Quasi-two-body $B_{(s)} \to V\pi\pi$ decays with resonance $f_0(980)$ in the PQCD approach

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Motivated by the measurements of the branching fractions of the quasi-two-body decays $B^0 \to K^{*0}(f_0(980) \to)\pi^+\pi^-$ and $B^0 \to \rho^0(f_0(980) \to)\pi^+\pi^-$, we study the charmless $B \to V(f_0(980) \to)\pi^+\pi^-$ decays in the perturbative QCD approach. Supposing that $f_0(980)$ is a two-quark state and mixture of $n\bar{n} = (u\bar{u} + d\bar{d})/\sqrt{2}$ and $s\bar{s}$ with the mixing angle θ, we calculate the branching fractions of these decays with the new introduced s-wave $\pi^+\pi^-$-pair wave function. When the mixing angle θ lies in the range $[135°, 155°]$, the calculated branching fractions of the $B^0 \to K^{*0} f_0(980) \to K^{*0}\pi^+\pi^-$ and $B^0 \to \rho^0 f_0(980) \to \rho^0\pi^+\pi^-$ decays are in agreement with the experimental data. The branching fractions of other decays could be measured in the current LHCb and Belle II experiments. Considering the isospin symmetry, we also estimate the branching fractions of the quasi-two-body decays $B \to V f_0(980) \to V\pi^0\pi^0$, which are half of the corresponding decays $B \to V f_0(980) \to V\pi^+\pi^-$. Moreover, the direct CP asymmetries of these decays are also calculated, and some of them can be tested in the current experiments.

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I. INTRODUCTION

In recent years, scholars have paid more attention to the nonleptonic three-body decays of B mesons on both the experimental and theoretical sides, as these decays can be used to test the standard model, to extract the angles of the Cabibbo-Kobayashi-Maskawa (CKM) unitarity triangle, and to search for the sources of the CP violation. In the last century, a large number of three-body B decays have been measured by *BABAR* [1], Belle [2], CLEO [3], and LHCb [4–10]. Meanwhile, on the factorization hypothesis, a few of theoretical methods have been proposed to study these decays, such as approaches based on the symmetry principle [11], the QCD factorization approach (QCDF) [12–16], the perturbative QCD approach (PQCD) [17–22], and other theoretical methods [23].

Unlike the two-body decays where the kinematics is fixed, three-body decay amplitudes depend on two kinematic variables. For a decay $B(p_B) \to M_1(p_1)M_2(p_2)M_3(p_3)$, it is generally used to define the variables as two invariant masses of two pairs of final state particles, for instance, s_{12} and s_{13} with the definition $s_{ij} = 2(p_i \cdot p_j)/m_B^2$. All physical kinematics configurations could define a two-dimensional region in the $s_{12} - s_{13}$ plane, and the density plot of the differential decay rate $d\Gamma/ds_{12}ds_{13}$ in this region is called a Dalitz plot. Specially, when the final states are the light mesons such as the π and K mesons, the corresponding configuration reduces to a triangle region. In general, the Dalitz plot has three typical regions according to the characteristic kinematics. The central region so-called Mercedes Star configuration corresponds to the case where all the invariant masses are roughly the same and of order of m_B. In this region all three light mesons have a large energy in the B meson rest frame and fly apart at about 120° angles. The corners regions correspond to the cases where one final state is soft and the others fly back to back with a large amount of energy at about $m_B/2$. At the edges of the Dalitz plot one invariant mass is small and the other two are large, which implies that two particles move collinearly and the third bachelor particle recoils back. The interactions between two collinear mesons leads eventually to the resonances. Compared with two other regions, the physics picture at the edges of the Dalitz plot is very similar to a two-body decay by viewing the two-meson pair as a whole, and we thus call it quasi-two-body decay. In the past 20 years, the PQCD approach based on the k_T factorization has been used to study the B meson two-body decays

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successfully, therefore it can be generalized for studying the quasi-two-body decays.

In the past few years, a large number of charmless quasi-two-body $B/B_s \to V(f_0(980) \to)\pi^+\pi^-$ decays have been measured in experiments [24], and some branching fractions or upper limits of them are summarized as follows:

$$\begin{aligned} &B^0 \to K^*(892)^0 f_0(980),\\ &f_0(980) \to \pi^+\pi^- = (3.9^{+2.1}_{-1.8}) \times 10^{-6},\\ &B^0 \to \omega f_0(980),\\ &f_0(980) \to \pi^+\pi^- < 1.5 \times 10^{-6},\\ &B^0 \to \phi f_0(980),\\ &f_0(980) \to \pi^+\pi^- < 3.8 \times 10^{-7},\\ &B^0 \to \rho^0 f_0(980),\\ &f_0(980) \to \pi^+\pi^- = (7.8 \pm 2.5) \times 10^{-7},\\ &B^+ \to \rho^+ f_0(980),\\ &f_0(980) \to \pi^+\pi^- < 2.0 \times 10^{-6},\\ &B_s \to \phi f_0(980),\\ &f_0(980) \to \pi^+\pi^- = (1.12 \pm 0.21) \times 10^{-6}. \end{aligned} \tag{1}$$

Except for the decay $B_s \to \phi f_0$, $f_0 \to \pi^+\pi^-$, other decays have not been studied theoretically in the literature. Motivated by this, we shall study the above decays in the PQCD approach, so as to further check the reliability of PQCD in multibody decays and present more predictions. For the sake of convenience $f_0(980)$ is abbreviated to f_0 in the following context unless otherwise indicated.

II. FRAMEWORK

In the framework of PQCD, the decay amplitude $\mathcal{A}$ of $B \to V(f_0(980) \to)\pi^+\pi^-$ decay can be decomposed as the convolution

$$\begin{aligned} \mathcal{A} = {}& C(t) \otimes \mathcal{H}(x_i, b_i, t) \otimes \Phi_B(x_1, b_1) \otimes \Phi_V(x_2, b_2)\\ &\otimes \Phi_{\pi\pi}(x_3, b_3) \otimes e^{-S(t)}, \end{aligned} \tag{2}$$

where x_i are the momentum fractions of the light quarks and, b_i are the conjugate variables of the quarks' transverse momenta k_{iT}. Φ_B and Φ_V are the wave functions of the B mesons and vector mesons, while the $\Phi_{\pi\pi}$ is the S-wave $\pi\pi$-pair wave function. These wave functions are nonperturbative and universal. The exponential term is the so-called Sudakov form factor caused by the additional scale introduced by the intrinsic transverse momenta k_T, which suppresses the soft dynamics effectively [25,26]. $\mathcal{H}(x_i, b_i, t)$ is the hard kernel, which can be calculated perturbatively. The parameter t is the largest scale in the hard kernel, which ensures that the higher order corrections are as small as possible.

In PQCD, the most important inputs are the initial and final mesons' wave functions. For the B meson and the light vector mesons, their wave functions have been studied extensively and the inner parameters have been fixed by the well measured two-body B meson decays [27,28], so we will not discuss them in this work. For the $\pi\pi$ pair, its S-wave function can be written as [29–33]

$$\begin{aligned} \Phi_{\pi\pi} = \frac{1}{\sqrt{2N_c}} [&\not{P} \phi_S(z,\xi,\omega) + \omega \phi_S^s(z,\xi,\omega)\\ &+ \omega(\not{n}\not{v} - 1)\phi_S^t(z,\xi,\omega)], \end{aligned} \tag{3}$$

with z being the momentum fraction of the light quark in the $\pi\pi$ pair. The parameter ξ is the momentum fraction of one π meson in the $\pi\pi$ pair. The momentum of the $\pi\pi$-pair P satisfies the condition $P^2 = \omega^2$, ω being the invariant mass of the $\pi\pi$ pair. $n = (1, 0, \vec{0})$ and $v = (0, 1, \vec{0})$ are the lightlike vectors. For the explicit expressions of the light-core distribution amplitudes (LCDAs) $\phi_S^{(s,t)}$, we adopt the form [34,35]

$$\begin{aligned} \phi_S(z,\xi,\omega) &= \frac{F_S(\omega)}{\sqrt{6}} 9a_s z(1-z)(2z-1),\\ \phi_S^s(z,\xi,\omega) &= \frac{F_S(\omega)}{2\sqrt{6}},\\ \phi_S^t(z,\xi,\omega) &= \frac{F_S(\omega)}{2\sqrt{6}}(1-2z), \end{aligned} \tag{4}$$

with Gegenbauer moment $a_s = 0.3 \pm 0.2$ [32]. $F_S(\omega)$ is the timelike form factor. In particular, for a narrow intermediate resonance, the timelike form factor $F_S(\omega)$ can be well described by the relative Breit-Wigner line shape [36]. However, due to the remarkable interference between two decays $f_0 \to \pi\pi$ and $f_0 \to K\bar{K}$, the relative Breit-Wigner line shape cannot work well for the timelike form factor of f_0. In this case, the Flatté line shape is proposed to describe that of f_0 [36,37], which is given as

$$F_S(\omega) = \frac{m_{f_0}^2}{m_{f_0}^2 - \omega^2 - i m_{f_0}(g_{\pi\pi}\rho_{\pi\pi} + g_{KK}\rho_{KK}F_{KK}^2)}, \tag{5}$$

with

$$\rho_{\pi\pi} = \sqrt{1 - \frac{4m_\pi^2}{\omega^2}}, \qquad \rho_{KK} = \sqrt{1 - \frac{4m_K^2}{\omega^2}}, \tag{6}$$

where $g_{\pi\pi}$ and g_{KK} are the coupling constants corresponding to $f_0 \to \pi\pi$ and $f_0 \to K\bar{K}$ decays, respectively, whose values are taken as $g_{\pi\pi} = (0.165 \pm 0.018)$ GeV^2 and $g_{KK}/g_{\pi\pi} = 4.21 \pm 0.33$ [36]. In addition, the factor $F_{KK} = e^{-\alpha q^2}$ is introduced to suppress the f_0 width above the KK threshold. The parameter α is taken as

$2.0 \pm 1.0\ \mathrm{GeV}^{-2}$, which does not affect the predictions remarkably [38]. It is noted that this line shape has been also adopted extensively in analyzing data in the LHCb experiment [38].

Although the quark model has achieved great successes, the underlying structures of the scalar mesons are not well established so far. There are many scenarios for the classification of the scalar mesons. One scenario is the naïve two-quark model, and the light scalar mesons below or near 1 GeV are identified as the lowest lying states. Another consistent picture [39] provided by the data implies that light scalar mesons below or near 1 GeV can be described by the $q^2\bar{q}^2$, while scalars above 1 GeV will form a conventional $q\bar{q}$ nonet with some possible glue content [40–42]. This picture can be used to interpret the mass degeneracy of f_0 and $a_0(980)$, the reason why the widths of $\kappa(800)$ and $\sigma(600)$ is broader than those of $a_0(980)$ and f_0, and the large couplings of f_0 and $a_0(980)$ to $K\bar{K}$. However, in practice it is hard for us to make quantitative predictions on B decays based on the four-quark picture for light scalar mesons as it involves the unknown form factors and decay constants that are beyond the conventional quark model. Hence, we here only discuss the two-quark scenario for f_0. Moreover, some experimental evidences indicate the existence of the nonstrange and strange quark contents in f_0, we therefore regard it as a mixture of $s\bar{s}$ and $n\bar{n} = (u\bar{u} + d\bar{d})/\sqrt{2}$

$$|f_0\rangle = |n\bar{n}\rangle \sin\theta + |s\bar{s}\rangle \cos\theta, \tag{7}$$

where θ is the mixing angle. Recent studies [43–45] show that the mixing angle θ lies in the ranges of $25° < \theta < 40°$ and $140° < \theta < 165°$, and studies based on the B decays favor the later range.

With the initial and final wave functions, we can calculate the whole amplitude of each decay mode in PQCD approach. In the leading order, the diagrams contributing to the decay $B^+ \to \rho^+\pi^+\pi^-$ are shown in Fig. 1. The first two diagrams are the emission type diagrams with the first one emitting the $\pi\pi$ pair and the second one with the vector meson emitted. The last two are the annihilation type diagrams. Because the decay amplitudes are very similar to those presented in the Ref. [22], for the sake of simplicity, we shall not present them in this work. The other parameters used in the numerical calculations, such as the mass of the mesons, CKM matrix elements and the life times of B mesons, are taken from the Particle Data Group [24].

III. NUMERICAL RESULTS

As aforementioned, the mixing angle of f_0 have not yet been determined. At first, we set θ as a free parameter and plot the branching fractions of $B_{u,d,s} \to Vf_0 \to V\pi^+\pi^-$ decays dependent on it in Fig. 2, where the green bands

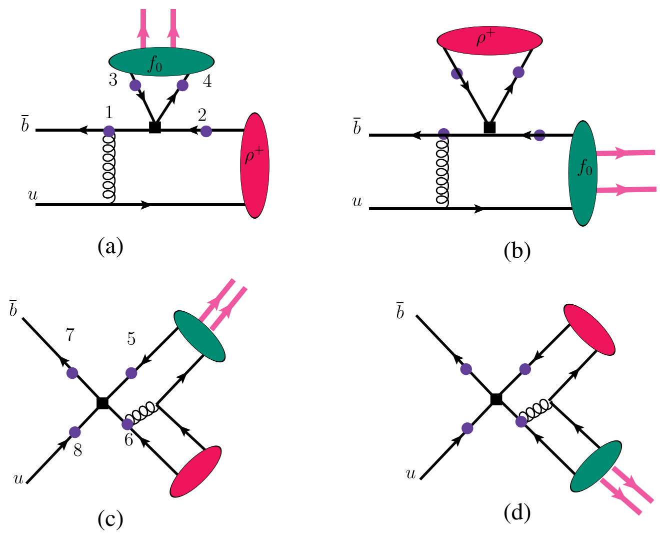

FIG. 1. Typical Feynman diagrams for the quasi-two-body decay $B^+ \to \rho^+\pi^+\pi^-$ in PQCD, where the black squares stand for the weak vertices, and large (purple) spots on the quark lines denote possible attachments of hard gluons. The green ellipse represent $\pi\pi$ pair and the red one is the light bachelor ρ^+ meson.

are the allowed regions in the experiments. Combining the experimental data of $B^0 \to \rho^0(f_0 \to)\pi^+\pi^-$ and $B^0 \to K^*(892)^0(f_0 \to)\pi^+\pi^-$ decays, we get the range of the mixing angle $135° \leq \theta \leq 155°$, which is consistent with the results obtained from $\phi \to f_0\gamma$ and $f_0 \to \gamma\gamma$. In our previous work [46], the decays $B \to K^*_{0,2}(1430)f_0/\sigma$ have been investigated, and we obtained $\theta \approx 145°$ after comparing with experimental result [47]. If the mixing angle $\theta = 145°$ is adopted, the branching ratios of the $B^0 \to \rho^0(f_0 \to)\pi^+\pi^-$ and $B^0 \to K^*(892)^0(f_0 \to)\pi^+\pi^-$ decays are given as

$$\mathcal{B}(B^0 \to \rho^0(770)(f_0 \to)\pi^+\pi^-) = 8.25 \times 10^{-7}, \tag{8}$$

$$\mathcal{B}(B^0 \to K^*(892)^0(f_0 \to)\pi^+\pi^-) = 2.45 \times 10^{-6}, \tag{9}$$

which well match the experimental measurements:

$$\mathcal{B}(B^0 \to \rho^0(770)(f_0 \to)\pi^+\pi^-) = (7.8 \pm 2.5) \times 10^{-7}, \tag{10}$$

$$\mathcal{B}(B^0 \to K^*(892)^0(f_0 \to)\pi^+\pi^-) = (2.6^{+1.4}_{-1.2}) \times 10^{-6}. \tag{11}$$

In view of this, we present all calculated results of the CP-averaged branching fractions and the local direct CP asymmetries of the concerned decay modes with $\theta = 145°$ in Table I. For comparison, the available experimental data are also listed. One can find that adopting the appropriate wave functions of initial and final states, our predictions are in good agreement with the current experimental data, although there are only upper limits for the $B^0 \to \omega(f_0 \to)\pi^+\pi^-$ and $B^+ \to \rho^+(f_0 \to)\pi^+\pi^-$ decays.

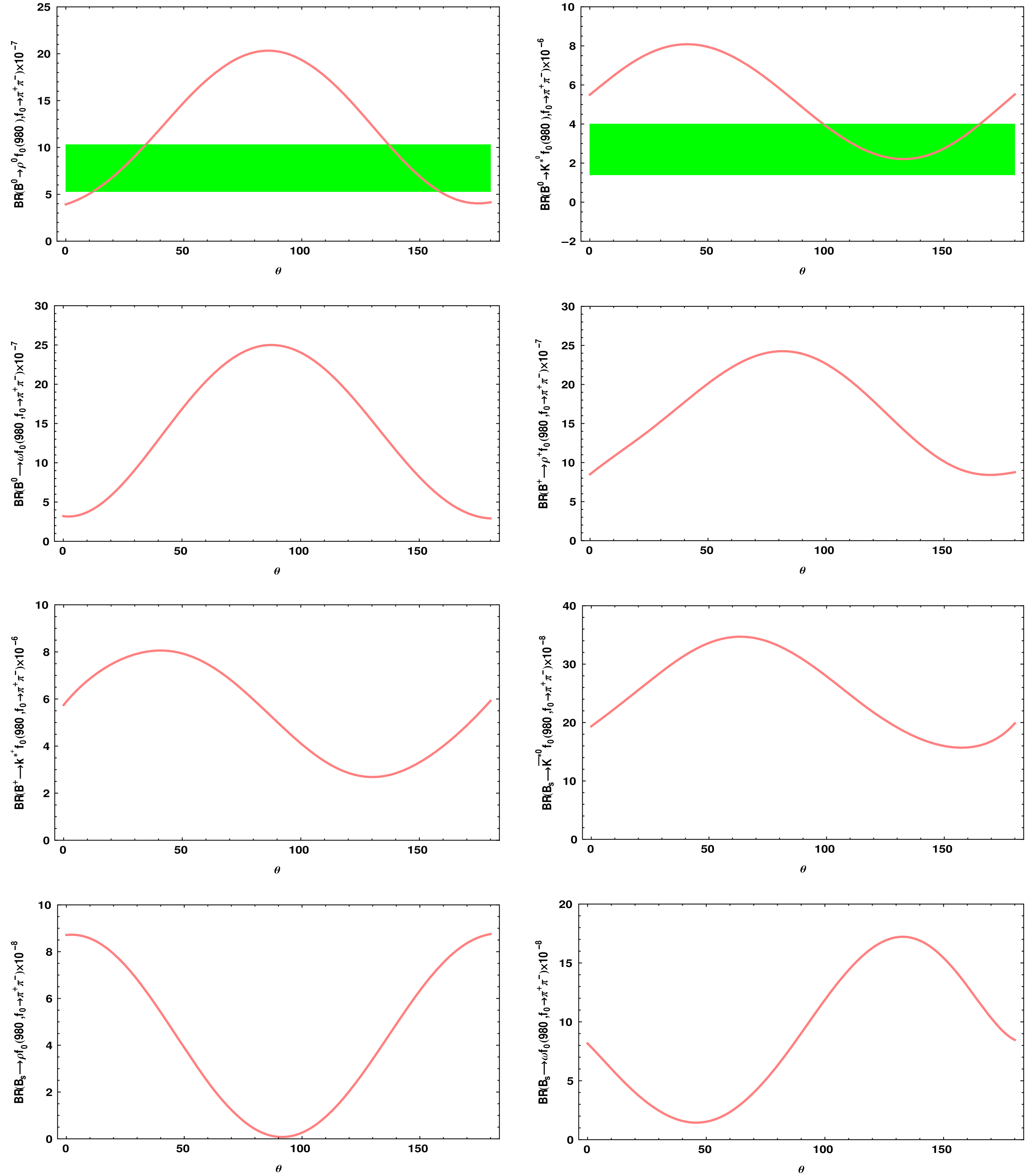

FIG. 2. Dependence of the CP-averaged branching fractions of the quasi-two-body $B_{(s)} \to V\pi^+\pi^-$ decays on the mixing angle θ in the PQCD approach. The θ dependence of the branching fractions. The green bands are the allowed regions in the experiments.

Finally, we plot all the branching fractions dependent on the mixing angle θ, which may shed light on the mixing angle by combining the ongoing experimental measurements.

We acknowledge that there are many uncertainties in our calculations. In this work, we mainly evaluate three kinds of errors, namely the parameters in wave functions of the initial and final states, the higher order and power

TABLE I. The results of CP averaged branching fractions (in 10^{-6}) and the direct CP asymmetries (%) in PQCD approach.

Decay Modes	Br(PQCD)	Br(EXP) [24]	A_{CP}^{dir}
$B^0 \to \rho^0(f_0 \to)\pi^+\pi^-$	$0.82^{+0.36+0.02+0.05}_{-0.34-0.16-0.10}$	0.78 ± 0.25	$-11.4^{+14.0+23.6+9.77}_{-3.54-8.72-0.00}$
$B^0 \to K^{*0}(f_0 \to)\pi^+\pi^-$	$2.45^{+0.66+0.56+0.00}_{-1.28-1.04-0.35}$	$2.6^{+1.4}_{-1.2}$	$-5.97^{+13.7+6.72+2.82}_{-0.57-0.00-0.00}$
$B^0 \to \omega(f_0 \to)\pi^+\pi^-$	$0.97^{+0.51+0.16+0.13}_{-0.39-0.19-0.10}$	< 1.5	$-13.7^{+6.10+11.5+1.99}_{-3.15-14.9-0.00}$
$B^+ \to \rho^+(f_0 \to)\pi^+\pi^-$	$1.23^{+0.50+0.25+0.00}_{-0.76-0.38-0.12}$	< 2.0	$-55.9^{+15.4+31.5+2.50}_{-19.5-4.34-7.33}$
$B^+ \to K^{*+}(f_0 \to)\pi^+\pi^-$	$3.18^{+0.94+0.76+0.00}_{-1.48-1.11-0.39}$	…	$-26.4^{+9.87+4.18+3.05}_{-6.34-1.21-3.05}$
$B_s \to \rho^0(f_0 \to)\pi^+\pi^-$	$0.06^{+0.02+0.01+0.00}_{-0.02-0.01-0.00}$	…	$8.82^{+4.23+1.68+0.00}_{-3.51-0.72-0.02}$
$B_s \to \omega(f_0 \to)\pi^+\pi^-$	$0.17^{+0.10+0.02+0.00}_{-0.09-0.04-0.02}$	…	$14.8^{+9.43+11.1+2.56}_{-13.1-8.17-0.53}$
$B_s \to \bar{K}^{*0}(f_0 \to)\pi^+\pi^-$	$0.15^{+0.09+0.05+0.01}_{-0.04-0.02-0.00}$	…	$87.7^{+0.00+0.00+0.00}_{-22.1-14.3-7.51}$

corrections, and the CKM matrix elements, respectively. In fact, the first kind of errors come from the uncertainties of nonperturbative parameters, such as the $B_{(s)}$ meson decay constants $f_B/f_{B_s} = 0.19 \pm 0.02/0.23 \pm 0.02$ GeV, the sharp parameters $\omega/\omega_s = 0.4 \pm 0.04/0.5 \pm 0.05$ GeV in the distribution amplitudes of B mesons, the Gegenbauer moments in the LCDAs of vector mesons, and the Gagenbauer moment a_S in the S-wave LCDAs of the $\pi\pi$ pair, etc. We can find from the table that these kinds of uncertainties are dominant. Fortunately, these kinds of uncertainties could be reduced with the developments of the experiments or other nonperturbative theoretical approaches in future. The second kind of errors arise from the unknown higher order of α_s and higher power corrections, which are reflected by varying the $\Lambda_{\rm QCD} = 0.25 \pm 0.05$ GeV and the factorization scale t from $0.8t$ to $1.2t$, respectively. The last ones are caused by the uncertainties of the CKM matrix elements.

If the narrow-width approximation holds in these decays, then the branching fraction of the quasi-two-body B meson decay can be decomposed as

$$\mathcal{B}(B \to M_1(R \to)M_2M_3) \simeq \mathcal{B}(B \to M_1R) \times \mathcal{B}(R \to M_2M_3), \tag{12}$$

where R represents a resonance. If two decays have a same resonance, we then define a ratio as

$$R_{V_1/V_2} = \frac{\mathcal{B}(B \to V_1R)}{\mathcal{B}(B \to V_2R)} = \frac{\mathcal{B}(B \to V_1R) \times \mathcal{B}(R \to \pi^+\pi^-)}{\mathcal{B}(B \to V_2R) \times \mathcal{B}(R \to \pi^+\pi^-)} \simeq \frac{\mathcal{B}(B \to V_1(R \to)\pi^+\pi^-)}{\mathcal{B}(B \to V_2(R \to)\pi^+\pi^-)}. \tag{13}$$

Based on the predictions in Table I, the ratio between the $B^0 \to \rho^0 f_0$ and $B^0 \to \omega f_0$ is given as

$$R_{\rho^0/\omega} = \frac{\mathcal{B}(B \to \rho^0 f_0)}{\mathcal{B}(B \to \omega f_0)} \sim 1, \tag{14}$$

which is in agreement with the results of QCDF [48]. Within the isospin relation

$$r = \frac{\mathcal{B}(f_0 \to \pi^+\pi^-)}{\mathcal{B}(f_0 \to \pi^0\pi^0)} = 2, \tag{15}$$

we obtain the relation

$$\mathcal{B}(B \to V(f_0 \to)\pi^0\pi^0) = \frac{1}{2}\mathcal{B}(B \to V(f_0 \to)\pi^+\pi^-), \tag{16}$$

which can be used to predict the branching fractions of the corresponding quasi-two-body $B \to V(f_0 \to)\pi^0\pi^0$ decays.

Now we turn to discuss the local direct CP asymmetries of these decays. In the quark level, $B \to K^*(f_0 \to)\pi^+\pi^-$, $B_s \to \rho^0/\omega(f_0 \to)\pi^+\pi^-$, and $B_s \to \phi(f_0(980) \to)\pi^+\pi^-$ are induced by $\bar{b} \to \bar{s}q\bar{q}$ transition, while $B \to (\rho, \omega)(f_0 \to)\pi^+\pi^-$ and $B_s \to K^*(f_0(980) \to)\pi^+\pi^-$ are controlled by $\bar{b} \to \bar{d}q\bar{q}$ transition. From Table I, it is found that the local CP asymmetries of decays $B^0 \to K^{*0}(f_0 \to)\pi^+\pi^-$ and $B_s \to \rho^0/\omega(f_0 \to)\pi^+\pi^-$ are very small, and the reason is that the tree diagrams contributions are both color and CKM elements suppressed. However, for the decay $B^+ \to K^{*+}(f_0 \to)\pi^+\pi^-$, because the spectator u quark enters into not only K^{*+} meson but also $\pi\pi$ pair, the contributions from tree and penguin operators are comparable, leading to a large CP asymmetry. For the decays $B^0 \to \rho^0/\omega(f_0 \to)\pi^+\pi^-$, although the contributions from tree operators are color suppressed, the destructive interference between the diagrams with the vector meson emitted and ones with the $\pi\pi$-pair emitted decreases the effects of penguin operators remarkably, therefore the CP asymmetries are as small as about 10%. For $B^+ \to \rho^+(f_0 \to)\pi^+\pi^-$ decay, its amplitude is more complicated. In Eq. (7), if the mixing angle θ of $f_0(980)$ is an obtuse angle, the sign of $n\bar{n}$ is negative. The spectator u quark of $B^+ \to \rho^+(f_0 \to)\pi^+\pi^-$ can enter into both f_0 and $\pi\pi$ pairs, so the negative sign leads to the cancellation between two tree operators contributions. With the sizable contributions of penguin operators, the CP asymmetry of this decay is as large as −55%. The decay of $B_s \to \bar{K}^{*0}(f_0 \to)\pi^+\pi^-$ is very similar to $B^+ \to \rho^+(f_0 \to)\pi^+\pi^-$, but the spectator is a strange quark. When the spectator enters into the kaon, both the tree and penguin operators contribute, and the tree operators are color suppressed. However, when it enters into the $\pi\pi$ pair, only penguin operators play roles. Due to large interference between two kinds of above contributions, the large CP asymmetry in $B_s \to \bar{K}^{*0}(f_0 \to)\pi^+\pi^-$ is reasonable. On the experimental side, these CP asymmetries have not been measured, and we hope these predictions can be tested in future.

IV. SUMMARY

In this work, we investigated the quasi-two-body $B/B_s \to V(f_0(980) \to)\pi^+\pi^-$ decays in PQCD approach, assuming that $f_0(980)$ is a mixture of $n\bar{n} = (u\bar{u} + d\bar{d})/\sqrt{2}$ and $s\bar{s}$ with the mixing angle θ. Within the S-wave two-pion wave function, both the branching fractions and the located CP asymmetries have been calculated. When the mixing angle θ is around 145°, the obtained branching fractions of the $B^0 \to \rho^0(f_0(980) \to)\pi^+\pi^-$ and $B^0 \to K^{*0}(f_0(980) \to)\pi^+\pi^-$ are in good agreement with the experimental data, and other results could be tested in the future experiments. In addition, the branching fractions of $B/B_s \to V(f_0 \to)\pi^0\pi^0$ could be predicted based on the isospin symmetry, which can be measured in the LHCb and Belle II experiments. We note that the calculated CP asymmetries of the $B^+ \to \rho^+(f_0 \to)\pi^+\pi^-$ and $B_s \to \bar{K}^{*0}(f_0 \to)\pi^+\pi^-$ decays are very large, which can be tested in the ongoing experiments. We acknowledge that there are many uncertainties in the calculation, and the dominant one is the two-meson wave function. Therefore, more precise multimeson wave functions from the nonperturbative approach are needed.

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