, keeping only a spin value $S = 0$ or $S = 1$. The amplitude squared for the spin-singlet state with $S = 0$ (1^1D_2) is given by the following equation:

$$\mathbb{P} = \left(\frac{5}{16\pi}\right)|R_D''(0)|^2 \times \left[\left(\left|\frac{\partial^2 M_{S=0}}{\partial q_x^2}\right|^2 + \left|\frac{\partial^2 M_{S=0}}{\partial q_y^2}\right|^2 + \left|\frac{\partial^2 M_{S=0}}{\partial q_z^2}\right|^2\right) + 3\left(\left|\frac{\partial^2 M_{S=0}}{\partial q_x\partial q_y}\right|^2 + \left|\frac{\partial^2 M_{S=0}}{\partial q_x\partial q_z}\right|^2 + \left|\frac{\partial^2 M_{S=0}}{\partial q_y\partial q_z}\right|^2\right)\right.$$
$$\left. - \mathrm{Re}\left(\frac{\partial^2 M_{S=0}}{\partial q_x^2}\frac{\partial^2 M_{S=0}^*}{\partial q_y^2} + \frac{\partial^2 M_{S=0}}{\partial q_x^2}\frac{\partial^2 M_{S=0}^*}{\partial q_z^2} + \frac{\partial^2 M_{S=0}}{\partial q_y^2}\frac{\partial^2 M_{S=0}^*}{\partial q_z^2}\right)\right]. \tag{9}$$

The sum of amplitudes squared for the spin-triplet states with $S = 1$ (1^3D_1, 1^3D_2, 1^3D_3) is presented below:

$$\mathbb{V} = \left(\frac{5}{16\pi}\right)|R_D''(0)|^2 \times \sum_{s_z}^{-1,0,1}\left[\left(\left|\frac{\partial^2 M_{S=1,s_z}}{\partial q_x^2}\right|^2 + \left|\frac{\partial^2 M_{S=1,s_z}}{\partial q_y^2}\right|^2 + \left|\frac{\partial^2 M_{S=1,s_z}}{\partial q_z^2}\right|^2\right)\right.$$
$$+ 3\left(\left|\frac{\partial^2 M_{S=1,s_z}}{\partial q_x\partial q_y}\right|^2 + \left|\frac{\partial^2 M_{S=1,s_z}}{\partial q_x\partial q_z}\right|^2 + \left|\frac{\partial^2 M_{S=1,s_z}}{\partial q_y\partial q_z}\right|^2\right)$$
$$\left. - \mathrm{Re}\left(\frac{\partial^2 M_{S=1,s_z}}{\partial q_x^2}\frac{\partial^2 M_{S=1,s_z}^*}{\partial q_y^2} + \frac{\partial^2 M_{S=1,s_z}}{\partial q_x^2}\frac{\partial^2 M_{S=1,s_z}^*}{\partial q_z^2} + \frac{\partial^2 M_{S=1,s_z}}{\partial q_y^2}\frac{\partial^2 M_{S=1,s_z}^*}{\partial q_z^2}\right)\right]. \tag{10}$$

A more rigorous consideration of this process within NRQCD [25] implies that the final meson is no longer a $\bar{b}c$ pair rather a superposition of Fock states:

$$|B_c(1^1D_2)\rangle = O(1)|\bar{b}c(^1D_2,\mathbf{1})\rangle + O(v)|\bar{b}c(^1P_1,\mathbf{8})g\rangle + O(v^2)|\bar{b}c(^1S_0,\mathbf{8}\text{ or }\mathbf{1})gg\rangle + \cdots \tag{11}$$

$$|B_c(1^3D_j)\rangle = O(1)|\bar{b}c(^3D_j,\mathbf{1})\rangle + O(v)|\bar{b}c(^3P_{j'},\mathbf{8})g\rangle + O(v^2)|\bar{b}c(^3S_1,\mathbf{8}\text{ or }\mathbf{1})gg\rangle + \cdots \tag{12}$$

where **1** and **8** refer to color singlet and color octet states of the quark pair.[1] The NRQCD local 4-fermion operators $\mathcal{O}_1^{B_c(1^1D_2)}(^1D_2)$ and $\mathcal{O}_1^{B_c(1^3D_j)}(^3D_j)$ relevant for the fist terms in the expansions (11) and (12) are related to the quarkonium wave function $R''_D(0)^2$:

$$\langle \mathcal{O}_1^{B_c(1^1D_2)}(^1D_2) \rangle \approx \frac{75 N_c}{4\pi} |R''_D(0)|^2, \qquad (13)$$

$$\langle \mathcal{O}_1^{B_c(1^3D_j)}(^3D_j) \rangle \approx \frac{15(2j+1)N_c}{4\pi} |R''_D(0)|^2. \quad (14)$$

Therefore production of the first components in the expansions (11) and (12) can be described by formulas obtained from (1).

By analogy with the fragmentation case [7], it can be shown that within NRQCD the contributions to the gluonic production of the second and third terms in (11) and (12) are of the same order on α_s and v, as the contribution of the first terms, and therefore they should be also included. Having an experience in calculation of gluonic production of *S*- and *P*-wave $\bar{b}c$ color singlets, it is not difficult to calculate the hard parts of appropriate production amplitudes for *S*- and *P*-wave $\bar{b}c$ color octets. Unfortunately the soft part of such amplitudes can not be accurately estimated, because values of the relevant NRQCD operators are unknown. However understanding the kinematic behavior of such contributions even without knowing the exact normalization could help in the experimental search of *D*-wave B_c states. We estimate here three additional NRQCD contributions enumerated in Eqs. (11) and (12) using the fairly defined hard parts of the processes normalized by coefficients which are extracted using a naive velocity scaling, as explained below.

To model the contributions of *P*-wave color octet states $|\bar{b}c(^1P_1, \mathbf{8})g\rangle$ and $|\bar{b}c(^3P_j, \mathbf{8})g\rangle$ to the discussed cross sections we use our tools for calculation of *P*-wave B_c color singlet states. We replace the color singlet wave function to color octet one and multiply the wave function derivative squared at origin $|R'_P(0)|^2$ by the coefficient $K_{P\mathbf{8}}$ which is of order $O(v_{\text{eff}}^2)$, where v_{eff}^2 is an effective squared velocity of quarks in the B_c meson:

$$\frac{\delta_{\bar{b}c}}{\sqrt{3}} \to \sqrt{2} t_{\bar{b}c}^a,$$

$$|R'_P(0)|^2 \to K_{P\mathbf{8}} \cdot |R'_P(0)|^2. \qquad (15)$$

To model the contributions of *S*-wave color octet states $|\bar{b}c(^1S_0, \mathbf{8})gg\rangle$ and $|\bar{b}c(^3S_1, \mathbf{8})gg\rangle$ we use our tools for calculation of *S*-wave B_c color singlet states, where we replace the color singlet wave function to color octet one and multiply the coordinate wave function squared at origin $|R_S(0)|^2$ by the coefficient $K_{S\mathbf{8}}$ which is of order $O([v_{\text{eff}}^2]^2)$:

$$\frac{\delta_{\bar{b}c}}{\sqrt{3}} \to \sqrt{2} t_{\bar{b}c}^a,$$

$$|R_S(0)|^2 \to K_{S\mathbf{8}} \cdot |R_S(0)|^2. \qquad (16)$$

Constructing the contributions of *S*-wave color singlet states $|\bar{b}c(^1S_0, \mathbf{1})gg\rangle$ and $|\bar{b}c(^3S_1, \mathbf{1})gg\rangle$, as in the previous case (16) we just rescale the wave function squared at origin $|R_S(0)|^2$ by the coefficient $K_{S\mathbf{1}}$ which is of order $O([v_{\text{eff}}^2]^2)$:

$$|R_S(0)|^2 \to K_{S\mathbf{1}} \cdot |R_S(0)|^2. \qquad (17)$$

The very similar approach was applied to estimate the color octet contribution to the B_c *P*-wave production in the work [29]. Also in [29] the properties of color matrix for the gluonic $\bar{b}c$ color octet production were studied in details.

It is worth to remind that for gluonic $b\bar{b}c\bar{c}$ production the replacement of the color singlet wave function of $\bar{b}c$-pair to the color octet wave function cannot be reduced to a simple scaling of the matrix element, because it essentially changes the relative contributions of different Feynman diagrams to the total amplitude.[3]

The v_{eff}^2 value we estimate as:

$$v_{\text{eff}}^2 = \frac{\langle E \rangle}{2\mu}, \qquad (18)$$

where $\langle E \rangle$ is the averaged kinematic energy of quark inside the B_c-meson and $\mu = \frac{m_c m_b}{m_c + m_b}$. Using the value $\langle E \rangle \approx 0.35$ GeV estimated in [31], we obtain that $v_{\text{eff}}^2 \approx 0.15$.

To estimate the additional NRQCD contributions numerically we choose the following central values for *K* coefficients in Eqs. (15) to (17):

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[1]As noted in [7], in the above Fock state expansion there are also other $O(v^2)$ states, but their production will be further suppressed by powers of v.

[2]There are two widely used normalizations for $\mathcal{O}_1$ matrix elements. One normalization method (BBL) inherits from the study [25]. Another one (PCGMM) is based on the work [26]. These two normalization methods relate to each other as follows: $\mathcal{O}_1^{\text{PCGMM}} = \frac{1}{2N_c} \mathcal{O}_1^{\text{BBL}}$. Since we consider our study to be a continuation of work [7] that uses the BBL normalization, we also use it in Eqs. (13) and (14). The exact determination of the discussed operators one can find for example in [27] and [28].

[3]For the first time the color matrix for the process $gg \to b\bar{b}c\bar{c}$ was investigated in [30], where it was shown that such color matrix has 13 nonzero eigenvalues. Three of them correspond to the cases, where the $\bar{b}c$-pair is in a color singlet state: $(\bar{b}c)_\mathbf{1} \otimes (b\bar{c})_\mathbf{1}$, $(\bar{b}c)_\mathbf{1} \otimes (b\bar{c})_{\mathbf{8-symmetric}}$ and $(\bar{b}c)_\mathbf{1} \otimes (b\bar{c})_{\mathbf{8-antisymmetric}}$. The remaining ten eigenvalues correspond to the cases, where the $\bar{b}c$-pair is in a color octet. We refer to the studies [20,29] for details.

TABLE I. B_c-meson parameters involved in calculations.

B_c-states	m_b	m_c	$|R(0)|^2$, $|R''(0)|^2$ [5]	$|R(0)|^2_{\mathrm{eff}}$, $|R''(0)|^2_{\mathrm{eff}}$ [33,34]	$|R(0)|^2_{\mathrm{eff}}$, $|R''(0)|^2$ [35,36]
1S	4.80 GeV	1.50 GeV	1.994 GeV^3	1.49 GeV^3	0.74 GeV^3
1D	5.20 GeV	1.80 GeV	0.0986 GeV^7	0.116 GeV^7	0.055 GeV^7

$$K_{P8} = v^2_{\mathrm{eff}} = 0.15,$$
$$K_{S8} = K_{S1} = [v^2_{\mathrm{eff}}]^2 = 0.0225. \qquad (19)$$

In order to estimate the uncertainties of the additional contributions to hadronic production we vary the K_{P8} value from 0.1 to 0.2, and the $K_{S8,1}$ value from 0.015 to 0.03.

It should be emphasized that the normalization values proposed in Eqs. (15) to (19) cannot be regarded as reliable, and may drastically differ from values, which be will measured experimentally or predicted within a more rigorous approach. Nevertheless, we think that it is useful to demonstrate in this study, how the color octet contribution could influence the kinematic behavior of the D-wave B_c meson yield at LHC experiments.

The calculation results have been tested for Lorentz invariance and gauge invariance. As it was already noted, the calculations were conducted within the method very close to ones applied to the study of S- and P-wave states. As in our previous researches the integration over phase space was carried out within the RAMBO algorithm [32].

While results of the latest calculations have been verified many times by other research groups [11–13,16–19, 21,22,24] we have tried to minimize the possibility of error in our new work.

III. ESTIMATIONS OF RELATIVE YIELD

For the numerical estimations of cross sections we involve the wave functions and masses listed in Table I
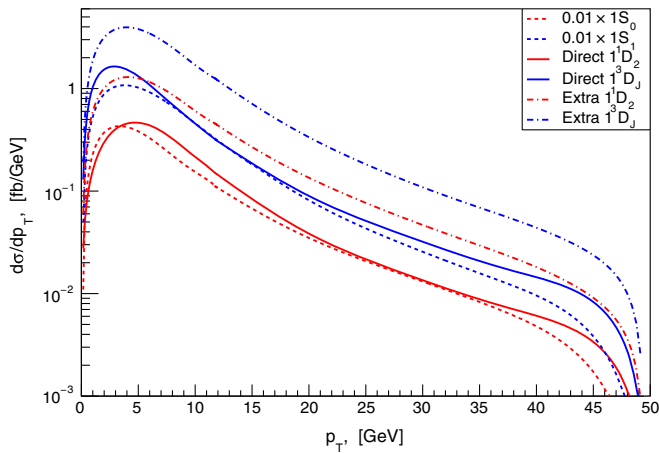

FIG. 1. $\sigma(gg \to B_c + X)$ dependence on transverse momentum at $\sqrt{s}_{gg} = 100$ GeV. Solid lines: direct D-wave states; dashed lines: S-wave states scaled by 0.01; dashed-dotted lines: extra D-wave states.

(see also Table IV where the predictions for masses of D-wave excitations are presented). The parameters of radial wave functions for 1S and 1D states are taken from works [5], [33,34], and [35,36]. Following most of the previous research on this topic, we choose the mass values of quarks in such a way as the mass of final $\bar{b}c$ quarkonium is correct. We are motivated by the fact, that relative yield of 2S excitations was described quite well within this choice.

Using the wave function parameters from [5] and choosing the quark masses as in Table I we predict, that the relative yield of $B_c(1D)$ with respect to the direct $B_c(1S)$ in the gluonic fusion is about $0.5 \div 1.3\%$, as seen from Table II, where the cross section values for the gluonic production are presented at different gluonic energies. As shown in Figure 1 the distributions over transverse momentum for D-wave states are quite similar to ones for S-wave states. It is worth mentioning, that the predicted ratio of states with spin $S = 1$ (1^3D_1, 1^3D_2, 1^3D_3) v.s. states with spin $S = 0$ (1^1D_2) is in approximate accordance with a simple spin counting rule (see ratios in Table III):

$$\frac{\sigma(1^3D_1 + 1^3D_2 + 1^3D_3)}{\sigma(1^1D_2)} \sim \frac{3 + 5 + 7}{5} = 3. \qquad (20)$$

This feature allows us to use the prediction of quasipotential model [33,34] where wave functions are essentially different for 1^3D_1, 1^3D_2, 1^3D_3 and 1^1D_2 states, as well as for 1^3S_1 and 1^1S_0 states, even if contributions of different D states are not estimated separately. For this case we can approximately estimate the cross section ratio averaging the wave function values according to spin counting rules (see Table I):

TABLE II.   Gluonic cross sections at different energies; values are performed with $\alpha_S = 0.1$ and wave functions from [5].

$\sqrt{s}_{gg}$, GeV	σ_{gg}, pb				
	1S	1D	$|\bar{b}c(P,\mathbf{8})g\rangle$	$|\bar{b}c(S,\mathbf{8})gg\rangle$	$|\bar{b}c(S,\mathbf{1})gg\rangle$
20	1.97	0.009	0.051	0.053	0.016
30	2.90	0.023	0.080	0.068	0.031
50	2.64	0.028	0.055	0.042	0.032
70	1.98	0.024	0.035	0.026	0.026
100	1.44	0.018	0.019	0.015	0.019
150	0.904	0.012	0.010	0.007	0.013

TABLE III.   Relative yields for D-wave B_c mesons for forward and central kinematic regions at LHC; the wave functions set [5] is applied.

Kinematic region	$\sigma(1^3S_1)/\sigma(1^3S_0)$	$\sigma(1^3D_j)/\sigma(1^1D_2)$	$\sigma(1D)/\sigma(1S)$, %
$2 < \eta < 4.5$, $p_T < 10$ GeV	2.4	3.0	$0.6 \div 0.7$
$|\eta| < 2.5$, 10 GeV $< p_T < 50$ GeV	2.4	2.3	$1.0 \div 1.1$

$|R''_D(0)|^2_{\text{eff}}$

$$= \frac{3|R''_{1^3D_1}(0)|^2 + 5|R''_{1^3D_2}(0)|^2 + 7|R''_{1^3D_3}(0)|^2 + 5|R''_{1^1D_2}(0)|^2}{20},$$

(21)

$$|R_S(0)|^2_{\text{eff}} = \frac{|R''_{1^1S_0}(0)|^2 + 3|R_{1^3S_1}(0)|^2}{4}.$$ (22)

Usage of (21) and (22) calculated within the approach [33,34] leads to a little bit more optimistic values: the discussed relative yield is approximately 1.6 times higher. Another model, based on the quasipotential approach [35,36], predicts essentially lower values for wave functions at origin (see Table I and Table V of Appendix). However this difference almost disappears when estimating the relative yield. Using the wave functions from [35,36] increases the final value by 1.5 times comparing to [5].

As it is seen from Table II and Figs. 1 and 2 the additional NRQCD contributions $|\bar{b}c(P, \mathbf{8})g\rangle$, $|\bar{b}c(S, \mathbf{8})gg\rangle$ and $|\bar{b}c(S, \mathbf{1})gg\rangle$ extracted within naive velocity scaling rules (15), (16), (17) are able to crucially change the production properties. While the p_T distribution shapes are more or less the same, the energy dependencies for the color octet contributions and for the color singlet contributions essentially differ: the color octet contributions decrease faster with energy, than the color singlet ones.
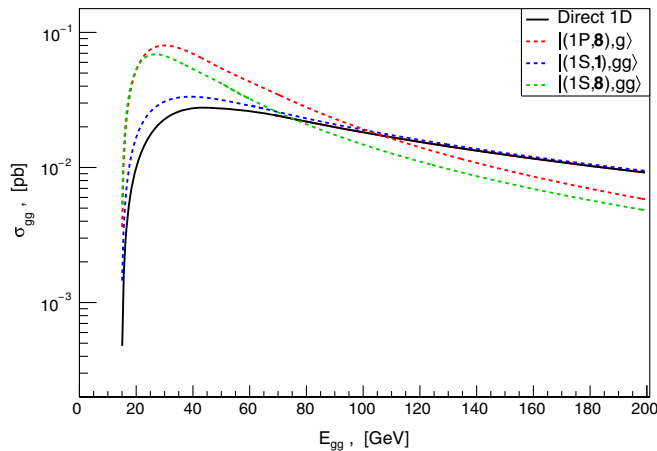

FIG. 2. $\sigma(gg \to B_c + X)$ dependence on gluon-gluon energy. Black line: direct D-wave production; red line: $|(1P, \mathbf{8})g\rangle$ contribution; blue and green lines: $|(1S, \mathbf{1})gg\rangle$ and $|(1S, \mathbf{8})gg\rangle$ contributions correspondingly.

Moreover, seems, that the shape of energy dependence is mostly determined by the color state of $\bar{b}c$-pair and practically does not depend on its orbital momentum.

Concerning the numerical values of the additional NRQCD contributions one can conclude that each of such contributions is of order of the direct color singlet production, as expected. This circumstance testifies to the self-consistency of our calculations. Since there are three such additional contributions, they crucially increase the total cross section.

To obtain the proton-proton cross sections the gluonic cross sections are convoluted with CT14 PDFs [37]:

$$\sigma_{pp} = \int \sigma_{gg}(\hat{s}_{gg}, \mu) f_{g1}(x_1, \mu) f_{g2}(x_2, \mu) dx_1 dx_2.$$ (23)

To decrease uncertainties due to the scale choice and QCD corrections we present a relative yield of $1D$ states with respect to $1S$ states. The calculations are performed for forward and central kinematic regions. The forward one is restricted by cuts $2 < \eta < 4.5$, $p_T < 10$ GeV and nearly corresponds to LHCb conditions, while the central one is restricted by cuts $2 < \eta < 4.5$, $p_T < 10$ GeV, $|\eta| < 2.5$, 10 GeV $< p_T < 50$ GeV and approximately corresponds to CMS or ATLAS conditions. The proton-proton energy of collision is chosen equal to 13 TeV.

When following the collinear approximation, one should always keep in mind the problem of transverse momentum of the initial gluons. Indeed, in some cases accounting the initial gluon transverse momenta crucially changes the production features (see, for example [38] or [39]). However, we believe that in our case the dependence on the initial gluonic transverse momenta is more or less eliminated in the ratio $\sigma(B_c(D))/\sigma(B_c(S))$.

The systematic uncertainty of the calculations is estimated with variation of the scale in the range $E_T/2 < \mu < 2E_T$. It is well seen in Figures 3 and 4 that the relative yields are hardly dependent on scale variation. As it is seen in Table III within the applied model (the color singlet production in the gluon fusion subprocess) the relative yield value depends on kinematics: for the central region it is approximately twice as large. The use of the wave functions set [33,34] or [35,36] increases the predicted yield of D-wave states to $1 \div 1.8\%$.

Accounting naively estimated contributions of $|\bar{b}c(P, \mathbf{8})g\rangle$, $|\bar{b}c(S, \mathbf{8})gg\rangle$ and $|\bar{b}c(S, \mathbf{1})gg\rangle$ can increase the relative yield of D-wave states by an order of
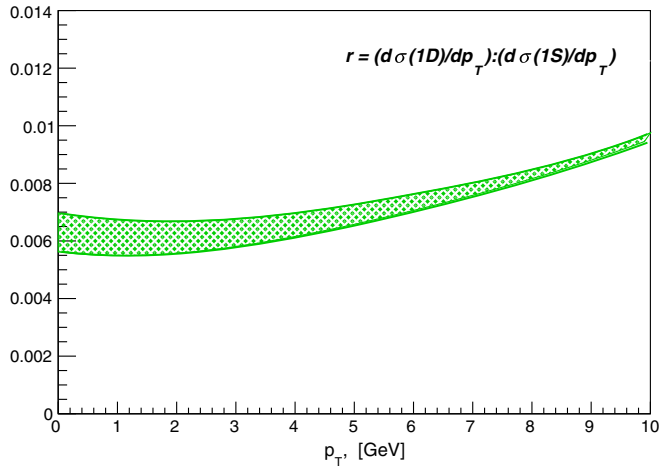
FIG. 3. r dependence on p_T at different scales for forward kinematics: $2 < \eta < 4.5, p_T < 10$ GeV.
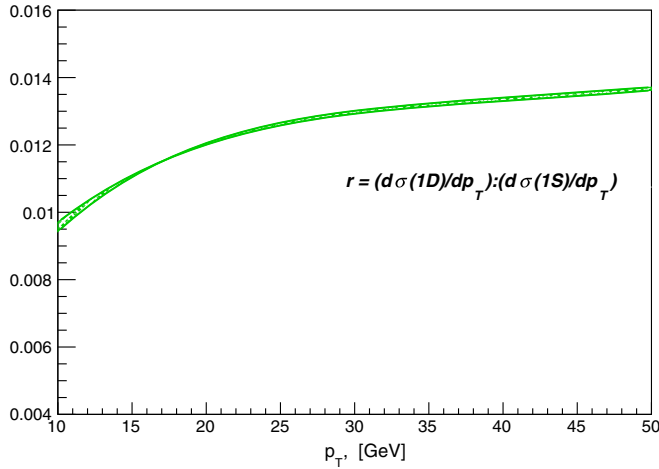

FIG. 4. r dependence on p_T at different scales for central kinematics: $|\eta| < 2.5, 10$ GeV $< p_T < 50$ GeV.
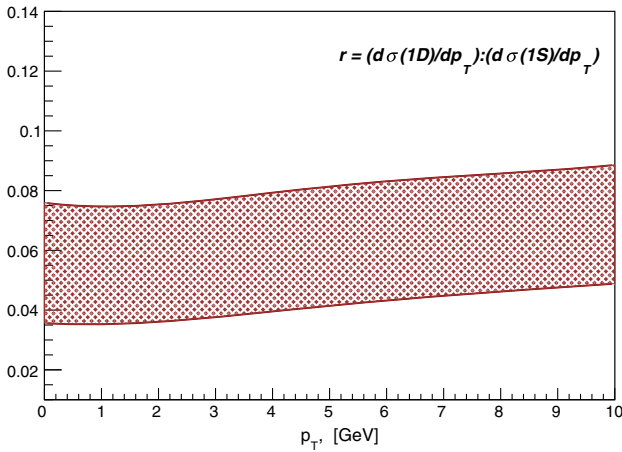

FIG. 5. r dependence on p_T at different scales for forward kinematics: $2 < \eta < 4.5, p_T < 10$ GeV. The contributions of $|\bar{b}c(P,\mathbf{8})g\rangle$, $|\bar{b}c(S,\mathbf{8})gg\rangle$ and $|\bar{b}c(S,\mathbf{1})gg\rangle$ states are included.
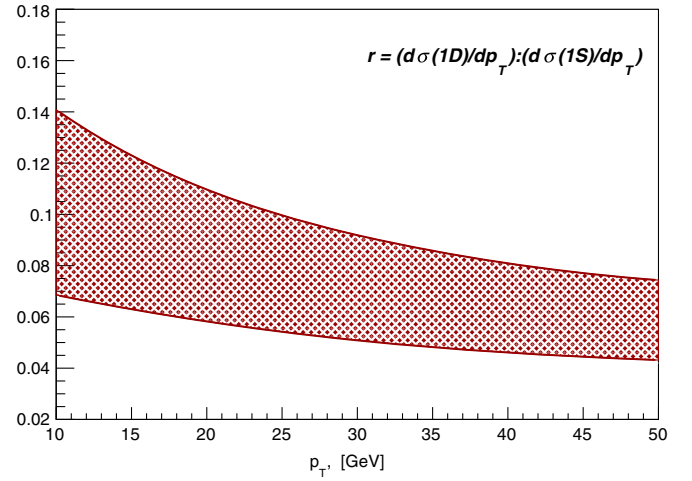

FIG. 6. r dependence on p_T at different scales for central kinematics: $|\eta| < 2.5, 10$ GeV $< p_T < 50$ GeV. The contributions of $|\bar{b}c(P,\mathbf{8})g\rangle$, $|\bar{b}c(S,\mathbf{8})gg\rangle$ and $|\bar{b}c(S,\mathbf{1})gg\rangle$ states are included.

magnitude, as it is shown in Figs. 5 and 6. Despite the fact that each additional contribution is comparable to the main one, their number provides such an increase in yield. As noted in the previous section we estimate the uncertainties for these contributions varying $K_{P\mathbf{8}}$ from 0.1 to 0.2, and $K_{S\mathbf{8,1}}$ from 0.015 to 0.03 [see (15), (16), (17)].

We emphasize ones again, that the naive normalization used in this research cannot be regarded as reliable, and may be drastically far from values, which will be measured experimentally or predicted within a more rigorous approach.

IV. CONCLUSIONS

The $B_c(2S)$ excitations have been observed at LHC in the $B_c\pi^+\pi^-$ spectrum [1–4], and this result stimulated us to estimate possibilities to search for $B_c(D)$ excitations in the same spectrum. At very large statistics it would be possible to distinguish two peaks in the $B_c\pi^+\pi^-$ mass spectrum: one peak near 7000 GeV formed by 1^1D_2 state and another one near 6930 GeV formed by 1^1D_1, 1^1D_2 and 1^1D_3 states decaying to $B_c^*\pi^+\pi^-$ with further radiative decay $B_c^* \xrightarrow{\gamma} B_c$. Also the D-wave B_c excitations could be found in cascade radiative decays $B_c(1D) \xrightarrow{\gamma} B_c(1P) \xrightarrow{\gamma} B_c(1S)$.

Taking into account the main color singlet contribution, we estimate $B_c(D)$ states yield in the hadronic production as $0.6 \div 1.8\%$ with respect to the direct production of $1S$ states for the chosen mass values. To convert this ratio into the more representative ratio of D-wave states yield to the yield of all B_c mesons, one should divide it by a factor of about 1.5, that leads to the values $0.4 \div 1.1\%$. Our estimations of the relative yield of D-wave B_c states in the hadronic production do not contradict the analogous estimations within the fragmentation approach [7].

TABLE IV.   Predictions for masses of *D*-wave B_c meson states in MeV.

State	EQ [6]	GKLT [31]	ZVR [40]	FUI [41]	EFG [42]	GI [43]	MBV [44]	SJSCP [45]	LLLGZ [46]
1^1D_2	7009	$\cdots$	7020	7023	$\cdots$	$\cdots$	$\cdots$	6994	$\cdots$
1^3D_1	7012	7008	7010	7024	7072	7028	6973	6998	7020
1^3D_2	7012	$\cdots$	7030	7025	$\cdots$	$\cdots$	$\cdots$	6997	$\cdots$
1^3D_3	7005	7007	7040	7022	7081	7045	7004	6990	7030
$1D_2'$	$\cdots$	7016	$\cdots$	$\cdots$	7079	7036	7003	$\cdots$	7032
$1D_2$	$\cdots$	7001	$\cdots$	$\cdots$	7077	7041	6974	$\cdots$	7024

Accounting contributions of $|\bar{b}c(P,\mathbf{8})g\rangle$, $|\bar{b}c(S,\mathbf{8})gg\rangle$ and $|\bar{b}c(S,\mathbf{1})gg\rangle$ states extracted using the naive velocity scaling rules increases the relative yield of *D*-wave states by an order of magnitude. Therefore the significant experimental excess of the relative yield of *D*-wave mesons over the value $0.4 \div 1.1\%$ will indicate an essential contribution of the color octet states to the production.

We have to conclude that an observation of the discussed states at LHC is a quite challenging experimental task due to the small relative yield.

TABLE V. B_c meson wave functions within the quasipotential models [33,34] and [35,36].

B_c-state	$|R(0)|^2$, $|R''(0)|^2$ [33,34]	$|R(0)|^2$, $|R''(0)|^2$ [35,36]
1^1S_0	2.68 GeV^3	0.97 GeV^3
1^3S_1	1.09 GeV^3	0.66 GeV^3
1^1D_2	0.078 GeV^7	
1^3D_1	0.314 GeV^7	0.055 GeV^3
1^3D_2	0.098 GeV^7	
1^3D_3	0.061 GeV^7	

APPENDIX: WAVE FUNCTIONS AND SPECTROSCOPY OF THE *D*-WAVE B_c STATES

In this Appendix we present the masses of *D*-wave states of B_c meson predicted within different models [6,31,40–46]. As well we present the wave function parameters obtained within the quasipotential approaches [33,34] and [35,36].

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