Radiative corrections of order $O(\alpha E_e/m_N)$ to Sirlin's radiative corrections of order $O(\alpha/\pi)$, induced by the hadronic structure of the neutron

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We investigate the contributions of the hadronic structure of the neutron to radiative $O(\alpha E_e/m_N)$ corrections [or the inner $O(\alpha E_e/m_N)$ RC] to the neutron beta decay, where α, E_e, and m_N are the fine-structure constant, the electron energy, and the nucleon mass, respectively. We perform the calculation within the effective quantum field theory of strong low-energy pion-nucleon interactions described by the linear σ model with chiral $SU(2)\times SU(2)$ symmetry and electroweak hadron-hadron, hadron-lepton, and lepton-lepton interactions for the electron-lepton family with $SU(2)_L\times U(1)_Y$ symmetry of the standard electroweak theory [Ivanov *et al.*, Phys. Rev. D **99**, 093006 (2019)]. We show that after renormalization, carried out in accordance with Sirlin's prescription [Sirlin, Phys. Rev. **164**, 1767 (1967)], the inner $O(\alpha E_e/m_N)$ RC are of the order of a few parts of 10^{-5}–10^{-4}. This agrees well with the results obtained in [Ivanov *et al.*, Phys. Rev. D **99**, 093006 (2019)].

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I. INTRODUCTION

According to Sirlin [1,2], the contribution of the hadronic structure of the neutron to the radiative $O(\alpha/\pi)$ corrections [or the *inner* $O(\alpha/\pi)$ radiative corrections (RC) [3]] to the neutron lifetime is a constant, calculated to leading order (LO) in the large nucleon mass m_N expansion, where α is the fine-structure constant [4]. Because of the divergent contribution, this constant has been removed by renormalization of the Fermi coupling constant G_V and the axial coupling constant g_A [1,2]. This result has been confirmed by Shann [5] for the calculation of the $O(\alpha/\pi)$ RC to the correlation coefficients of the neutron beta decay with a polarized neutron and unpolarized electron and proton (see also [6–8]). However, as has been shown in [9], the contributions of the inner $O(\alpha^2/\pi^2)$ RC to the neutron radiative beta decay should have a nontrivial dependence on the electron and photon energies, even these RC are calculated to LO in the large nucleon mass m_N expansion.

Recently [10], we have calculated the $O(\alpha E_e/m_N)$ RC as next-to-leading order (NLO) corrections in the large nucleon mass m_N expansion to Sirlin's $O(\alpha/\pi)$ RC [1] (or to the *outer* model-independent RC [3]), where E_e is an electron energy, We have carried out the calculation within the effective quantum field theory of strong and electroweak low-energy interactions LσM & SET. In this theory, strong low-energy pion-nucleon interactions are described by the linear σ model (LσM) with chiral $SU(2)\times SU(2)$ symmetry [11–13]. For the description of electroweak hadron-hadron, hadron-lepton, and lepton-lepton interactions for the electron-lepton family, we have used the standard electroweak theory (SET) with $SU(2)_L\times U(1)_Y$ symmetry [14]. This effective quantum field theory is some kind of a hadronized version of the Standard Model (SM) [4,15]. From a gauge invariant set of the Feynman diagrams with a one-photon exchange, where the contribution of strong low-energy interactions is presented by the axial coupling constant g_A only (see Fig. 7 in Ref. [10]), we have reproduced outer $O(\alpha/\pi)$ RC [1] and calculated NLO $O(\alpha E_e/m_N)$ terms. This confirms Sirlin's confidence level for this kind of $O(\alpha E_e/m_N)$ RC.

We have calculated the contributions of strong low-energy interactions to $O(\alpha E_e/m_N)$ RC within the LσM in the limit $m_\sigma\to\infty$ of the σ-meson mass. In such a limit and

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in the tree-approximation, the LσM reproduces all results of the current algebra in the form of effective chiral Lagrangians of pion-nucleon interactions with nonlinear realization of chiral $SU(2)\times SU(2)$ symmetry and different parametrizations of the pion field [16–18].

For the exponential parametrization of the pion field, the Lagrangian $\mathcal{L}_{\mathrm{L}\sigma\mathrm{M}}|_{m_\sigma\to\infty}$ of the LσM, taken at $m_\sigma\to\infty$, reduces to the Lagrangian of the chiral quantum field theory with the structure of low-energy interactions agreeing well with Gasser-Leutwyler's chiral perturbation theory (ChPT) or the heavy baryon chiral perturbation theory (HBχPT) [19–36] with chiral $SU(2)\times SU(2)$ symmetry (see, for example, Ecker [24]). We denote the Lagrangian of the HBχPT as $\mathcal{L}_{\mathrm{HB}\chi\mathrm{PT}}$. At the tree level, the Lagrangians $\mathcal{L}_{\mathrm{L}\sigma\mathrm{M}}|_{m_\sigma\to\infty}$ and $\mathcal{L}_{\mathrm{HB}\chi\mathrm{PT}}$ differ only by the value of the *bare* axial coupling constant $g_A^{(0)}$. Indeed, it is $g_A^{(0)}=1$ in $\mathcal{L}_{\mathrm{L}\sigma\mathrm{M}}|_{m_\sigma\to\infty}$ and $g_A^{(0)}\neq 1$ in $\mathcal{L}_{\mathrm{HB}\chi\mathrm{PT}}$ (see also [16]). However, as has been shown in [10], a deviation of the axial coupling constant from unity $g_A>1$ can be obtained in the LσM in the one-hadron-loop approximation. We get $g_A>1$, taking the limit $m_\sigma\to\infty$ and renormalizing the contribution of the hadronic axial-vector current. In turn, hadron-loop corrections (or chiral-hadron-loop corrections), calculated in the HBχPT, lead to appearance effective low-energy interactions proportional to low-energy constants (LECs) [19–36]. These LECs play an important role for the correct description of the dynamics of low-energy processes within the HBχPT [19–36]. Unfortunately, LECs do not appear in the observables of low-energy processes described by the LσM. Nevertheless, fortunately, it turns out that the LECs of the HBχPT do not contribute to the inner $O(\alpha E_e/m_N)$ RC. This should in principle allow us to apply the LσM for the description of strong low-energy interactions in the inner $O(\alpha E_e/m_N)$ RC. To confirm this, we propose to discuss the studies carried out by Alvarez *et al.* [32] and Ando *et al.* [6]. We will focus our attention on using them to analyze the applicability of the LσM to the computation of the inner $O(\alpha E_e/m_N)$ RC.

In our study of the inner $O(\alpha/\pi)$ and $O(\alpha E_e/m_N)$ RC to the neutron beta decay the contributions of strong low-energy interactions, calculated in the LσM, are proportional to $g_{\pi N}^2$ and $g_{\pi N}^2/m_N$, where $g_{\pi N}$ is the pion-nucleon coupling constant. Alvarez *et al.* [32] have analyzed the amplitude of the low-energy πN scattering. They have compared the contributions of the LσM with chiral $SU(2)\times SU(2)$ symmetry, taken in its extended version—the extended linear σ model (ELσM) and the HBχPT. Recall that the ELσM differs from the LσM by the phenomenological local current-current interaction [12] and the interaction proportional to the phenomenological parameter ε_3 [37]. The local current-current phenomenological interaction is intended to introduce $g_A>1$ into the LσM at the tree level. The parameter ε_3 defines a correction to the Goldberger-Treiman relation [38] and leads to a deviation of the divergence of the hadronic axial-vector current from its canonical form [11].

As has been shown by Alvarez *et al.* [32], the contributions of the ELσM and HBχPT coincide fully at the tree approximation (see [13,28]). Then, taking results obtained by Alvarez *et al.* [32] in the limit $m_\sigma\to\infty$ and setting $\varepsilon_3=0$, one may show that the difference between the contributions of these two theories appears only in the terms dependent on LECs. These terms are proportional to $g_{\pi N}^2/m_N^2$ and $g_{\pi N}^2/m_N^3$, respectively. Since in the neutron beta decay we compute the contributions of strong low-energy interactions proportional to $g_{\pi N}^2$ and $g_{\pi N}^2/m_N$ only, the problematic terms $g_{\pi N}^2/m_N^2$ and $g_{\pi N}^2/m_N^3$, which can depend on LECs, do not appear at all.

A correctness of the application of the LσM without LECs to the computation of the inner $O(\alpha E_e/\pi)$ RC can be also confirmed by the results obtained by Ando *et al.* [6]. They have studied the $O(\alpha/\pi)$ RC and the $O(E_e/m_N)$ corrections, caused by weak magnetism and proton recoil, to the neutron beta decay within the HBχPT. As has been shown by Ando *et al.* [6], only two LECs, namely $(\alpha/2\pi)e_V^R$ and $(\alpha/2\pi)e_A^R$, are needed for the consistent analyzes of these corrections. The value of $(\alpha/2\pi)e_V^R$ has been fixed in terms of the inner $O(\alpha/\pi)$ RC, induced by the γW^- box and calculated by Marciano and Sirlin [39]. In turn, the difference of LECs $(\alpha/2\pi)(e_A^R-e_V^R)$ has been removed by renormalization of the axial coupling constant g_A. Then, no LECs proportional to $1/m_N$ have been found in [6] for the calculation of RC to the neutron lifetime and correlation coefficients of the neutron beta decay. This should indicate that the inner $O(\alpha E_e/m_N)$ RC, which we calculate in this paper within the LσM & SET, should not contradict the results, that can be, in principle, obtained describing strong low-energy interactions within the HBχPT.

This paper is addressed to the calculation of the contribution of the hadronic structure of the neutron to the inner $O(\alpha/\pi)$ and $O(\alpha E_e/m_N)$ RC to the neutron beta decay. We calculate them in the two-loop approximation within the effective quantum field theory LσM & SET [10]. The complete set of the two-loop Feynman diagrams is shown in Figs. 1–4. They are conditioned by one-photon exchange and the contributions of strong low-energy interactions, which do not reduce to the axial coupling constant g_A after renormalization [10]. We treat the inner $O(\alpha E_e/m_N)$ RC as next-to-leading order (NLO) corrections in the large nucleon mass expansion to the inner $O(\alpha/\pi)$ RC [1]. They can be observable after the removal of the inner $O(\alpha/\pi)$ RC by renormalization of the Fermi weak coupling constant and the axial coupling constant (see [1]).

The paper is organized as follows. In Sec. II, we describe in outline the approach to the analytical calculation of the

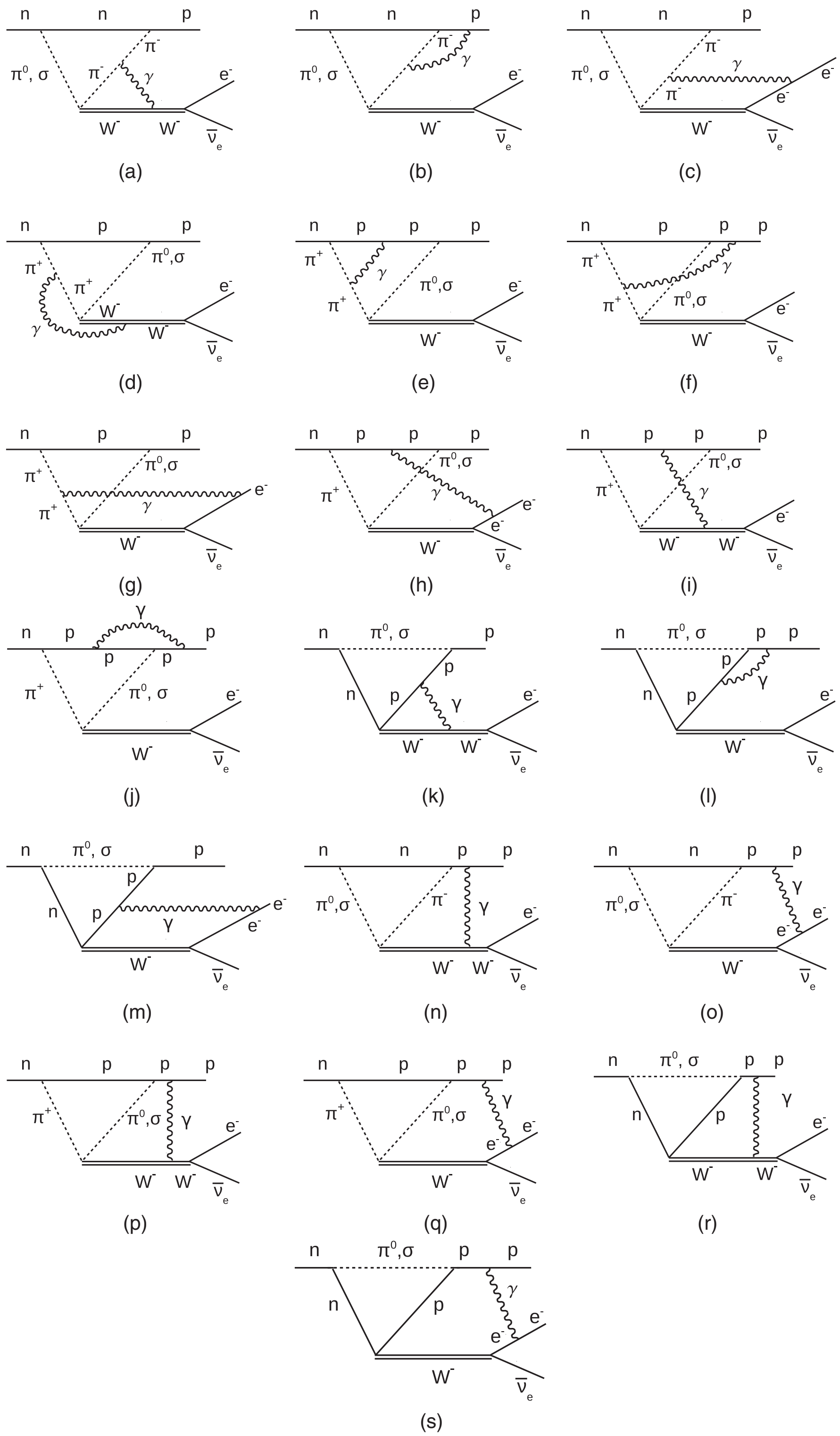

FIG. 1. The two-loop Feynman diagrams of the inner RC with a virtual photon coupled to the hadronic structure of the neutron, charged decay particles, and the electroweak W^- boson.

two-loop Feynman diagrams in Figs. 1–4 in agreement with the applicability of the LσM for the calculation of strong low-energy interactions. In Sec. III, we define the general expression for the contribution of the one-virtual photon exchanges to the amplitude of the neutron beta decay within the LσM & SET. In Sec. IV, we present the contributions of the inner $O(\alpha E_e/m_N)$ RC (i) to the amplitude of the neutron beta decay, (ii) to the electron-energy and

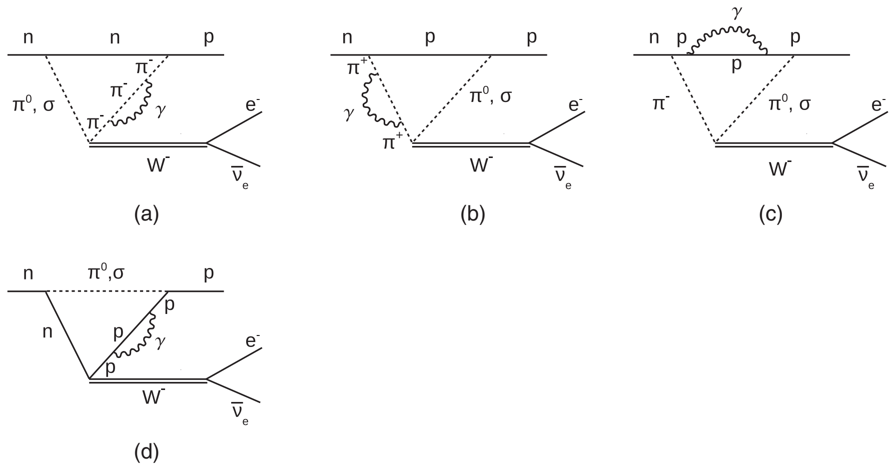

FIG. 2. The two-loop Feynman diagrams of the inner RC with self-energy corrections to the virtual charged hadrons.

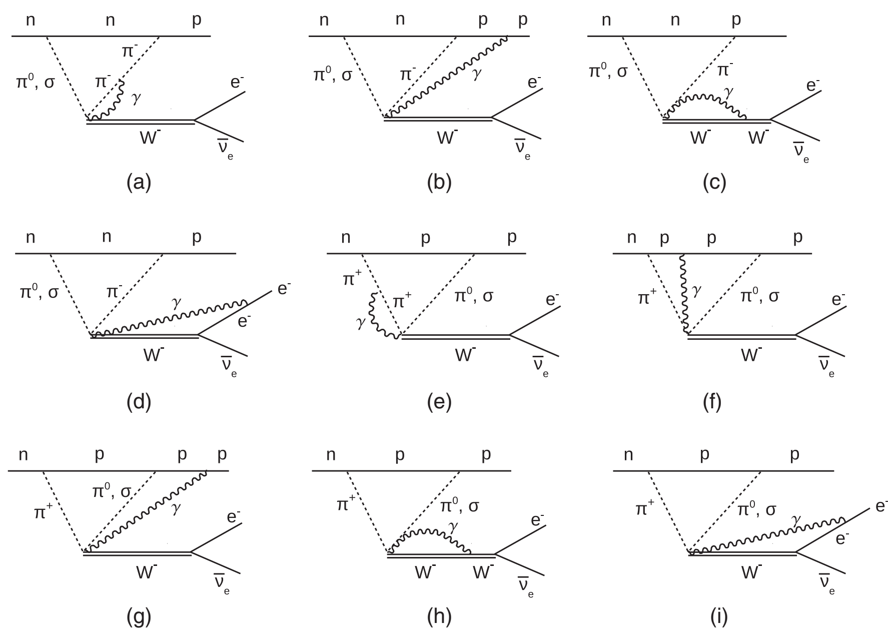

FIG. 3. The two-loop Feynman diagrams of the inner RC, induced by the interactions of the γW^{-}-pair with the $\pi\pi$ and $\pi\sigma$ pairs.

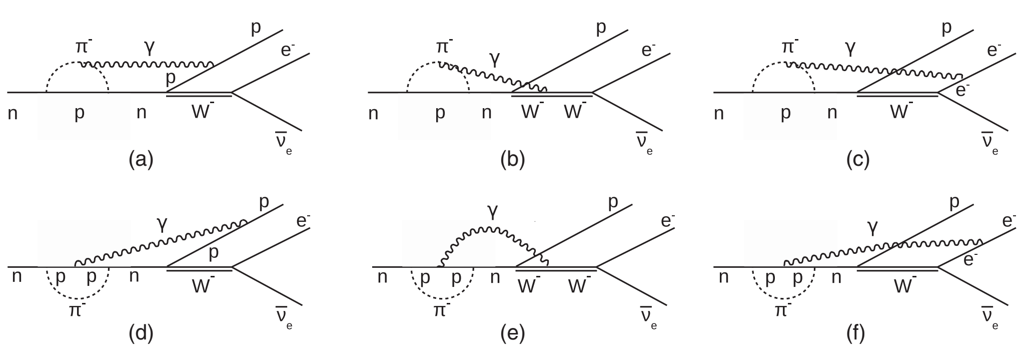

FIG. 4. The two-loop Feynman diagrams of the inner RC, induced by a virtual photon emitted by virtual hadrons from the self-energy hadronic corrections to the neutron.

angular distribution of the neutron beta decay with unpolarized massive fermions, and (iii) to the rate of the neutron beta decay. We show that in the total electron-energy region $m_e \le E_e \le E_0$ the inner $O(\alpha E_e/m_N)$ RC are of the order of a few parts of 10^{-5}–10^{-4}. In Sec. V, we discuss the obtained results and perspectives of further development of the effective quantum field theory of strong and electroweak low-energy interactions LσM & SET, where strong low-energy interactions are described by the HBχPT.

In the Supplemental Material [40] in Appendixes A, B, C, D, E, and F, we give (i) the analytical expressions for the Feynman diagrams in Figs. 1–4, obtained by using the Lagrangian Eq. (44) in Ref. [10], (ii) the analysis of gauge invariance of the Feynman diagrams in Figs. 1–4, and (iii) the analytical calculation of the Feynman diagrams using the standard procedure [41–53]. The numerical values of the structure constants of the inner $O(\alpha E_e/m_N)$ RC to the amplitude of the neutron beta decay are evaluated by using Wolfram *Mathematica* 12.0.

II. THE APPROACH TO ANALYTICAL CALCULATIONS OF THE FEYNMAN DIAGRAMS IN FIGS. 1–4

An important role of strong low-energy interactions in decay processes has been pointed by Weinberg [54]. In this connection, according to Sirlin [2], the current algebra is a nice tool for the analysis of contributions of strong low-energy interactions in the $O(\alpha/\pi)$ RC to semileptonic and leptonic decays of hadrons. The method of the current algebra is model independent. It is based on the use of equal-time commutators of the hadronic currents and their divergences imposed by the $SU(2)\times SU(2)$ or $SU(3)\times SU(3)$ symmetries of strong low-energy interactions [55,56] (see also [15]). Indeed, as has been pointed out by Sirlin [2]: "In fact, a current algebra formulation is probably our only hope of controlling the effects of the strong interactions in a clear and logical manner." He has shown [2,57] that the contributions of strong low-energy interactions to the inner $O(\alpha/\pi)$ RC have the standard $V-A$ structure [58–60] in the amplitude of the neutron beta decay.

In our analysis, we treat the inner $O(\alpha E_e/m_N)$ RC as NLO corrections in the large nucleon mass m_N expansion to the inner $O(\alpha/\pi)$ RC. This causes the inner $O(\alpha E_e/m_N)$ RC to have the $V-A$ structure as well. To reproduce the inner $O(\alpha/\pi)$ RC with the $V-A$ structure and to calculate NLO corrections $O(\alpha E_e/m_N)$, we use the *leading logarithmic approximation* (LLA) (see, for example, [61,62]) for the analytical calculation of the Feynman diagrams in Figs. 1–4. As has been pointed out by Bissegger and Fuhrer [62], the linear σ model without a nucleon is equivalent to the ChPT by Gasser and Leutwyler [19] in the LLA. The application of the LLA to the calculation of the inner $O(\alpha E_e/m_N)$ RC within the effective field theory LσM & SET can also be justified as follows. It is well-known by example of the bosonization of the extended Nambu-Jona-Lasinio (ENJL) model [26] that the divergent parts of the Feynman diagrams or the counterterms preserve fully the symmetry of the dynamical quark system. Indeed, the effective local Lagrangian of the bound quark-antiquark pairs, induced by the divergent parts of one-quark-loop diagrams, preserves the symmetry of the ENJL quark model [26] with a local four-quark interaction, invariant under chiral $SU(2)\times SU(2)$ or $SU(3)\times SU(3)$ symmetries.

Thus, keeping only the divergent contributions of the Feynman diagrams in Figs. 1–4, we preserve the chiral $SU(2)\times SU(2)$ symmetry of the LσM. The divergent contributions of the Feynman diagrams in Figs. 1–4, calculated by following the standard procedure [41–53] with the n-dimensional regularization [43–53], are proportional to $\Gamma(2-n/2)(Q/m_N^2)^{-4+n}/U^{n/2}$. In this product, Q is a function of the Feynman parameters, momenta, and squares of the masses of interacting particles in the dependence of the structure of the Feynman diagram, and U is the determinant of the Feynman diagram depending on the Feynman parameters and the structure of the Feynman diagram [43,44,51–53]. Then, taking the limit $n\to 4$ and keeping the divergent contributions proportional to $\Gamma(2-n/2)$, we get $(\Gamma(2-n/2) - 2\ell n Q/m_N^2)/U^2$. Expanding $(\Gamma(2-n/2) - 2\ell n Q/m_N^2)/U^2$ in powers of $k_n\cdot q/m_N^2$, $k_n\cdot k_e/m_N^2$, and m_N^2/M_W^2 and integrating over the Feynman parameters, we obtain the leading order contributions, determined by $(\Gamma(2-n/2) - 2\ell n Q/m_N^2)/U^2|_{q=k_e=0}$, and the NLO contributions proportional to $k_n\cdot q/m_N^2$, $k_n\cdot k_e/m_N^2$, and m_N^2/M_W^2. The sum of these contributions preserve the $V-A$ structure of the amplitude of the neutron beta decay (see Appendix F of [40]).

III. GENERATING FUNCTIONAL OF INNER RADIATIVE CORRECTIONS IN THE ONE-VIRTUAL PHOTON EXCHANGE APPROXIMATION

The general expression for contributions to the amplitude of the neutron beta decay, taken in the one-virtual photon exchange approximation with a photon coupled to the hadronic structure of the neutron, is defined by [10,63,64]

$$M(n\to pe^-\bar\nu_e)_{\rm st} = \Big\langle {\rm in}, \bar\nu_e\Big(\vec k_{\bar\nu}, +\frac{1}{2}\Big), e^-(\vec k_e,\sigma_e), \times p(\vec k_p,\sigma_p)|{\rm T}e^{i\int d^4x \mathcal{L}_{{\rm L}\sigma{\rm M\&SET}}(x)}|n(\vec k_n,\sigma_n),{\rm in}\Big\rangle_{\text{one-photon-approx}}, \quad (1)$$

where $\mathcal{L}_{\mathrm{L}\sigma\mathrm{M}\&\mathrm{SET}}$ is the Lagrangian of the effective quantum field theory LσM & SET defined by Eq. (44) in Ref. [10], and T is the time-ordering operator [65].

The wave functions of fermions in the initial and final states are determined in terms of operators of creation (annihilation) [65,66] (see also [10,63,64]),

$$|n(\vec{k}_n,\sigma_n),\mathrm{in}\rangle = a^{\dagger}_{n,\mathrm{in}}(\vec{k}_n,\sigma_n)|0\rangle,$$
$$\left\langle \mathrm{in},\bar{\nu}_e\left(\vec{k}_{\bar{\nu}},+\frac{1}{2}\right),e^-(\vec{k}_e,\sigma_e),p(\vec{k}_p,\sigma_p)\right| = \langle 0|b_{\bar{\nu}_e,\mathrm{in}}\left(\vec{k}_{\bar{\nu}},+\frac{1}{2}\right)a_{e,\mathrm{in}}(\vec{k}_e,\sigma_e)a_{p,\mathrm{in}}(\vec{k}_p,\sigma_p). \quad (2)$$

The operators of creation (annihilation) obey standard anticommutation relations [65,66]. The required contribution of the hadronic structure of the neutron to the inner $O(\alpha/\pi)$ RC appears in the two-loop approximation with one-virtual photon exchange. It is defined by the Feynman diagrams in Figs. 1–4. Since, as has been shown in [63,64], the RC to the one-pion-pole exchanges are of the order of 10^{-9} (see also [67]), we omit them from consideration. The analytical expressions for the Feynman diagrams in Figs. 1–4 and their properties under gauge transformation of the virtual photon propagator are given and investigated in Appendixes A, B, C, D, E, and F [40]. For the calculation of the Feynman diagrams in Figs. 1–4, we have used the standard technique [41–53]. We have carried out the numerical evaluation of the structure constants of the analytical expressions for the Feynman diagrams in Figs. 1–4 with Wolfram *Mathematica* 12.0.

IV. THE INNER $O(\alpha/\pi)$ AND $O(\alpha E_e/m_N)$ RADIATIVE CORRECTIONS

The contributions of the inner $O(\alpha/\pi)$ and $O(\alpha E_e/m_N)$ RC are calculated to the amplitude of the neutron beta decay in Appendixes A, B, C, D, E, and F of [40]. These corrections are described by the complete set of the Feynman diagrams in Figs. 1–4. We have given the analytical expressions for the Feynman diagrams in Figs. 1–4 in Appendix A in [40]. As we have shown in Appendix B in [40], this set of Feynman diagrams is gauge invariant. In other words, it does not depend on longitudinal polarization states of a virtual photon. The inner $O(\alpha E_e/m_N)$ RC are obtained as NLO terms for the inner $O(\alpha/\pi)$ RC, calculated to LO in the large nucleon mass m_N expansion [1,2]. Following [1], we have absorbed the terms of order $O(\alpha/\pi)$ by renormalization of the Fermi weak couping constant G_V and the axial coupling constant g_A. As a result, the contribution of the Feynman diagrams in Figs. 1–4, calculated to NLO in the large nucleon mass m_N expansion and the electroweak W^--boson mass M_W expansion, is given by (see Appendix F in [40])

$$M(n\to pe^-\bar{\nu}_e)^{(\mathrm{NLO})}_{\mathrm{st}} = -\frac{\alpha}{2\pi}G_V\Big\{\Big(G^{(V)}_{\mathrm{st}}\frac{k_n\cdot q}{m_N^2} + H^{(V)}_{\mathrm{st}}\frac{k_n\cdot k_e}{m_N^2} + (G^{(W)}_{\mathrm{st}} + F^{(W)}_{\mathrm{st}})\frac{m_N^2}{M_W^2}\Big)[\bar{u}_e\gamma^{\mu}(1-\gamma^5)v_{\bar{\nu}}][\bar{u}_p\gamma_{\mu}u_n] + \Big(G^{(A)}_{\mathrm{st}}\frac{k_n\cdot q}{m_N^2} + H^{(A)}_{\mathrm{st}}\frac{k_n\cdot k_e}{m_N^2} + H^{(W)}_{\mathrm{st}}\frac{m_N^2}{M_W^2}\Big)[\bar{u}_e\gamma^{\mu}(1-\gamma^5)v_{\bar{\nu}}][\bar{u}_p\gamma_{\mu}\gamma^5u_n]\Big\}, \quad (3)$$

where $q = k_p - k_n = -k_e - k_{\bar{\nu}}$ is a four-momentum transfer and $E_e/m_N \sim 10^{-3}$ and $m_N^2/M_W^2 \sim 10^{-4}$, respectively.

The contribution of the inner $O(\alpha E_e/m_N)$ RC in Eq. (3) agrees well with the assertion that the inner $O(\alpha/\pi)$ RC do not depend on the electron energy [1,2]. The structure constants in Eq. (3) are equal to $G^{(V)}_{\mathrm{st}} = -70.71$, $H^{(V)}_{\mathrm{st}} = 67.75$, $G^{(W)}_{st} = 8.94$, $G^{(A)}_{\mathrm{st}} = 41.95$, $H^{(A)}_{\mathrm{st}} = -40.78$, $H^{(W)}_{\mathrm{st}} = 2.10$, and $F^{(W)}_{\mathrm{st}} = -1.64$ (see Appendix F in [40]). The Lorentz structure of Eq. (3) is obtained at the neglect of the contributions of order $O(m_em_N/M_W^2) \sim O(k_n\cdot q/M_W^2) \sim O(k_n\cdot k_e/M_W^2) \sim 10^{-7}$, and $O(E_0^2/m_N^2) \sim O(m_{\pi}^2/M_W^2) \sim 10^{-6}$, respectively (see [40]). We would like to emphasize that all structure constants in the matrix element Eq. (3) are induced by the contributions of the first class currents [68], which are G even [69] (see also [66]).

In the rest frame of the neutron and in the nonrelativistic approximation for the proton, the amplitude of the neutron beta decay with the contribution of the inner $O(\alpha E_e/m_N)$ RC is given by

$$M(n\to pe^-\bar{\nu}_e) = -2m_NG_V\Big\{\Big(1+\frac{\alpha}{2\pi}\bar{g}_{\mathrm{st}}(E_e)\Big)[\varphi_p^{\dagger}\varphi_n][\bar{u}_e\gamma^0(1-\gamma^5)v_{\bar{\nu}}] + \Big(g_A + \frac{\alpha}{2\pi}\bar{f}_{\mathrm{st}}(E_e)\Big)[\varphi_p^{\dagger}\vec{\sigma}\varphi_n]\cdot[\bar{u}_e\vec{\gamma}(1-\gamma^5)v_{\bar{\nu}}] + \ldots\Big\}, \quad (4)$$

where the ellipsis implies the contributions of other terms (see, for example, [8,10]), which we do not take into account here. The functions $\bar{g}_{\mathrm{st}}(E_e)$ and $\bar{f}_{\mathrm{st}}(E_e)$ are defined in terms of the structure constants as follows:

$$\bar{g}_{\rm st}(E_e) = -G^{(V)}_{\rm st}\,\frac{E_0}{m_N} + \Big(G^{(W)}_{\rm st} + F^{(W)}_{\rm st}\Big)\,\frac{m^2_N}{M^2_W} + H^{(V)}_{\rm st}\,\frac{E_e}{m_N} = 0.098\Big(1 + 0.95\,\frac{E_e}{E_0}\Big),$$
$$\bar{f}_{\rm st}(E_e) = +G^{(A)}_{\rm st}\,\frac{E_0}{m_N} - H^{(W)}_{\rm st}\,\frac{m^2_N}{M^2_W} - H^{(A)}_{\rm st}\,\frac{E_e}{m_N} = 0.057\Big(1 + \frac{E_e}{E_0}\Big), \qquad (5)$$

where $E_0 = (m^2_n - m^2_p + m^2_e)/2m_n = 1.2926$ MeV is the end point energy of the electron-energy spectrum [70,71]. The electron-energy and angular distribution of the neutron beta decay for unpolarized massive fermions, taking into account the RC Eq. (4), is given by

$$\frac{d^5\lambda_n(E_e,\vec{k}_e,\vec{k}_{\bar{\nu}})}{dE_e d\Omega_e d\Omega_{\bar{\nu}}} = (1+3g^2_A)\,\frac{|G_V|^2}{16\pi^5}\Big\{1 + \frac{\alpha}{\pi}\,(g_{\rm st}(E_e) + 3f_{\rm st}(E_e)) + \Big[a_0 + \frac{\alpha}{\pi}\,(g_{\rm st}(E_e) - f_{\rm st}(E_e))\Big]\,\frac{\vec{k}_e\cdot\vec{k}_{\bar{\nu}}}{E_e E_{\bar{\nu}}} + \dots\Big\}$$
$$\times\sqrt{E^2_e - m^2_e}\,E_e F(E_e, Z=1), \qquad (6)$$

where $d\Omega_e$ and $d\Omega_{\bar{\nu}}$ are infinitesimal solid angles in the directions of the electron and antineutrino three momenta, $a_0 = (1-g^2_A)/(1+3g^2_A)$ [70,71], and $F(E_e, Z=1)$ is the well-known relativistic Fermi function, describing electron-proton Coulomb final-state interaction [72,73]. The ellipsis denote the contributions of other terms (see, for example, [8]). The functions $g_{st}(E_e)$ and $f_{st}(E_e)$ are related to the functions $\bar{g}_{\rm st}(E_e)$ and $\bar{f}_{\rm st}(E_e)$ as follows:

$$g_{st}(E_e) = \frac{1}{1+3g^2_A}\,\bar{g}_{\rm st}(E_e) = \frac{0.098}{1+3g^2_A}\Big(1 + 0.95\,\frac{E_e}{E_0}\Big) = 0.017\Big(1 + 0.95\,\frac{E_e}{E_0}\Big),$$
$$f_{st}(E_e) = \frac{g_A}{1+3g^2_A}\,\bar{f}_{\rm st}(E_e) = \frac{0.057g_A}{1+3g^2_A}\Big(1 + \frac{E_e}{E_0}\Big) = 0.012\Big(1 + \frac{E_e}{E_0}\Big). \qquad (7)$$

These corrections depend strongly on the axial coupling constant g_A. The numerical values are evaluated for $g_A = 1.2764$ [74] (see also [75]). The rate of the neutron beta decay is defined by the integral,

$$\lambda_n = (1+3g^2_A)\,\frac{|G_V|^2}{\pi^3}\int^{E_0}_{m_e}\Big(1 + \frac{\alpha}{\pi}\,h_{\rm st}(E_e) + \dots\Big)\sqrt{E^2_e - m^2_e}\,E_e F(E_e, Z=1)dE_e, \qquad (8)$$

where the function $h_{\rm st}(E_e)$ is equal to

$$h_{\rm st}(E_e) = g_{\rm st}(E_e) + 3f_{\rm st}(E_e) = 0.053\Big(1 + \frac{E_e}{E_0}\Big). \qquad (9)$$

The functions $(\alpha/\pi)g_{\rm st}(E_e)$, $(\alpha/\pi)f_{\rm st}(E_e)$, and $(\alpha/\pi)h_{\rm st}(E_e)$ are calculated at the neglect of the terms of the order 10^{-6}. In Fig. 5, we plot these functions for $g_A = 1.2764$. The values of the functions $(\alpha/\pi)g_{\rm st}(E_e)$ are $(\alpha/\pi)f_{\rm st}(E_e)$ are of the same order and of the order of a few parts of 10^{-5}. An increase of the values of the function $(\alpha/\pi)h_{\rm st}(E_e)$ to a few parts of 10^{-4} is caused by the hadronic axial-vector current. Its contribution to the neutron lifetime is enhanced by a factor of 3 with respect to the contribution of the hadronic vector current.

In order to calculate correctly the relative contribution of the inner $O(\alpha E_e/m_N)$ RC, described by the function $(\alpha/\pi)h_{\rm st}(E_e)$, to the neutron lifetime we make a replacement $1 + \dots \to \zeta(E_e)$ in Eq. (8). We take the correlation function $\zeta(E_e)$ in the form, calculated in [8]. It contains a complete set of outer $O(\alpha/\pi)$ RC [1] and the $O(E_e/m_N)$ corrections, caused by weak magnetism and proton recoil. In addition, the correlation function $\zeta(E_e)$ includes the contributions of the inner $O(\alpha/\pi)$ RC defined by Δ^V_R and Δ^A_R. They are induced by the Feynman γW^--box diagrams and calculated to LO in the large nucleon mass m_N expansion in [39,76–81]. Having integrated over the electron energy, we get

$$\lambda_n = (1+3g^2_A)\,\frac{|G_V|^2}{\pi^3}\,(6,136\times 10^{-2} + 1.18\times 10^{-5})$$
$$\propto 1 + 1.92\times 10^{-4}, \qquad (10)$$

where the term 1.18×10^{-5} is the contribution of the function $(\alpha/\pi)h_{\rm st}(E_e)$ defining the inner $O(\alpha E_e/m_N)$ RC in the neutron lifetime. In turn, the relative contribution of these RC is of the order 10^{-4}.

V. DISCUSSION

We have calculated the inner $O(\alpha E_e/m_N)$ RC, induced by the hadronic structure of the neutron (i) to the amplitude of the neutron beta decay, (ii) to the electron-energy and angular distribution of the neutron beta decay for

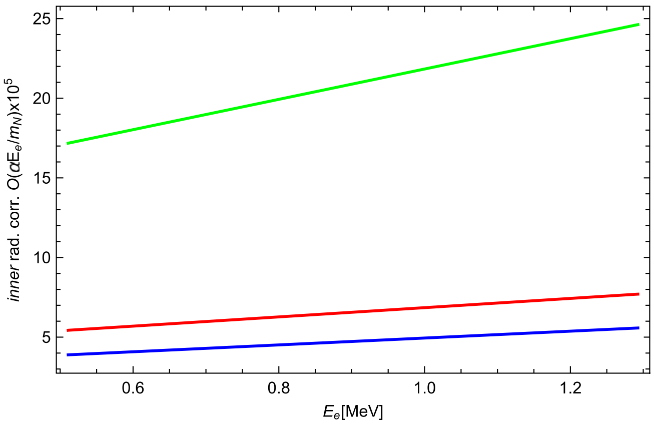

FIG. 5. The RC $(\alpha/\pi)g_{\rm st}(E_e)$ (red), $(\alpha/\pi)f_{\rm st}(E_e)$ (blue), and $(\alpha/\pi)h_{\rm st}(E_e)$ (green) caused by the hadronic structure of the neutron, to the electron-energy and angular distribution of the neutron beta decay, and to the neutron lifetime in the electron energy region $m_e \le E_e < E_0$. These are next-to-leading order corrections in the large nucleon mass m_N expansion to Sirlin's inner RC, which have been absorbed by renormalization of the Fermi weak coupling constant G_V and the axial coupling constant g_A [1].

unpolarized massive fermions, and (iii) to the neutron lifetime. We treat them as NLO terms in the large nucleon mass m_N expansion to Sirlin's inner $O(\alpha/\pi)$ RC [1,2], calculated to LO in the large nucleon mass m_N expansion.

For the calculation of the inner $O(\alpha/\pi)$ and $O(\alpha E_e/m_N)$ RC, we have used the effective quantum field theory LσM & SET of strong and electroweak low-energy interactions, proposed in [10]. In such an effective quantum field theory, strong low-energy interactions are described by the linear σ model (LσM) with chiral $SU(2)\times SU(2)$ symmetry [11]. In turn, electroweak interactions are described by the standard electroweak theory (SET) with $SU(2)_L \times U_Y$ symmetry [14,15] (see also [4]). The hadronic and leptonic sectors are represented by the nucleon coupled to pions and the scalar isoscalar σ-meson and the electron-lepton family, respectively. The application of this effective quantum field theory to the calculation of the inner $O(\alpha E_e/m_N)$ RC is well motivated and justified by the results, obtained in [10].

The contributions of the inner $O(\alpha/\pi)$ and $O(\alpha E_e/m_N)$ RC are described in the LσM & SET by the two-loop Feynman diagrams. They are shown in Figs. 1–4. We have calculated these Feynman diagrams in the LLA [61,62]. This approximation is justified as follows: (i) the linear σ model without the nucleon is equivalent to the ChPT by Gasser and Leutwyler [19] in the LLA (see [62]), (ii) the leading divergent and finite logarithms preserve the chiral $SU(2)\times SU(2)$ symmetry of strong low-energy interactions, described by the LσM (see also [26] for bosonization of the ENJL quark model), and (iii) the contributions of the inner $O(\alpha/\pi)$ and $O(\alpha E_e/m_N)$ RC to the amplitude of the neutron beta decay have the standard $V-A$ structure in agreement with [1,2]. Such a $V-A$ structure of the inner $O(\alpha/\pi)$ RC has been pointed out by Sirlin [1,2] within the current algebra approach [55,56]. The latter has allowed to remove the inner $O(\alpha/\pi)$ RC by renormalization of the Fermi weak couping constant G_V and the axial coupling constant g_A [1,2].

After renormalization, we have got a set of the inner $O(\alpha E_e/m_N)$ RC of the order of a few parts of $10^{-5}-10^{-4}$. We would like to emphasize that these corrections are calculated in agreement with the constraints on the applicability of the LσM for the description of strong low-energy interactions [32,62]. It agrees also with analysis of the RC in the neutron beta decay, performed in [6] within the HBχPT.

The inner $O(\alpha E_e/m_N)$ RC are represented by two functions $(\alpha/\pi)g_{\rm st}(E_e)$ and $(\alpha/\pi)f_{\rm st}(E_e)$ in the electron-energy and angular distribution of the neutron beta decay. In turn, the contribution of the inner $O(\alpha E_e/m_N)$ to the neutron lifetime is determined by the function $(\alpha/\pi)h_{\rm st}(E_e)$. It is a linear superposition of the functions $g_{\rm st}(E_e)$ and $f_{\rm st}(E_e)$, i.e., $h_{\rm st}(E_e) = g_{\rm st}(E_e) + 3f_{\rm st}(E_e) = 0.053(1+E_e/E_0)$. We have plotted the functions $(\alpha/\pi)g_{\rm st}(E_e)$, $(\alpha/\pi)f_{\rm st}(E_e)$, and $(\alpha/\pi)h_{\rm st}(E_e)$ in Fig. 5. In the electron-energy region $m_e \le E_e \le E_0$, the numerical values of the functions $(\alpha/\pi)g_{\rm st}(E_e)$ and $(\alpha/\pi)f_{\rm st}(E_e)$ are of the order of a few parts of 10^{-5}. Nevertheless, the function $(\alpha/\pi)h_{\rm st}(E_e)$ is of the order of 10^{-4} and varies over the region $1.72\times 10^{-4} \le (\alpha/\pi)h_{\rm st}(E_e) \le 2.46\times 10^{-4}$. The contribution of the function $(\alpha/\pi)h_{\rm st}(E_e)$, integrated over the phase volume of the neutron beta decay, is of the order of 10^{-5} to the rate of the neutron beta decay. In turn, its relative contribution is of the order of 10^{-4}.

As has been shown in [82], the inner $O(\alpha E_e/m_N)$ RC Eq. (7) provide the SM theoretical description of the neutron beta decay at the level of 10^{-5}–10^{-4} together with (i) the $O(\alpha E_e/m_N)$ RC, calculated in [10], (ii) the $O(E_e^2/m_N^2) \sim 10^{-5}$ corrections, caused by weak magnetism and proton recoil [83], and (iii) Wilkinson's corrections [73] (see also [8,84,85]). The theoretical accuracy of these corrections is of a few parts of 10^{-6} [82]. Such a SM theoretical background of the neutron beta decay should be very important for experimental searches of interactions beyond the SM [86–88] with experimental uncertainties of a few part of 10^{-5} and even better.

Of course, a very challenging extension of our approach to the calculation of the inner $O(\alpha E_e/m_N)$ RC is the use of the effective quantum field theory of strong and electroweak low-energy interactions HBχPT & SET. In this effective theory, the hadronic part is described by the HBχPT, which is accepted as an effective low-energy dynamics of QCD (see, for example, Ecker [24,25]). In such an effective theory, we could do calculations beyond the LLA and take into account a variety of Lorentz structures that differ from the standard $V-A$ structure. We would like to emphasize that the problem of the reformulation of the effective quantum field theory of strong and electroweak low-energy interactions LσM & SET, where the LσM is replaced by the HBχPT, is not straightforward. We are planning to devote to the analysis and solution of this problem our subsequent researches.

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