Pole skipping and Rarita-Schwinger fields

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In this note we analyze the equations of motion of a minimally coupled Rarita-Schwinger field near the horizon of an anti–de Sitter-Schwarzschild geometry. We find that at special complex values of the frequency and momentum there exist two independent regular solutions that are ingoing at the horizon. These special points in Fourier space are associated with the "pole skipping" phenomenon in thermal two-point functions of operators that are holographically dual to the bulk fields. We find that the leading pole-skipping point is located at a positive imaginary frequency with the distance from the origin being equal to half of the Lyapunov exponent for maximally chaotic theories.

I. INTRODUCTION

Retarded Green's functions are one of the main objects of interest in field theories as they encode the response of a system in equilibrium to small perturbations. In a theory with a dual gravitational description [1] one can calculate such correlators at finite temperature by studying the equations of motion of bulk fields in a black hole geometry which asymptotes to anti–de Sitter (AdS) spacetime [2,3]. In Lorentzian signature, one is prescribed to take the solution that is ingoing at the horizon [4] which uniquely determines the retarded Green's function of the dual boundary operators. Finding the precise form of the correlation function is generically difficult as the full solutions to the to the bulk equations of motion depend on the details of the background geometry.

However, some features of the Green's functions are encoded in the behavior of the bulk fields in the near-horizon region of the geometry. The most prominent example is the hydrodynamic description of holographic theories, described by the low-frequency and low-momentum limit (see for example [5,6]), where one finds that the radial evolution of the bulk fields becomes trivial [7] and leads to universal results such as the ratio of the shear viscosity and the entropy density [8].

Interestingly, the horizon region also contains information about the correlator away from the origin of Fourier space. Namely, at certain imaginary values of the frequency and (generically) complex values of the momentum, all solutions to the equations of motion become ingoing at the horizon. This phenomenon was called "pole skipping" as at such points in momentum space the associated boundary Green's function effectively skips a pole: at these locations a zero and a pole of the correlator coincide.

Pole skipping was initially observed in the energy density correlator which is obtained by solving the linearized Einstein's equations in the bulk. More generally, one finds that for the sound channel of the energy-momentum tensor [9] the leading pole-skipping point, that is the pole-skipping point at the most positive imaginary frequency (see Fig. 1), is located at [10–14]

$$\omega_* = +i\lambda_L, \qquad k_* = i\frac{\lambda_L}{v_B}, \qquad (1)$$

where $\lambda_L = 2\pi T$ is the Lyapunov exponent for holographic theories and v_B is the butterfly velocity. Thus this location seems to be connected to the chaotic properties of holographic systems [15–17],[1] which is the case even in the presence of stringy corrections [20] (see [21–25] for further examples).

Correlators associated to the remaining channels of the energy-momentum tensor and correlators of lower-spin

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[1]Pole skipping in theories which are not maximally chaotic has been discussed in [18,19]. In such theories the relation to chaos is not straightforward and the applicability of pole skipping as a diagnostic for chaos might be limited to only imposing bounds on chaos.
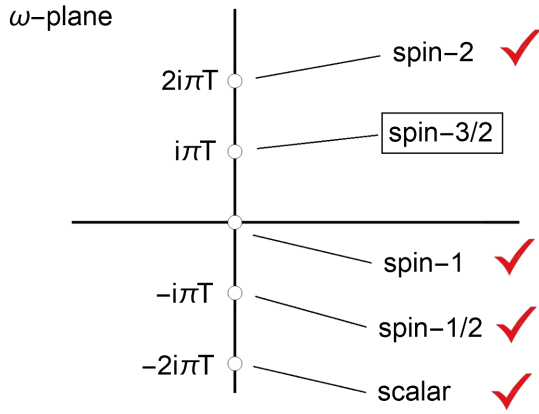
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FIG. 1. Location of the leading pole-skipping point on the frequency plane for bulk fields of various spin. If the fields decompose into multiple channels, we depict the pole-skipping point of the channel with the most positive imaginary value. This paper focuses on the spin-$\frac{3}{2}$ case.

bulk fields also exhibit pole-skipping, albeit at different imaginary frequencies. In the case of scalar fields all pole-skipping points are located in the lower-half of the complex frequency plane at (imaginary) bosonic Matsubara frequencies $\omega_n^B = -2\pi i T n$, where $n = 1, 2, \ldots$. The same applies for the shear and scalar channels of the energy-momentum tensor and the transverse channel of the $U(1)$ gauge field, while in the longitudinal channel of the gauge field there exists an additional "hydrodynamic" pole-skipping point located at $\omega = 0$ and $k = 0$ [13,14] (see also [26–30]). Note that such an infinite tower of pole-skipping points at negative imaginary Matsubara frequencies can also be found in the energy density correlator [14]. Similarly, for fermion fields all pole-skipping points are on the lower-half of the complex frequency plane [31], but they are located at fermionic Matsubara frequencies $\omega_n^F = -2\pi i T(n - 1/2)$ where $n = 1, 2, \ldots$.

Note that there is a relationship between the spins of bulk fields and the frequency values of the leading pole-skipping points as depicted in Fig. 1, where we always pick the channel whose leading pole-skipping point has the largest imaginary frequency. The locations on the complex ω-plane are given by $\omega_{0,s} = 2\pi i T(s - 1)$, where s denotes the spin of the bulk field.[2] We note that Fig. 1 is a compilation of several independent results. However, the analysis for spin-$\frac{3}{2}$ fields has been missing from the

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literature. In this note, by analyzing the Rarita-Schwinger field in the near-horizon region of an AdS-Schwarzschild black hole, we show that indeed the relation between the spin and value of the frequency of the leading pole-skipping point holds for any (half)-integer spin with $s \leq 2$.

In addition to the locations of the pole-skipping points, the near-horizon analysis also predicts the form of the Green's function near such special points. Typically, the correlator near a generic pole-skipping location $(\omega_{\rm ps}, k_{\rm ps})$ takes the form [12,14,31,34]

$$G^R(\omega_{\rm ps} + \delta\omega, k_{\rm ps} + \delta k) \propto \frac{\delta\omega - \left(\frac{\delta\omega}{\delta k}\right)_z \delta k}{\delta\omega - \left(\frac{\delta\omega}{\delta k}\right)_p \delta k}, \qquad (2)$$

where $(\delta\omega/\delta k)_{p,z}$ denote the slope of the line of *poles* and line of *zeros* passing through the pole-skipping point.[3] The Green's function of all leading pole-skipping points depicted in Fig. 1 has the form (2) and similarly we show that the near-horizon analysis predicts that the corresponding correlator at the leading pole-skipping point of the Rarita-Schwinger field takes on the same form.

II. GRAVITATIONAL SETUP

Let the bulk theory be described by the Einstein-Hilbert action with a negative cosmological constant

$$S = \int d^{d+2}x \sqrt{-g}(R - 2\Lambda), \qquad (3)$$

where $\Lambda = -d(d+1)/2L^2$ and L denotes the radius of AdS which we set to $L = 1$ in all further expressions. The resulting equations of motion admit a planar black hole solution that is asymptotically AdS and is described by the line element

$$ds^2 = -r^2 f(r) dv^2 + 2dv dr + h(r) dx^i dx^i. \qquad (4)$$

We are using the ingoing Eddington-Finkelstein coordinates where r denotes the radial direction with the boundary of AdS located at $r \to \infty$. The usual time coordinate can be recovered using the relation $v = t + r_*$, where $dr_* = dr/(r^2 f(r))$, in which case one can see that

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$\{t, x^i\}$, with $i = 1\dots d$, are the coordinates of the $d+1$ dimensional Minkowski space at a fixed value of r.

The two functions appearing in the metric are given by

$$f(r) = 1 - \left(\frac{r_0}{r}\right)^{d+1}, \qquad h(r) = r^2, \qquad (5)$$

meaning that there is a nondegenerate horizon at a finite radius $r = r_0$ where $f(r)$ vanishes. However as we are using the ingoing Eddington-Finkelstein coordinates, such a point is a regular point of the metric. The associated (non-vanishing) Hawking temperature is given by $4\pi T = r_0^2 f'(r_0) = (d+1)r_0$. In what follows, we keep both $f(r)$ and $h(r)$ generic which allows us to identify the source of the contributions when analysing the location of pole-skipping points. Furthermore, it allows for an easier generalization of our results to a larger class of background geometries, such as geometries with an additional matter content deforming the background [36,37], even though our derivation applies only when $f(r)$ and $h(r)$ take the form (5).

The aim of this note is the analysis of the near-horizon behavior of a minimally coupled spin-$\frac{3}{2}$ field. In an AdS/CFT context, such fields were first considered in [38–42] and have later been used to study the properties of the charged current in the dual boundary theory (see for example [43–46]). For a summary and discussion of recent results see [47].

The action describing the massive Rarita-Schwinger field Ψ_M is given by[4]

$$S_{RS} \propto \int d^{d+2}x \sqrt{-g}\,\bar{\Psi}_M(\Gamma^{MNP}\nabla_N - m\Gamma^{MP})\Psi_P. \quad (6)$$

Since we are only interested in the bulk equations of motion, we do not need the precise details of the overall normalization factor or the additional boundary terms.[5] The antisymmetrized products of curved space gamma matrices Γ^{MP} and Γ^{MNP} act on the spinor index of the Rarita-Schwinger field, which we suppress throughout the note. The covariant derivative acting on the spin-$\frac{3}{2}$ field is given by $\nabla_M\Psi_P = \partial_M\Psi_P - \tilde{\Gamma}^N_{MP}\Psi_N + \frac{1}{4}(\omega_{ab})_M\Gamma^{ab}\Psi_P$, where $\tilde{\Gamma}^N_{MP}$ are the Christoffel symbols and ω_M is the spin connection one form.

The equation of motion derived from (6) is

$$\Gamma^{MNP}\nabla_N\Psi_P - m\Gamma^{MN}\Psi_N = 0, \qquad (7)$$

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[4]Throughout this note we use upper case Latin letters $(M, N, \dots)$ to denote the curved spacetime indices, whereas lower case Latin letters $(a, b, \dots)$ to denote the flat space indices.

[5]We note that another mass term $m'g^{MN}\bar{\Psi}_M\Psi_N$ can be added to the action. This introduces a spin-$\frac{1}{2}$ degree of freedom in the Rarita-Schwinger field (see e.g., [47]) which may change the structure of pole skipping. We do not consider such a term here.

however since the background metric satisfies the vacuum Einstein's equation one can show (see for example [45]) that the above equation of motion is equivalent to

$$(\slashed{\nabla} + m)\Psi_N = 0, \qquad (8)$$

with additional constraints

$$\Gamma^M\Psi_M = 0, \qquad \nabla^M\Psi_M = 0, \qquad (9)$$

where we defined $\slashed{\nabla} = \Gamma^M\nabla_M$. Note that while (8) looks like a Dirac equation, it actually couples different vector components of the field due to the term involving the Christoffel symbols in the covariant derivative, which is not present if the covariant derivative is acting on a Dirac field. In deriving (8) we assume that the mass takes on a generic positive value in which case the constraints (9) follow naturally from the equations of motion (7). If $m = d/2$, the spin-$\frac{3}{2}$ field is physically massless [48] (the nonzero value of m is a consequence of the curvature of spacetime) and the conditions (9) are not imposed by the equations of motion, but can be considered as a choice of gauge.

III. POLE SKIPPING

We now show that at a specific complex value of the frequency and momentum the equations of motion (8) and the associated constraints admit two independent solutions that are regular at the horizon. We use the following orthonormal frame[6]

$$E^{\underline{v}} = \frac{1+f(r)}{2}rdv - \frac{dr}{r}, \qquad E^{\underline{r}} = \frac{1-f(r)}{2}rdv + \frac{dr}{r},$$
$$E^{\underline{i}} = \sqrt{h(r)}dx^i, \qquad (10)$$

in which case the line element (4) can be written as $ds^2 = \eta_{ab}E^aE^b$ with $\eta_{ab} = \text{diag}(-1, 1, 1, \dots, 1)$.

Next we use the fact that the metric components depend only on the r coordinate and introduce a plane wave ansatz $\Psi_M(v, r, x^i) = \psi_M(r)e^{-i\omega v + ik_ix^i}$ in which case the equations of motion reduce to a system of coupled first order ordinary differential equations for $\psi_M(r)$. To separate these equations into tractable subsystems we decompose the components of the field based on its eigenvalues under the action of $\Gamma^{\underline{r}}$ and $\Gamma^{(2)} \equiv \hat{k}_i\Gamma^{\underline{vi}}$, where $\hat{k}_i = k_i/k$ is the normalized Euclidean vector in d-dimensional flat space and $k \equiv \sqrt{\sum_{i=1}^d k_i^2}$ its magnitude. These two matrices commute and their eigenvalues are equal to ± 1, hence all vector components of the Rarita-Schwinger field can be decomposed as

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[6]We use underlined indices to indicate specific flat space indices ($a = \underline{v}, \underline{r}, \dots$) and distinguish them from particular curved space indices which are not underlined ($M = v, r, \dots$).

$$\psi_M = \sum_{\alpha_1=\pm} \sum_{\alpha_2=\pm} \psi_M^{(\alpha_1,\alpha_2)} \qquad (11)$$

where the indices in the bracket denote the eigenvalues under the action of the gamma matrices as

$$\Gamma^{\underline{r}} \psi_M^{(\alpha_1,\alpha_2)} = \alpha_1 \psi_M^{(\alpha_1,\alpha_2)}, \qquad \Gamma^{(2)} \psi_M^{(\alpha_1,\alpha_2)} = \alpha_2 \psi_M^{(\alpha_1,\alpha_2)}, \quad (12)$$

with $\alpha_{1,2} = \pm$. Each of the components in the decomposition contains a quarter of the total degrees of freedom of the spinor.

The leading pole-skipping point for the energy-density correlator is found by considering the vv-component of the dual bulk excitation [10–12]. Similarly, the location of the leading pole-skipping point for the $U(1)$ gauge field is uncovered by considering the v-component of the bulk field [14]. Expecting similar behavior, we focus on the ψ_v component of the Rarita-Schwinger field. By using the constraints (9), we find that only ψ_r couples to ψ_v so we focus on that subsystem of equations. This subsystem then separates into two decoupled sets of differential equation—one containing components whose $\Gamma^{\underline{r}}$ and $\Gamma^{(2)}$ eigenvalues are equal, and one with components whose eigenvalues under the action of the aforementioned matrices are opposite. But the two decoupled systems of differential equations are related by $k \to -k$. We can thus focus only on one and obtain the results for the other by a reversal of the momentum k. For concreteness, we choose the subsystem dealing with the components whose eigenvalues under $\Gamma^{\underline{r}}$ and $\Gamma^{(2)}$ are equal. These equations are given in full detail in the Appendix [see (A5)], together with some additional information about their derivation.

In our analysis the detailed form of the equations of motion is not needed as we are merely interested in their near-horizon expansion. Because in the ingoing Eddington-Finkelstein coordinates the horizon is a regular point, we can expand the field components in a series as

$$\psi_M^{(\alpha_1,\alpha_2)}(r) = \sum_{l=0}^{\infty} \psi_{M,l}^{(\alpha_1,\alpha_2)}(r-r_0)^l, \qquad (13)$$

where $\psi_{M,l}^{(\alpha_1,\alpha_2)}$ are constant coefficients. Similarly, the equations of motion themselves can be expanded in a series at the horizon after which solving the differential equations translates into solving a system of algebraic equations at each order of the series expansion.

At a generic point in Fourier space these algebraic equations halve the number of free parameters in the Rarita-Schwinger field which corresponds to choosing the solution to the equations of motion that is ingoing at the horizon. For example, by evaluating the equations of motion directly at the horizon, one finds, among others, the following equation [see (A7)]

$$\left[-2mr_0 - 4i\omega + \frac{2ikr_0}{\sqrt{h(r_0)}} - r_0^2 f'(r_0)\right] \Gamma^{\underline{v}} \psi_{v,0}^{(-,-)}$$

$$+ \left[2mr_0 - 4i\omega - \frac{2ikr_0}{\sqrt{h(r_0)}} - r_0^2 f'(r_0)\right] \psi_{v,0}^{(+,+)} = 0,$$

$$(14)$$

which for generic ω and k relates $\psi_{v,0}^{(-,-)}$ to $\psi_{v,0}^{(+,+)}$. Similarly, equations of motion at higher order in the near-horizon expansion relate all other coefficients (including those involving ψ_r components) to the ones appearing in (14) allowing us to perturbatively construct a solution with half of the total number of degrees of freedom.

Equation (14) is trivially satisfied if both coefficients in the square brackets vanish, which is the case if the frequency and momentum are precisely

$$\omega \equiv \omega_0 = \frac{ir_0^2 f'(r_0)}{4} = i\pi T, \qquad (15a)$$

$$k \equiv k_0 = -im\sqrt{h(r_0)} = -\frac{4m}{d+1} i\pi T. \qquad (15b)$$

Then both $\psi_{v,0}^{(+,+)}$ and $\psi_{v,0}^{(-,-)}$ remain unconstrained and by using other algebraic equations from the near horizon expansion of the equations of motion we can construct two linearly independent solutions that are regular at the horizon.[7]

The plots in Fig. 2 show the location of poles of the boundary Green's function in the complex ω-plane at different values of the momentum, calculated using the Leaver method [49]. We see that at the special point (15b) there is a pole located exactly at (15a)—this is the pole predicted by the near-horizon analysis.

Finally, recall that there exists an additional pole-skipping point at the same frequency (15a) but at the opposite momentum to (15b) that is obtained from the near-horizon analysis of the components, whose $\Gamma^{\underline{r}}$ and $\Gamma^{(2)}$ eigenvalues are opposite.

As is known from previous results [11,12,14,29,31,34], a location in Fourier space where there exist multiple independent ingoing solutions at the horizon corresponds to a point where a pole and a zero of the boundary retarded Green's function coincide. As such, (15) is the first location at which we observe pole skipping for the Rarita-Schwinger field. Most notably, this point is located

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[7]If we set $\omega = i\pi T$ but leave k generic, the complete near-horizon expansions, such as (13), contain additional logarithmic terms $\log(r-r_0)$ similar to the terms appearing in the scalar [14] or fermion [31] field expansions. However, one finds that by imposing the equations of motion, all coefficients multiplying terms with a logarithmic divergence at the horizon have to vanish, unless the momentum is finely tuned to the pole-skipping value (15b).
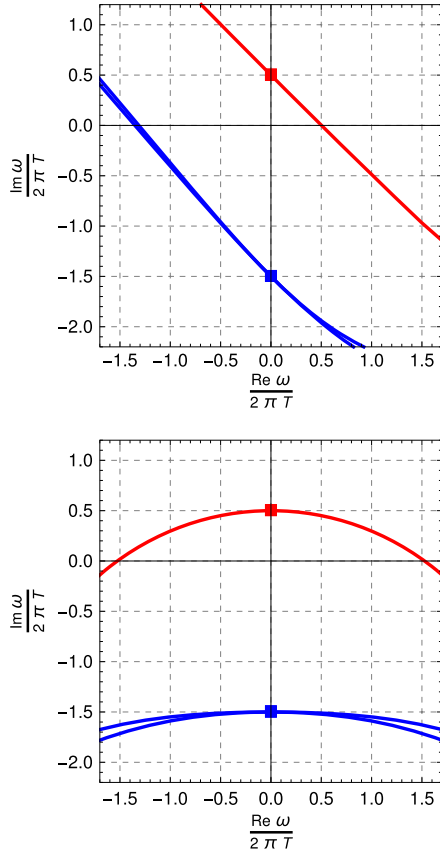
FIG. 2. Numerical analysis of the poles of the retarded Green's function in AdS$_4$ for a Rarita-Schwinger field with mass $m = 3.4$. The poles are calculated using the methods presented in [49] adapted to the spin-3/2 case. The top figure depicts the motion of the poles on the complex ω-plane near the value of the momentum where pole-skipping is observed. The momentum is varied linearly as $k = k_0 + \epsilon e^{\frac{i\pi}{4}}$, where $\epsilon \in \mathbb{R}$. We notice that the red curve passes through ω_0 as $\epsilon = 0$ which we denote with a red square. In blue we depict two additional poles which pass through $\omega = -3\pi i$ as $\epsilon \to 0$. These correspond to higher order pole-skipping points which are discussed in the Appendix. The bottom figure shows the location of the poles as the momentum is varied as $k = k_0 e^{i\epsilon}$. Pole skipping occurs at the filled squares.

at a *positive* imaginary frequency. Up to now, the only other example of a pole-skipping point being found on the upper complex ω half-plane is the leading pole-skipping location for the energy-density correlator, in which case the precise location in Fourier space has been conjectured to be related to the chaotic properties of the theory, as seen in (1).

For the Rarita-Schwinger field, the modulus (15a) is exactly half the value of the maximal Lyapunov exponent bound of [17]. In our opinion this suggests that there is little relation to chaos, despite the seeming exponential growth of the solution, if the values (15) are inserted into the plane-wave ansatz. This is further substantiated by the fact that the value of the momentum at which pole skipping occurs

depends on the mass of the field and thus the location is sensitive both to the background *and* the probe.

Finally, recall that the equations of motion near the horizon also predict the form of the Green's function near the pole-skipping point. To that end evaluate (14) at a nearby point in Fourier space, which we denote by $\omega = \omega_0 + \epsilon\delta\omega$ and $k = k_0 + \epsilon\delta k$, where $\epsilon > 0$ is a small number used as a bookkeeping device and we keep the direction of the momentum $\hat{k}$ fixed. The analysis of [31] applies to this case and we refer the reader to that reference for a detailed calculation. The final result of [31] applies here as well: working at linear order in ϵ, one finds that near (15) the retarded Green's function takes the pole-skipping form (2).

IV. DISCUSSION

It has recently been shown that quantum chaos in field theories with holographic duals manifests itself in the thermal energy density two-point functions. These correlators exhibit the pole-skipping phenomenon at special (imaginary) values of the frequency and momentum. At such points poles and zeroes collide and the Green's function becomes ill-defined. It was found [10–12] that the pole-skipping frequency lies at $\omega = +2\pi iT$. The modulus $|\omega| = 2\pi T$ is thought to be related to the maximal Lyapunov exponent [17].

For bulk fields with spins other than two, the corresponding boundary two-point functions also show pole skipping at certain Matsubara frequencies. Scalar fields first exhibit poleskipping at $\omega = -2\pi iT$, while for conserved currents this occurs at $\omega = 0$ [14]. Fermionic fields show pole skipping at $\omega = -\pi iT$, i.e., at the first negative fermionic Matsubara frequency [31]. The calculations rely on an analysis of the near-horizon region of the bulk geometry and give nontrivial constraints on Green's functions at frequencies $\omega \sim T$.

Although the relevant pole-skipping momenta do not seem to be universal, there is a definite structure in pole-skipping frequencies; see Fig. 1. Namely, there is a relationship between the frequency of the first pole-skipping point and the spin of the bulk field.

In this paper we have dealt with the spin-$\frac{3}{2}$ case which has been missing from previous analyses. In particular, we have shown from a bulk perspective that a Rarita-Schwinger field in an AdS-Schwarzschild background exhibits pole-skipping precisely at the expected first fermionic Matsubara frequency on the upper-half plane. Since the analysis only concerned the near-horizon region, we expect that the results can be extended to more general spacetimes with regular horizons. With this result, we complete the hierarchy of pole-skipping locations for various fields of different spin.

It is important to note that the results for individual fields were calculated independently and in no way relied on supersymmetry. Furthermore, the above mentioned

locations are merely the first pole-skipping points and in some sense are the simplest ones—the locations in momentum space depend only on the values of the background metric at the horizon and/or the nearby region through first derivatives, whereas higher order pole-skipping points generically depend on higher order derivatives as well [14]. Thus it is enticing to conjecture that the hierarchy is the property of the near-horizon region of spacetime itself.

It would be interesting to investigate the origin of this hierarchy (for a CFT analysis, see [35,50,51]). Note that the pole-skipping points are located at (imaginary) Matsubara frequencies which means that the static bulk has a Euclidean counterpart (obtained by Wick-rotation). Since the geometry smoothly caps off, a shift along the Euclidean time circle translates into a rotation at the tip (which is the Euclidean analog of the event horizon). Fourier modes are therefore connected to spin, providing an explanation for the observed hierarchy of pole-skipping points.

In the above analysis we have merely shown the existence of a single pole-skipping point, but previous results on other fields would suggest that there exists an entire tower of higher-order pole-skipping points at negative imaginary frequencies, which we have not focused on. Furthermore, the above analysis can be expanded to include matter fields or allow for an additional mass term in the action of the Rarita-Schwinger field. It would be interesting to see if there exists a configuration at which the pole-skipping points vanish.

Finally, the Rarita-Schwinger fields in backgrounds with simple horizons have been used in calculations of fermionic currents [43–46]. It would be interesting to study the pole-skipping points in that context and see the consequences on the spectral function of the dual fermionic currents in the boundary theory.

ACKNOWLEDGMENTS

APPENDIX: EQUATIONS OF MOTION

In this Appendix we spell out some of the details that have been omitted in the main part of this note. We keep the same notational conventions as in the main text. In addition, throughout the note we use the following convention for the flat space Clifford algebra

$$\{\Gamma^a, \Gamma^b\} = 2\eta^{ab}, \qquad (A1)$$

where $\eta^{ab} = \mathrm{diag}(-1, +1, \ldots, +1)$, which most importantly implies that $\Gamma^{\underline{v}}$ squares to -1 while all others square to 1.

We start at the rewritten equations of motion (8). We again want to point out that we keep $h(r)$ and $f(r)$ unevaluated to keep track of the origin of individual terms in the final expressions. However, in deriving (8) from (7), one assumes that the Ricci tensor and the Ricci scalar satisfy

$$R_{MN} = -(d+1)g_{MN}, \qquad R = -(d+1)(d+2), \quad (A2)$$

which is the case if the two aforementioned functions take the form (5), as for example for a black brane solution. So while the results presented in this note hold only for backgrounds that satisfy (A2), we believe that the generalization of our results to more complicated backgrounds should be straightforward and that the findings would resemble those of this note.

Recall that due to the presence of the Christoffel terms in the covariant derivative, different vector components of the Rarita-Schwinger field are coupled. We can make this explicit by putting such terms on the right-hand side, resulting in

$$(\not{D} + m)\Psi_M = \Gamma^N \tilde{\Gamma}^P_{MN} \Psi_P, \qquad (A3)$$

where $D_M = \partial_M + \frac{1}{4}(\omega_{ab})_M \Gamma^{ab}$ denotes the covariant derivative acting on a spinor field. The left-hand side of Eq. (A3) with the choice of vielbein (10) has been worked out in [31]. One can then use the gamma-traceless condition (9) to arrive at two coupled equations containing only the components Ψ_v and Ψ_r

$$(\not{D} + m)\Psi_v = \frac{\partial_r(r^2 f(r))}{2r} \left\{ [\Gamma^{\underline{v}} + \Gamma^{\underline{r}}]\Psi_v \right.$$
$$\left. + \frac{r^2}{2}[(1 + f(r))\Gamma^{\underline{v}} - (1 - f(r))\Gamma^{\underline{r}}]\Psi_r \right\},$$
$$(A4a)$$

$$(\not{D} + m)\Psi_r = -\frac{\partial_r(r^2 f(r))}{2r}[\Gamma^{\underline{v}} + \Gamma^{\underline{r}}]\Psi_r - \frac{\partial_r h(r)}{2rh(r)}$$
$$\times \left\{ [\Gamma^{\underline{v}} + \Gamma^{\underline{r}}]\Psi_v + \frac{r^2}{2}[(1 + f(r))\Gamma^{\underline{r}} \right.$$
$$\left. - (1 - f(r))\Gamma^{\underline{v}}]\Psi_r \right\}.$$
$$(A4b)$$

Since the components of the metric depend only on the coordinate r, one can write the field in the form of a plane wave $\Psi_M(r) = \psi_M(r)e^{-i\omega v + ik_i x^i}$, and insert this ansatz into the above equations. After decomposing the spinors in terms of their behavior under the action of $\Gamma^{\underline{r}}$ and $\Gamma^{(2)} \equiv \hat{k}_i \Gamma^{\underline{v}i}$ matrices, as described in (11) and (12), one obtains eight coupled first order ordinary differential equations for the different components of ψ_v and ψ_r. The system of equations can be split into two decoupled subsystems of 4 equations, with one describing the

components whose $\Gamma^{\underline{r}}$ and $\Gamma^{(2)}2$ eigenvalues are equal and the other containing the components with opposite eigenvalues under the action of the two matrices. The two subsystems of equations are related by $k \to -k$, so it is sufficient to analyze only one of the subsystems and

infer the results of the other by simply reversing the momentum.

Hence in what follows we focus at the system of equations that involve $\psi_v^{(\pm,\pm)}$ and $\psi_r^{(\pm,\pm)}$ which are given, after some algebra, by equations

$$
\begin{aligned}
\mathcal{S}_1 = & \left[r^2 f(r)\partial_r - i\omega - \frac{2rf(r) + r^2 f'(r)}{4} + \frac{dr^2 f(r)h'(r)}{4h(r)} + \frac{mr(1+f(r))}{2} - \frac{ikr(1-f(r))}{2\sqrt{h(r)}} \right] \psi_v^{(+,+)} \\
& + \Gamma^{\underline{r}} \left[-i\omega - \frac{4rf(r) + r^2 f'(r)}{4} - \frac{mr(1-f(r))}{2} + \frac{ikr(1+f(r))}{2\sqrt{h(r)}} \right] \psi_v^{(-,-)} - \frac{r^2 f(r)}{2}\partial_r(r^2 f(r))\Gamma^{\underline{r}}\psi_r^{(-,-)}, \quad (A5a)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{S}_2 = & \Gamma^{\underline{r}} \left[r^2 f(r)\partial_r - i\omega - \frac{2rf(r) + r^2 f'(r)}{4} + \frac{dr^2 f(r)h'(r)}{4h(r)} - \frac{mr(1+f(r))}{2} + \frac{ikr(1-f(r))}{2\sqrt{h(r)}} \right] \psi_v^{(-,-)} \\
& + \left[-i\omega - \frac{4rf(r) + r^2 f'(r)}{4} + \frac{mr(1-f(r))}{2} - \frac{ikr(1+f(r))}{2\sqrt{h(r)}} \right] \psi_v^{(+,+)} - \frac{r^2 f(r)}{2}\partial_r(r^2 f(r))\psi_r^{(+,+)}, \quad (A5b)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{S}_3 = & \left[r^2 f(r)\partial_r - i\omega + \frac{3(2rf(r) + r^2 f'(r))}{4} + \frac{(d+2)r^2 f(r)h'(r)}{4h(r)} + \frac{mr(1+f(r))}{2} - \frac{ikr(1-f(r))}{2\sqrt{h(r)}} \right] \psi_r^{(+,+)} \\
& + \Gamma^{\underline{r}} \left[-i\omega + \frac{4rf(r) + 3r^2 f'(r)}{4} - \frac{mr(1-f(r))}{2} + \frac{ikr(1+f(r))}{2\sqrt{h(r)}} \right] \psi_r^{(-,-)} + \frac{h'(r)}{2h(r)}(\psi_v^{(+,+)} + \Gamma^{\underline{r}}\psi_v^{(-,-)}), \quad (A5c)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{S}_4 = & \Gamma^{\underline{r}} \left[r^2 f(r)\partial_r - i\omega + \frac{3(2rf(r) + r^2 f'(r))}{4} + \frac{(d+2)r^2 f(r)h'(r)}{4h(r)} - \frac{mr(1+f(r))}{2} + \frac{ikr(1-f(r))}{2\sqrt{h(r)}} \right] \psi_r^{(-,-)} \\
& + \left[-i\omega + \frac{4rf(r) + 3r^2 f'(r)}{4} + \frac{mr(1-f(r))}{2} - \frac{ikr(1+f(r))}{2\sqrt{h(r)}} \right] \psi_r^{(+,+)} + \frac{h'(r)}{2h(r)}(\psi_v^{(+,+)} + \Gamma^{\underline{r}}\psi_v^{(-,-)}). \quad (A5d)
\end{aligned}
$$

For later convenience we have labelled them as $\mathcal{S}_i$, with $i = 1, 2, 3, 4$, so that the equations of motion can be summarized in a compact way as $\mathcal{S}_i = 0$.

We are interested in the near-horizon expansion of these equations. As we are working with the ingoing Eddington-Finkelstein coordinates in which the horizon is a regular point, we assume that all functions and fields can be expanded in a series around the horizon at $r = r_0$. The expansion of (5) is trivial and we use the field expansion (13). Then (A5) also get expanded around the horizon

$$
\mathcal{S}_i = \sum_{l=0}^{\infty} \mathcal{S}_i^{(l)}(r - r_0)^l = 0, \Rightarrow \mathcal{S}_i^{(l)} = 0, \quad i = 1, 2, 3, 4,
$$

(A6)

in which case the equations of motion become an infinite set of algebraic equations that we can solve order by order.

Our main interest lies in evaluating the equations of motion directly at the horizon, or in other words, the zeroth order equations $\mathcal{S}_i^{(0)} = 0$. While the four equations in (A5) are independent in general, one finds that directly at the

horizon there are only two independent equations as $\mathcal{S}_1^{(0)} = \mathcal{S}_2^{(0)}$ and $\mathcal{S}_3^{(0)} = \mathcal{S}_4^{(0)}$, with

$$
\begin{aligned}
\mathcal{S}_1^{(0)} = & \left[-2mr_0 - 4i\omega + \frac{2ikr_0}{\sqrt{h(r_0)}} - r_0^2 f'(r_0) \right] \Gamma^{\underline{r}}\psi_{v,0}^{(-,-)} \\
& + \left[2mr_0 - 4i\omega - \frac{2ikr_0}{\sqrt{h(r_0)}} - r_0^2 f'(r_0) \right] \psi_{v,0}^{(+,+)} = 0,
\end{aligned}
$$

(A7a)

$$
\begin{aligned}
\mathcal{S}_3^{(0)} = & \left[-2mr_0 - 4i\omega + \frac{2ikr_0}{\sqrt{h(r_0)}} + 3r_0^2 f'(r_0) \right] \Gamma^{\underline{r}}\psi_{r,0}^{(-,-)} \\
& + \left[2mr_0 - 4i\omega - \frac{2ikr_0}{\sqrt{h(r_0)}} + 3r_0^2 f'(r_0) \right] \psi_{r,0}^{(+,+)} \\
& + \frac{2h'(r_0)}{h(r_0)}(\psi_{v,0}^{(+,+)} + \Gamma^{\underline{r}}\psi_{v,0}^{(-,\#1)}) = 0.
\end{aligned}
$$

(A7b)

In fact (A7a) is presented in the main text as (14).

In order to find the locations of pole-skipping points, we need to look for the values of ω and k at which (A7) and their higher-order analogues do not impose enough

constraints on the solutions of the equations of motion to uniquely determine the retarded Green's function of the boundary theory. The determination of the leading pole-skipping point is described in the main text [see the discussion around Eq. (15)], but in fact there also exists an infinite tower of pole-skipping points at *negative* imaginary fermionic Matsubara frequencies $\omega_n^F = -2\pi i T(n - 1/2)$ where $n = 1, 2, \ldots$.

The procedure of identifying such locations mimics the analysis presented for the fermion case [31], but with some additional caveats, and will not be discussed in detail here. We just report some partial results which explain the additional poles found in the numerical analysis that we present in Fig. 2. In order to find the pole-skipping points at $\omega_1^F = -i\pi T$, one needs to analyze the equations at linear order in the expansion (A6) and the locations are obtained by looking for points at which a linear combination of $\psi_{v,1}^{(+,+)}$ and $\psi_{v,1}^{(-,-)}$ represents an additional free parameter of the solutions to the equations of motion. We find that for a generic value of the mass m there are three pole-skipping points at this value of the frequency.

Pole-skipping points at $\omega_2^F = -3i\pi T$ are perhaps more interesting as they arise due to both $\psi_{v,2}^{(\alpha_1,\alpha_2)}$ and $\psi_{r,0}^{(\alpha_1,\alpha_2)}$. In fact, one finds that two special points coincide at

$$\omega = -\frac{3i}{4} r_0^2 f'(r_0) = -3i\pi T, \tag{A8a}$$

$$k = -im\sqrt{h(r_0)} = -\frac{4i\pi}{d+1} mT, \tag{A8b}$$

and are thus responsible for the additional two poles that can be seen in Fig. 2. A simple way to confirm that this is indeed the case is to insert these values into (A7) and observe that the prefactors in the square brackets (A7b) vanish, hence leaving $\psi_{r,0}^{(+,+)}$ and $\psi_{r,0}^{(-,-)}$ unconstrained. Furthermore, one finds that at these values of the frequency and momentum (A7a) and (A7b) give equivalent constraints. We have not investigated the significance of this "double" pole-skipping point (see the analysis of [34]), but we attribute this occurrence to the interplay between the ψ_r and ψ_v components in the equations of motion.

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